Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/173-6.3.7-d-hyper-^m-a+b-c-tanh-^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [263]. This is test number [173].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (263)	0.00 (0)
Mathematica	100.00 (263)	0.00 (0)
Fricas	98.86 (260)	1.14 (3)
Maple	94.68 (249)	5.32 (14)
Giac	87.45 (230)	12.55 (33)
Mupad	70.34 (185)	29.66 (78)
Maxima	67.30 (177)	32.70 (86)
Sympy	15.97 (42)	84.03 (221)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

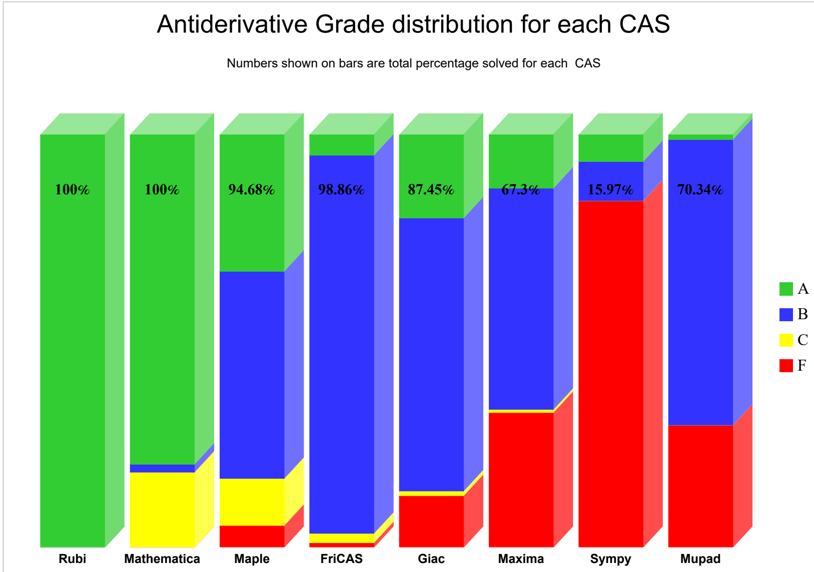
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

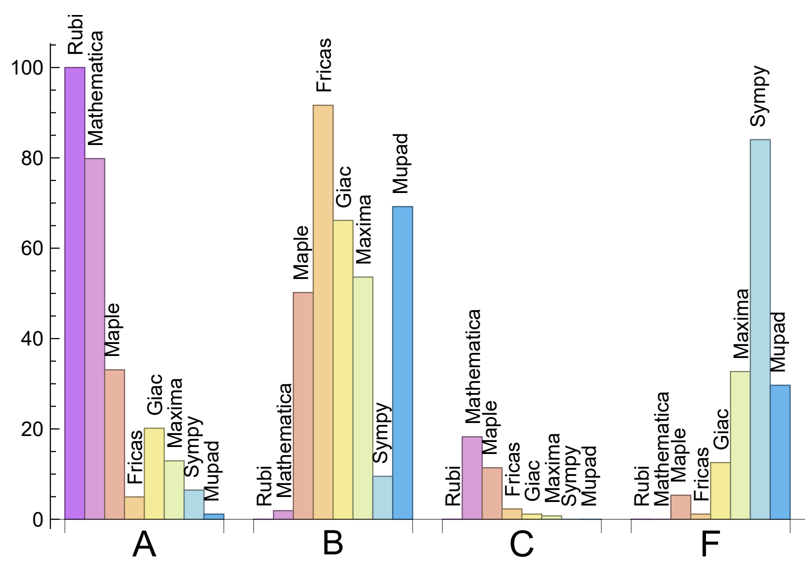
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	79.85	1.90	18.25	0.00
Maple	33.08	50.19	11.41	5.32
Giac	20.15	66.16	1.14	12.55
Maxima	12.93	53.61	0.76	32.70
Sympy	6.46	9.51	0.00	84.03
Fricas	4.94	91.63	2.28	1.14
Mupad	N/A	69.20	0.00	29.66

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	14	100.00 %	0.00 %	0.00 %
Fricas	3	100.00 %	0.00 %	0.00 %
Giac	33	15.15 %	0.00 %	84.85 %
Maxima	86	98.84 %	0.00 %	1.16 %
Sympy	221	85.97 %	13.12 %	0.90 %
Mupad	78	98.72 %	1.28 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

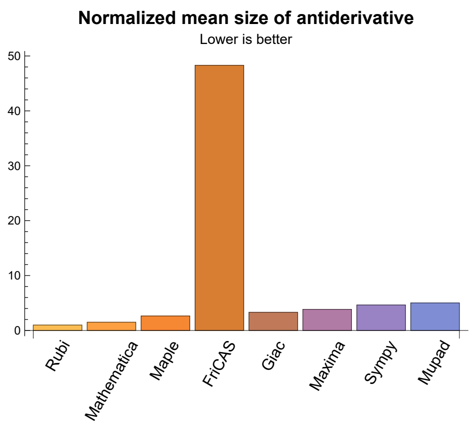
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

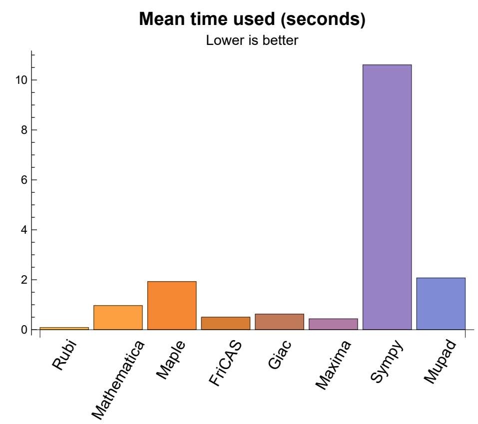
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	91.19	0.99	78.00	1.00
Mathematica	0.97	121.86	1.50	86.00	1.00
Maple	1.93	231.10	2.64	187.00	2.40
Maxima	0.44	410.79	3.84	231.00	2.82
Fricas	0.50	4006.01	48.30	1952.00	23.76
Sympy	10.61	381.71	4.64	119.50	1.93
Giac	0.62	280.82	3.31	216.50	2.52
Mupad	2.07	389.15	5.02	177.00	2.67

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
50
40
30
20
10
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
10
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{74, 76, 77, 79\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {74, 76, 77, 79}

Maple {74, 76, 77, 79}

Maxima {}

Fricas {74, 76, 77, 79}

Sympy {}

Giac {74, 76, 77, 79}

Mupad {76, 77, 79}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {93, 95, 98, 101, 217, 236, 243, 250, 252}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

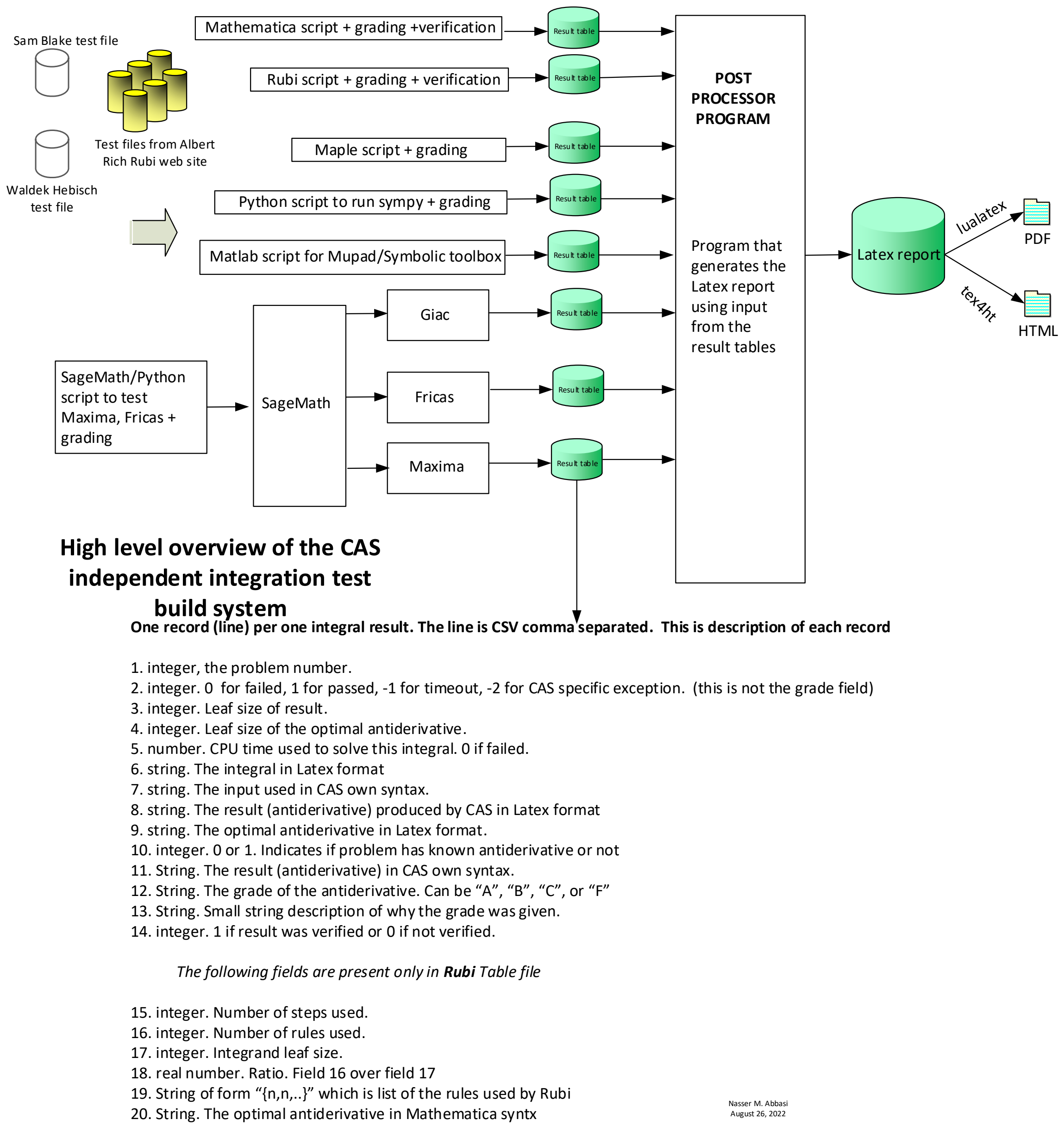
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 100, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 141, 143, 145, 147, 148, 149, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 210, 212, 213, 214, 216, 218, 219, 221, 222, 223, 225, 226, 227, 228, 229, 231, 233, 234, 235, 237, 240, 248, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade: { 104, 144, 146, 202, 241 }

C grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 73, 75, 78, 80, 93, 95, 98, 101, 133, 140, 142, 150, 154, 209, 211, 215, 217, 220, 224, 230, 232, 236, 238, 239, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254 }

F grade: { }

2.1.3 Maple

A grade: { 3, 4, 5, 11, 13, 19, 21, 29, 31, 37, 39, 47, 49, 51, 59, 67, 74, 76, 77, 79, 81, 82, 83, 84, 89, 134, 135, 136, 137, 139, 140, 141, 142, 143, 145, 147, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 181, 182, 183, 184, 185, 186, 188, 190, 191, 192, 193, 194, 195, 196, 199, 201, 202, 203, 204, 205, 206, 207, 257, 258, 261 }

B grade: { 1, 2, 6, 7, 8, 9, 10, 12, 14, 15, 16, 17, 18, 20, 22, 23, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 40, 41, 42, 43, 44, 45, 46, 48, 54, 56, 57, 62, 64, 65, 70, 72, 86, 88, 91, 94, 96, 97, 99, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 138, 144, 146, 148, 156, 157, 158, 168, 177, 179, 187, 189, 197, 198, 200, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 255, 256 }

C grade: { 50, 52, 53, 55, 58, 60, 61, 63, 66, 68, 69, 71, 73, 75, 78, 80, 85, 87, 90, 92, 93, 95, 98, 100, 101, 103, 259, 260, 262, 263 }

F grade: { 214, 215, 216, 217, 218, 223, 224, 235, 236, 237, 244, 245, 253, 254 }

2.1.4 Maxima

A grade: { 5, 6, 30, 49, 50, 51, 52, 53, 54, 66, 68, 74, 76, 77, 79, 81, 86, 94, 102, 110, 112, 121, 138, 139, 140, 150, 172, 174, 175, 176, 178, 202, 204, 206 }

B grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 32, 33, 35, 38, 40, 41, 43, 46, 48, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 107, 114, 116, 119, 123, 125, 128, 130, 132, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 207, 257, 258 }

C grade: { 203, 205 }

F grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 73, 75, 78, 80, 106, 108, 109, 111, 113, 115, 117, 118, 120, 122, 124, 126, 127, 129, 131, 133, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 74, 76, 77, 79, 81, 82, 83, 89, 138, 206 }

B grade: { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 78, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263 }

C grade: { 73, 75, 80, 226, 228, 256 }

F grade: { 203, 205, 207 }

2.1.6 Sympy

A grade: { 74, 76, 77, 79, 134, 136, 138, 148, 160, 207, 208, 210, 212, 233, 242, 251, 257 }

B grade: { 135, 137, 140, 144, 145, 146, 147, 156, 157, 158, 159, 168, 169, 170, 171, 172, 173, 174, 175, 191, 193, 195, 219, 221, 258 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 139, 141, 142, 143, 149, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 167, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

2.1.7 Giac

A grade: { 6, 8, 14, 23, 30, 32, 49, 50, 51, 52, 53, 54, 58, 60, 61, 63, 66, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 93, 110, 112, 137, 138, 139, 140, 141, 150, 153, 171, 173, 174, 175, 176, 177, 178, 189, 202, 203, 205, 206, 207, 257, 258 }

B grade: { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 27, 33, 35, 38, 40, 41, 43, 46, 48, 55, 56, 57, 59, 62, 64, 65, 67, 69, 70, 71, 72, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 114, 116, 119, 121, 123, 125, 128, 130, 132, 134, 135, 136, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

C grade: { 226, 228, 256 }

F grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 106, 108, 109, 111, 113, 115, 117, 118, 120, 122, 124, 126, 127, 129, 131, 133, 259, 260, 261, 262, 263 }

2.1.8 Mupad

A grade: { 76, 77, 79 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 190, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 205, 206, 207, 208, 210, 212, 219, 221, 225, 226, 227, 229, 231, 233, 234, 238, 240, 242, 247, 249, 251, 255, 256, 257, 258 }

C grade: { }

F grade: { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 74, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 186, 187, 188, 189, 197, 198, 199, 200, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 228, 230, 232, 235, 236, 237, 239, 241, 243, 244, 245, 246, 248, 250, 252, 253, 254, 259, 260, 261, 262, 263 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	149	154	120	0	142	101
N.S.	1	1.00	0.77	2.04	2.11	1.64	0.00	1.95	1.38
time (sec)	N/A	0.058	0.281	1.940	0.270	0.375	0.000	0.426	0.245

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	73	141	136	91	0	105	99
N.S.	1	1.00	1.55	3.00	2.89	1.94	0.00	2.23	2.11
time (sec)	N/A	0.045	0.057	1.749	0.278	0.367	0.000	0.436	1.174

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	66	101	71	0	107	64
N.S.	1	1.00	0.93	1.50	2.30	1.61	0.00	2.43	1.45
time (sec)	N/A	0.038	0.168	1.235	0.272	0.346	0.000	0.424	0.145

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	45	44	67	42	0	63	27
N.S.	1	1.00	1.80	1.76	2.68	1.68	0.00	2.52	1.08
time (sec)	N/A	0.024	0.050	1.201	0.274	0.331	0.000	0.406	0.118

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	52	27	40	167	0	68	64
N.S.	1	1.00	2.00	1.04	1.54	6.42	0.00	2.62	2.46
time (sec)	N/A	0.024	0.027	1.174	0.265	0.329	0.000	0.430	0.118

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	59	39	88	0	45	43
N.S.	1	1.00	1.00	2.46	1.62	3.67	0.00	1.88	1.79
time (sec)	N/A	0.023	0.024	1.954	0.280	0.374	0.000	0.416	1.050

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	87	150	152	924	0	142	156
N.S.	1	1.00	1.71	2.94	2.98	18.12	0.00	2.78	3.06
time (sec)	N/A	0.044	0.047	2.402	0.279	0.368	0.000	0.434	1.133

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	61	87	113	244	0	80	173
N.S.	1	1.00	1.39	1.98	2.57	5.55	0.00	1.82	3.93
time (sec)	N/A	0.033	0.060	2.127	0.286	0.326	0.000	0.442	1.097

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	94	293	295	394	0	293	293
N.S.	1	1.00	0.80	2.48	2.50	3.34	0.00	2.48	2.48
time (sec)	N/A	0.099	1.031	2.042	0.284	0.333	0.000	0.495	0.309

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	71	273	265	259	0	205	215
N.S.	1	1.00	0.92	3.55	3.44	3.36	0.00	2.66	2.79
time (sec)	N/A	0.069	0.395	2.157	0.312	0.336	0.000	0.495	0.287

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	70	118	217	291	0	213	248
N.S.	1	1.00	0.89	1.49	2.75	3.68	0.00	2.70	3.14
time (sec)	N/A	0.076	0.666	1.572	0.269	0.343	0.000	0.466	1.177

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	46	98	171	167	0	139	154
N.S.	1	1.00	0.94	2.00	3.49	3.41	0.00	2.84	3.14
time (sec)	N/A	0.038	0.246	1.394	0.272	0.332	0.000	0.458	0.191

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	63	196	890	0	123	160
N.S.	1	1.00	0.98	1.24	3.84	17.45	0.00	2.41	3.14
time (sec)	N/A	0.044	0.130	1.614	0.274	0.340	0.000	0.483	0.154

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	169	136	264	0	86	209
N.S.	1	1.00	0.93	3.67	2.96	5.74	0.00	1.87	4.54
time (sec)	N/A	0.040	0.361	2.379	0.286	0.347	0.000	0.460	1.157

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	96	239	181	2462	0	153	261
N.S.	1	1.00	1.17	2.91	2.21	30.02	0.00	1.87	3.18
time (sec)	N/A	0.085	1.034	2.744	0.271	0.361	0.000	0.455	0.162

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	157	210	393	0	143	143
N.S.	1	1.00	0.82	2.18	2.92	5.46	0.00	1.99	1.99
time (sec)	N/A	0.055	0.382	2.547	0.275	0.337	0.000	0.463	1.080

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	125	506	480	879	0	505	730
N.S.	1	1.00	0.69	2.78	2.64	4.83	0.00	2.77	4.01
time (sec)	N/A	0.152	2.725	2.474	0.302	0.375	0.000	0.645	1.390

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	474	439	540	0	330	361
N.S.	1	1.00	0.87	4.51	4.18	5.14	0.00	3.14	3.44
time (sec)	N/A	0.090	0.226	2.422	0.303	0.350	0.000	0.588	0.412

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	139	95	180	377	725	0	393	668
N.S.	1	1.14	0.78	1.48	3.09	5.94	0.00	3.22	5.48
time (sec)	N/A	0.128	1.538	1.723	0.271	0.371	0.000	0.569	0.315

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	170	321	383	0	236	308
N.S.	1	1.00	0.90	2.43	4.59	5.47	0.00	3.37	4.40
time (sec)	N/A	0.048	0.613	1.465	0.282	0.342	0.000	0.520	1.245

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	118	560	2277	0	196	317
N.S.	1	1.00	0.94	1.40	6.67	27.11	0.00	2.33	3.77
time (sec)	N/A	0.062	0.225	1.580	0.294	0.394	0.000	0.518	0.246

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	70	344	348	572	0	202	590
N.S.	1	1.00	1.09	5.38	5.44	8.94	0.00	3.16	9.22
time (sec)	N/A	0.046	0.501	2.525	0.280	0.384	0.000	0.539	1.235

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	127	410	403	5037	0	206	412
N.S.	1	1.00	0.84	2.70	2.65	33.14	0.00	1.36	2.71
time (sec)	N/A	0.163	6.151	2.915	0.304	0.367	0.000	0.554	1.256

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	87	403	493	925	0	257	622
N.S.	1	1.00	0.89	4.11	5.03	9.44	0.00	2.62	6.35
time (sec)	N/A	0.066	0.855	2.615	0.300	0.346	0.000	0.542	0.279

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	93	434	514	2024	0	276	250
N.S.	1	1.00	0.79	3.68	4.36	17.15	0.00	2.34	2.12
time (sec)	N/A	0.118	0.196	2.470	0.545	0.398	0.000	1.707	1.678

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	135	202	0	1367	0	0	955
N.S.	1	1.00	1.80	2.69	0.00	18.23	0.00	0.00	12.73
time (sec)	N/A	0.094	0.395	2.316	0.000	0.361	0.000	0.000	2.655

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	67	310	316	916	0	159	198
N.S.	1	1.00	0.86	3.97	4.05	11.74	0.00	2.04	2.54
time (sec)	N/A	0.072	0.116	2.381	0.515	0.410	0.000	0.959	1.512

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	107	104	0	666	0	0	520
N.S.	1	1.00	2.02	1.96	0.00	12.57	0.00	0.00	9.81
time (sec)	N/A	0.046	0.164	1.537	0.000	0.416	0.000	0.000	2.002

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	123	67	0	587	0	0	284
N.S.	1	1.00	2.24	1.22	0.00	10.67	0.00	0.00	5.16
time (sec)	N/A	0.053	0.158	2.395	0.000	0.405	0.000	0.000	1.921

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	188	62	618	0	69	136
N.S.	1	1.00	1.00	3.92	1.29	12.88	0.00	1.44	2.83
time (sec)	N/A	0.045	0.082	2.527	0.509	0.386	0.000	0.649	1.311

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	170	111	0	1790	0	0	787
N.S.	1	1.00	2.00	1.31	0.00	21.06	0.00	0.00	9.26
time (sec)	N/A	0.086	0.471	2.844	0.000	0.430	0.000	0.000	1.802

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	247	134	1628	0	119	254
N.S.	1	1.00	1.01	3.53	1.91	23.26	0.00	1.70	3.63
time (sec)	N/A	0.062	0.226	2.800	0.514	0.391	0.000	0.641	1.439

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	132	512	1690	7366	0	476	-1
N.S.	1	1.00	0.69	2.67	8.80	38.36	0.00	2.48	-0.01
time (sec)	N/A	0.179	0.783	2.568	0.710	0.462	0.000	2.058	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	160	267	0	5025	0	0	-1
N.S.	1	1.00	1.29	2.15	0.00	40.52	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.954	2.612	0.000	0.415	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	105	387	840	3918	0	367	-1
N.S.	1	1.00	0.80	2.93	6.36	29.68	0.00	2.78	-0.01
time (sec)	N/A	0.116	0.494	2.549	0.640	0.405	0.000	1.215	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	133	167	0	2252	0	0	-1
N.S.	1	1.00	1.45	1.82	0.00	24.48	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.543	2.163	0.000	0.390	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	175	161	0	2614	0	0	-1
N.S.	1	1.00	1.70	1.56	0.00	25.38	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.540	3.210	0.000	0.404	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	86	266	212	2562	0	227	-1
N.S.	1	1.00	1.05	3.24	2.59	31.24	0.00	2.77	-0.01
time (sec)	N/A	0.051	0.346	2.808	0.596	0.405	0.000	0.724	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	203	187	0	6335	0	0	-1
N.S.	1	1.00	1.44	1.33	0.00	44.93	0.00	0.00	-0.01
time (sec)	N/A	0.147	2.841	3.198	0.000	0.457	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	338	282	5062	0	209	-1
N.S.	1	1.00	1.01	2.99	2.50	44.80	0.00	1.85	-0.01
time (sec)	N/A	0.112	0.678	3.181	0.597	0.411	0.000	0.670	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	184	610	3392	18818	0	861	-1
N.S.	1	1.00	0.77	2.54	14.13	78.41	0.00	3.59	-0.00
time (sec)	N/A	0.251	0.623	2.813	1.024	0.548	0.000	3.179	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	227	341	0	13095	0	0	-1
N.S.	1	1.00	1.37	2.05	0.00	78.89	0.00	0.00	-0.01
time (sec)	N/A	0.213	1.368	2.661	0.000	0.515	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	158	488	1806	12965	0	535	-1
N.S.	1	1.00	0.85	2.64	9.76	70.08	0.00	2.89	-0.01
time (sec)	N/A	0.179	0.892	2.711	0.852	0.532	0.000	2.131	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	157	252	0	7119	0	0	-1
N.S.	1	1.00	1.25	2.00	0.00	56.50	0.00	0.00	-0.01
time (sec)	N/A	0.072	1.274	2.201	0.000	0.457	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	236	304	0	10716	0	0	-1
N.S.	1	1.00	1.51	1.95	0.00	68.69	0.00	0.00	-0.01
time (sec)	N/A	0.188	1.045	3.197	0.000	0.558	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	109	306	478	8312	0	351	-1
N.S.	1	1.00	0.97	2.73	4.27	74.21	0.00	3.13	-0.01
time (sec)	N/A	0.062	0.740	2.921	0.663	0.426	0.000	1.038	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	269	304	0	21301	0	0	-1
N.S.	1	1.00	1.37	1.55	0.00	108.68	0.00	0.00	-0.01
time (sec)	N/A	0.235	3.135	3.412	0.000	0.601	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	149	397	615	14334	0	395	-1
N.S.	1	1.00	0.99	2.63	4.07	94.93	0.00	2.62	-0.01
time (sec)	N/A	0.146	0.982	3.206	0.703	0.482	0.000	1.021	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	92	183	194	1530	0	203	156
N.S.	1	1.00	0.70	1.39	1.47	11.59	0.00	1.54	1.18
time (sec)	N/A	0.128	0.166	2.361	0.498	0.363	0.000	0.444	0.269

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	104	186	174	1070	0	132	171
N.S.	1	1.00	1.06	1.90	1.78	10.92	0.00	1.35	1.74
time (sec)	N/A	0.086	0.230	2.207	0.486	0.360	0.000	0.441	0.228

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	69	68	141	924	0	140	115
N.S.	1	1.00	0.69	0.68	1.41	9.24	0.00	1.40	1.15
time (sec)	N/A	0.086	0.087	1.530	0.485	0.362	0.000	0.443	1.147

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	72	125	105	528	0	86	128
N.S.	1	1.00	1.14	1.98	1.67	8.38	0.00	1.37	2.03
time (sec)	N/A	0.057	0.095	2.187	0.477	0.355	0.000	0.421	0.139

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	75	101	83	522	0	74	233
N.S.	1	1.00	1.53	2.06	1.69	10.65	0.00	1.51	4.76
time (sec)	N/A	0.053	0.027	2.889	0.477	0.375	0.000	0.438	2.471

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	80	44	141	0	45	79
N.S.	1	1.00	1.00	2.76	1.52	4.86	0.00	1.55	2.72
time (sec)	N/A	0.026	0.025	2.872	0.266	0.332	0.000	0.439	0.165

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	95	177	156	1188	0	143	173
N.S.	1	1.00	1.34	2.49	2.20	16.73	0.00	2.01	2.44
time (sec)	N/A	0.066	0.025	3.198	0.487	0.379	0.000	0.445	2.481

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	74	156	184	1739	0	149	162
N.S.	1	1.00	1.32	2.79	3.29	31.05	0.00	2.66	2.89
time (sec)	N/A	0.041	0.157	3.091	0.475	0.342	0.000	0.443	1.235

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	206	156	365	379	5034	0	374	359
N.S.	1	1.21	0.92	2.15	2.23	29.61	0.00	2.20	2.11
time (sec)	N/A	0.208	1.884	2.843	0.510	0.413	0.000	0.580	0.444

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	121	345	348	3341	0	289	397
N.S.	1	1.00	0.66	1.90	1.91	18.36	0.00	1.59	2.18
time (sec)	N/A	0.162	0.517	2.781	0.499	0.363	0.000	0.557	1.436

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	159	137	130	301	3649	0	296	306
N.S.	1	1.23	1.06	1.01	2.33	28.29	0.00	2.29	2.37
time (sec)	N/A	0.144	1.144	1.631	0.495	0.364	0.000	0.551	1.360

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	90	244	253	2230	0	203	338
N.S.	1	1.00	0.73	1.98	2.06	18.13	0.00	1.65	2.75
time (sec)	N/A	0.103	0.311	2.816	0.483	0.374	0.000	0.495	1.290

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	106	187	447	2498	0	178	522
N.S.	1	1.00	1.08	1.91	4.56	25.49	0.00	1.82	5.33
time (sec)	N/A	0.101	0.124	3.298	0.472	0.375	0.000	0.495	3.283

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	94	234	256	518	0	122	483
N.S.	1	1.00	2.00	4.98	5.45	11.02	0.00	2.60	10.28
time (sec)	N/A	0.040	0.194	3.199	0.274	0.346	0.000	0.516	1.264

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	138	312	378	4642	0	192	561
N.S.	1	1.00	1.29	2.92	3.53	43.38	0.00	1.79	5.24
time (sec)	N/A	0.116	0.132	3.723	0.487	0.390	0.000	0.524	2.896

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	147	302	468	4125	0	249	344
N.S.	1	1.00	1.52	3.11	4.82	42.53	0.00	2.57	3.55
time (sec)	N/A	0.069	0.134	3.400	0.497	0.369	0.000	0.520	0.261

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	306	294	679	647	12323	0	694	682
N.S.	1	1.11	1.07	2.47	2.35	44.81	0.00	2.52	2.48
time (sec)	N/A	0.345	6.201	3.138	0.501	0.485	0.000	0.927	1.703

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	291	675	604	8462	0	580	757
N.S.	1	1.00	0.83	1.92	1.72	24.11	0.00	1.65	2.16
time (sec)	N/A	0.247	6.445	2.842	0.492	0.438	0.000	0.822	1.598

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	241	244	206	544	9862	0	577	617
N.S.	1	1.10	1.11	0.94	2.47	44.83	0.00	2.62	2.80
time (sec)	N/A	0.219	6.188	1.950	0.509	0.448	0.000	0.777	0.556

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	168	535	484	6410	0	463	707
N.S.	1	1.00	0.62	1.99	1.80	23.83	0.00	1.72	2.63
time (sec)	N/A	0.178	5.791	2.822	0.524	0.433	0.000	0.675	1.487

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	154	440	654	7127	0	414	671
N.S.	1	1.00	0.70	2.01	2.99	32.54	0.00	1.89	3.06
time (sec)	N/A	0.197	2.111	3.671	0.497	0.466	0.000	0.663	6.399

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	113	508	679	1192	0	311	1515
N.S.	1	1.00	1.59	7.15	9.56	16.79	0.00	4.38	21.34
time (sec)	N/A	0.046	0.534	3.446	0.285	0.334	0.000	0.687	1.378

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	243	645	586	10985	0	426	731
N.S.	1	1.00	1.05	2.78	2.53	47.35	0.00	1.84	3.15
time (sec)	N/A	0.223	6.271	5.023	0.494	0.440	0.000	0.672	8.048

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	213	565	997	9459	0	437	646
N.S.	1	1.00	1.54	4.09	7.22	68.54	0.00	3.17	4.68
time (sec)	N/A	0.078	0.141	3.691	0.505	0.475	0.000	0.668	0.540

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	645	452	0	17123	0	338	2500
N.S.	1	1.00	1.31	0.92	0.00	34.87	0.00	0.69	5.09
time (sec)	N/A	0.649	3.208	4.425	0.000	1.491	0.000	0.669	4.355

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	826	289	0	62017	0	303	-1
N.S.	1	0.00	25.03	8.76	0.00	1879.30	0.00	9.18	-0.03
time (sec)	N/A	0.029	0.388	7.433	0.000	4.534	0.000	1.530	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	423	301	0	10695	0	206	2100
N.S.	1	1.00	1.10	0.78	0.00	27.85	0.00	0.54	5.47
time (sec)	N/A	0.444	2.511	4.000	0.000	1.345	0.000	0.547	2.815

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	409	159	0	40923	0	169	2500
N.S.	1	0.00	13.19	5.13	0.00	1320.10	0.00	5.45	80.65
time (sec)	N/A	0.018	0.165	5.369	0.000	1.990	0.000	1.059	87.988

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	319	96	0	20085	0	146	2500
N.S.	1	0.00	10.29	3.10	0.00	647.90	0.00	4.71	80.65
time (sec)	N/A	0.026	0.125	4.535	0.000	2.034	0.000	0.848	16.503

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	190	116	0	640	0	21	669
N.S.	1	1.00	1.21	0.74	0.00	4.08	0.00	0.13	4.26
time (sec)	N/A	0.093	0.134	4.229	0.000	0.398	0.000	0.468	8.707

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	201	136	0	24389	0	68	2500
N.S.	1	0.00	6.09	4.12	0.00	739.06	0.00	2.06	75.76
time (sec)	N/A	0.032	0.280	4.852	0.000	5.914	0.000	0.651	26.921

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	322	173	0	1954	0	180	2500
N.S.	1	1.00	1.50	0.80	0.00	9.09	0.00	0.84	11.63
time (sec)	N/A	0.163	2.443	4.516	0.000	1.498	0.000	0.492	3.218

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	46	104	63	0	104	74
N.S.	1	1.00	0.70	0.73	1.65	1.00	0.00	1.65	1.17
time (sec)	N/A	0.036	0.114	2.382	0.258	0.337	0.000	0.443	0.212

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	44	40	83	45	0	84	74
N.S.	1	1.00	1.47	1.33	2.77	1.50	0.00	2.80	2.47
time (sec)	N/A	0.025	0.015	2.211	0.271	0.409	0.000	0.428	0.210

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	54	69	30	0	78	27
N.S.	1	1.00	0.97	1.64	2.09	0.91	0.00	2.36	0.82
time (sec)	N/A	0.029	0.047	1.525	0.273	0.403	0.000	0.430	0.146

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	47	32	55	102	0	47	66
N.S.	1	1.00	1.74	1.19	2.04	3.78	0.00	1.74	2.44
time (sec)	N/A	0.023	0.029	1.454	0.472	0.380	0.000	0.425	0.139

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	106	80	323	0	84	125
N.S.	1	1.00	1.20	2.65	2.00	8.08	0.00	2.10	3.12
time (sec)	N/A	0.022	0.022	1.455	0.483	0.388	0.000	0.427	0.132

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	60	34	159	0	59	59
N.S.	1	1.00	1.00	2.14	1.21	5.68	0.00	2.11	2.11
time (sec)	N/A	0.020	0.012	1.536	0.274	0.353	0.000	0.439	1.218

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	93	172	181	1046	0	153	280
N.S.	1	1.00	1.41	2.61	2.74	15.85	0.00	2.32	4.24
time (sec)	N/A	0.035	0.026	1.814	0.478	0.391	0.000	0.436	1.251

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	86	96	371	345	0	95	304
N.S.	1	1.00	1.79	2.00	7.73	7.19	0.00	1.98	6.33
time (sec)	N/A	0.029	0.041	1.678	0.273	0.407	0.000	0.445	0.165

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	63	73	171	95	0	186	102
N.S.	1	1.00	0.74	0.86	2.01	1.12	0.00	2.19	1.20
time (sec)	N/A	0.062	0.223	1.759	0.265	0.349	0.000	0.487	0.266

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	71	231	161	519	0	152	130
N.S.	1	1.00	1.31	4.28	2.98	9.61	0.00	2.81	2.41
time (sec)	N/A	0.047	0.305	1.849	0.481	0.385	0.000	0.479	0.257

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	96	140	105	0	167	77
N.S.	1	1.00	1.06	1.88	2.75	2.06	0.00	3.27	1.51
time (sec)	N/A	0.056	0.284	1.450	0.278	0.360	0.000	0.474	1.289

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	203	152	774	0	160	182
N.S.	1	1.00	0.97	3.38	2.53	12.90	0.00	2.67	3.03
time (sec)	N/A	0.060	0.168	1.834	0.469	0.422	0.000	0.478	0.237

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	427	218	199	1373	0	170	303
N.S.	1	1.00	4.69	2.40	2.19	15.09	0.00	1.87	3.33
time (sec)	N/A	0.062	6.965	2.044	0.470	0.362	0.000	0.465	0.162

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	170	53	391	0	169	482
N.S.	1	1.00	1.00	3.47	1.08	7.98	0.00	3.45	9.84
time (sec)	N/A	0.037	0.151	2.072	0.281	0.432	0.000	0.459	1.240

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	792	358	345	2824	0	267	572
N.S.	1	1.00	6.34	2.86	2.76	22.59	0.00	2.14	4.58
time (sec)	N/A	0.101	8.022	2.300	0.479	0.387	0.000	0.488	0.181

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	83	239	928	677	0	238	732
N.S.	1	1.00	1.09	3.14	12.21	8.91	0.00	3.13	9.63
time (sec)	N/A	0.049	0.388	2.196	0.287	0.339	0.000	0.466	1.220

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	81	293	267	227	0	282	133
N.S.	1	1.00	0.89	3.22	2.93	2.49	0.00	3.10	1.46
time (sec)	N/A	0.094	0.501	1.856	0.272	0.375	0.000	0.624	1.407

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	494	386	284	1840	0	264	232
N.S.	1	1.00	5.68	4.44	3.26	21.15	0.00	3.03	2.67
time (sec)	N/A	0.075	6.688	2.000	0.474	0.384	0.000	0.589	0.355

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	148	256	369	0	265	243
N.S.	1	1.00	0.88	1.90	3.28	4.73	0.00	3.40	3.12
time (sec)	N/A	0.065	0.602	1.528	0.277	0.367	0.000	0.561	0.289

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	353	295	2411	0	272	355
N.S.	1	1.00	0.94	3.57	2.98	24.35	0.00	2.75	3.59
time (sec)	N/A	0.087	0.379	2.108	0.485	0.473	0.000	0.558	0.309

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	1341	403	362	3465	0	310	535
N.S.	1	1.00	9.00	2.70	2.43	23.26	0.00	2.08	3.59
time (sec)	N/A	0.108	10.719	2.169	0.503	0.390	0.000	0.525	1.394

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	348	71	786	0	347	1050
N.S.	1	1.00	1.00	5.19	1.06	11.73	0.00	5.18	15.67
time (sec)	N/A	0.042	0.130	2.132	0.264	0.369	0.000	0.561	1.333

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	158	611	553	6114	0	485	951
N.S.	1	1.00	0.80	3.09	2.79	30.88	0.00	2.45	4.80
time (sec)	N/A	0.159	11.157	2.520	0.486	0.414	0.000	0.547	1.364

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	218	448	1847	1185	0	447	1424
N.S.	1	1.00	2.14	4.39	18.11	11.62	0.00	4.38	13.96
time (sec)	N/A	0.064	0.618	2.385	0.292	0.364	0.000	0.547	1.333

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	115	438	514	2180	0	301	967
N.S.	1	1.00	0.96	3.65	4.28	18.17	0.00	2.51	8.06
time (sec)	N/A	0.121	0.216	2.739	0.559	0.493	0.000	2.014	1.930

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	79	307	0	1850	0	0	2194
N.S.	1	1.00	0.99	3.84	0.00	23.12	0.00	0.00	27.42
time (sec)	N/A	0.079	0.308	2.746	0.000	0.418	0.000	0.000	2.877

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	311	316	948	0	155	880
N.S.	1	1.00	1.00	4.04	4.10	12.31	0.00	2.01	11.43
time (sec)	N/A	0.070	0.115	2.541	0.509	0.498	0.000	1.045	1.933

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	208	0	766	0	0	154
N.S.	1	1.00	1.02	3.92	0.00	14.45	0.00	0.00	2.91
time (sec)	N/A	0.051	0.162	2.678	0.000	0.392	0.000	0.000	1.676

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	152	0	511	0	0	147
N.S.	1	1.00	1.00	4.22	0.00	14.19	0.00	0.00	4.08
time (sec)	N/A	0.031	0.036	2.098	0.000	0.416	0.000	0.000	0.335

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	160	36	455	0	44	81
N.S.	1	1.00	1.00	5.00	1.12	14.22	0.00	1.38	2.53
time (sec)	N/A	0.037	0.050	2.300	0.510	0.343	0.000	0.559	1.459

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	175	0	540	0	0	449
N.S.	1	1.00	1.00	3.18	0.00	9.82	0.00	0.00	8.16
time (sec)	N/A	0.051	0.150	2.498	0.000	0.453	0.000	0.000	1.705

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	197	63	649	0	70	176
N.S.	1	1.00	1.00	3.94	1.26	12.98	0.00	1.40	3.52
time (sec)	N/A	0.047	0.106	2.181	0.519	0.394	0.000	0.588	1.615

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	236	0	1584	0	0	1012
N.S.	1	1.00	0.92	2.74	0.00	18.42	0.00	0.00	11.77
time (sec)	N/A	0.085	0.460	2.499	0.000	0.423	0.000	0.000	2.096

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	252	140	2032	0	135	252
N.S.	1	1.00	0.95	3.36	1.87	27.09	0.00	1.80	3.36
time (sec)	N/A	0.064	0.276	2.493	0.506	0.382	0.000	0.628	1.654

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	111	378	0	6934	0	0	-1
N.S.	1	1.00	0.87	2.95	0.00	54.17	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.801	2.997	0.000	0.441	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	110	386	840	4324	0	416	-1
N.S.	1	1.00	0.79	2.76	6.00	30.89	0.00	2.97	-0.01
time (sec)	N/A	0.131	0.593	2.761	0.610	0.400	0.000	1.297	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	89	286	0	3502	0	0	-1
N.S.	1	1.00	0.88	2.83	0.00	34.67	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.595	2.878	0.000	0.428	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	252	0	2041	0	0	-1
N.S.	1	1.00	0.94	3.04	0.00	24.59	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.260	2.494	0.000	0.430	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	63	232	125	1515	0	138	-1
N.S.	1	1.00	0.95	3.52	1.89	22.95	0.00	2.09	-0.02
time (sec)	N/A	0.043	0.198	2.451	0.530	0.379	0.000	0.703	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	228	0	1555	0	0	-1
N.S.	1	1.00	0.96	3.17	0.00	21.60	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.097	2.537	0.000	0.361	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	83	259	127	1443	0	143	-1
N.S.	1	1.00	1.08	3.36	1.65	18.74	0.00	1.86	-0.01
time (sec)	N/A	0.054	0.238	2.407	0.532	0.395	0.000	0.712	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	203	273	0	2140	0	0	-1
N.S.	1	1.00	1.99	2.68	0.00	20.98	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.379	2.447	0.000	0.387	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	102	306	209	2869	0	232	-1
N.S.	1	1.00	1.05	3.15	2.15	29.58	0.00	2.39	-0.01
time (sec)	N/A	0.093	0.456	2.329	0.589	0.495	0.000	0.726	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	265	351	0	6396	0	0	-1
N.S.	1	1.00	1.71	2.26	0.00	41.26	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.973	2.734	0.000	0.436	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	164	501	1806	13887	0	546	-1
N.S.	1	1.00	0.83	2.53	9.12	70.14	0.00	2.76	-0.01
time (sec)	N/A	0.222	0.936	3.144	0.837	0.511	0.000	1.462	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	145	375	0	11392	0	0	-1
N.S.	1	1.00	0.94	2.44	0.00	73.97	0.00	0.00	-0.01
time (sec)	N/A	0.155	1.982	3.257	0.000	0.496	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	134	394	0	7909	0	0	-1
N.S.	1	1.00	0.93	2.74	0.00	54.92	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.807	2.738	0.000	0.431	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	77	284	366	5840	0	320	-1
N.S.	1	1.00	0.80	2.96	3.81	60.83	0.00	3.33	-0.01
time (sec)	N/A	0.050	0.587	2.544	0.626	0.414	0.000	1.013	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	123	342	0	6614	0	0	-1
N.S.	1	1.00	0.95	2.65	0.00	51.27	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.556	2.950	0.000	0.470	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	334	360	5659	0	319	-1
N.S.	1	1.00	1.00	2.90	3.13	49.21	0.00	2.77	-0.01
time (sec)	N/A	0.066	0.720	2.641	0.646	0.414	0.000	0.878	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	276	0	5077	0	0	-1
N.S.	1	1.00	0.85	2.65	0.00	48.82	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.214	2.695	0.000	0.421	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	128	376	332	5233	0	322	-1
N.S.	1	1.00	0.98	2.87	2.53	39.95	0.00	2.46	-0.01
time (sec)	N/A	0.088	0.676	2.817	0.657	0.459	0.000	0.906	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	317	389	0	8070	0	0	-1
N.S.	1	1.00	2.03	2.49	0.00	51.73	0.00	0.00	-0.01
time (sec)	N/A	0.158	2.214	2.724	0.000	0.486	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	97	85	199	339	82	134	50
N.S.	1	1.00	1.80	1.57	3.69	6.28	1.52	2.48	0.93
time (sec)	N/A	0.036	0.036	0.375	0.287	0.390	0.154	0.459	1.218

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	71	168	1205	88	92	53
N.S.	1	1.00	0.88	1.45	3.43	24.59	1.80	1.88	1.08
time (sec)	N/A	0.042	0.188	0.373	0.481	0.341	0.121	0.440	0.118

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	65	63	105	160	54	86	34
N.S.	1	1.00	1.81	1.75	2.92	4.44	1.50	2.39	0.94
time (sec)	N/A	0.028	0.024	0.373	0.291	0.326	0.096	0.428	1.170

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	41	49	76	399	60	57	37
N.S.	1	1.00	1.32	1.58	2.45	12.87	1.94	1.84	1.19
time (sec)	N/A	0.022	0.022	0.367	0.506	0.373	0.082	0.432	1.168

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	47	31	37	20	29	18
N.S.	1	1.00	1.47	2.47	1.63	1.95	1.05	1.53	0.95
time (sec)	N/A	0.011	0.007	0.284	0.285	0.347	0.059	0.419	0.073

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	24	35	70	0	46	228
N.S.	1	1.00	1.32	0.96	1.40	2.80	0.00	1.84	9.12
time (sec)	N/A	0.031	0.035	1.720	0.290	0.472	0.000	0.436	1.312

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	32	28	31	38	100	30	25
N.S.	1	1.00	1.78	1.56	1.72	2.11	5.56	1.67	1.39
time (sec)	N/A	0.020	0.022	1.454	0.274	0.338	6.246	0.444	1.267

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	35	106	407	0	58	76
N.S.	1	1.00	1.26	1.13	3.42	13.13	0.00	1.87	2.45
time (sec)	N/A	0.030	0.088	1.803	0.276	0.349	0.000	0.449	1.264

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	61	46	105	156	0	86	162
N.S.	1	1.00	1.69	1.28	2.92	4.33	0.00	2.39	4.50
time (sec)	N/A	0.030	0.034	1.716	0.277	0.378	0.000	0.454	1.168

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	51	56	206	1216	0	93	177
N.S.	1	1.00	1.04	1.14	4.20	24.82	0.00	1.90	3.61
time (sec)	N/A	0.043	0.242	1.709	0.288	0.369	0.000	0.467	1.246

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	190	158	369	796	165	300	91
N.S.	1	1.00	2.29	1.90	4.45	9.59	1.99	3.61	1.10
time (sec)	N/A	0.059	0.065	0.383	0.304	0.356	0.241	0.485	0.180

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	66	130	333	3441	170	191	100
N.S.	1	1.00	0.87	1.71	4.38	45.28	2.24	2.51	1.32
time (sec)	N/A	0.080	0.318	0.385	0.499	0.415	0.218	0.500	1.315

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	137	120	231	483	117	218	67
N.S.	1	1.00	2.17	1.90	3.67	7.67	1.86	3.46	1.06
time (sec)	N/A	0.054	0.042	0.365	0.289	0.386	0.166	0.479	1.300

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	92	186	1638	122	116	76
N.S.	1	1.00	0.88	1.61	3.26	28.74	2.14	2.04	1.33
time (sec)	N/A	0.056	0.235	0.366	0.493	0.356	0.133	0.450	1.206

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	65	84	114	201	68	103	47
N.S.	1	1.00	1.51	1.95	2.65	4.67	1.58	2.40	1.09
time (sec)	N/A	0.024	0.520	0.300	0.272	0.359	0.114	0.421	1.243

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	50	104	668	0	141	210
N.S.	1	1.00	0.98	1.02	2.12	13.63	0.00	2.88	4.29
time (sec)	N/A	0.055	0.101	1.856	0.507	0.355	0.000	0.463	1.319

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	64	49	64	97	0	71	59
N.S.	1	1.00	1.78	1.36	1.78	2.69	0.00	1.97	1.64
time (sec)	N/A	0.049	0.079	1.480	0.276	0.366	0.000	0.491	1.244

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	50	134	677	0	141	211
N.S.	1	1.00	0.96	0.96	2.58	13.02	0.00	2.71	4.06
time (sec)	N/A	0.066	0.121	1.827	0.286	0.368	0.000	0.492	1.415

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	65	59	114	197	0	103	175
N.S.	1	1.00	1.51	1.37	2.65	4.58	0.00	2.40	4.07
time (sec)	N/A	0.052	0.447	1.612	0.293	0.388	0.000	0.499	0.175

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	58	71	236	1649	0	118	197
N.S.	1	1.00	0.81	0.99	3.28	22.90	0.00	1.64	2.74
time (sec)	N/A	0.075	0.311	1.724	0.281	0.372	0.000	0.509	1.272

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	98	87	231	473	0	218	529
N.S.	1	1.00	1.56	1.38	3.67	7.51	0.00	3.46	8.40
time (sec)	N/A	0.057	0.071	1.836	0.277	0.380	0.000	0.523	0.204

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	74	102	390	3454	0	192	362
N.S.	1	1.00	0.80	1.11	4.24	37.54	0.00	2.09	3.93
time (sec)	N/A	0.080	0.352	1.807	0.281	0.391	0.000	0.564	0.274

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	123	247	583	1563	260	534	138
N.S.	1	1.00	1.08	2.17	5.11	13.71	2.28	4.68	1.21
time (sec)	N/A	0.074	1.056	0.396	0.290	0.350	0.373	0.600	0.260

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	98	205	540	7502	279	309	155
N.S.	1	1.00	0.92	1.92	5.05	70.11	2.61	2.89	1.45
time (sec)	N/A	0.111	0.217	0.405	0.535	0.433	0.334	0.557	1.241

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	108	193	400	1036	192	418	106
N.S.	1	1.00	1.15	2.05	4.26	11.02	2.04	4.45	1.13
time (sec)	N/A	0.067	1.189	0.398	0.293	0.395	0.247	0.547	1.218

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	76	151	351	4298	211	216	123
N.S.	1	1.00	0.92	1.82	4.23	51.78	2.54	2.60	1.48
time (sec)	N/A	0.076	0.169	0.388	0.506	0.463	0.211	0.526	1.253

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	95	141	239	567	126	241	86
N.S.	1	1.00	1.28	1.91	3.23	7.66	1.70	3.26	1.16
time (sec)	N/A	0.032	0.423	0.313	0.285	0.337	0.173	0.432	1.282

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	86	214	2381	0	267	380
N.S.	1	1.00	0.93	1.19	2.97	33.07	0.00	3.71	5.28
time (sec)	N/A	0.072	0.378	1.991	0.482	0.385	0.000	0.511	0.401

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	80	147	341	0	135	218
N.S.	1	1.00	1.37	1.36	2.49	5.78	0.00	2.29	3.69
time (sec)	N/A	0.057	1.631	1.634	0.277	0.352	0.000	0.537	1.308

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	63	76	203	1686	0	274	327
N.S.	1	1.00	0.88	1.06	2.82	23.42	0.00	3.81	4.54
time (sec)	N/A	0.079	0.329	1.971	0.492	0.381	0.000	0.576	2.476

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	82	80	147	341	0	135	219
N.S.	1	1.00	1.39	1.36	2.49	5.78	0.00	2.29	3.71
time (sec)	N/A	0.066	0.883	1.599	0.308	0.347	0.000	0.583	1.287

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	86	264	2393	0	267	381
N.S.	1	1.00	0.81	1.04	3.18	28.83	0.00	3.22	4.59
time (sec)	N/A	0.085	0.394	1.732	0.284	0.399	0.000	0.653	0.395

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	100	100	239	557	0	241	568
N.S.	1	1.00	1.35	1.35	3.23	7.53	0.00	3.26	7.68
time (sec)	N/A	0.064	1.243	1.799	0.303	0.361	0.000	0.631	1.299

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	76	117	420	4305	0	217	380
N.S.	1	1.00	0.74	1.14	4.08	41.80	0.00	2.11	3.69
time (sec)	N/A	0.092	0.180	1.783	0.296	0.389	0.000	0.704	0.318

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	128	214	410	1176	209	447	133
N.S.	1	1.00	1.16	1.95	3.73	10.69	1.90	4.06	1.21
time (sec)	N/A	0.053	1.245	0.312	0.304	0.375	0.284	0.457	0.200

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	170	303	624	2133	308	721	188
N.S.	1	1.00	1.06	1.89	3.90	13.33	1.92	4.51	1.18
time (sec)	N/A	0.070	1.581	0.337	0.312	0.398	0.380	0.468	1.321

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	85	133	742	415	132	72
N.S.	1	1.00	0.91	1.29	2.02	11.24	6.29	2.00	1.09
time (sec)	N/A	0.083	0.132	0.542	0.497	0.410	12.227	0.477	0.269

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	66	87	509	777	428	87	56
N.S.	1	1.00	1.12	1.47	8.63	13.17	7.25	1.47	0.95
time (sec)	N/A	0.077	0.144	0.736	0.644	0.369	6.210	0.465	1.230

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	70	82	118	306	96	46
N.S.	1	1.00	0.91	1.52	1.78	2.57	6.65	2.09	1.00
time (sec)	N/A	0.071	0.029	0.690	0.494	0.394	4.281	0.462	1.236

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	72	215	486	253	65	38
N.S.	1	1.00	1.02	1.57	4.67	10.57	5.50	1.41	0.83
time (sec)	N/A	0.056	0.022	0.744	0.523	0.392	3.458	0.440	0.106

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	66	58	82	146	61	43
N.S.	1	1.00	0.83	1.57	1.38	1.95	3.48	1.45	1.02
time (sec)	N/A	0.043	0.027	0.533	0.297	0.359	3.378	0.439	1.170

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	65	71	57	484	240	63	37
N.S.	1	1.00	1.44	1.58	1.27	10.76	5.33	1.40	0.82
time (sec)	N/A	0.052	0.060	0.895	0.514	0.382	3.407	0.407	0.091

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	113	101	118	0	97	194
N.S.	1	1.00	0.90	1.88	1.68	1.97	0.00	1.62	3.23
time (sec)	N/A	0.073	0.052	2.903	0.273	0.420	0.000	0.436	1.468

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	67	232	329	784	0	89	402
N.S.	1	1.00	1.12	3.87	5.48	13.07	0.00	1.48	6.70
time (sec)	N/A	0.076	0.137	2.740	0.516	0.391	0.000	0.459	1.616

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	60	155	159	747	0	133	313
N.S.	1	1.00	0.71	1.82	1.87	8.79	0.00	1.56	3.68
time (sec)	N/A	0.107	0.113	3.001	0.280	0.495	0.000	0.475	1.539

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	91	288	1038	2368	0	147	519
N.S.	1	1.00	1.11	3.51	12.66	28.88	0.00	1.79	6.33
time (sec)	N/A	0.127	0.486	2.735	0.620	0.400	0.000	0.459	1.614

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	69	91	217	1141	0	194	170
N.S.	1	1.00	0.83	1.10	2.61	13.75	0.00	2.34	2.05
time (sec)	N/A	0.107	0.383	0.733	0.516	0.460	0.000	0.531	1.609

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	90	104	1010	1950	0	195	1655
N.S.	1	1.00	1.01	1.17	11.35	21.91	0.00	2.19	18.60
time (sec)	N/A	0.081	0.412	0.808	0.748	0.440	0.000	0.505	1.692

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	57	84	170	629	0	149	210
N.S.	1	1.00	0.79	1.17	2.36	8.74	0.00	2.07	2.92
time (sec)	N/A	0.081	0.360	0.707	0.284	0.370	0.000	0.490	0.427

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	86	100	614	2025	0	177	106
N.S.	1	1.00	1.01	1.18	7.22	23.82	0.00	2.08	1.25
time (sec)	N/A	0.073	0.319	0.793	0.623	0.424	0.000	0.490	0.660

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	86	170	623	0	149	129
N.S.	1	1.00	0.81	1.26	2.50	9.16	0.00	2.19	1.90
time (sec)	N/A	0.060	0.315	0.720	0.283	0.366	0.000	0.467	1.466

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	97	103	206	1942	0	195	110
N.S.	1	1.00	1.09	1.16	2.31	21.82	0.00	2.19	1.24
time (sec)	N/A	0.060	0.386	1.119	0.526	0.420	0.000	0.446	1.541

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	183	235	1148	0	195	-1
N.S.	1	1.00	0.87	1.93	2.47	12.08	0.00	2.05	-0.01
time (sec)	N/A	0.103	1.466	3.202	0.299	0.443	0.000	0.496	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	329	976	3725	0	336	-1
N.S.	1	1.00	0.93	2.76	8.20	31.30	0.00	2.82	-0.01
time (sec)	N/A	0.127	1.230	2.855	0.683	0.431	0.000	0.531	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	93	226	402	3468	0	323	-1
N.S.	1	1.00	0.75	1.82	3.24	27.97	0.00	2.60	-0.01
time (sec)	N/A	0.137	0.611	3.158	0.283	0.518	0.000	0.520	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	139	382	2345	8482	0	281	-1
N.S.	1	1.00	0.87	2.40	14.75	53.35	0.00	1.77	-0.01
time (sec)	N/A	0.193	1.118	3.001	0.868	0.446	0.000	0.557	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	144	157	3354	7528	0	408	2669
N.S.	1	1.00	1.00	1.09	23.29	52.28	0.00	2.83	18.53
time (sec)	N/A	0.145	0.887	0.887	1.495	0.478	0.000	0.628	0.922

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	91	127	376	2584	3907	245	416
N.S.	1	1.00	0.83	1.17	3.45	23.71	35.84	2.25	3.82
time (sec)	N/A	0.121	0.785	0.723	0.332	0.383	114.452	0.592	0.825

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	135	148	2432	7757	0	384	2574
N.S.	1	1.00	0.99	1.08	17.75	56.62	0.00	2.80	18.79
time (sec)	N/A	0.136	0.804	0.899	1.177	0.468	0.000	0.580	1.856

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	115	384	2611	3403	245	397
N.S.	1	1.00	0.82	1.17	3.92	26.64	34.72	2.50	4.05
time (sec)	N/A	0.097	0.363	0.762	0.336	0.413	113.439	0.549	1.805

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	137	152	1472	7791	0	388	255
N.S.	1	1.00	1.00	1.11	10.74	56.87	0.00	2.83	1.86
time (sec)	N/A	0.111	0.826	0.859	0.816	0.490	0.000	0.562	3.471

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	77	116	378	2554	3412	245	235
N.S.	1	1.00	0.82	1.23	4.02	27.17	36.30	2.61	2.50
time (sec)	N/A	0.076	0.433	0.961	0.328	0.413	113.167	0.527	2.255

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	147	156	507	7496	0	409	260
N.S.	1	1.00	1.04	1.10	3.57	52.79	0.00	2.88	1.83
time (sec)	N/A	0.115	0.238	1.273	0.598	0.449	0.000	0.450	0.897

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	117	273	498	4800	0	295	-1
N.S.	1	1.00	0.85	1.98	3.61	34.78	0.00	2.14	-0.01
time (sec)	N/A	0.144	1.250	3.260	0.332	0.668	0.000	0.588	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	166	428	1944	11865	0	437	-1
N.S.	1	1.00	0.93	2.40	10.92	66.66	0.00	2.46	-0.01
time (sec)	N/A	0.205	4.268	2.919	0.879	0.557	0.000	0.588	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	138	316	770	10720	0	474	-1
N.S.	1	1.00	0.81	1.85	4.50	62.69	0.00	2.77	-0.01
time (sec)	N/A	0.187	1.285	3.272	0.357	0.766	0.000	0.642	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	194	481	4285	22038	0	493	-1
N.S.	1	1.00	0.85	2.11	18.79	96.66	0.00	2.16	-0.00
time (sec)	N/A	0.258	2.522	3.171	1.346	0.576	0.000	0.677	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	203	219	925	20020	0	750	2500
N.S.	1	1.00	1.01	1.09	4.60	99.60	0.00	3.73	12.44
time (sec)	N/A	0.196	0.445	1.283	0.769	0.589	0.000	0.503	1.380

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	19	4	5	8	0	5	3
N.S.	1	1.00	6.33	1.33	1.67	2.67	0.00	1.67	1.00
time (sec)	N/A	0.012	0.007	0.467	0.507	0.348	0.000	0.410	0.045

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	F	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	15	5	0	0	1	14
N.S.	1	1.00	1.31	0.94	0.31	0.00	0.00	0.06	0.88
time (sec)	N/A	0.014	0.006	0.741	0.503	0.000	0.000	0.419	0.248

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	29	21	28	140	0	45	20
N.S.	1	1.00	1.32	0.95	1.27	6.36	0.00	2.05	0.91
time (sec)	N/A	0.015	0.014	0.383	0.491	0.335	0.000	0.401	0.100

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	F	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	28	32	0	0	41	27
N.S.	1	1.00	0.80	0.80	0.91	0.00	0.00	1.17	0.77
time (sec)	N/A	0.016	0.016	0.691	0.514	0.000	0.000	0.420	1.173

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	2	0	11	2
N.S.	1	1.00	1.00	1.27	1.00	0.18	0.00	1.00	0.18
time (sec)	N/A	0.013	0.006	0.296	0.478	0.383	0.000	0.413	0.145

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	25	0	12	21	14
N.S.	1	1.00	1.00	0.92	1.92	0.00	0.92	1.62	1.08
time (sec)	N/A	0.015	0.006	0.612	0.473	0.000	0.218	0.402	0.095

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	85	288	0	4529	97	980	119
N.S.	1	1.00	0.98	3.31	0.00	52.06	1.11	11.26	1.37
time (sec)	N/A	0.113	0.349	0.930	0.000	0.586	3.798	1.102	9.603

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	242	338	0	9360	0	938	-1
N.S.	1	1.00	2.00	2.79	0.00	77.36	0.00	7.75	-0.01
time (sec)	N/A	0.130	4.671	0.801	0.000	0.698	0.000	1.005	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	253	0	2329	71	630	66
N.S.	1	1.00	0.95	4.02	0.00	36.97	1.13	10.00	1.05
time (sec)	N/A	0.079	0.115	0.681	0.000	0.518	2.494	0.820	3.465

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	193	276	0	4825	0	554	-1
N.S.	1	1.00	2.27	3.25	0.00	56.76	0.00	6.52	-0.01
time (sec)	N/A	0.080	2.362	0.637	0.000	0.541	0.000	0.797	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	238	0	1543	51	349	51
N.S.	1	1.00	1.00	5.41	0.00	35.07	1.16	7.93	1.16
time (sec)	N/A	0.052	0.020	0.692	0.000	0.425	1.270	0.611	1.694

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	81	238	0	3443	0	253	-1
N.S.	1	1.00	1.35	3.97	0.00	57.38	0.00	4.22	-0.02
time (sec)	N/A	0.032	0.219	0.757	0.000	0.466	0.000	0.560	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	3467	0	255	-1
N.S.	1	1.00	1.00	0.00	0.00	61.91	0.00	4.55	-0.02
time (sec)	N/A	0.079	0.022	1.779	0.000	0.452	0.000	0.580	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	0	0	1539	0	348	-1
N.S.	1	1.00	0.88	0.00	0.00	32.06	0.00	7.25	-0.02
time (sec)	N/A	0.061	0.093	1.493	0.000	0.388	0.000	0.616	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	4891	0	557	-1
N.S.	1	1.00	1.00	0.00	0.00	58.93	0.00	6.71	-0.01
time (sec)	N/A	0.107	0.159	1.585	0.000	0.514	0.000	0.750	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	161	0	0	2355	0	629	-1
N.S.	1	1.00	2.06	0.00	0.00	30.19	0.00	8.06	-0.01
time (sec)	N/A	0.097	4.230	1.664	0.000	0.467	0.000	0.817	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	111	0	0	9642	0	947	-1
N.S.	1	1.00	0.92	0.00	0.00	79.69	0.00	7.83	-0.01
time (sec)	N/A	0.149	0.453	1.566	0.000	0.669	0.000	1.001	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	86	488	0	4941	175	1063	112
N.S.	1	1.00	1.05	5.95	0.00	60.26	2.13	12.96	1.37
time (sec)	N/A	0.103	0.315	0.579	0.000	0.586	17.081	1.384	10.989

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	247	529	0	10046	0	949	-1
N.S.	1	1.00	2.01	4.30	0.00	81.67	0.00	7.72	-0.01
time (sec)	N/A	0.167	5.062	0.601	0.000	0.707	0.000	1.150	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	473	0	2385	128	662	64
N.S.	1	1.00	0.94	7.51	0.00	37.86	2.03	10.51	1.02
time (sec)	N/A	0.069	0.112	0.525	0.000	0.445	10.015	1.028	3.711

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	110	473	0	4841	0	584	-1
N.S.	1	1.00	1.25	5.38	0.00	55.01	0.00	6.64	-0.01
time (sec)	N/A	0.059	0.451	0.721	0.000	0.547	0.000	0.847	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	4039	0	433	-1
N.S.	1	1.00	1.00	0.00	0.00	56.89	0.00	6.10	-0.01
time (sec)	N/A	0.097	0.051	1.407	0.000	0.539	0.000	0.939	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	197	0	0	3913	0	430	-1
N.S.	1	1.00	2.56	0.00	0.00	50.82	0.00	5.58	-0.01
time (sec)	N/A	0.084	2.027	1.373	0.000	0.510	0.000	0.907	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	51	97	0	679	0	104	68
N.S.	1	1.00	1.65	3.13	0.00	21.90	0.00	3.35	2.19
time (sec)	N/A	0.019	0.044	0.932	0.000	0.366	0.000	0.441	0.235

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	53	142	0	226	0	104	43
N.S.	1	1.00	1.18	3.16	0.00	5.02	0.00	2.31	0.96
time (sec)	N/A	0.024	0.024	0.946	0.000	0.375	0.000	0.426	1.343

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	74	158	0	1027	0	202	78
N.S.	1	1.00	1.48	3.16	0.00	20.54	0.00	4.04	1.56
time (sec)	N/A	0.025	0.098	0.761	0.000	0.395	0.000	0.433	0.287

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	76	211	0	361	0	204	-1
N.S.	1	1.00	1.13	3.15	0.00	5.39	0.00	3.04	-0.01
time (sec)	N/A	0.032	0.051	0.822	0.000	0.367	0.000	0.420	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	68	164	0	2827	0	592	65
N.S.	1	1.00	0.97	2.34	0.00	40.39	0.00	8.46	0.93
time (sec)	N/A	0.094	0.358	0.778	0.000	0.516	0.000	0.757	2.166

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	208	178	0	5494	0	559	-1
N.S.	1	1.00	2.36	2.02	0.00	62.43	0.00	6.35	-0.01
time (sec)	N/A	0.085	3.231	0.795	0.000	0.598	0.000	0.700	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	129	0	1625	0	345	39
N.S.	1	1.00	1.00	2.74	0.00	34.57	0.00	7.34	0.83
time (sec)	N/A	0.074	0.069	0.733	0.000	0.439	0.000	0.623	1.688

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	101	137	0	3361	0	252	-1
N.S.	1	1.00	1.68	2.28	0.00	56.02	0.00	4.20	-0.02
time (sec)	N/A	0.063	0.291	0.700	0.000	0.504	0.000	0.591	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	114	0	1361	31	188	23
N.S.	1	1.00	1.00	3.93	0.00	46.93	1.07	6.48	0.79
time (sec)	N/A	0.045	0.012	0.908	0.000	0.421	0.692	0.500	1.625

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	114	0	1287	0	188	25
N.S.	1	1.00	1.00	3.68	0.00	41.52	0.00	6.06	0.81
time (sec)	N/A	0.019	0.017	0.791	0.000	0.440	0.000	0.507	1.571

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	3527	0	254	-1
N.S.	1	1.00	1.00	0.00	0.00	62.98	0.00	4.54	-0.02
time (sec)	N/A	0.078	0.029	1.664	0.000	0.481	0.000	0.592	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	123	0	0	1565	0	343	-1
N.S.	1	1.00	2.41	0.00	0.00	30.69	0.00	6.73	-0.02
time (sec)	N/A	0.064	3.968	1.720	0.000	0.418	0.000	0.594	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	107	0	0	5711	0	565	-1
N.S.	1	1.00	1.22	0.00	0.00	64.90	0.00	6.42	-0.01
time (sec)	N/A	0.115	0.342	1.720	0.000	0.636	0.000	0.697	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	322	0	3991	0	486	70
N.S.	1	1.00	0.93	4.47	0.00	55.43	0.00	6.75	0.97
time (sec)	N/A	0.113	0.084	0.691	0.000	0.594	0.000	0.678	2.521

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	188	328	0	6973	0	410	-1
N.S.	1	1.00	2.24	3.90	0.00	83.01	0.00	4.88	-0.01
time (sec)	N/A	0.087	1.690	0.719	0.000	0.608	0.000	0.661	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	287	0	2525	0	313	45
N.S.	1	1.00	1.00	5.52	0.00	48.56	0.00	6.02	0.87
time (sec)	N/A	0.081	0.088	0.638	0.000	0.444	0.000	0.554	2.060

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	112	289	0	2281	0	319	-1
N.S.	1	1.00	2.11	5.45	0.00	43.04	0.00	6.02	-0.02
time (sec)	N/A	0.072	1.288	0.651	0.000	0.459	0.000	0.550	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	273	0	2277	51	318	41
N.S.	1	1.00	0.84	5.57	0.00	46.47	1.04	6.49	0.84
time (sec)	N/A	0.064	0.021	0.663	0.000	0.452	11.055	0.547	1.942

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	223	272	0	2509	0	314	-1
N.S.	1	1.00	3.98	4.86	0.00	44.80	0.00	5.61	-0.02
time (sec)	N/A	0.029	3.021	0.720	0.000	0.464	0.000	0.535	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	6955	0	411	-1
N.S.	1	1.00	0.90	0.00	0.00	89.17	0.00	5.27	-0.01
time (sec)	N/A	0.106	0.060	1.546	0.000	0.669	0.000	0.647	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	230	0	0	3929	0	485	-1
N.S.	1	1.00	2.71	0.00	0.00	46.22	0.00	5.71	-0.01
time (sec)	N/A	0.107	7.216	1.689	0.000	0.568	0.000	0.680	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	231	706	0	19265	0	829	-1
N.S.	1	1.00	1.96	5.98	0.00	163.26	0.00	7.03	-0.01
time (sec)	N/A	0.153	1.448	0.733	0.000	1.131	0.000	0.765	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	68	599	0	7033	0	735	92
N.S.	1	1.00	0.81	7.13	0.00	83.73	0.00	8.75	1.10
time (sec)	N/A	0.122	0.087	0.720	0.000	0.672	0.000	0.613	4.013

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	132	642	0	5719	0	708	-1
N.S.	1	1.00	1.47	7.13	0.00	63.54	0.00	7.87	-0.01
time (sec)	N/A	0.090	2.287	0.724	0.000	0.686	0.000	0.618	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	565	0	6621	0	749	82
N.S.	1	1.00	0.85	7.64	0.00	89.47	0.00	10.12	1.11
time (sec)	N/A	0.095	0.065	0.663	0.000	0.707	0.000	0.626	3.824

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	193	584	0	6507	0	752	-1
N.S.	1	1.00	2.19	6.64	0.00	73.94	0.00	8.55	-0.01
time (sec)	N/A	0.086	5.560	0.656	0.000	0.719	0.000	0.629	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	43	550	0	5779	73	707	76
N.S.	1	1.00	0.61	7.86	0.00	82.56	1.04	10.10	1.09
time (sec)	N/A	0.074	0.030	0.655	0.000	0.664	14.814	0.639	3.556

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	943	550	0	6933	0	738	-1
N.S.	1	1.00	10.14	5.91	0.00	74.55	0.00	7.94	-0.01
time (sec)	N/A	0.061	6.223	0.717	0.000	0.723	0.000	0.624	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	73	0	0	19305	0	832	-1
N.S.	1	1.00	0.68	0.00	0.00	178.75	0.00	7.70	-0.01
time (sec)	N/A	0.147	0.057	1.569	0.000	1.226	0.000	0.733	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	246	0	0	10671	0	922	-1
N.S.	1	1.00	1.88	0.00	0.00	81.46	0.00	7.04	-0.01
time (sec)	N/A	0.166	6.893	1.762	0.000	1.189	0.000	0.803	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	35	62	0	543	0	58	63
N.S.	1	1.00	1.40	2.48	0.00	21.72	0.00	2.32	2.52
time (sec)	N/A	0.014	0.020	0.931	0.000	0.361	0.000	0.422	0.171

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	66	0	175	0	58	22
N.S.	1	1.00	1.37	2.44	0.00	6.48	0.00	2.15	0.81
time (sec)	N/A	0.015	0.016	0.957	0.000	0.354	0.000	0.411	1.200

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	112	95	99	194	2074	100	142	91
N.S.	1	1.26	1.07	1.11	2.18	23.30	1.12	1.60	1.02
time (sec)	N/A	0.054	0.503	0.320	0.485	0.347	0.161	0.434	1.157

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	49	41	73	95	102	25	38
N.S.	1	1.00	1.29	1.08	1.92	2.50	2.68	0.66	1.00
time (sec)	N/A	0.045	0.074	0.345	0.473	0.360	0.253	0.411	0.097

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	166	788	0	11528	0	0	-1
N.S.	1	1.00	1.34	6.35	0.00	92.97	0.00	0.00	-0.01
time (sec)	N/A	0.168	3.280	2.737	0.000	0.553	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	333	0	5136	0	0	-1
N.S.	1	1.00	0.97	3.74	0.00	57.71	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.044	2.366	0.000	0.512	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	0	1286	0	0	-1
N.S.	1	1.00	1.00	0.92	0.00	32.15	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.014	3.003	0.000	0.511	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	431	0	3914	0	0	-1
N.S.	1	1.00	0.99	5.82	0.00	52.89	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.407	2.428	0.000	0.557	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	113	637	0	16463	0	0	-1
N.S.	1	1.00	0.96	5.40	0.00	139.52	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.599	2.540	0.000	1.410	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [9] had the largest ratio of [23]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	21	0.238
2	A	3	2	1.00	21	0.095
3	A	4	4	1.00	21	0.190
4	A	3	2	1.00	19	0.105
5	A	3	3	1.00	19	0.158
6	A	3	2	1.00	21	0.095
7	A	4	4	1.00	21	0.190
8	A	3	2	1.00	21	0.095
9	A	6	5	1.00	23	0.217
10	A	3	2	1.00	23	0.087
11	A	5	5	1.00	23	0.217
12	A	3	2	1.00	21	0.095
13	A	4	3	1.00	21	0.143
14	A	3	2	1.00	23	0.087
15	A	5	5	1.00	23	0.217
16	A	3	2	1.00	23	0.087
17	A	6	5	1.00	23	0.217
18	A	3	2	1.00	23	0.087
19	A	6	5	1.14	23	0.217
20	A	3	2	1.00	21	0.095
21	A	4	3	1.00	21	0.143
22	A	3	2	1.00	23	0.087
23	A	6	5	1.00	23	0.217
24	A	3	2	1.00	23	0.087
25	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	4	1.00	23	0.174
27	A	5	5	1.00	23	0.217
28	A	3	3	1.00	21	0.143
29	A	4	4	1.00	21	0.190
30	A	3	3	1.00	23	0.130
31	A	5	5	1.00	23	0.217
32	A	4	4	1.00	23	0.174
33	A	7	6	1.00	23	0.261
34	A	5	4	1.00	23	0.174
35	A	6	6	1.00	23	0.261
36	A	4	4	1.00	21	0.190
37	A	5	5	1.00	21	0.238
38	A	4	4	1.00	23	0.174
39	A	6	6	1.00	23	0.261
40	A	5	4	1.00	23	0.174
41	A	8	6	1.00	23	0.261
42	A	6	5	1.00	23	0.217
43	A	7	6	1.00	23	0.261
44	A	5	4	1.00	21	0.190
45	A	6	6	1.00	21	0.286
46	A	5	4	1.00	23	0.174
47	A	7	6	1.00	23	0.261
48	A	6	5	1.00	23	0.217
49	A	8	5	1.00	21	0.238
50	A	9	6	1.00	21	0.286
51	A	7	5	1.00	21	0.238
52	A	7	6	1.00	19	0.316
53	A	5	3	1.00	19	0.158
54	A	3	2	1.00	21	0.095
55	A	6	3	1.00	21	0.143
56	A	3	2	1.00	21	0.095
57	A	8	5	1.21	23	0.217
58	A	12	8	1.00	23	0.348
59	A	7	5	1.23	23	0.217
60	A	10	8	1.00	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	8	5	1.00	21	0.238
62	A	3	2	1.00	23	0.087
63	A	9	5	1.00	23	0.217
64	A	3	2	1.00	23	0.087
65	A	8	5	1.11	23	0.217
66	A	20	8	1.00	23	0.348
67	A	7	5	1.10	23	0.217
68	A	17	8	1.00	21	0.381
69	A	13	5	1.00	21	0.238
70	A	3	2	1.00	23	0.087
71	A	14	6	1.00	23	0.261
72	A	3	2	1.00	23	0.087
73	A	11	10	1.00	23	0.435
74	A	0	0	0.00	0	0.000
75	A	11	10	1.00	23	0.435
76	A	0	0	0.00	0	0.000
77	A	0	0	0.00	0	0.000
78	A	8	8	1.00	23	0.348
79	A	0	0	0.00	0	0.000
80	A	12	11	1.00	23	0.478
81	A	4	4	1.00	21	0.190
82	A	2	1	1.00	21	0.048
83	A	3	3	1.00	21	0.143
84	A	3	3	1.00	19	0.158
85	A	3	3	1.00	19	0.158
86	A	2	1	1.00	21	0.048
87	A	4	4	1.00	21	0.190
88	A	3	2	1.00	21	0.095
89	A	4	4	1.00	23	0.174
90	A	4	3	1.00	23	0.130
91	A	5	4	1.00	23	0.174
92	A	5	4	1.00	21	0.190
93	A	4	4	1.00	21	0.190
94	A	3	2	1.00	23	0.087
95	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	2	1.00	23	0.087
97	A	6	5	1.00	23	0.217
98	A	5	4	1.00	23	0.174
99	A	5	4	1.00	23	0.174
100	A	6	5	1.00	21	0.238
101	A	5	5	1.00	21	0.238
102	A	3	2	1.00	23	0.087
103	A	6	6	1.00	23	0.261
104	A	3	2	1.00	23	0.087
105	A	6	6	1.00	23	0.261
106	A	4	3	1.00	23	0.130
107	A	5	5	1.00	23	0.217
108	A	3	3	1.00	21	0.143
109	A	2	2	1.00	21	0.095
110	A	2	2	1.00	23	0.087
111	A	4	4	1.00	23	0.174
112	A	3	3	1.00	23	0.130
113	A	5	5	1.00	23	0.217
114	A	4	3	1.00	23	0.130
115	A	5	4	1.00	23	0.174
116	A	6	6	1.00	23	0.261
117	A	5	4	1.00	21	0.190
118	A	3	3	1.00	21	0.143
119	A	3	3	1.00	23	0.130
120	A	3	3	1.00	23	0.130
121	A	3	3	1.00	23	0.130
122	A	5	5	1.00	23	0.217
123	A	5	4	1.00	23	0.174
124	A	6	6	1.00	23	0.261
125	A	7	6	1.00	23	0.261
126	A	6	5	1.00	21	0.238
127	A	4	4	1.00	21	0.190
128	A	4	3	1.00	23	0.130
129	A	4	4	1.00	23	0.174
130	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	3	1.00	23	0.130
132	A	4	4	1.00	23	0.174
133	A	6	6	1.00	23	0.261
134	A	4	3	1.00	21	0.143
135	A	3	3	1.00	21	0.143
136	A	3	3	1.00	21	0.143
137	A	2	2	1.00	19	0.105
138	A	3	2	1.00	12	0.167
139	A	3	2	1.00	19	0.105
140	A	2	2	1.00	21	0.095
141	A	3	3	1.00	21	0.143
142	A	4	4	1.00	21	0.190
143	A	4	4	1.00	21	0.190
144	A	4	3	1.00	23	0.130
145	A	4	3	1.00	23	0.130
146	A	4	3	1.00	23	0.130
147	A	4	3	1.00	21	0.143
148	A	4	3	1.00	14	0.214
149	A	4	3	1.00	21	0.143
150	A	4	3	1.00	23	0.130
151	A	4	3	1.00	23	0.130
152	A	4	3	1.00	23	0.130
153	A	4	3	1.00	23	0.130
154	A	4	3	1.00	23	0.130
155	A	4	3	1.00	23	0.130
156	A	4	3	1.00	23	0.130
157	A	4	3	1.00	23	0.130
158	A	4	3	1.00	23	0.130
159	A	4	3	1.00	21	0.143
160	A	4	3	1.00	14	0.214
161	A	4	3	1.00	21	0.143
162	A	4	3	1.00	23	0.130
163	A	4	3	1.00	23	0.130
164	A	4	3	1.00	23	0.130
165	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	3	1.00	23	0.130
167	A	4	3	1.00	23	0.130
168	A	4	3	1.00	14	0.214
169	A	4	3	1.00	14	0.214
170	A	4	3	1.00	23	0.130
171	A	5	5	1.00	23	0.217
172	A	4	3	1.00	23	0.130
173	A	4	4	1.00	23	0.174
174	A	5	4	1.00	21	0.190
175	A	3	3	1.00	14	0.214
176	A	4	3	1.00	21	0.143
177	A	5	5	1.00	23	0.217
178	A	4	3	1.00	23	0.130
179	A	6	6	1.00	23	0.261
180	A	4	3	1.00	23	0.130
181	A	5	5	1.00	23	0.217
182	A	4	3	1.00	23	0.130
183	A	5	5	1.00	23	0.217
184	A	4	3	1.00	21	0.143
185	A	5	5	1.00	14	0.357
186	A	4	3	1.00	21	0.143
187	A	6	6	1.00	23	0.261
188	A	4	3	1.00	23	0.130
189	A	7	6	1.00	23	0.261
190	A	6	6	1.00	23	0.261
191	A	4	3	1.00	23	0.130
192	A	6	6	1.00	23	0.261
193	A	4	3	1.00	23	0.130
194	A	6	6	1.00	23	0.261
195	A	4	3	1.00	21	0.143
196	A	6	6	1.00	14	0.429
197	A	4	3	1.00	21	0.143
198	A	7	7	1.00	23	0.304
199	A	4	3	1.00	23	0.130
200	A	8	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	7	6	1.00	14	0.429
202	A	3	3	1.00	12	0.250
203	A	4	4	1.00	10	0.400
204	A	4	4	1.00	12	0.333
205	A	5	5	1.00	10	0.500
206	A	3	3	1.00	12	0.250
207	A	3	3	1.00	10	0.300
208	A	7	6	1.00	17	0.353
209	A	8	7	1.00	17	0.412
210	A	6	6	1.00	17	0.353
211	A	7	6	1.00	17	0.353
212	A	5	5	1.00	15	0.333
213	A	6	5	1.00	12	0.417
214	A	7	5	1.00	15	0.333
215	A	5	5	1.00	17	0.294
216	A	8	6	1.00	17	0.353
217	A	6	6	1.00	17	0.353
218	A	9	7	1.00	17	0.412
219	A	7	6	1.00	17	0.353
220	A	8	7	1.00	17	0.412
221	A	6	5	1.00	15	0.333
222	A	7	6	1.00	12	0.500
223	A	8	6	1.00	15	0.400
224	A	7	6	1.00	17	0.353
225	A	5	5	1.00	10	0.500
226	A	6	5	1.00	12	0.417
227	A	6	6	1.00	10	0.600
228	A	7	6	1.00	12	0.500
229	A	6	5	1.00	17	0.294
230	A	7	6	1.00	17	0.353
231	A	5	5	1.00	17	0.294
232	A	6	5	1.00	17	0.294
233	A	4	4	1.00	15	0.267
234	A	3	3	1.00	12	0.250
235	A	7	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	5	5	1.00	17	0.294
237	A	8	6	1.00	17	0.353
238	A	6	5	1.00	17	0.294
239	A	7	6	1.00	17	0.353
240	A	5	5	1.00	17	0.294
241	A	4	4	1.00	17	0.235
242	A	5	5	1.00	15	0.333
243	A	4	4	1.00	12	0.333
244	A	8	6	1.00	15	0.400
245	A	6	6	1.00	17	0.353
246	A	8	7	1.00	17	0.412
247	A	6	5	1.00	17	0.294
248	A	6	6	1.00	17	0.353
249	A	6	6	1.00	17	0.353
250	A	6	6	1.00	17	0.353
251	A	6	5	1.00	15	0.333
252	A	6	6	1.00	12	0.500
253	A	9	7	1.00	15	0.467
254	A	7	7	1.00	17	0.412
255	A	3	3	1.00	10	0.300
256	A	3	3	1.00	12	0.250
257	A	6	4	1.26	14	0.286
258	A	6	5	1.00	8	0.625
259	A	9	8	1.00	15	0.533
260	A	8	7	1.00	15	0.467
261	A	4	4	1.00	15	0.267
262	A	6	6	1.00	15	0.400
263	A	7	7	1.00	15	0.467

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=73

$$\frac{3}{8}(a+5b)x-\frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{b\tanh(c+dx)}{d}$$

```
[Out] 3/8*(a+5*b)*x-1/8*(5*a+9*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)*cosh(d*x+c)
^3*sinh(d*x+c)/d-b*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3744, 466, 1171, 396, 212}

$$\frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)}{4d}-\frac{(5a+9b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3}{8}x(a+5b)-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (3*(a + 5*b)*x)/8 - ((5*a + 9*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a +
 b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) - (b*Tanh[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-a-b-4(a+b)x^2-4b}{(1-x^2)^2}\right.}{4c}$$

$$= -\frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)}{4d}$$

$$= -\frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)}{4d}$$

$$= \frac{3}{8}(a+5b)x - \frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\,\text{cc}}{}$$

Mathematica [A]
time = 0.28, size = 56, normalized size = 0.77

$$\frac{12(a+5b)(c+dx)-8(a+2b)\sinh(2(c+dx))+(a+b)\sinh(4(c+dx))-32b\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
[Out] (12*(a + 5*b)*(c + d*x) - 8*(a + 2*b)*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(c
 + d*x)] - 32*b*Tanh[c + d*x])/(32*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(67) = 134$.
time = 1.94, size = 149, normalized size = 2.04

method	result
risch	$\frac{3ax}{8}+\frac{15bx}{8}+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}-\frac{e^{2dx+2c}b}{4d}-\frac{e^{2dx+2c}a}{8d}+\frac{e^{-2dx-2c}b}{4d}+\frac{e^{-2dx-2c}a}{8d}-\frac{e^{-4dx-4c}a}{64d}-\frac{e^{-4dx-4c}b}{64d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a*x+15/8*b*x+1/64/d*exp(4*d*x+4*c)*a+1/64/d*exp(4*d*x+4*c)*b-1/4/d*exp(
2*d*x+2*c)*b-1/8/d*exp(2*d*x+2*c)*a+1/4/d*exp(-2*d*x-2*c)*b+1/8/d*exp(-2*d*
x-2*c)*a-1/64/d*exp(-4*d*x-4*c)*a-1/64/d*exp(-4*d*x-4*c)*b+2*b/d/(1+exp(2*d
*x+2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(67) = 134$.
time = 0.27, size = 154, normalized size = 2.11

$$\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{64}b\left(\frac{120(dx+c)}{d}+\frac{16e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}-\frac{15e^{(-2dx-2c)}+144e^{(-4dx-4c)}-1}{d(e^{(-4dx-4c)}+e^{(-6dx-6c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) -
 e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*
(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
```

Fricas [A]
time = 0.37, size = 120, normalized size = 1.64

$$\frac{(a+b)\sinh(dx+c)^5+(10(a+b)\cosh(dx+c)^2-7a-15b)\sinh(dx+c)^3+8(3(a+5b)dx+8b)\cosh(dx+c)+(5(a+b)\cosh(dx+c)^4-3(7a+15b)\cosh(dx+c)^2-8a-80b)\sinh(dx+c)}{64d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/64*((a + b)*sinh(d*x + c)^5 + (10*(a + b)*cosh(d*x + c)^2 - 7*a - 15*b)*s
inh(d*x + c)^3 + 8*(3*(a + 5*b)*d*x + 8*b)*cosh(d*x + c) + (5*(a + b)*cosh(
d*x + c)^4 - 3*(7*a + 15*b)*cosh(d*x + c)^2 - 8*a - 80*b)*sinh(d*x + c))/(d
*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^2(c+dx)\right)\sinh^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(67) = 134.
time = 0.43, size = 142, normalized size = 1.95

$$\frac{24\,(dx+c)(a+5\,b)+ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-16\,be^{(2\,dx+2\,c)}-\left(18\,ae^{(4\,dx+4\,c)}+90\,be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-16\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-4\,dx-4\,c)}+\frac{128\,b}{e^{(2\,dx+2\,c)}+1}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*(a + 5*b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a*
e^(2*d*x + 2*c) - 16*b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) + 90*b*e^(4*
d*x + 4*c) - 8*a*e^(2*d*x + 2*c) - 16*b*e^(2*d*x + 2*c) + a + b)*e^(-4*d*x
- 4*c) + 128*b/(e^(2*d*x + 2*c) + 1))/d
```

Mupad [B]
time = 0.24, size = 101, normalized size = 1.38

$$x\left(\frac{3\,a}{8}+\frac{15\,b}{8}\right)+\frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,\left(a+b\right)}{64\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,\left(a+b\right)}{64\,d}+\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,\left(a+2\,b\right)}{8\,d}-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,\left(a+2\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*((3*a)/8 + (15*b)/8) + (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (exp(- 4*c - 4*
d*x)*(a + b))/(64*d) + (exp(4*c + 4*d*x)*(a + b))/(64*d) + (exp(- 2*c - 2*d
*x)*(a + 2*b))/(8*d) - (exp(2*c + 2*d*x)*(a + 2*b))/(8*d)
```

3.2 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=47

$$-\frac{(a+2b)\cosh(c+dx)}{d}+\frac{(a+b)\cosh^3(c+dx)}{3d}-\frac{b\text{sech}(c+dx)}{d}$$

```
[Out] -(a+2*b)*cosh(d*x+c)/d+1/3*(a+b)*cosh(d*x+c)^3/d-b*sech(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3745, 459}

$$\frac{(a+b)\cosh^3(c+dx)}{3d}-\frac{(a+2b)\cosh(c+dx)}{d}-\frac{b\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(((a + 2*b)*Cosh[c + d*x])/d) + ((a + b)*Cosh[c + d*x]^3)/(3*d) - (b*Sech[
c + d*x])/d
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx&=\frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b-bx^2)}{x^4}\,dx,x,\text{sech}(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int \left(-b+\frac{-a-b}{x^4}+\frac{a+2b}{x^2}\right)\,dx,x,\text{sech}(c+dx)\right)}{d}\\&=-\frac{(a+2b)\cosh(c+dx)}{d}+\frac{(a+b)\cosh^3(c+dx)}{3d}-\frac{b\text{sech}(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 73, normalized size = 1.55

$$-\frac{3a\cosh(c+dx)}{4d}-\frac{7b\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}+\frac{b\cosh(3(c+dx))}{12d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) - (7*b*Cosh[c + d*x])/(4*d) + (a*Cosh[3*(c + d*x
)])/(12*d) + (b*Cosh[3*(c + d*x)])/(12*d) - (b*Sech[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(45) = 90$.
time = 1.75, size = 141, normalized size = 3.00

method	result
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}-\frac{3a\,e^{dx+c}}{8d}-\frac{7b\,e^{dx+c}}{8d}-\frac{3\,e^{-dx-c}a}{8d}-\frac{7\,e^{-dx-c}b}{8d}+\frac{e^{-3dx-3c}a}{24d}+\frac{e^{-3dx-3c}b}{24d}-\frac{2b\,e^{dx+c}}{d(1+e^{2dx+2c})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*a+1/24/d*exp(3*d*x+3*c)*b-3/8*a/d*exp(d*x+c)-7/8*b/d*
exp(d*x+c)-3/8/d*exp(-d*x-c)*a-7/8/d*exp(-d*x-c)*b+1/24/d*exp(-3*d*x-3*c)*a
+1/24/d*exp(-3*d*x-3*c)*b-2/d*b*exp(d*x+c)/(1+exp(2*d*x+2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(45) = 90$.
time = 0.28, size = 136, normalized size = 2.89

$$-\frac{1}{24}\,b\left(\frac{21\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{20\,e^{(-2\,dx-2\,c)}+69\,e^{(-4\,dx-4\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)}\right)}\right)+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*b*((21*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x - 2*c) + 69
*e^(-4*d*x - 4*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 1/24*a*
(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/
d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.
time = 0.37, size = 91, normalized size = 1.94

$$\frac{(a+b)\cosh\left(dx+c\right)^{4}+(a+b)\sinh\left(dx+c\right)^{4}-4\,(2\,a+5\,b)\cosh\left(dx+c\right)^{2}+2\,\left(3\,(a+b)\cosh\left(dx+c\right)^{2}-4\,a-10\,b\right)\sinh\left(dx+c\right)^{2}-9\,a-45\,b}{24\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/24*((a + b)*cosh(d*x + c)^4 + (a + b)*sinh(d*x + c)^4 - 4*(2*a + 5*b)*cos
h(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 4*a - 10*b)*sinh(d*x + c)^2 -
 9*a - 45*b)/(d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh^2 \left(c + dx\right)\right) \sinh^3 \left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(45) = 90.
time = 0.44, size = 105, normalized size = 2.23

$$\frac{a\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 + b\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 - 12\,a\left(e^{(dx+c)} + e^{(-dx-c)}\right) - 24\,b\left(e^{(dx+c)} + e^{(-dx-c)}\right) - \frac{48\,b}{e^{(dx+c)} + e^{(-dx-c)}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*(e^(d*x + c) + e^(-d*x - c))^3 + b*(e^(d*x + c) + e^(-d*x - c))^3 -
 12*a*(e^(d*x + c) + e^(-d*x - c)) - 24*b*(e^(d*x + c) + e^(-d*x - c)) - 48
*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B]
time = 1.17, size = 99, normalized size = 2.11

$$\frac{e^{-3\,c-3\,d\,x}\,\left(a+b\right)}{24\,d} + \frac{e^{3\,c+3\,d\,x}\,\left(a+b\right)}{24\,d} - \frac{e^{c+d\,x}\,\left(3\,a+7\,b\right)}{8\,d} - \frac{e^{-c-d\,x}\,\left(3\,a+7\,b\right)}{8\,d} - \frac{2\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (exp(- 3*c - 3*d*x)*(a + b))/(24*d) + (exp(3*c + 3*d*x)*(a + b))/(24*d) - (
exp(c + d*x)*(3*a + 7*b))/(8*d) - (exp(- c - d*x)*(3*a + 7*b))/(8*d) - (2*b
*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.3 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}(a+3b)x+\frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b\tanh(c+dx)}{d}$$

```
[Out] -1/2*(a+3*b)*x+1/2*(a+b)*cosh(d*x+c)*sinh(d*x+c)/d+b*tanh(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3744, 466, 396, 212}

$$\frac{(a+b)\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{1}{2}x(a+3b)+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((a + 3*b)*x) + ((a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b*Tanh[
c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{a+b+2bx^2}{1-x^2}\,dx, x, \tan$$
$$= \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b\tanh(c+dx)}{d} - \frac{(a+3b)\text{S}$$
$$= -\frac{1}{2}(a+3b)x + \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b\tanh(c+$$

Mathematica [A]
time = 0.17, size = 41, normalized size = 0.93

$$\frac{-2(a+3b)(c+dx)+(a+b)\sinh(2(c+dx))+4b\tanh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
[Out] (-2*(a + 3*b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)] + 4*b*Tanh[c + d*x])/(4
*d)
```

Maple [A]
time = 1.24, size = 66, normalized size = 1.50

method	result	size
derivativedivides	$\frac{a\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$	66
default	$\frac{a\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$	66
risch	$-\frac{ax}{2}-\frac{3bx}{2}+\frac{e^{2dx+2c}a}{8d}+\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}a}{8d}-\frac{e^{-2dx-2c}b}{8d}-\frac{2b}{d\left(1+e^{2dx+2c}\right)}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sinh(d*x+c)*cosh(d*x+c)-1/2*d*x-1/2*c)+b*(1/2*sinh(d*x+c)^3/cos
h(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(40) = 80$.
time = 0.27, size = 101, normalized size = 2.30

$$-\frac{1}{8}a\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{8}b\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b*(12*(d*x + c)
/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) +
e^(-4*d*x - 4*c))))
```

Fricas [A]
time = 0.35, size = 71, normalized size = 1.61

$$\frac{(a+b)\sinh(dx+c)^3-4((a+3b)dx+2b)\cosh(dx+c)+\left(3(a+b)\cosh(dx+c)^2+a+9b\right)\sinh(dx+c)}{8d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((a + b)*sinh(d*x + c)^3 - 4*((a + 3*b)*d*x + 2*b)*cosh(d*x + c) + (3*(
a + b)*cosh(d*x + c)^2 + a + 9*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tanh^2(c+dx))\sinh^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(40) = 80$.
time = 0.42, size = 107, normalized size = 2.43

$$-\frac{4(dx+c)(a+3b)-ae^{(2dx+2c)}-be^{(2dx+2c)}-\frac{ae^{(4dx+4c)}+3be^{(4dx+4c)}-14be^{(2dx+2c)}-a-b}{e^{(4dx+4c)}+e^{(2dx+2c)}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*(a + 3*b) - a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - (a*e^
(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 14*b*e^(2*d*x + 2*c) - a - b)/(e^(4*d
*x + 4*c) + e^(2*d*x + 2*c)))/d
```

Mupad [B]
time = 0.15, size = 64, normalized size = 1.45

$$\frac{e^{2c+2dx}(a+b)}{8d}-\frac{2b}{d\,(e^{2c+2dx}+1)}-\frac{e^{-2c-2dx}(a+b)}{8d}-x\left(\frac{a}{2}+\frac{3b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b))/(8*d) - (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (exp(
- 2*c - 2*d*x)*(a + b))/(8*d) - x*(a/2 + (3*b)/2)
```

3.4 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=25

$$\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}$$

```
[Out] (a+b)*cosh(d*x+c)/d+b*sech(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3745, 14}

$$\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int \frac{a+b-bx^2}{x^2}\,dx,x,\text{sech}(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(-b+\frac{a+b}{x^2}\right)\,dx,x,\text{sech}(c+dx)\right)}{d}\\ &= \frac{(a+b)\cosh(c+dx)}{d}+\frac{b\text{sech}(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 1.80

$$\frac{a\cosh(c)\cosh(dx)}{d}+\frac{b\cosh(c+dx)}{d}+\frac{b\operatorname{sech}(c+dx)}{d}+\frac{a\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d + (b*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d + (a*Si
nh[c]*Sinh[d*x])/d
```

Maple [A]
time = 1.20, size = 44, normalized size = 1.76

method	result	size
derivativedivides	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	44
default	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	44
risch	$\frac{a\,e^{dx+c}}{2d}+\frac{b\,e^{dx+c}}{2d}+\frac{e^{-dx-c}a}{2d}+\frac{e^{-dx-c}b}{2d}+\frac{2b\,e^{dx+c}}{d\left(1+e^{2dx+2c}\right)}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*cosh(d*x+c)+b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(25) = 50$.
time = 0.27, size = 67, normalized size = 2.68

$$\frac{1}{2}\,b\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x - c) + e^(-3*d
*x - 3*c)))) + a*cosh(d*x + c)/d
```

Fricas [A]
time = 0.33, size = 42, normalized size = 1.68

$$\frac{(a+b)\cosh\left(dx+c\right)^2+(a+b)\sinh\left(dx+c\right)^2+a+3\,b}{2\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a + 3*b)/(d*cosh(d
*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)\sinh\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(25) = 50.
time = 0.41, size = 63, normalized size = 2.52

$$\frac{a\left(e^{(dx+c)} + e^{(-dx-c)}\right) + b\left(e^{(dx+c)} + e^{(-dx-c)}\right) + \frac{4\,b}{e^{(dx+c)} + e^{(-dx-c)}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*(e^(d*x + c) + e^(-d*x - c)) + b*(e^(d*x + c) + e^(-d*x - c)) + 4*b/
(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B]
time = 0.12, size = 27, normalized size = 1.08

$$\frac{b}{d\cosh\left(c + d\,x\right)} + \frac{\cosh(c + d\,x)\ (a + b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] b/(d*cosh(c + d*x)) + (cosh(c + d*x)*(a + b))/d
```

3.5 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=26

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[Out] -a*arctanh(cosh(d*x+c))/d-b*sech(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3745, 396, 213}

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) - (b*Sech[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+b-bx^2}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$
$$= -\frac{b\operatorname{sech}(c+dx)}{d} + \frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$
$$= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}(c+dx)}{d}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 2.00

$$-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a*Log[Cosh[c/2 + (d*x)/2]])/d) + (a*Log[Sinh[c/2 + (d*x)/2]])/d - (b*Sec
h[c + d*x])/d
```

Maple [A]
time = 1.17, size = 27, normalized size = 1.04

method	result	size
derivativedivides	$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)-\frac{b}{\cosh(dx+c)}}{d}$	27
default	$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)-\frac{b}{\cosh(dx+c)}}{d}$	27
risch	$-\frac{2b\,\mathrm{e}^{dx+c}}{d\left(1+\mathrm{e}^{2dx+2c}\right)}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))-b/cosh(d*x+c))
```

Maxima [A]
time = 0.27, size = 40, normalized size = 1.54

$$\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d} - \frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*log(tanh(1/2*d*x + 1/2*c))/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(26) = 52$.
time = 0.33, size = 167, normalized size = 6.42

$$-\frac{2b\cosh(dx+c) + \left(a\cosh(dx+c)^2 + 2a\cosh(dx+c)\sinh(dx+c) + a\sinh(dx+c)^2 + a\right)\log(\cosh(dx+c)+\sinh(dx+c)+1) - \left(a\cosh(dx+c)^2 + 2a\cosh(dx+c)\sinh(dx+c) + a\sinh(dx+c)^2 + a\right)\log(\cosh(dx+c)+\sinh(dx+c)-1) + 2b\sinh(dx+c)}{d\cosh(dx+c)^2 + 2d\cosh(dx+c)\sinh(dx+c) + d\sinh(dx+c)^2 + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(2*b*cosh(d*x + c) + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c)
+ a*sinh(d*x + c)^2 + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d
*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*log(co
sh(d*x + c) + sinh(d*x + c) - 1) + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^2 +
2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)\operatorname{csch}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.
time = 0.43, size = 68, normalized size = 2.62

$$-\frac{a\log\left(e^{(dx+c)} + e^{(-dx-c)} + 2\right) - a\log\left(e^{(dx+c)} + e^{(-dx-c)} - 2\right) + \frac{4b}{e^{(dx+c)} + e^{(-dx-c)}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(e^(d*x + c) + e^(-d*x - c) + 2) - a*log(e^(d*x + c) + e^(-d*x -
 c) - 2) + 4*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B]
time = 0.12, size = 64, normalized size = 2.46

$$-\frac{2\,\mathrm{atan}\left(\frac{a\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}} - \frac{2\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x),x)
```

```
[Out] - (2*atan((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-
d^2)^(1/2) - (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.6 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=24

$$-\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

```
[Out] -a*coth(d*x+c)/d+b*tanh(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3744, 14}

$$\frac{b\tanh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(b+\frac{a}{x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= -\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$-\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(24) = 48$.
time = 1.95, size = 59, normalized size = 2.46

method	result	size
risch	$-\frac{2\left(a\,\mathrm{e}^{2dx+2c}+b\,\mathrm{e}^{2dx+2c}+a-b\right)}{d\left(1+\mathrm{e}^{2dx+2c}\right)\left(\mathrm{e}^{2dx+2c}-1\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(a*exp(2*d*x+2*c)+b*exp(2*d*x+2*c)+a-b)/d/(1+exp(2*d*x+2*c))/(exp(2*d*x+
2*c)-1)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 1.62

$$\frac{2\,b}{d(e^{(-2\,dx-2\,c)}+1)}+\frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*b/(d*(e^(-2*d*x - 2*c) + 1)) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(24) = 48$.
time = 0.37, size = 88, normalized size = 3.67

$$-\frac{4\,(a\cosh{(dx+c)}+b\sinh{(dx+c)})}{d\cosh{(dx+c)}^3+3\,d\cosh{(dx+c)}\sinh{(dx+c)}^2+d\sinh{(dx+c)}^3-d\cosh{(dx+c)}+\left(3\,d\cosh{(dx+c)}^2+d\right)\sinh{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4*(a*cosh(d*x + c) + b*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d*x +
c)^2 + d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)\operatorname{csch}^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**2, x)
```

Giac [A]
time = 0.42, size = 45, normalized size = 1.88

$$-\frac{2\left(ae^{(2\,dx+2\,c)} + be^{(2\,dx+2\,c)} + a - b\right)}{d(e^{(4\,dx+4\,c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/(d*(e^(4*d*x + 4*c) - 1)
)
```

Mupad [B]
time = 1.05, size = 43, normalized size = 1.79

$$-\frac{\frac{2\,(a-b)}{d} + \frac{2\,e^{2\,c+2\,d\,x}\,(a+b)}{d}}{e^{4\,c+4\,d\,x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^2,x)
```

```
[Out] -((2*(a - b))/d + (2*exp(2*c + 2*d*x)*(a + b))/d)/(exp(4*c + 4*d*x) - 1)
```

3.7 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=51

$$\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[Out] 1/2*(a-2*b)*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d+b*sech(d
*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3745, 466, 396, 213}

$$\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a - 2*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/
(2*d) + (b*Sech[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2(a+b-bx^2)}{(-1+x^2)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$
$$= -\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{\operatorname{Subst}\left(\int \frac{-a+2bx^2}{-1+x^2}\,dx, x, \operatorname{sech}(c+\right.}{2d}$$
$$= -\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{b\operatorname{sech}(c+dx)}{d} - \frac{(a-2b)\operatorname{Subst}}{}$$
$$= \frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} +$$

Mathematica [A]
time = 0.05, size = 87, normalized size = 1.71

$$-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/8*(a*Csch[(c + d*x)/2]^2)/d - (a*Log[Tanh[(c + d*x)/2]])/(2*d) + (b*Log[
Tanh[(c + d*x)/2]])/d - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sech[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(47) = 94$.
time = 2.40, size = 150, normalized size = 2.94

method	result
risch	$-\frac{e^{dx+c}\left(a\,e^{4dx+4c}-2b\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a-2b\right)}{d\left(1+e^{2dx+2c}\right)\left(e^{2dx+2c}-1\right)^2} + \frac{\ln\left(e^{dx+c}-1\right)b}{d} - \frac{a\ln\left(e^{dx+c}-1\right)}{2d} - \frac{\ln\left(e^{dx+c}+1\right)b}{d} + \frac{a\ln(e^{dx}}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -exp(d*x+c)*(a*exp(4*d*x+4*c)-2*b*exp(4*d*x+4*c)+2*a*exp(2*d*x+2*c)+4*b*exp
(2*d*x+2*c)+a-2*b)/d/(1+exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)^2+1/d*ln(exp(d*x
+c)-1)*b-1/2*a/d*ln(exp(d*x+c)-1)-1/d*ln(exp(d*x+c)+1)*b+1/2*a/d*ln(exp(d*x
+c)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(47) = 94$.
time = 0.28, size = 152, normalized size = 2.98

$$\frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c)
+ e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b*(l
og(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2
*d*x - 2*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(47) = 94$.
time = 0.37, size = 924, normalized size = 18.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a - 2*b)*cosh(d*x + c)^5 + 10*(a - 2*b)*cosh(d*x + c)*sinh(d*x + c
)^4 + 2*(a - 2*b)*sinh(d*x + c)^5 + 4*(a + 2*b)*cosh(d*x + c)^3 + 4*(5*(a -
 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 4*(5*(a - 2*b)*cosh(d*x
+ c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a - 2*b)*cosh(d*x
+ c) - ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(d*x + c)*sinh(d*x + c)
^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x + c)^4 + (15*(a - 2*b)*
cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(a - 2*b)*cosh(d*x + c)^3
 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c)^2 + (
15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(
d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*(a - 2*b)*cosh(d*x + c)^3 -
 (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*b)*log(cosh(d*x + c) + sinh
(d*x + c) + 1) + ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(d*x + c)*sin
h(d*x + c)^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x + c)^4 + (15*
(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(a - 2*b)*cosh(
d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a - 2*b)*cosh(d*x
+ c)^2 + (15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d*x + c)^2 - a +
2*b)*sinh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*(a - 2*b)*cosh(d*
```

```
x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*b)*log(cosh(d*x +
 c) + sinh(d*x + c) - 1) + 2*(5*(a - 2*b)*cosh(d*x + c)^4 + 6*(a + 2*b)*cos
h(d*x + c)^2 + a - 2*b)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x +
c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - d*cosh(d*x + c)^4 + (15*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*si
nh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 6*d*cosh(d*x +
c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 -
d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)\operatorname{csch}^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(47) = 94.
time = 0.43, size = 142, normalized size = 2.78

$$\frac{(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-2\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+8\,b\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-4\,e^{(dx+c)}-4\,e^{(-dx-c)}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] 1/4*((a - 2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a - 2*b)*log(e^(d*x +
 c) + e^(-d*x - c) - 2) - 4*(a*(e^(d*x + c) + e^(-d*x - c))^2 - 2*b*(e^(d*x
 + c) + e^(-d*x - c))^2 + 8*b)/((e^(d*x + c) + e^(-d*x - c))^3 - 4*e^(d*x +
 c) - 4*e^(-d*x - c)))/d
```

Mupad [B]
time = 1.13, size = 156, normalized size = 3.06

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a\,\sqrt{-d^2}-2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^2-4\,a\,b+4\,b^2}}\right)\,\sqrt{a^2-4\,a\,b+4\,b^2}}{\sqrt{-d^2}}-\frac{a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^3,x)
[Out] (atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) - 2*b*(-d^2)^(1/2)))/(d*(a^2 - 4*a*b
 + 4*b^2)^(1/2)))*(a^2 - 4*a*b + 4*b^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c + d*
x))/(d*(exp(2*c + 2*d*x) - 1)) + (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) +
1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.8 $\int \mathrm{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=44

$$\frac{(a-b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}$$

```
[Out] (a-b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d-b*tanh(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3744, 459}

$$\frac{(a-b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a - b)*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x])/d
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \mathrm{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\mathrm{Subst}\left(\int \frac{(1-x^2)(a+bx^2)}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \left(-b+\frac{a}{x^4}+\frac{-a+b}{x^2}\right)dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{(a-b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 1.39

$$\frac{2a\coth(c+dx)}{3d}-\frac{b\coth(c+dx)}{d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c +
 d*x]^2)/(3*d) - (b*Tanh[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(42) = 84.
time = 2.13, size = 87, normalized size = 1.98

method	result	size
risch	$-\frac{4\left(3a\,{\mathrm e}^{4dx+4c}+3b\,{\mathrm e}^{4dx+4c}+2a\,{\mathrm e}^{2dx+2c}-6b\,{\mathrm e}^{2dx+2c}-a+3b\right)}{3d\left({\mathrm e}^{2dx+2c}-1\right)^{3}\left(1+{\mathrm e}^{2dx+2c}\right)}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(3*a*exp(4*d*x+4*c)+3*b*exp(4*d*x+4*c)+2*a*exp(2*d*x+2*c)-6*b*exp(2*d*
x+2*c)-a+3*b)/d/(exp(2*d*x+2*c)-1)^3/(1+exp(2*d*x+2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(42) = 84.
time = 0.29, size = 113, normalized size = 2.57

$$\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+\frac{4\,b}{d(e^{(-4\,dx-4\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1))) + 4*b/(d*(e^(-4*d*x - 4*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(42) = 84.
time = 0.33, size = 244, normalized size = 5.55

$$-\frac{8\left((a+3\,b)\cosh\left(dx+c\right)^2+4\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a+3\,b)\sinh\left(dx+c\right)^2+a-3\,b\right)}{3\left(d\cosh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6-2\,d\cosh\left(dx+c\right)^4+\left(15\,d\cosh\left(dx+c\right)^2-2\,d\right)\sinh\left(dx+c\right)^4+4\left(5\,d\cosh\left(dx+c\right)^3-2\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3-d\cosh\left(dx+c\right)^2+\left(15\,d\cosh\left(dx+c\right)^4-12\,d\cosh\left(dx+c\right)^2-d\right)\sinh\left(dx+c\right)^2+2\left(3\,d\cosh\left(dx+c\right)^5-4\,d\cosh\left(dx+c\right)^3+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/3*((a + 3*b)*cosh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c) + (a + 3*
b)*sinh(d*x + c)^2 + a - 3*b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d
*x + c)^5 + d*sinh(d*x + c)^6 - 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2
 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(
d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)
^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 + d*
cosh(d*x + c))*sinh(d*x + c) + 2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh^{2}\left(c + dx\right)\right) \operatorname{csch}^{4}\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**4, x)
```

Giac [A]
time = 0.44, size = 80, normalized size = 1.82

$$\frac{2\left(\frac{3\,b}{e^{(2\,dx+2\,c)}+1} - \frac{3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-2\,a+3\,b}{\left(e^{(2\,dx+2\,c)}-1\right)^{3}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2/3*(3*b/(e^(2*d*x + 2*c) + 1) - (3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c)
 - 6*b*e^(2*d*x + 2*c) - 2*a + 3*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B]
time = 1.10, size = 173, normalized size = 3.93

$$\frac{2\,b}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{\frac{2\,\left(2\,a-b\right)}{3\,d}+\frac{2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{2\,b}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{\frac{2\,b}{3\,d}+\frac{2\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{3\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a-b\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^4,x)
```

```
[Out] (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - ((2*(2*a - b))/(3*d) + (2*b*exp(2*c + 2*
d*x))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (2*b)/(3*d*(exp(
2*c + 2*d*x) - 1)) - ((2*b)/(3*d) + (2*b*exp(4*c + 4*d*x))/(3*d) + (4*exp(2
*c + 2*d*x)*(2*a - b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + ex
p(6*c + 6*d*x) - 1)
```

3.9 $\int \sinh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal. Leaf size=118

$$\frac{1}{8}\left(3a^2+30ab+35b^2\right)x-\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d}-\frac{\left(a^2+10ab+13b^2\right)\tanh(c+dx)}{4d}+\frac{(a+$$

```
[Out] 1/8*(3*a^2+30*a*b+35*b^2)*x-1/8*(a+b)*(a+9*b)*cosh(d*x+c)*sinh(d*x+c)/d-1/4
*(a^2+10*a*b+13*b^2)*tanh(d*x+c)/d+1/4*(a+b)^2*sinh(d*x+c)^4*tanh(d*x+c)/d-
1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 474, 466, 1167, 212}

$$-\frac{\left(a^2+10ab+13b^2\right)\tanh(c+dx)}{4d}+\frac{1}{8}x\left(3a^2+30ab+35b^2\right)+\frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d}-\frac{(a+b)(a+9b)\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 30*a*b + 35*b^2)*x)/8 - ((a + b)*(a + 9*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*d) - ((a^2 + 10*a*b + 13*b^2)*Tanh[c + d*x])/(4*d) + ((a + b)^2*S
inh[c + d*x]^4*Tanh[c + d*x])/(4*d) - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
```

```
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{x^4(a+bx^2)^2}{(1-x^2)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d}-\frac{\text{Subst}\left(\int\frac{x^4(a^2+10ab+5b^2\cdots}{(1-x^2)^2}\right.}{\cdots}$$

$$=-\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)^2\sinh^4(\cdots}{\cdots}$$

$$=-\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)^2\sinh^4(\cdots}{\cdots}$$

$$=-\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d}-\frac{(a^2+10ab+1\cdots}{\cdots}$$

$$=\frac{1}{8}\left(3a^2+30ab+35b^2\right)x-\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(\cdots}{8d}$$

Mathematica [A]
time = 1.03, size = 94, normalized size = 0.80

$$\frac{12(3a^2+30ab+35b^2)(c+dx)-24(a^2+4ab+3b^2)\sinh(2(c+dx))+3(a+b)^2\sinh(4(c+dx))+32b(-6a-10b+b\,\text{sech}^2(c+dx))\tanh(c+dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (12*(3*a^2 + 30*a*b + 35*b^2)*(c + d*x) - 24*(a^2 + 4*a*b + 3*b^2)*Sinh[2*(
c + d*x)] + 3*(a + b)^2*Sinh[4*(c + d*x)] + 32*b*(-6*a - 10*b + b*Sech[c +
d*x]^2)*Tanh[c + d*x])/(96*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(108) = 216$.
time = 2.04, size = 293, normalized size = 2.48

method	result
risch	$\frac{3a^2x}{8} + \frac{15abx}{4} + \frac{35b^2x}{8} + \frac{e^{4dx+4c}a^2}{64d} + \frac{e^{4dx+4c}ab}{32d} + \frac{e^{4dx+4c}b^2}{64d} - \frac{e^{2dx+2c}a^2}{8d} - \frac{e^{2dx+2c}ab}{2d} - \frac{3\,e^{2dx+2c}b^2}{8d} + \frac{e^{-2dx-2c}a^2}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^2*x+15/4*a*b*x+35/8*b^2*x+1/64/d*exp(4*d*x+4*c)*a^2+1/32/d*exp(4*d*x+
4*c)*a*b+1/64/d*exp(4*d*x+4*c)*b^2-1/8/d*exp(2*d*x+2*c)*a^2-1/2/d*exp(2*d*x
+2*c)*a*b-3/8/d*exp(2*d*x+2*c)*b^2+1/8/d*exp(-2*d*x-2*c)*a^2+1/2/d*exp(-2*d
*x-2*c)*a*b+3/8/d*exp(-2*d*x-2*c)*b^2-1/64/d*exp(-4*d*x-4*c)*a^2-1/32/d*exp
(-4*d*x-4*c)*a*b-1/64/d*exp(-4*d*x-4*c)*b^2+4/3*b*(3*a*exp(4*d*x+4*c)+6*b*e
xp(4*d*x+4*c)+6*a*exp(2*d*x+2*c)+9*b*exp(2*d*x+2*c)+3*a+5*b)/d/(1+exp(2*d*x
+2*c))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(108) = 216$.
time = 0.28, size = 295, normalized size = 2.50

$$\frac{1}{64}a^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + \frac{1}{192}b^2\left(\frac{840(dx+c)}{d} + \frac{3\left(24e^{(-2dx-2c)} - e^{(-4dx-4c)}\right)}{d} - \frac{63e^{(-2dx-2c)} + 1487e^{(-4dx-4c)} + 2517e^{(-6dx-6c)} + 1608e^{(-8dx-8c)} - 3}{d\left(e^{(-4dx-4c)} + 3e^{(-6dx-6c)} + 3e^{(-8dx-8c)} + e^{(-10dx-10c)}\right)}\right) + \frac{1}{32}ab\left(\frac{120(dx+c)}{d} + \frac{16e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} - \frac{15e^{(-2dx-2c)} + 144e^{(-4dx-4c)} - 1}{d\left(e^{(-4dx-4c)} + e^{(-6dx-6c)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/192*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x -
 2*c) - e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*c)
+ 2517*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*c) +
 3*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 1/32*a*b
*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*
d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6
*c))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(108) = 216$.
time = 0.33, size = 394, normalized size = 3.34

$$\frac{3\left(a^2+2ab+b^2\right)\sinh(dx+c)^7 + 3\left(21\left(a^2+2ab+b^2\right)\cosh(dx+c)^2 - 5a^2 - 26ab - 21b^2\right)\sinh(dx+c)^5 + 8\left(3\left(3a^2+30ab+35b^2\right)dx + 48ab + 80b^2\right)\cosh(dx+c)^3 + 24\left(3\left(3a^2+30ab+35b^2\right)dx + 48ab + 80b^2\right)\cosh(dx+c)\sinh(dx+c)^2 + \left(105\left(a^2+2ab+b^2\right)\cosh(dx+c)^4 - 30\left(5a^2+26ab+21b^2\right)\cosh(dx+c)^2 - 63a^2 - 654ab - 847b^2\right)\sinh(dx+c)^3 + 24\left(3\left(3a^2+30ab+35b^2\right)dx + 48ab + 80b^2\right)\cosh(dx+c) + 3\left(7\left(a^2+2ab+b^2\right)\cosh(dx+c)^6 - 5\left(5a^2+26ab+21b^2\right)\cosh(dx+c)^4 - \left(63a^2+654ab+847b^2\right)\cosh(dx+c)^2 - 15a^2 - 190ab - 175b^2\right)\sinh(dx+c)}{192\left(d\cosh(dx+c)^3 + 3d\cosh(dx+c)\sinh(dx+c)^2 + 3d\cosh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^7 + 3*(21*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 - 5*a^2 - 26*a*b - 21*b^2)*sinh(d*x + c)^5 + 8*(3*(3*a^2 + 30
*a*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c)^3 + 24*(3*(3*a^2 + 30*a
*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (105*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 30*(5*a^2 + 26*a*b + 21*b^2)*cosh(d*x +
 c)^2 - 63*a^2 - 654*a*b - 847*b^2)*sinh(d*x + c)^3 + 24*(3*(3*a^2 + 30*a*b
 + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c) + 3*(7*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^6 - 5*(5*a^2 + 26*a*b + 21*b^2)*cosh(d*x + c)^4 - (63*a^2 + 6
54*a*b + 847*b^2)*cosh(d*x + c)^2 - 15*a^2 - 190*a*b - 175*b^2)*sinh(d*x +
c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x +
 c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^{2}\left(c + dx\right)\right)^{2}\sinh^{4}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(108) = 216.
time = 0.50, size = 293, normalized size = 2.48

$$\frac{3\,a^2e^{(4\,dx+4\,c)} + 6\,abe^{(4\,dx+4\,c)} + 3\,b^2e^{(4\,dx+4\,c)} - 24\,a^2e^{(2\,dx+2\,c)} - 96\,abe^{(2\,dx+2\,c)} - 72\,b^2e^{(2\,dx+2\,c)} + 24\,(3\,a^2 + 30\,ab + 35\,b^2)(dx + c) - 3\,\left(18\,a^2e^{(4\,dx+4\,c)} + 180\,abe^{(4\,dx+4\,c)} + 210\,b^2e^{(4\,dx+4\,c)} - 8\,a^2e^{(2\,dx+2\,c)} - 32\,abe^{(2\,dx+2\,c)} - 24\,b^2e^{(2\,dx+2\,c)} + a^2 + 2\,ab + b^2\right)e^{(-4\,dx-4\,c)} + \frac{256\,\left(3\,abe^{(4\,dx+4\,c)}+6\,b^2e^{(4\,dx+4\,c)}+6\,abe^{(2\,dx+2\,c)}+9\,b^2e^{(2\,dx+2\,c)}+3\,ab+5\,b^2\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*e^(4*d*x + 4*c) + 6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x + 4*c
) - 24*a^2*e^(2*d*x + 2*c) - 96*a*b*e^(2*d*x + 2*c) - 72*b^2*e^(2*d*x + 2*c
) + 24*(3*a^2 + 30*a*b + 35*b^2)*(d*x + c) - 3*(18*a^2*e^(4*d*x + 4*c) + 18
0*a*b*e^(4*d*x + 4*c) + 210*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 3
2*a*b*e^(2*d*x + 2*c) - 24*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d
*x - 4*c) + 256*(3*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) + 6*a*b*e^(2
*d*x + 2*c) + 9*b^2*e^(2*d*x + 2*c) + 3*a*b + 5*b^2)/(e^(2*d*x + 2*c) + 1)^
3)/d
```

Mupad [B]
time = 0.31, size = 293, normalized size = 2.48

$$\frac{\frac{4\,\left(b^2+a\,b\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,b^2+a\,b\right)}{3\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}+x\,\left(\frac{3\,a^2}{8}+\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)+\frac{\frac{4\,\left(2\,b^2+a\,b\right)}{3\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+a\,b\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,b^2+a\,b\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}+\frac{4\,\left(2\,b^2+a\,b\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{{\mathrm{e}}^{-2\,c-2\,d\,x}\,\left(a^2+4\,a\,b+3\,b^2\right)}{8\,d}-\frac{{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2+4\,a\,b+3\,b^2\right)}{8\,d}-\frac{{\mathrm{e}}^{-4\,c-4\,d\,x}\,{\left(a+b\right)}^2}{64\,d}+\frac{{\mathrm{e}}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^2}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] ((4*(a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(a*b + 2*b^2))/(3*d))/(2*exp(2
*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + x*((15*a*b)/4 + (3*a^2)/8 + (35*b^2)/
8) + ((4*(a*b + 2*b^2))/(3*d) + (8*exp(2*c + 2*d*x)*(a*b + b^2))/(3*d) + (4
*exp(4*c + 4*d*x)*(a*b + 2*b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4
*d*x) + exp(6*c + 6*d*x) + 1) + (4*(a*b + 2*b^2))/(3*d*(exp(2*c + 2*d*x) +
1)) + (exp(- 2*c - 2*d*x)*(4*a*b + a^2 + 3*b^2))/(8*d) - (exp(2*c + 2*d*x)*
(4*a*b + a^2 + 3*b^2))/(8*d) - (exp(- 4*c - 4*d*x)*(a + b)^2)/(64*d) + (exp
(4*c + 4*d*x)*(a + b)^2)/(64*d)
```

3.10 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=77

$$-\frac{(a+b)(a+3b)\cosh(c+dx)}{d}+\frac{(a+b)^2\cosh^3(c+dx)}{3d}-\frac{b(2a+3b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

```
[Out] -(a+b)*(a+3*b)*cosh(d*x+c)/d+1/3*(a+b)^2*cosh(d*x+c)^3/d-b*(2*a+3*b)*sech(d
*x+c)/d+1/3*b^2*sech(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3745, 459}

$$\frac{(a+b)^2\cosh^3(c+dx)}{3d}-\frac{(a+b)(a+3b)\cosh(c+dx)}{d}-\frac{b(2a+3b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(((a + b)*(a + 3*b)*Cosh[c + d*x])/d) + ((a + b)^2*Cosh[c + d*x]^3)/(3*d)
- (b*(2*a + 3*b)*Sech[c + d*x])/d + (b^2*Sech[c + d*x]^3)/(3*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b-bx^2)^2}{x^4} \, dx, x, \text{sech}(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(2a+3b) - \frac{(a+b)^2}{x^4} + \frac{(a+b)(a+3b)}{x^2} + b^2x^2\right) dx, x, \text{sec}}{d}$$
$$= -\frac{(a+b)(a+3b)\cosh(c+dx)}{d} + \frac{(a+b)^2\cosh^3(c+dx)}{3d} - \frac{b(2}{}$$

Mathematica [A]
time = 0.39, size = 71, normalized size = 0.92

$$\frac{-3(3a^2+14ab+11b^2)\cosh(c+dx) + (a+b)^2\cosh(3(c+dx)) + 4b\text{sech}(c+dx)\left(-6a-9b+b\text{sech}^2(c+dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*(3*a^2 + 14*a*b + 11*b^2)*Cosh[c + d*x] + (a + b)^2*Cosh[3*(c + d*x)] +
 4*b*Sech[c + d*x]*(-6*a - 9*b + b*Sech[c + d*x]^2))/(12*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(73) = 146.
time = 2.16, size = 273, normalized size = 3.55

method	result
risch	$\frac{e^{3dx+3c}a^2}{24d} + \frac{e^{3dx+3c}ab}{12d} + \frac{e^{3dx+3c}b^2}{24d} - \frac{3e^{dx+c}a^2}{8d} - \frac{7ab\,e^{dx+c}}{4d} - \frac{11e^{dx+c}b^2}{8d} - \frac{3e^{-dx-c}a^2}{8d} - \frac{7e^{-dx-c}ab}{4d} - \frac{11e^{-dx-c}b^2}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*a^2+1/12/d*exp(3*d*x+3*c)*a*b+1/24/d*exp(3*d*x+3*c)*b
^2-3/8/d*exp(d*x+c)*a^2-7/4*a*b/d*exp(d*x+c)-11/8/d*exp(d*x+c)*b^2-3/8/d*ex
p(-d*x-c)*a^2-7/4/d*exp(-d*x-c)*a*b-11/8/d*exp(-d*x-c)*b^2+1/24/d*exp(-3*d*
x-3*c)*a^2+1/12/d*exp(-3*d*x-3*c)*a*b+1/24/d*exp(-3*d*x-3*c)*b^2-2/3*b*exp(
d*x+c)*(6*a*exp(4*d*x+4*c)+9*b*exp(4*d*x+4*c)+12*a*exp(2*d*x+2*c)+14*b*exp(
2*d*x+2*c)+6*a+9*b)/d/(1+exp(2*d*x+2*c))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(73) = 146.
time = 0.31, size = 265, normalized size = 3.44

$$-\frac{1}{24}b^2\left(\frac{33e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{30e^{(-2dx-2c)} + 240e^{(-4dx-4c)} + 322e^{(-6dx-6c)} + 177e^{(-8dx-8c)} - 1}{d(e^{(-3dx-3c)} + 3e^{(-5dx-5c)} + 3e^{(-7dx-7c)} + e^{(-9dx-9c)})}\right) - \frac{1}{12}ab\left(\frac{21e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{20e^{(-2dx-2c)} + 69e^{(-4dx-4c)} - 1}{d(e^{(-3dx-3c)} + e^{(-5dx-5c)})}\right) + \frac{1}{24}a^2\left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*((33*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (30*e^(-2*d*x - 2*c) +
240*e^(-4*d*x - 4*c) + 322*e^(-6*d*x - 6*c) + 177*e^(-8*d*x - 8*c) - 1)/(d*
(e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c) + e^(-9*d*x - 9
*c)))) - 1/12*a*b*((21*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x -
 2*c) + 69*e^(-4*d*x - 4*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))
 + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3
*d*x - 3*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(73) = 146$.
time = 0.34, size = 259, normalized size = 3.36

$$\frac{(a^2 + 2\,ab + b^2)\cosh\left(dx + c\right)^6 + (a^2 + 2\,ab + b^2)\sinh\left(dx + c\right)^6 - 6\,(a^2 + 6\,ab + 5\,b^2)\cosh\left(dx + c\right)^4 + 3\left(5\,(a^2 + 2\,ab + b^2)\cosh\left(dx + c\right)^2 - 2\,a^2 - 12\,ab - 10\,b^2\right)\sinh\left(dx + c\right)^4 - 3\,(11\,a^2 + 86\,ab + 91\,b^2)\cosh\left(dx + c\right)^2 + 3\left(5\,(a^2 + 2\,ab + b^2)\cosh\left(dx + c\right)^4 - 12\,(a^2 + 6\,ab + 5\,b^2)\cosh\left(dx + c\right)^2 - 11\,a^2 - 86\,ab - 91\,b^2\right)\sinh\left(dx + c\right)^2 - 26\,a^2 - 220\,ab - 210\,b^2}{24\left(d\cosh\left(dx + c\right)^3 + 3\,d\cosh\left(dx + c\right)\sinh\left(dx + c\right)^2 + 3\,d\cosh\left(dx + c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + (a^2 + 2*a*b + b^2)*sinh(d*x +
c)^6 - 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 - 2*a^2 - 12*a*b - 10*b^2)*sinh(d*x + c)^4 - 3*(11*a^2 + 86*
a*b + 91*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 -
12*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - 11*a^2 - 86*a*b - 91*b^2)*sinh(d
*x + c)^2 - 26*a^2 - 220*a*b - 210*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^2\sinh^3\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(73) = 146$.
time = 0.49, size = 205, normalized size = 2.66

$$\frac{a^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 + 2\,ab\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 + b^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 - 12\,a^2\left(e^{(dx+c)} + e^{(-dx-c)}\right) - 48\,ab\left(e^{(dx+c)} + e^{(-dx-c)}\right) - 36\,b^2\left(e^{(dx+c)} + e^{(-dx-c)}\right) - \frac{16\left(6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+9\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\,b^2\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*(e^(d*x + c) + e^(-d*x - c))^3 + 2*a*b*(e^(d*x + c) + e^(-d*x - c
))^3 + b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a^2*(e^(d*x + c) + e^(-d*x -
 c)) - 48*a*b*(e^(d*x + c) + e^(-d*x - c)) - 36*b^2*(e^(d*x + c) + e^(-d*x
- c)) - 16*(6*a*b*(e^(d*x + c) + e^(-d*x - c))^2 + 9*b^2*(e^(d*x + c) + e^(
-d*x - c))^2 - 4*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

Mupad [B]
time = 0.29, size = 215, normalized size = 2.79

$$\frac{e^{-3c-3dx}(a+b)^2}{24d}-\frac{e^{c+dx}(3a^2+14ab+11b^2)}{8d}+\frac{e^{3c+3dx}(a+b)^2}{24d}-\frac{e^{-c-dx}(3a^2+14ab+11b^2)}{8d}-\frac{8b^2e^{c+dx}}{3d(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1)}-\frac{2e^{c+dx}(3b^2+2ab)}{d(e^{2c+2dx}+1)}+\frac{8b^2e^{c+dx}}{3d(2e^{2c+2dx}+e^{4c+4dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(- 3*c - 3*d*x)*(a + b)^2)/(24*d) - (exp(c + d*x)*(14*a*b + 3*a^2 + 11*
b^2))/(8*d) + (exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(- c - d*x)*(14*a*b
 + 3*a^2 + 11*b^2))/(8*d) - (8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) +
 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (2*exp(c + d*x)*(2*a*b + 3*b
^2))/(d*(exp(2*c + 2*d*x) + 1)) + (8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*
d*x) + exp(4*c + 4*d*x) + 1))
```

3.11 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=79

$$-\frac{1}{2}(a+b)(a+5b)x+\frac{(a+b)(a+5b)\tanh(c+dx)}{2d}+\frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -1/2*(a+b)*(a+5*b)*x+1/2*(a+b)*(a+5*b)*tanh(d*x+c)/d+1/2*(a+b)^2*sinh(d*x+c
)^2*tanh(d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {3744, 474, 470, 327, 212}

$$\frac{(a+b)(a+5b)\tanh(c+dx)}{2d}+\frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d}-\frac{1}{2}x(a+b)(a+5b)+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a + b)*(a + 5*b)*x) + ((a + b)*(a + 5*b)*Tanh[c + d*x])/(2*d) + ((a
+ b)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(2*d) + (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{x^2(a^2+6ab+3b^2+2b}{1-x^2}\right.}{2}$$

$$= \frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d} + \frac{b^2\tanh^3(c+dx)}{3d} - \frac{((a-}{}$$

$$= \frac{(a+b)(a+5b)\tanh(c+dx)}{2d} + \frac{(a+b)^2\sinh^2(c+dx)\tanh(c+}{2d}$$

$$= -\frac{1}{2}(a+b)(a+5b)x + \frac{(a+b)(a+5b)\tanh(c+dx)}{2d} + \frac{(a+b)^2}{}$$

Mathematica [**A**]
time = 0.67, size = 70, normalized size = 0.89

$$\frac{-6(a^2+6ab+5b^2)(c+dx)+3(a+b)^2\sinh(2(c+dx))+4b\left(6a+7b-b\,\text{sech}^2(c+dx)\right)\tanh(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (-6*(a^2 + 6*a*b + 5*b^2)*(c + d*x) + 3*(a + b)^2*Sinh[2*(c + d*x)] + 4*b*(
6*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

Maple [**A**]
time = 1.57, size = 118, normalized size = 1.49

method	result
derivativedivides	$\frac{a^2\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^2\left(\frac{\sinh^5(dx+c)}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tanh(d}{2}\right.}{d}$
default	$\frac{a^2\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^2\left(\frac{\sinh^5(dx+c)}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tanh(d}{2}\right.}{d}$
risch	$-\frac{a^2x}{2}-3abx-\frac{5b^2x}{2}+\frac{e^{2dx+2c}a^2}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}-\frac{e^{-2dx-2c}ab}{4d}-\frac{e^{-2dx-2c}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sinh(d*x+c)*cosh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(1/2*sinh(d*x+c)
^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^2*(1/2*sinh(d*x+c)^5/cosh(d
*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(71) = 142$.
time = 0.27, size = 217, normalized size = 2.75

$$-\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{24}b^2\left(\frac{60(dx+c)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx-2c)}+201e^{(-4dx-4c)}+147e^{(-6dx-6c)}+3}{d(e^{(-2dx-2c)}+3e^{(-4dx-4c)}+3e^{(-6dx-6c)}+e^{(-8dx-8c)})}\right)-\frac{1}{4}ab\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/24*b^2*(60*(d*x
 + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x - 4*
c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) +
3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) - 1/4*a*b*(12*(d*x + c)/d + e^(-2*
d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x -
 4*c))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(71) = 142$.
time = 0.34, size = 291, normalized size = 3.68

$$\frac{3(a^2+2ab+b^2)\sinh(dx+c)^5-4(3(a^2+6ab+5b^2)dx+12ab+14b^2)\cosh(dx+c)^3-12(3(a^2+6ab+5b^2)dx+12ab+14b^2)\cosh(dx+c)\sinh(dx+c)^2+\left(30(a^2+2ab+b^2)\cosh(dx+c)^2+9a^2+66ab+65b^2\right)\sinh(dx+c)^3-12(3(a^2+6ab+5b^2)dx+12ab+14b^2)\cosh(dx+c)+3\left(5(a^2+2ab+b^2)\cosh(dx+c)^4+(9a^2+66ab+65b^2)\cosh(dx+c)^2+2a^2+20ab+10b^2\right)\sinh(dx+c)}{24\left(d\cosh(dx+c)^3+3d\cosh(dx+c)\sinh(dx+c)^2+3d\cosh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^5 - 4*(3*(a^2 + 6*a*b + 5*b^2)*d*
x + 12*a*b + 14*b^2)*cosh(d*x + c)^3 - 12*(3*(a^2 + 6*a*b + 5*b^2)*d*x + 12
*a*b + 14*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*(a^2 + 2*a*b + b^2)*cosh
```

```
(d*x + c)^2 + 9*a^2 + 66*a*b + 65*b^2)*sinh(d*x + c)^3 - 12*(3*(a^2 + 6*a*b
 + 5*b^2)*d*x + 12*a*b + 14*b^2)*cosh(d*x + c) + 3*(5*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + (9*a^2 + 66*a*b + 65*b^2)*cosh(d*x + c)^2 + 2*a^2 + 20*a*b
 + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x +
 c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\sinh^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(71) = 142.
time = 0.47, size = 213, normalized size = 2.70

$$\frac{3\,a^2e^{(2\,dx+2\,c)}+6\,abe^{(2\,dx+2\,c)}+3\,b^2e^{(2\,dx+2\,c)}-12\,(a^2+6\,ab+5\,b^2)(dx+c)+3\left(2\,a^2e^{(2\,dx+2\,c)}+12\,abe^{(2\,dx+2\,c)}+10\,b^2e^{(2\,dx+2\,c)}-a^2-2\,ab-b^2\right)e^{(-2\,dx-2\,c)}-\frac{16\,(6\,abe^{(4\,dx+4\,c)}+9\,b^2e^{(4\,dx+4\,c)}+12\,abe^{(2\,dx+2\,c)}+12\,b^2e^{(2\,dx+2\,c)}+6\,ab+7\,b^2)}{(e^{(2\,dx+2\,c)}+1)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*e^(2*d*x + 2*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c)
 - 12*(a^2 + 6*a*b + 5*b^2)*(d*x + c) + 3*(2*a^2*e^(2*d*x + 2*c) + 12*a*b*e
^(2*d*x + 2*c) + 10*b^2*e^(2*d*x + 2*c) - a^2 - 2*a*b - b^2)*e^(-2*d*x - 2*
c) - 16*(6*a*b*e^(4*d*x + 4*c) + 9*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x +
2*c) + 12*b^2*e^(2*d*x + 2*c) + 6*a*b + 7*b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B]
time = 1.18, size = 248, normalized size = 3.14

$$\frac{e^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2}{8\,d}-x\,\left(\frac{a^2}{2}+3\,a\,b+\frac{5\,b^2}{2}\right)-\frac{\frac{2\,\left(3\,b^2+2\,a\,b\right)}{3\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,\left(b^2+2\,a\,b\right)}{3\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,\left(3\,b^2+2\,a\,b\right)}{3\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}-\frac{2\,\left(3\,b^2+2\,a\,b\right)}{3\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{e^{-2\,c-2\,d\,x}\,{\left(a+b\right)}^2}{8\,d}-\frac{\frac{2\,\left(b^2+2\,a\,b\right)}{3\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,\left(3\,b^2+2\,a\,b\right)}{3\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - x*(3*a*b + a^2/2 + (5*b^2)/2) - ((2*(2
*a*b + 3*b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d) + (2*exp(4*
c + 4*d*x)*(2*a*b + 3*b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
 + exp(6*c + 6*d*x) + 1) - (2*(2*a*b + 3*b^2))/(3*d*(exp(2*c + 2*d*x) + 1))
 - (exp(- 2*c - 2*d*x)*(a + b)^2)/(8*d) - ((2*(2*a*b + b^2))/(3*d) + (2*exp
(2*c + 2*d*x)*(2*a*b + 3*b^2))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x
) + 1)
```

3.12 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{2b(a+b)\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^3(c+dx)}{3d}$$

```
[Out] (a+b)^2*cosh(d*x+c)/d+2*b*(a+b)*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3745, 276}

$$\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{2b(a+b)\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Cosh[c + d*x])/d + (2*b*(a + b)*Sech[c + d*x])/d - (b^2*Sech[c +
 d*x]^3)/(3*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{x^2}\,dx,x,\text{sech}(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(-2b(a+b)+\frac{(a+b)^2}{x^2}+b^2x^2\right)\,dx,x,\text{sech}(c+dx)\right)}{d}\\ &= \frac{(a+b)^2\cosh(c+dx)}{d}+\frac{2b(a+b)\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 46, normalized size = 0.94

$$\frac{3(a+b)^2\cosh(c+dx)+b\,\text{sech}(c+dx)\left(6(a+b)-b\,\text{sech}^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (3*(a + b)^2*Cosh[c + d*x] + b*Sech[c + d*x]*(6*(a + b) - b*Sech[c + d*x]^2
))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(47) = 94$.
time = 1.39, size = 98, normalized size = 2.00

method	result
derivativedivides	$\frac{a^2\cosh(dx+c)+2ab\left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+b^2\left(\frac{\sinh^4(dx+c)}{\cosh(dx+c)^3}+\frac{4\left(\sinh^2(dx+c)\right)}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)}{d}$
default	$\frac{a^2\cosh(dx+c)+2ab\left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+b^2\left(\frac{\sinh^4(dx+c)}{\cosh(dx+c)^3}+\frac{4\left(\sinh^2(dx+c)\right)}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)}{d}$
risch	$\frac{e^{dx+c}a^2}{2d}+\frac{ab\,e^{dx+c}}{d}+\frac{e^{dx+c}b^2}{2d}+\frac{e^{-dx-c}a^2}{2d}+\frac{e^{-dx-c}ab}{d}+\frac{e^{-dx-c}b^2}{2d}+\frac{4b\,e^{dx+c}\left(3a\,e^{4dx+4c}+3b\,e^{4dx+4c}+6a\,e\right.}{3d\left(1+e^{2dx+2\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*cosh(d*x+c)+2*a*b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c))+b^2*(s
inh(d*x+c)^4/cosh(d*x+c)^3+4*sinh(d*x+c)^2/cosh(d*x+c)^3+8/3/cosh(d*x+c)^3)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(47) = 94$.
time = 0.27, size = 171, normalized size = 3.49

$$\frac{1}{6}b^2\left(\frac{3\,e^{(-dx-c)}}{d}+\frac{33\,e^{(-2\,dx-2\,c)}+41\,e^{(-4\,dx-4\,c)}+27\,e^{(-6\,dx-6\,c)}+3}{d(e^{(-dx-c)}+3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right)+ab\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-dx-c)}+e^{(-3\,dx-3\,c)})}\right)+\frac{a^2\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 1/6*b^2*(3*e^(-d*x - c)/d + (33*e^(-2*d*x - 2*c) + 41*e^(-4*d*x - 4*c) + 27
*e^(-6*d*x - 6*c) + 3)/(d*(e^(-d*x - c) + 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x
- 5*c) + e^(-7*d*x - 7*c)))) + a*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) +
1)/(d*(e^(-d*x - c) + e^(-3*d*x - 3*c)))) + a^2*cosh(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(47) = 94$.
time = 0.33, size = 167, normalized size = 3.41

$$\frac{3\,(a^2+2\,ab+b^2)\cosh\left(dx+c\right)^4+3\,(a^2+2\,ab+b^2)\sinh\left(dx+c\right)^4+12\,(a^2+4\,ab+3\,b^2)\cosh\left(dx+c\right)^2+6\left(3\,(a^2+2\,ab+b^2)\cosh\left(dx+c\right)^2+2\,a^2+8\,ab+6\,b^2\right)\sinh\left(dx+c\right)^2+9\,a^2+42\,ab+25\,b^2}{6\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+3\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 12*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + 6*(3*(a^2 + 2*a*b + b^
2)*cosh(d*x + c)^2 + 2*a^2 + 8*a*b + 6*b^2)*sinh(d*x + c)^2 + 9*a^2 + 42*a*
b + 25*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*co
sh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^2\left(c+dx\right)\right)^2\sinh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(47) = 94$.
time = 0.46, size = 139, normalized size = 2.84

$$\frac{3\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)+3\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{8\left(3\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-2\,b^2\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*(e^(d*x + c) + e^(-d*x - c)) + 6*a*b*(e^(d*x + c) + e^(-d*x - c)
) + 3*b^2*(e^(d*x + c) + e^(-d*x - c)) + 8*(3*a*b*(e^(d*x + c) + e^(-d*x -
c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 2*b^2)/(e^(d*x + c) + e^(-d*
x - c))^3)/d
```

Mupad [B]
time = 0.19, size = 154, normalized size = 3.14

$$\frac{e^{c+d\,x}\,{\left(a+b\right)}^2}{2\,d}+\frac{e^{-c-d\,x}\,{\left(a+b\right)}^2}{2\,d}+\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}+\frac{4\,e^{c+d\,x}\,\left(b^2+a\,b\right)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(c + d*x)*(a + b)^2)/(2*d) + (exp(- c - d*x)*(a + b)^2)/(2*d) + (8*b^2*
exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d
*x) + 1)) + (4*exp(c + d*x)*(a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8*b^
2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.13 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=51

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[Out] -a^2*arctanh(cosh(d*x+c))/d-b*(2*a+b)*sech(d*x+c)/d+1/3*b^2*sech(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3745, 398, 213}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) - (b*(2*a + b)*Sech[c + d*x])/d + (b^2*Se
ch[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^2}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-b(2a+b)+b^2x^2+\frac{a^2}{-1+x^2}\right)\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$
$$= -\frac{b(2a+b)\operatorname{sech}(c+dx)}{d} + \frac{b^2\operatorname{sech}^3(c+dx)}{3d} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\right.}{}$$
$$= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d} - \frac{b(2a+b)\operatorname{sech}(c+dx)}{d} + \frac{b^2\operatorname{sech}^3(c}{3d}$$

Mathematica [A]
time = 0.13, size = 50, normalized size = 0.98

$$\frac{3a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-3b(2a+b)\operatorname{sech}(c+dx)+b^2\operatorname{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (3*a^2*Log[Tanh[(c + d*x)/2]] - 3*b*(2*a + b)*Sech[c + d*x] + b^2*Sech[c +
d*x]^3)/(3*d)
```

Maple [A]
time = 1.61, size = 63, normalized size = 1.24

method	result	size
derivativedivides	$\frac{-2a^2\operatorname{arctanh}\left(e^{dx+c}\right)-\frac{2ab}{\cosh(dx+c)}+b^2\left(-\frac{\sinh^2(dx+c)}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)}{d}$	63
default	$\frac{-2a^2\operatorname{arctanh}\left(e^{dx+c}\right)-\frac{2ab}{\cosh(dx+c)}+b^2\left(-\frac{\sinh^2(dx+c)}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)}{d}$	63
risch	$-\frac{2b\,e^{dx+c}\left(6a\,e^{4dx+4c}+3b\,e^{4dx+4c}+12a\,e^{2dx+2c}+2b\,e^{2dx+2c}+6a+3b\right)}{3d\left(1+e^{2dx+2c}\right)^3}+\frac{a^2\ln\left(e^{dx+c}-1\right)}{d}-\frac{a^2\ln\left(e^{dx+c}+1\right)}{d}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))-2*a*b/cosh(d*x+c)+b^2*(-sinh(d*x+c)^2/cosh(
d*x+c)^3-2/3/cosh(d*x+c)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(49) = 98$.
time = 0.27, size = 196, normalized size = 3.84

$$-\frac{2}{3}b^2\left(\frac{3e^{(-dx-c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{2e^{(-3dx-3c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{3e^{(-5dx-5c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+\frac{a^2\log\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}-\frac{4ab}{d(e^{(dx+c)}+e^{(-dx-c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -2/3*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-
6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x - 5*c)/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^2*log(tanh(1/2*d*x +
1/2*c))/d - 4*a*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(49) = 98$.
time = 0.34, size = 890, normalized size = 17.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + b^2)*cosh(d*x +
c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + b^2)*sinh(d*x + c)^3 +
 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + b^2)*cosh(d*x + c))*sinh(d*
x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) + 3*(a^2*cosh(d*x + c)^6 + 6*a^2*c
osh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)^4
+ 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 +
 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^2
*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 6*(
a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x +
 c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(a^2*cosh(d*x + c)^6 + 6*a^
2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)
^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^
2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*
a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 +
6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(5*(2*a*b + b^2)*cosh(d*
x + c)^4 + 2*(6*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c))/(d
*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 +
3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*
cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2
+ 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*
cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^2 \operatorname{csch}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(49) = 98.
time = 0.48, size = 123, normalized size = 2.41

$$-\frac{3\,a^2\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\,a^2\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{4\left(6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\,b^2\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*a^2*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*a^2*log(e^(d*x + c) + e
^(-d*x - c) - 2) + 4*(6*a*b*(e^(d*x + c) + e^(-d*x - c))^2 + 3*b^2*(e^(d*x
+ c) + e^(-d*x - c))^2 - 4*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

Mupad [B]
time = 0.15, size = 160, normalized size = 3.14

$$\frac{8\,b^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{8\,b^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}\,\left(b^2+2\,a\,b\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x),x)
```

```
[Out] (8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (8
*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
+ 6*d*x) + 1)) - (2*exp(c + d*x)*(2*a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1))
- (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/
(-d^2)^(1/2)
```

3.14 $\int \mathrm{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -a^2*coth(d*x+c)/d+2*a*b*tanh(d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3744, 276}

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -((a^2*Coth[c + d*x])/d) + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x]^3)/
(3*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \mathrm{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{\mathrm{Subst}\left(\int \frac{(a+bx^2)^2}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= -\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 43, normalized size = 0.93

$$\frac{-3a^2\coth(c+dx)+b\big(6a+b-b\mathrm{sech}^2(c+dx)\big)\tanh(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*a^2*Coth[c + d*x] + b*(6*a + b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(44) = 88.
time = 2.38, size = 169, normalized size = 3.67

method	result
risch	$-\frac{2\left(3a^2 e^{6dx+6c}+6ab\, e^{6dx+6c}+3b^2 e^{6dx+6c}+9a^2 e^{4dx+4c}+6ab\, e^{4dx+4c}-3b^2 e^{4dx+4c}+9a^2 e^{2dx+2c}-6ab\, e^{2dx+2c}+b^2 e^{2dx+2c}+3a^2-6ab-\right.}{3d\left(1+e^{2dx+2c}\right)^3\left(e^{2dx+2c}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(3*a^2*exp(6*d*x+6*c)+6*a*b*exp(6*d*x+6*c)+3*b^2*exp(6*d*x+6*c)+9*a^2*
exp(4*d*x+4*c)+6*a*b*exp(4*d*x+4*c)-3*b^2*exp(4*d*x+4*c)+9*a^2*exp(2*d*x+2*
c)-6*a*b*exp(2*d*x+2*c)+b^2*exp(2*d*x+2*c)+3*a^2-6*a*b-b^2)/d/(1+exp(2*d*x+
2*c))^3/(exp(2*d*x+2*c)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(44) = 88.
time = 0.29, size = 136, normalized size = 2.96

$$\frac{2}{3}\,b^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)+\frac{4\,ab}{d(e^{(-2\,dx-2\,c)}+1)}+\frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) + 1)) + 2*a^2/(d*(e^(-2*d
*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(44) = 88.
time = 0.35, size = 264, normalized size = 5.74

$$-\frac{4\left((3\,a^2+b^2)\cosh\left(dx+c\right)^3+3\,(3\,a^2+b^2)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+2\,(3\,ab+b^2)\sinh\left(dx+c\right)^3+(9\,a^2-b^2)\cosh\left(dx+c\right)+2\left(3\,(3\,ab+b^2)\cosh\left(dx+c\right)^2+3\,ab-b^2\right)\sinh\left(dx+c\right)\right)}{3\left(d\cosh\left(dx+c\right)^5+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+d\sinh\left(dx+c\right)^5+d\cosh\left(dx+c\right)^3+\left(10\,d\cosh\left(dx+c\right)^2+3\,d\right)\sinh\left(dx+c\right)^3+\left(10\,d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2-2\,d\cosh\left(dx+c\right)+\left(5\,d\cosh\left(dx+c\right)^4+9\,d\cosh\left(dx+c\right)^2+2\,d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/3*((3*a^2 + b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 2*(3*a*b + b^2)*sinh(d*x + c)^3 + (9*a^2 - b^2)*cosh(d*x + c) +
2*(3*(3*a*b + b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c))/(d*cosh(d*
x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + d*cosh(d
*x + c)^3 + (10*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^3 + (10*d*cosh(d*x +
 c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 - 2*d*cosh(d*x + c) + (5*d*cosh(
d*x + c)^4 + 9*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^2 \operatorname{csch}^2\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**2, x)
```

Giac [A]
time = 0.46, size = 86, normalized size = 1.87

$$-\frac{2\left(\frac{3a^2}{e^{(2dx+2c)}-1} + \frac{6abe^{(4dx+4c)}+3b^2e^{(4dx+4c)}+12abe^{(2dx+2c)}+6ab+b^2}{\left(e^{(2dx+2c)}+1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2/(e^(2*d*x + 2*c) - 1) + (6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x
 + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 6*a*b + b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B]
time = 1.16, size = 209, normalized size = 4.54

$$-\frac{\frac{2(2ab-b^2)}{3d} + \frac{2e^{2c+2dx}(b^2+2ab)}{3d}}{2e^{2c+2dx}+e^{4c+4dx}+1} - \frac{\frac{2(b^2+2ab)}{3d} + \frac{2e^{4c+4dx}(b^2+2ab)}{3d} + \frac{4e^{2c+2dx}(2ab-b^2)}{3d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1} - \frac{2a^2}{d\left(e^{2c+2dx}-1\right)} - \frac{2\left(b^2+2ab\right)}{3d\left(e^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^2,x)
```

```
[Out] - ((2*(2*a*b - b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d))/(2*e
xp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(2*a*b + b^2))/(3*d) + (2*exp
(4*c + 4*d*x)*(2*a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b - b^2))/(3*
d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*a
^2)/(d*(exp(2*c + 2*d*x) - 1)) - (2*(2*a*b + b^2))/(3*d*(exp(2*c + 2*d*x) +
 1))
```

3.15 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=82

$$\frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a(a-4b)\operatorname{sech}(c+dx)}{2d}-\frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[Out] 1/2*a*(a-4*b)*arctanh(cosh(d*x+c))/d-1/2*a*(a-4*b)*sech(d*x+c)/d-1/2*a^2*cs
ch(d*x+c)^2*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 474, 470, 327, 213}

$$-\frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d}-\frac{a(a-4b)\operatorname{sech}(c+dx)}{2d}+\frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*(a - 4*b)*Sech[c + d*x])/(2
*d) - (a^2*Csch[c + d*x]^2*Sech[c + d*x])/(2*d) - (b^2*Sech[c + d*x]^3)/(3*
d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{x^2(a+b-bx^2)^2}{(-1+x^2)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= -\frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{x^2(3a^2-2(a+b)^2+2b^2x^2}{-1+x^2}\right.}{2d}$$

$$= -\frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d} - \frac{b^2\operatorname{sech}^3(c+dx)}{3d} - \frac{(a(a-4}{}$$

$$= -\frac{a(a-4b)\operatorname{sech}(c+dx)}{2d} - \frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d} - \frac{b^2}{}$$

$$= \frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a(a-4b)\operatorname{sech}(c+dx)}{2d} - \frac{a}{}$$

Mathematica [A]
time = 1.03, size = 96, normalized size = 1.17

$$-\frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+12a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-48ab\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+3a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)-48ab\operatorname{sech}(c+dx)+8b^2\operatorname{sech}^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/24*(3*a^2*Csch[(c + d*x)/2]^2 + 12*a^2*Log[Tanh[(c + d*x)/2]] - 48*a*b*L
og[Tanh[(c + d*x)/2]] + 3*a^2*Sech[(c + d*x)/2]^2 - 48*a*b*Sech[c + d*x] +
8*b^2*Sech[c + d*x]^3)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(74) = 148$.
time = 2.74, size = 239, normalized size = 2.91

method	result
risch	$-\frac{e^{dx+c}\left(3a^2 e^{8dx+8c}-12ab\,e^{8dx+8c}+12a^2 e^{6dx+6c}+8b^2 e^{6dx+6c}+18a^2 e^{4dx+4c}+24ab\,e^{4dx+4c}-16b^2 e^{4dx+4c}+12a^2 e^{2dx+2c}+8b^2 e^{2dx+2c}\right.}{3d\left(e^{2dx+2c}-1\right)^2\left(1+e^{2dx+2c}\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*exp(d*x+c)*(3*a^2*exp(8*d*x+8*c)-12*a*b*exp(8*d*x+8*c)+12*a^2*exp(6*d*
x+6*c)+8*b^2*exp(6*d*x+6*c)+18*a^2*exp(4*d*x+4*c)+24*a*b*exp(4*d*x+4*c)-16*
b^2*exp(4*d*x+4*c)+12*a^2*exp(2*d*x+2*c)+8*b^2*exp(2*d*x+2*c)+3*a^2-12*a*b)
/d/(exp(2*d*x+2*c)-1)^2/(1+exp(2*d*x+2*c))^3+1/2*a^2/d*ln(exp(d*x+c)+1)-2*a
*b/d*ln(exp(d*x+c)+1)-1/2*a^2/d*ln(exp(d*x+c)-1)+2*a*b/d*ln(exp(d*x+c)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(74) = 148$.
time = 0.27, size = 181, normalized size = 2.21

$$\frac{1}{2}a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)-\frac{8\,b^2}{3\,d(e^{(dx+c)}+e^{(-dx-c)})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c
) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*
a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*
(e^(-2*d*x - 2*c) + 1))) - 8/3*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2462 vs. $2(74) = 148$.
time = 0.36, size = 2462, normalized size = 30.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 54*(a^2 - 4*a*b)*cosh(d*x + c)*sinh
(d*x + c)^8 + 6*(a^2 - 4*a*b)*sinh(d*x + c)^9 + 8*(3*a^2 + 2*b^2)*cosh(d*x
+ c)^7 + 8*(27*(a^2 - 4*a*b)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x + c)
^7 + 56*(9*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (3*a^2 + 2*b^2)*cosh(d*x + c))*s
inh(d*x + c)^6 + 4*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^5 + 4*(189*(a^2 -
```

4*a*b)*cosh(d*x + c)^4 + 42*(3*a^2 + 2*b^2)*cosh(d*x + c)^2 + 9*a^2 + 12*a *b - 8*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 70*(3* a^2 + 2*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c))*si nh(d*x + c)^4 + 8*(3*a^2 + 2*b^2)*cosh(d*x + c)^3 + 8*(63*(a^2 - 4*a*b)*cos h(d*x + c)^6 + 35*(3*a^2 + 2*b^2)*cosh(d*x + c)^4 + 5*(9*a^2 + 12*a*b - 8*b ^2)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x + c)^3 + 8*(27*(a^2 - 4*a*b)* cosh(d*x + c)^7 + 21*(3*a^2 + 2*b^2)*cosh(d*x + c)^5 + 5*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(a^2 - 4*a*b)*cosh(d*x + c) - 3*((a^2 - 4*a*b)*cosh(d*x + c)^10 + 10*(a^ 2 - 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 - 4*a*b)*sinh(d*x + c)^10 + (a^2 - 4*a*b)*cosh(d*x + c)^8 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^8 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 2*(105 *(a^2 - 4*a*b)*cosh(d*x + c)^4 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4 *a*b)*sinh(d*x + c)^6 + 4*(63*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 14*(a^2 - 4*a *b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a ^2 - 4*a*b)*cosh(d*x + c)^4 + 2*(105*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 35*(a^ 2 - 4*a*b)*cosh(d*x + c)^4 - 15*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^4 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^7 + 7*(a^2 - 4*a*b)*c osh(d*x + c)^5 - 5*(a^2 - 4*a*b)*cosh(d*x + c)^3 - (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a^2 - 4*a*b)*cosh(d*x + c)^2 + (45*(a^2 - 4*a*b)*co sh(d*x + c)^8 + 28*(a^2 - 4*a*b)*cosh(d*x + c)^6 - 30*(a^2 - 4*a*b)*cosh(d* x + c)^4 - 12*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^2 + a^2 - 4*a*b + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 4*(a^2 - 4*a*b)*cosh(d *x + c)^7 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^5 - 4*(a^2 - 4*a*b)*cosh(d*x + c) ^3 + (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d *x + c) + 1) + 3*((a^2 - 4*a*b)*cosh(d*x + c)^10 + 10*(a^2 - 4*a*b)*cosh(d* x + c)*sinh(d*x + c)^9 + (a^2 - 4*a*b)*sinh(d*x + c)^10 + (a^2 - 4*a*b)*cos h(d*x + c)^8 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^8 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b)*cosh(d*x + c))*s inh(d*x + c)^7 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 2*(105*(a^2 - 4*a*b)*cos h(d*x + c)^4 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^6 + 4*(63*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^ 3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b)*cosh(d *x + c)^4 + 2*(105*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 35*(a^2 - 4*a*b)*cosh(d* x + c)^4 - 15*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^4 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^7 + 7*(a^2 - 4*a*b)*cosh(d*x + c)^5 - 5 *(a^2 - 4*a*b)*cosh(d*x + c)^3 - (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c) ^3 + (a^2 - 4*a*b)*cosh(d*x + c)^2 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^8 + 28 *(a^2 - 4*a*b)*cosh(d*x + c)^6 - 30*(a^2 - 4*a*b)*cosh(d*x + c)^4 - 12*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^2 + a^2 - 4*a*b + 2* (5*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 4*(a^2 - 4*a*b)*cosh(d*x + c)^7 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^5 - 4*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b) *cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(27*(a^2 - 4*a*b)*cosh(d*x + c)^8 + 28*(3*a^2 + 2*b^2)*cosh(d*x + c)^6 + 10*

```
(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 + 2*b^2)*cosh(d*x + c)
^2 + 3*a^2 - 12*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c
)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + d*cosh(d*x + c)^8 + (45*d*cosh(d*x
 + c)^2 + d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*s
inh(d*x + c)^7 - 2*d*cosh(d*x + c)^6 + 2*(105*d*cosh(d*x + c)^4 + 14*d*cosh
(d*x + c)^2 - d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 + 14*d*cosh(d*x
+ c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 2*d*cosh(d*x + c)^4 + 2*(105*
d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 - 15*d*cosh(d*x + c)^2 - d)*sinh(d
*x + c)^4 + 8*(15*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 - 5*d*cosh(d*x +
c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c)^2 + (45*d*cosh(d*
x + c)^8 + 28*d*cosh(d*x + c)^6 - 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 - 6*
d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) +
d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh^{2}{\left(c + d x \right)}\right)^{2} \operatorname{csch}^{3}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(74) = 148.
time = 0.45, size = 153, normalized size = 1.87

$$-\frac{\frac{12\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4} - 3\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right) + 3\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right) - \frac{16\left(3\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-2\,b^2\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c) + e^(-d*x - c))^2
- 4) - 3*(a^2 - 4*a*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 3*(a^2 - 4*a*b
)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 16*(3*a*b*(e^(d*x + c) + e^(-d*x -
c))^2 - 2*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

Mupad [B]
time = 0.16, size = 261, normalized size = 3.18

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^2\,\sqrt{-d^2}-4\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}\right)\,\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}{\sqrt{-d^2}}+\frac{8\,b^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}-\frac{a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,b^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}+\frac{4\,a\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) - 4*a*b*(-d^2)^(1/2)))/(d*(a^4 - 8
*a^3*b + 16*a^2*b^2)^(1/2)))*(a^4 - 8*a^3*b + 16*a^2*b^2)^(1/2))/(-d^2)^(1/
2) + (8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) + 1)) - (a^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*
a^2*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*b^2*
exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (4*a*b*ex
p(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.16 $\int \mathbf{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=72

$$\frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{(2a-b)b\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] a*(a-2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d-(2*a-b)*b*tanh(d*x+c)/d-1/3
*b^2*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3744, 459}

$$-\frac{a^2\coth^3(c+dx)}{3d}-\frac{b(2a-b)\tanh(c+dx)}{d}+\frac{a(a-2b)\coth(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a*(a - 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) - ((2*a - b)*b*
Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+bx^2)^2}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(b(-2a+b)+\frac{a^2}{x^4}-\frac{a(a-2b)}{x^2}-b^2x^2\right)dx, x, \tanh(c+\right.}{d}$$
$$= \frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{(2a-b)b\tanh(}{d}$$

Mathematica [A]
time = 0.38, size = 59, normalized size = 0.82

$$\frac{-a\coth(c+dx)\left(-2a+6b+a\operatorname{csch}^2(c+dx)\right)+b\left(-6a+2b+b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-(a*Coth[c + d*x]*(-2*a + 6*b + a*Csch[c + d*x]^2)) + b*(-6*a + 2*b + b*Se
ch[c + d*x]^2)*Tanh[c + d*x])/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(68) = 136.
time = 2.55, size = 157, normalized size = 2.18

method	result	si
risch	$-\frac{4\left(3a^2 e^{8dx+8c}+6ab\,e^{8dx+8c}+3b^2 e^{8dx+8c}+8a^2 e^{6dx+6c}-8b^2 e^{6dx+6c}+6a^2 e^{4dx+4c}-12ab\,e^{4dx+4c}+6b^2 e^{4dx+4c}-a^2+6ab-b^2\right)}{3d\left(e^{2dx+2c}-1\right)^3\left(1+e^{2dx+2c}\right)^3}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(3*a^2*exp(8*d*x+8*c)+6*a*b*exp(8*d*x+8*c)+3*b^2*exp(8*d*x+8*c)+8*a^2*
exp(6*d*x+6*c)-8*b^2*exp(6*d*x+6*c)+6*a^2*exp(4*d*x+4*c)-12*a*b*exp(4*d*x+4
*c)+6*b^2*exp(4*d*x+4*c)-a^2+6*a*b-b^2)/d/(exp(2*d*x+2*c)-1)^3/(1+exp(2*d*x
+2*c))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(68) = 136.
time = 0.28, size = 210, normalized size = 2.92

$$\frac{4}{3}b^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+\frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)+\frac{8ab}{d(e^{(-4dx-4c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 4/3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) -
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 8*a*b/(d*(e^(-4*d*x - 4*c) -
1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(68) = 136$.
time = 0.34, size = 393, normalized size = 5.46

$$-\frac{8\left(\left(a^{2}+6\,ab+b^{2}\right)\cosh\left(dx+c\right)^{4}+8\left(a^{2}+b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\left(a^{2}+6\,ab+b^{2}\right)\sinh\left(dx+c\right)^{4}+4\left(a^{2}-b^{2}\right)\cosh\left(dx+c\right)^{2}+2\left(3\left(a^{2}+6\,ab+b^{2}\right)\cosh\left(dx+c\right)^{2}+2\,a^{2}-2\,b^{2}\right)\sinh\left(dx+c\right)^{2}+3\,a^{2}-6\,ab+3\,b^{2}+8\left(\left(a^{2}+b^{2}\right)\cosh\left(dx+c\right)^{3}+\left(a^{2}-b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}{3\left(d\cosh\left(dx+c\right)^{8}+56\,d\cosh\left(dx+c\right)^{3}\sinh\left(dx+c\right)^{5}+28\,d\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)^{6}+8\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{7}+d\sinh\left(dx+c\right)^{8}-4\,d\cosh\left(dx+c\right)^{4}+2\left(35\,d\cosh\left(dx+c\right)^{4}-2\,d\right)\sinh\left(dx+c\right)^{4}+8\left(7\,d\cosh\left(dx+c\right)^{5}-d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+4\left(7\,d\cosh\left(dx+c\right)^{6}-6\,d\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{2}+8\left(d\cosh\left(dx+c\right)^{7}-d\cosh\left(dx+c\right)^{3}\right)\sinh\left(dx+c\right)+3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/3*((a^2 + 6*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + b^2)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^2 + 6*a*b + b^2)*sinh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x
 + c)^2 + 2*(3*(a^2 + 6*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - 2*b^2)*sinh(d*
x + c)^2 + 3*a^2 - 6*a*b + 3*b^2 + 8*((a^2 + b^2)*cosh(d*x + c)^3 + (a^2 -
b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^
3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c
)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^4 + 2*(35*d*cosh(
d*x + c)^4 - 2*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(7*d*cosh(d*x + c)^6 - 6*d*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + 3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^{2}\left(c+dx\right)\right)^{2}\operatorname{csch}^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(68) = 136$.
time = 0.46, size = 143, normalized size = 1.99

$$-\frac{4\left(3\,a^{2}e^{(8\,dx+8\,c)}+6\,abe^{(8\,dx+8\,c)}+3\,b^{2}e^{(8\,dx+8\,c)}+8\,a^{2}e^{(6\,dx+6\,c)}-8\,b^{2}e^{(6\,dx+6\,c)}+6\,a^{2}e^{(4\,dx+4\,c)}-12\,abe^{(4\,dx+4\,c)}+6\,b^{2}e^{(4\,dx+4\,c)}-a^{2}+6\,ab-b^{2}\right)}{3\,d\left(e^{(4\,dx+4\,c)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/3*(3*a^2*e^(8*d*x + 8*c) + 6*a*b*e^(8*d*x + 8*c) + 3*b^2*e^(8*d*x + 8*c)
 + 8*a^2*e^(6*d*x + 6*c) - 8*b^2*e^(6*d*x + 6*c) + 6*a^2*e^(4*d*x + 4*c) -
12*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) - a^2 + 6*a*b - b^2)/(d*(e^(
4*d*x + 4*c) - 1)^3)
```

Mupad [B]
time = 1.08, size = 143, normalized size = 1.99

$$-\frac{4\left(6\,a\,b-a^2-b^2+6\,a^2\,{\mathrm{e}}^{4\,c+4\,d\,x}+8\,a^2\,{\mathrm{e}}^{6\,c+6\,d\,x}+3\,a^2\,{\mathrm{e}}^{8\,c+8\,d\,x}+6\,b^2\,{\mathrm{e}}^{4\,c+4\,d\,x}-8\,b^2\,{\mathrm{e}}^{6\,c+6\,d\,x}+3\,b^2\,{\mathrm{e}}^{8\,c+8\,d\,x}-12\,a\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}+6\,a\,b\,{\mathrm{e}}^{8\,c+8\,d\,x}\right)}{3\,d\,{\left({\mathrm{e}}^{4\,c+4\,d\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^4,x)
```

```
[Out] -(4*(6*a*b - a^2 - b^2 + 6*a^2*exp(4*c + 4*d*x) + 8*a^2*exp(6*c + 6*d*x) +
3*a^2*exp(8*c + 8*d*x) + 6*b^2*exp(4*c + 4*d*x) - 8*b^2*exp(6*c + 6*d*x) +
3*b^2*exp(8*c + 8*d*x) - 12*a*b*exp(4*c + 4*d*x) + 6*a*b*exp(8*c + 8*d*x)))
/(3*d*(exp(4*c + 4*d*x) - 1)^3)
```

3.17 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=182

$$\frac{3}{8}(a+b)\left(a^2+14ab+21b^2\right)x-\frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d}-\frac{b\left(6a^2+35ab+21b^2\right)\tanh^3(c+dx)}{8d}$$

```
[Out] 3/8*(a+b)*(a^2+14*a*b+21*b^2)*x-3/8*(a+b)*(a^2+14*a*b+21*b^2)*tanh(d*x+c)/d
-1/8*b*(6*a^2+35*a*b+21*b^2)*tanh(d*x+c)^3/d-3/40*b^2*(5*a+21*b)*tanh(d*x+c
)^5/d-3/8*(a+3*b)*sinh(d*x+c)^2*tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2/d+1/4*cos
h(d*x+c)*sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 478, 591, 584, 212}

$$-\frac{b\left(6a^2+35ab+21b^2\right)\tanh^3(c+dx)}{8d}-\frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d}+\frac{3}{8}x(a+b)\left(a^2+14ab+21b^2\right)-\frac{3b^2(5a+21b)\tanh^5(c+dx)}{40d}-\frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d}+\frac{\sinh^3(c+dx)\cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a + b)*(a^2 + 14*a*b + 21*b^2)*x)/8 - (3*(a + b)*(a^2 + 14*a*b + 21*b^2
)*Tanh[c + d*x])/(8*d) - (b*(6*a^2 + 35*a*b + 21*b^2)*Tanh[c + d*x]^3)/(8*d
) - (3*b^2*(5*a + 21*b)*Tanh[c + d*x]^5)/(40*d) - (3*(a + 3*b)*Sinh[c + d*x
]^2*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2)/(8*d) + (Cosh[c + d*x]*Sinh[c
+ d*x]^3*(a + b*Tanh[c + d*x]^2)^3)/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^3}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d}$$

$$= -\frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d}$$

$$= -\frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d} - \frac{b(6a^2+35ab+}{}$$

$$= \frac{3}{8}(a+b)\left(a^2+14ab+21b^2\right)x - \frac{3(a+b)\left(a^2+14ab+21b^2\right)\text{t}}{8d}$$

Mathematica [A]

time = 2.73, size = 125, normalized size = 0.69

$$\frac{60\left(a^3+15a^2b+35ab^2+21b^3\right)(c+dx)-40(a+b)^2(a+4b)\sinh(2(c+dx))+5(a+b)^3\sinh(4(c+dx))-32b\left(15a^2+50ab+36b^2-b(5a+7b)\operatorname{sech}^2(c+dx)+b^2\operatorname{sech}^4(c+dx)\right)\tanh(c+dx)}{160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (60*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*(c + d*x) - 40*(a + b)^2*(a + 4*b)
*Sinh[2*(c + d*x)] + 5*(a + b)^3*Sinh[4*(c + d*x)] - 32*b*(15*a^2 + 50*a*b
+ 36*b^2 - b*(5*a + 7*b)*Sech[c + d*x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c + d*
x])/(160*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(170) = 340$.
time = 2.47, size = 506, normalized size = 2.78

method	result
risch	$\frac{3a^3x}{8}+\frac{45a^2bx}{8}+\frac{105a\,b^2x}{8}+\frac{63b^3x}{8}+\frac{e^{4dx+4c}a^3}{64d}+\frac{3\,e^{4dx+4c}a^2b}{64d}+\frac{3b^2e^{4dx+4c}a}{64d}+\frac{b^3e^{4dx+4c}}{64d}-\frac{e^{2dx+2c}a^3}{8d}-\frac{3b\,e^{2dx+2}}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^3*x+45/8*a^2*b*x+105/8*a*b^2*x+63/8*b^3*x+1/64/d*exp(4*d*x+4*c)*a^3+3
/64/d*exp(4*d*x+4*c)*a^2*b+3/64*b^2/d*exp(4*d*x+4*c)*a+1/64*b^3/d*exp(4*d*x
+4*c)-1/8/d*exp(2*d*x+2*c)*a^3-3/4*b/d*exp(2*d*x+2*c)*a^2-9/8/d*exp(2*d*x+2
*c)*a*b^2-1/2*b^3/d*exp(2*d*x+2*c)+1/8/d*exp(-2*d*x-2*c)*a^3+3/4*b/d*exp(-2
*d*x-2*c)*a^2+9/8/d*exp(-2*d*x-2*c)*a*b^2+1/2*b^3/d*exp(-2*d*x-2*c)-1/64/d*
exp(-4*d*x-4*c)*a^3-3/64/d*exp(-4*d*x-4*c)*a^2*b-3/64*b^2/d*exp(-4*d*x-4*c)
*a-1/64*b^3/d*exp(-4*d*x-4*c)+2/5*b*(15*a^2*exp(8*d*x+8*c)+60*a*b*exp(8*d*x
+8*c)+50*b^2*exp(8*d*x+8*c)+60*a^2*exp(6*d*x+6*c)+210*a*b*exp(6*d*x+6*c)+15
0*b^2*exp(6*d*x+6*c)+90*a^2*exp(4*d*x+4*c)+290*a*b*exp(4*d*x+4*c)+210*b^2*e
xp(4*d*x+4*c)+60*a^2*exp(2*d*x+2*c)+190*a*b*exp(2*d*x+2*c)+130*b^2*exp(2*d*
x+2*c)+15*a^2+50*a*b+36*b^2)/d/(1+exp(2*d*x+2*c))^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(170) = 340$.
time = 0.30, size = 480, normalized size = 2.64

$$\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{320}b^3\left(\frac{2520(dx+c)}{d}+\frac{5\left(32e^{(-2dx-2c)}-e^{(-4dx-4c)}\right)}{d}-\frac{135e^{(-2dx-2c)}+5358e^{(-4dx-4c)}+18190e^{(-6dx-6c)}+28455e^{(-8dx-8c)}+19995e^{(-10dx-10c)}+6560e^{(-12dx-12c)}-5}{d\left(e^{(-4dx-4c)}+5e^{(-6dx-6c)}+10e^{(-8dx-8c)}+10e^{(-10dx-10c)}+5e^{(-12dx-12c)}+e^{(-14dx-14c)}\right)}\right)+\frac{1}{64}ab^2\left(\frac{840(dx+c)}{d}+\frac{3\left(24e^{(-2dx-2c)}-e^{(-4dx-4c)}\right)}{d}-\frac{63e^{(-2dx-2c)}+1487e^{(-4dx-4c)}+2517e^{(-6dx-6c)}+1608e^{(-8dx-8c)}-3}{d\left(e^{(-4dx-4c)}+3e^{(-6dx-6c)}+3e^{(-8dx-8c)}+e^{(-10dx-10c)}\right)}\right)+\frac{3}{64}a^2b\left(\frac{120(dx+c)}{d}+\frac{16e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}-\frac{15e^{(-2dx-2c)}+144e^{(-4dx-4c)}-1}{d\left(e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^3*(2520*(d*x + c)/d + 5*(32*e^(-2*d*x
```

```
- 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4*c
) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x - 10
*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c)))) + 1/64*a*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x - 2*c)
- e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*c) + 2517
*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*c) + 3*e^(
-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 3/64*a^2*b*(12
0*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x
- 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))
))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(170) = 340$.
time = 0.38, size = 879, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^9 - 15*(a^3 + 11*a^2
*b + 19*a*b^2 + 9*b^3 - 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^7 + 8*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b
+ 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 + 288
*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)*sinh(d*x
+ c)^4 + (630*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 150*a^3 - 2
010*a^2*b - 4850*a*b^2 - 3054*b^3 - 315*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 + 288*b^3 + 1
5*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^3 + 5*(84*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 105*(a^3 + 11*a^2*b + 19*a*b^2 +
 9*b^3)*cosh(d*x + c)^4 - 62*a^3 - 978*a^2*b - 2282*a*b^2 - 1302*b^3 - 4*(7
5*a^3 + 1005*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
3 + 40*(2*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2
+ 21*b^3)*d*x)*cosh(d*x + c)^3 + 3*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a
^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 80
*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)
*d*x)*cosh(d*x + c) + 5*(9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8
- 21*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3)*cosh(d*x + c)^6 - 2*(75*a^3 + 1005
*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^4 - 36*a^3 - 612*a^2*b - 1372
*a*b^2 - 924*b^3 - 6*(31*a^3 + 489*a^2*b + 1141*a*b^2 + 651*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^
4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(
d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(170) = 340.
time = 0.65, size = 505, normalized size = 2.77

$$\frac{5\,a^{3}e^{(4\,dx+4\,c)} + 15\,a^{2}be^{(4\,dx+4\,c)} + 15\,ab^{2}e^{(4\,dx+4\,c)} + 5\,b^{3}e^{(4\,dx+4\,c)} - 40\,a^{3}e^{(2\,dx+2\,c)} - 240\,a^{2}be^{(2\,dx+2\,c)} - 360\,ab^{2}e^{(2\,dx+2\,c)} - 160\,b^{3}e^{(2\,dx+2\,c)} + 120\,(a^{3} + 15\,a^{2}b + 35\,ab^{2} + 21\,b^{3})(dx + c) - 5\,(18\,a^{3}e^{(4\,dx+4\,c)} + 270\,a^{2}be^{(4\,dx+4\,c)} + 630\,ab^{2}e^{(4\,dx+4\,c)} + 378\,b^{3}e^{(4\,dx+4\,c)} - 8\,a^{3}e^{(2\,dx+2\,c)} - 48\,a^{2}be^{(2\,dx+2\,c)} - 72\,ab^{2}e^{(2\,dx+2\,c)} - 32\,b^{3}e^{(2\,dx+2\,c)} + a^{3} + 3\,a^{2}b + 3\,ab^{2} + b^{3})e^{(-4\,dx-4\,c)} + \frac{128\,(15\,a^{2}be^{(8\,dx+8\,c)} + 60\,ab^{2}e^{(8\,dx+8\,c)} + 50\,b^{3}e^{(8\,dx+8\,c)} + 60\,a^{2}be^{(6\,dx+6\,c)} + 210\,ab^{2}e^{(6\,dx+6\,c)} + 150\,b^{3}e^{(6\,dx+6\,c)} + 90\,a^{2}be^{(4\,dx+4\,c)} + 290\,ab^{2}e^{(4\,dx+4\,c)} + 210\,b^{3}e^{(4\,dx+4\,c)} + 60\,a^{2}be^{(2\,dx+2\,c)} + 190\,ab^{2}e^{(2\,dx+2\,c)} + 130\,b^{3}e^{(2\,dx+2\,c)} + 15\,a^{2}b + 50\,ab^{2} + 36\,b^{3})}{(e^{(2\,dx+2\,c)} + 1)^{5}}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/320*(5*a^3*e^(4*d*x + 4*c) + 15*a^2*b*e^(4*d*x + 4*c) + 15*a*b^2*e^(4*d*x
 + 4*c) + 5*b^3*e^(4*d*x + 4*c) - 40*a^3*e^(2*d*x + 2*c) - 240*a^2*b*e^(2*d
*x + 2*c) - 360*a*b^2*e^(2*d*x + 2*c) - 160*b^3*e^(2*d*x + 2*c) + 120*(a^3
+ 15*a^2*b + 35*a*b^2 + 21*b^3)*(d*x + c) - 5*(18*a^3*e^(4*d*x + 4*c) + 270
*a^2*b*e^(4*d*x + 4*c) + 630*a*b^2*e^(4*d*x + 4*c) + 378*b^3*e^(4*d*x + 4*c
) - 8*a^3*e^(2*d*x + 2*c) - 48*a^2*b*e^(2*d*x + 2*c) - 72*a*b^2*e^(2*d*x +
2*c) - 32*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x -
4*c) + 128*(15*a^2*b*e^(8*d*x + 8*c) + 60*a*b^2*e^(8*d*x + 8*c) + 50*b^3*e^
(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 210*a*b^2*e^(6*d*x + 6*c) + 150*
b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 290*a*b^2*e^(4*d*x + 4*c)
+ 210*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 190*a*b^2*e^(2*d*x +
 2*c) + 130*b^3*e^(2*d*x + 2*c) + 15*a^2*b + 50*a*b^2 + 36*b^3)/(e^(2*d*x +
 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.39, size = 730, normalized size = 4.01

$$\frac{\frac{2\,(3\,a^2\,b+12\,a\,b^2+10\,b^3)}{5\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b+9\,a\,b^2+5\,b^3)}{5\,d}+\frac{12\,e^{4\,c+4\,d\,x}\,(3\,a^2\,b+8\,a\,b^2+6\,b^3)}{5\,d}+\frac{8\,e^{6\,c+6\,d\,x}\,(3\,a^2\,b+9\,a\,b^2+5\,b^3)}{5\,d}+\frac{2\,e^{8\,c+8\,d\,x}\,(3\,a^2\,b+12\,a\,b^2+10\,b^3)}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}+\frac{\frac{2\,(3\,a^2\,b+9\,a\,b^2+5\,b^3)}{5\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b+12\,a\,b^2+10\,b^3)}{5\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}+x\,\left(\frac{3\,a^3}{8}+\frac{45\,a^2\,b}{8}+\frac{105\,a\,b^2}{8}+\frac{63\,b^3}{8}\right)+\frac{\frac{2\,(3\,a^2\,b+9\,a\,b^2+5\,b^3)}{5\,d}+\frac{6\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b+8\,a\,b^2+6\,b^3)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(3\,a^2\,b+9\,a\,b^2+5\,b^3)}{5\,d}+\frac{2\,e^{6\,c+6\,d\,x}\,(3\,a^2\,b+12\,a\,b^2+10\,b^3)}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}+\frac{\frac{2\,(3\,a^2\,b+8\,a\,b^2+6\,b^3)}{5\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b+9\,a\,b^2+5\,b^3)}{5\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(3\,a^2\,b+12\,a\,b^2+10\,b^3)}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}+\frac{2\,(3\,a^2\,b+12\,a\,b^2+10\,b^3)}{5\,d\,(e^{2\,c+2\,d\,x}+1)}-\frac{e^{-4\,c-4\,d\,x}\,{(a+b)}^3}{64\,d}+\frac{e^{4\,c+4\,d\,x}\,{(a+b)}^3}{64\,d}+\frac{e^{-2\,c-2\,d\,x}\,{(a+b)}^2\,(a+4\,b)}{8\,d}-\frac{e^{2\,c+2\,d\,x}\,{(a+b)}^2\,(a+4\,b)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] ((2*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d) + (8*exp(2*c + 2*d*x)*(9*a*b^2 + 3
*a^2*b + 5*b^3))/(5*d) + (12*exp(4*c + 4*d*x)*(8*a*b^2 + 3*a^2*b + 6*b^3))/
(5*d) + (8*exp(6*c + 6*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(8*c
 + 8*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*ex
p(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d
```

```
*x) + 1) + ((2*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(12
*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) +
 1) + x*((105*a*b^2)/8 + (45*a^2*b)/8 + (3*a^3)/8 + (63*b^3)/8) + ((2*(9*a*
b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6*exp(2*c + 2*d*x)*(8*a*b^2 + 3*a^2*b + 6*
b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*e
xp(6*c + 6*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(4*exp(2*c + 2*d*x) +
 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + ((2*(8*a
*b^2 + 3*a^2*b + 6*b^3))/(5*d) + (4*exp(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5
*b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(3
*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (2*(12*a*b
^2 + 3*a^2*b + 10*b^3))/(5*d*(exp(2*c + 2*d*x) + 1)) - (exp(- 4*c - 4*d*x)*
(a + b)^3)/(64*d) + (exp(4*c + 4*d*x)*(a + b)^3)/(64*d) + (exp(- 2*c - 2*d*
x)*(a + b)^2*(a + 4*b))/(8*d) - (exp(2*c + 2*d*x)*(a + b)^2*(a + 4*b))/(8*d
)
```

3.18 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=105

$$-\frac{(a+b)^2(a+4b)\cosh(c+dx)}{d}+\frac{(a+b)^3\cosh^3(c+dx)}{3d}-\frac{3b(a+b)(a+2b)\text{sech}(c+dx)}{d}+\frac{b^2(3a+4b)\text{sech}^3(c}{3d}$$

```
[Out] -(a+b)^2*(a+4*b)*cosh(d*x+c)/d+1/3*(a+b)^3*cosh(d*x+c)^3/d-3*b*(a+b)*(a+2*b
)*sech(d*x+c)/d+1/3*b^2*(3*a+4*b)*sech(d*x+c)^3/d-1/5*b^3*sech(d*x+c)^5/d
```

Rubi [A]
time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3745, 459}

$$\frac{b^2(3a+4b)\text{sech}^3(c+dx)}{3d}+\frac{(a+b)^3\cosh^3(c+dx)}{3d}-\frac{(a+b)^2(a+4b)\cosh(c+dx)}{d}-\frac{3b(a+b)(a+2b)\text{sech}(c+dx)}{d}-\frac{b^3\text{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(((a + b)^2*(a + 4*b)*Cosh[c + d*x])/d) + ((a + b)^3*Cosh[c + d*x]^3)/(3*d
) - (3*b*(a + b)*(a + 2*b)*Sech[c + d*x])/d + (b^2*(3*a + 4*b)*Sech[c + d*x
]^3)/(3*d) - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b-bx^2)^3}{x^4}\,dx, x, \text{sech}(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(3(-a-2b)b(a+b) - \frac{(a+b)^3}{x^4} + \frac{(a+b)^2(a+4b)}{x^2} + b^2(3a\right.\right.}{d}$$
$$= -\frac{(a+b)^2(a+4b)\cosh(c+dx)}{d} + \frac{(a+b)^3\cosh^3(c+dx)}{3d} - \frac{3}{}$$

Mathematica [A]
time = 0.23, size = 91, normalized size = 0.87

$$\frac{-45(a+b)^2(a+5b)\cosh(c+dx)+5(a+b)^3\cosh(3(c+dx))-180b(a+b)(a+2b)\text{sech}(c+dx)+20b^2(3a+4b)\text{sech}^3(c+dx)-12b^3\text{sech}^5(c+dx)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-45*(a + b)^2*(a + 5*b)*Cosh[c + d*x] + 5*(a + b)^3*Cosh[3*(c + d*x)] - 18
0*b*(a + b)*(a + 2*b)*Sech[c + d*x] + 20*b^2*(3*a + 4*b)*Sech[c + d*x]^3 -
12*b^3*Sech[c + d*x]^5)/(60*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(99) = 198$.
time = 2.42, size = 474, normalized size = 4.51

method	result
risch	$\frac{e^{3dx+3c}a^3}{24d} + \frac{e^{3dx+3c}a^2b}{8d} + \frac{a\,b^2e^{3dx+3c}}{8d} + \frac{e^{3dx+3c}b^3}{24d} - \frac{3\,e^{dx+c}a^3}{8d} - \frac{21\,e^{dx+c}a^2b}{8d} - \frac{33a\,e^{dx+c}b^2}{8d} - \frac{15b^3e^{dx+c}}{8d} - \frac{3\,e^{-dx}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*a^3+1/8/d*exp(3*d*x+3*c)*a^2*b+1/8*a*b^2/d*exp(3*d*x+
3*c)+1/24/d*exp(3*d*x+3*c)*b^3-3/8/d*exp(d*x+c)*a^3-21/8/d*exp(d*x+c)*a^2*b
-33/8*a/d*exp(d*x+c)*b^2-15/8*b^3/d*exp(d*x+c)-3/8/d*exp(-d*x-c)*a^3-21/8/d
*exp(-d*x-c)*a^2*b-33/8*a/d*exp(-d*x-c)*b^2-15/8/d*exp(-d*x-c)*b^3+1/24/d*e
xp(-3*d*x-3*c)*a^3+1/8/d*exp(-3*d*x-3*c)*a^2*b+1/8*a*b^2/d*exp(-3*d*x-3*c)+
1/24/d*exp(-3*d*x-3*c)*b^3-2/15*b*exp(d*x+c)*(45*a^2*exp(8*d*x+8*c)+135*a*b
*exp(8*d*x+8*c)+90*b^2*exp(8*d*x+8*c)+180*a^2*exp(6*d*x+6*c)+480*a*b*exp(6*
d*x+6*c)+280*b^2*exp(6*d*x+6*c)+270*a^2*exp(4*d*x+4*c)+690*a*b*exp(4*d*x+4*
c)+428*b^2*exp(4*d*x+4*c)+180*a^2*exp(2*d*x+2*c)+480*a*b*exp(2*d*x+2*c)+280
*b^2*exp(2*d*x+2*c)+45*a^2+135*a*b+90*b^2)/d/(1+exp(2*d*x+2*c))^5
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(99) = 198$.
time = 0.30, size = 439, normalized size = 4.18

$$-\frac{1}{120}\,b^{3}\left(\frac{5\,\left(45\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}+\frac{200\,e^{(-2\,dx-2\,c)}+2515\,e^{(-4\,dx-4\,c)}+6680\,e^{(-6\,dx-6\,c)}+9073\,e^{(-8\,dx-8\,c)}+5600\,e^{(-10\,dx-10\,c)}+1665\,e^{(-12\,dx-12\,c)}-5}{d(e^{(-3\,dx-3\,c)}+5\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+10\,e^{(-9\,dx-9\,c)}+5\,e^{(-11\,dx-11\,c)}+e^{(-13\,dx-13\,c)})}\right)-\frac{1}{8}\,ab^{2}\left(\frac{33\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{30\,e^{(-2\,dx-2\,c)}+240\,e^{(-4\,dx-4\,c)}+322\,e^{(-6\,dx-6\,c)}+177\,e^{(-8\,dx-8\,c)}-1}{d(e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+3\,e^{(-7\,dx-7\,c)}+e^{(-9\,dx-9\,c)})}\right)-\frac{1}{8}\,a^{2}b\left(\frac{21\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{20\,e^{(-2\,dx-2\,c)}+69\,e^{(-4\,dx-4\,c)}-1}{d(e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right)+\frac{1}{24}\,a^{3}\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*b^3*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*c
) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c) +
 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x - 3*c
) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*e^(-
11*d*x - 11*c) + e^(-13*d*x - 13*c)))) - 1/8*a*b^2*((33*e^(-d*x - c) - e^(-
3*d*x - 3*c))/d + (30*e^(-2*d*x - 2*c) + 240*e^(-4*d*x - 4*c) + 322*e^(-6*d
*x - 6*c) + 177*e^(-8*d*x - 8*c) - 1)/(d*(e^(-3*d*x - 3*c) + 3*e^(-5*d*x -
5*c) + 3*e^(-7*d*x - 7*c) + e^(-9*d*x - 9*c)))) - 1/8*a^2*b*((21*e^(-d*x -
c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x - 2*c) + 69*e^(-4*d*x - 4*c) - 1)/
(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d -
9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(99) = 198$.
time = 0.35, size = 540, normalized size = 5.14

$$\frac{5\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})\cosh\left(dx+c\right)^{8}+5\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})\sinh\left(dx+c\right)^{8}-20\,(a^{3}+12\,a^{2}b+21\,ab^{2}+10\,b^{3})\cosh\left(dx+c\right)^{6}-20\,(a^{3}+12\,a^{2}b+21\,ab^{2}+10\,b^{3}-7\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})\cosh\left(dx+c\right)^{2})\sinh\left(dx+c\right)^{6}-20\,(11\,a^{3}+123\,a^{2}b+249\,ab^{2}+137\,b^{3})\cosh\left(dx+c\right)^{4}+10\,(35\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})\cosh\left(dx+c\right)^{4}-22\,a^{3}-246\,a^{2}b-498\,ab^{2}-274\,b^{3}-30\,(a^{3}+12\,a^{2}b+21\,ab^{2}+10\,b^{3})\cosh\left(dx+c\right)^{2})\sinh\left(dx+c\right)^{4}-425\,a^{3}-5235\,a^{2}b-10395\,ab^{2}-5649\,b^{3}-20\,(31\,a^{3}+372\,a^{2}b+747\,ab^{2}+390\,b^{3})\cosh\left(dx+c\right)^{2}+20\,(7\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})\cosh\left(dx+c\right)^{6}-15\,(a^{3}+12\,a^{2}b+21\,ab^{2}+10\,b^{3})\cosh\left(dx+c\right)^{4}-31\,a^{3}-372\,a^{2}b-747\,ab^{2}-390\,b^{3}-6\,(11\,a^{3}+123\,a^{2}b+249\,ab^{2}+137\,b^{3})\cosh\left(dx+c\right)^{2})\sinh\left(dx+c\right)^{2}}{120\,(d\cosh\left(dx+c\right)^{5}+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+5\,d\cosh\left(dx+c\right)^{3}+5\,(2\,d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right))\sinh\left(dx+c\right)^{2}+10\,d\cosh\left(dx+c\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sinh(d*x + c)^8 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)
*cosh(d*x + c)^6 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3 - 7*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 20*(11*a^3 + 123*a^2*
b + 249*a*b^2 + 137*b^3)*cosh(d*x + c)^4 + 10*(35*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^4 - 22*a^3 - 246*a^2*b - 498*a*b^2 - 274*b^3 - 30*(a^3
 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 425*a^3
 - 5235*a^2*b - 10395*a*b^2 - 5649*b^3 - 20*(31*a^3 + 372*a^2*b + 747*a*b^2
 + 390*b^3)*cosh(d*x + c)^2 + 20*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)^6 - 15*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^4 - 31*a^3
 - 372*a^2*b - 747*a*b^2 - 390*b^3 - 6*(11*a^3 + 123*a^2*b + 249*a*b^2 + 13
7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x
+ c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*c
osh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\sinh^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(99) = 198.
time = 0.59, size = 330, normalized size = 3.14

$$\frac{5\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+15\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+15\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+5\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-60\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-360\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)-540\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-240\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{16\left(45\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+135\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+90\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4-60\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-80\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+48\,b^3\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(5*a^3*(e^(d*x + c) + e^(-d*x - c))^3 + 15*a^2*b*(e^(d*x + c) + e^(-d
*x - c))^3 + 15*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 5*b^3*(e^(d*x + c) +
 e^(-d*x - c))^3 - 60*a^3*(e^(d*x + c) + e^(-d*x - c)) - 360*a^2*b*(e^(d*x
+ c) + e^(-d*x - c)) - 540*a*b^2*(e^(d*x + c) + e^(-d*x - c)) - 240*b^3*(e^
(d*x + c) + e^(-d*x - c)) - 16*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 1
35*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 90*b^3*(e^(d*x + c) + e^(-d*x - c
))^4 - 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 80*b^3*(e^(d*x + c) + e^(-
d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B]
time = 0.41, size = 361, normalized size = 3.44

$$\frac{e^{-3c-3dx}\,(a+b)^3}{24\,d}+\frac{e^{3c+3dx}\,(a+b)^3}{24\,d}+\frac{8\,e^{c+dx}\,(4\,b^3+3\,a\,b^2)}{3\,d\,(2\,e^{2c+2dx}+e^{4c+4dx}+1)}+\frac{64\,b^3\,e^{c+dx}}{5\,d\,(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1)}-\frac{8\,e^{c+dx}\,(32\,b^3+15\,a\,b^2)}{15\,d\,(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1)}-\frac{32\,b^3\,e^{c+dx}}{5\,d\,(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1)}-\frac{3\,e^{c+dx}\,(a+b)^2\,(a+5\,b)}{8\,d}-\frac{6\,e^{c+dx}\,(a^2\,b+3\,a\,b^2+2\,b^3)}{d\,(e^{2c+2dx}+1)}-\frac{3\,e^{-c-dx}\,(a+b)^2\,(a+5\,b)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (exp(- 3*c - 3*d*x)*(a + b)^3)/(24*d) + (exp(3*c + 3*d*x)*(a + b)^3)/(24*d)
 + (8*exp(c + d*x)*(3*a*b^2 + 4*b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c +
4*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c +
 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*exp(c + d*x)*(15
*a*b^2 + 32*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
+ 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*
c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x)
+ 1)) - (3*exp(c + d*x)*(a + b)^2*(a + 5*b))/(8*d) - (6*exp(c + d*x)*(3*a*b
^2 + a^2*b + 2*b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (3*exp(- c - d*x)*(a + b)
^2*(a + 5*b))/(8*d)
```

3.19 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=122

$$-\frac{1}{2}(a+b)^2(a+7b)x+\frac{(a+b)^3}{4d(1-\tanh(c+dx))}+\frac{3b(a+b)^2\tanh(c+dx)}{d}+\frac{b^2(3a+2b)\tanh^3(c+dx)}{3d}+\frac{b^3\tanh^5(c}{5d}$$

```
[Out] -1/2*(a+b)^2*(a+7*b)*x+1/4*(a+b)^3/d/(1-tanh(d*x+c))+3*b*(a+b)^2*tanh(d*x+c
)/d+1/3*b^2*(3*a+2*b)*tanh(d*x+c)^3/d+1/5*b^3*tanh(d*x+c)^5/d-1/4*(a+b)^3/d
/(1+tanh(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 478, 542, 396, 212}

$$\frac{b(81a^2+190ab+105b^2)\tanh(c+dx)}{30d}+\frac{7b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{10d}+\frac{b(33a+35b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{30d}+\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2d}-\frac{1}{2}x(a+b)^2(a+7b)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*((a + b)^2*(a + 7*b)*x) + (b*(81*a^2 + 190*a*b + 105*b^2)*Tanh[c + d*x
])/(30*d) + (b*(33*a + 35*b)*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2))/(30*d)
+ (7*b*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2)/(10*d) + (Cosh[c + d*x]*Sin
h[c + d*x]*(a + b*Tanh[c + d*x]^2)^3)/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
```

```
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^3}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2d} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{7b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{10d} + \frac{\cosh(c+dx)\sinh(c}{}$$

$$= \frac{b(33a+35b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{30d} + \frac{7b\tanh(c}{}$$

$$= \frac{b(81a^2+190ab+105b^2)\tanh(c+dx)}{30d} + \frac{b(33a+35b)\tanh(c}{}$$

$$= -\frac{1}{2}(a+b)^2(a+7b)x + \frac{b(81a^2+190ab+105b^2)\tanh(c+dx}{30d}$$

Mathematica [A]
time = 1.54, size = 95, normalized size = 0.78

$$\frac{-30(a+b)^2(a+7b)(c+dx)+15(a+b)^3\sinh(2(c+dx))+4b\left(45a^2+105ab+58b^2-b(15a+16b)\text{sech}^2(c+dx)+3b^2\text{sech}^4(c+dx)\right)\tanh(c+dx)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-30*(a + b)^2*(a + 7*b)*(c + d*x) + 15*(a + b)^3*Sinh[2*(c + d*x)] + 4*b*(
45*a^2 + 105*a*b + 58*b^2 - b*(15*a + 16*b)*Sech[c + d*x]^2 + 3*b^2*Sech[c
+ d*x]^4)*Tanh[c + d*x])/(60*d)
```

Maple [A]
time = 1.72, size = 180, normalized size = 1.48

method	result
derivativedivides	$\frac{a^3\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+3a\,b^2\left(\frac{\sinh^5(dx+c)}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tanh}{}\right.}{d}$
default	$\frac{a^3\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+3a\,b^2\left(\frac{\sinh^5(dx+c)}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tanh}{}\right.}{d}$
risch	$-\frac{a^3x}{2}-\frac{9a^2bx}{2}-\frac{15a\,b^2x}{2}-\frac{7b^3x}{2}+\frac{e^{2dx+2c}a^3}{8d}+\frac{3b\,e^{2dx+2c}a^2}{8d}+\frac{3\,e^{2dx+2c}a\,b^2}{8d}+\frac{b^3e^{2dx+2c}}{8d}-\frac{e^{-2dx-2c}a^3}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sinh(d*x+c)*cosh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(1/2*sinh(d*x+
c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+3*a*b^2*(1/2*sinh(d*x+c)^5/
cosh(d*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3)+b^3*(1/2*sin
h(d*x+c)^7/cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/
10*tanh(d*x+c)^5))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(110) = 220$.
time = 0.27, size = 377, normalized size = 3.09

$$-\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{120}b^3\left(\frac{420(dx+c)}{d}+\frac{15e^{(-2dx-2c)}}{d}-\frac{1003e^{(-2dx-2c)}+3350e^{(-4dx-4c)}+5590e^{(-6dx-6c)}+3915e^{(-8dx-8c)}+1455e^{(-10dx-10c)}+15}{d\left(e^{(-2dx-2c)}+5e^{(-4dx-4c)}+10e^{(-6dx-6c)}+10e^{(-8dx-8c)}+5e^{(-10dx-10c)}+e^{(-12dx-12c)}\right)}\right)-\frac{1}{8}ab^2\left(\frac{60(dx+c)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx-2c)}+201e^{(-4dx-4c)}+147e^{(-6dx-6c)}+3}{d\left(e^{(-2dx-2c)}+3e^{(-4dx-4c)}+3e^{(-6dx-6c)}+e^{(-8dx-8c)}\right)}\right)-\frac{3}{8}a^2b\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/120*b^3*(420*(d
*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d*x
 - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*x -
 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))) - 1/
8*a*b^2*(60*(d*x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 20
1*e^(-4*d*x - 4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(
-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) - 3/8*a^2*b*(12*(d
```

```
*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x -
2*c) + e^(-4*d*x - 4*c))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(110) = 220$.
time = 0.37, size = 725, normalized size = 5.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^7 - 4*(90*a^2*b + 2
10*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x +
c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 +
7*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (75*a^3 + 585*a^2*b + 1065*a*b^
2 + 539*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2
 + 7*b^3)*d*x)*cosh(d*x + c)^3 + 5*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^4 + 27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3 + 2*(75*a^3 + 585*a
^2*b + 1065*a*b^2 + 539*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 20*(2*(90*a
^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cos
h(d*x + c)^3 + 3*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a
*b^2 + 7*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 40*(90*a^2*b + 210*a*b^
2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x + c) + 5*
(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + (75*a^3 + 585*a^2*b +
 1065*a*b^2 + 539*b^3)*cosh(d*x + c)^4 + 15*a^3 + 189*a^2*b + 285*a*b^2 + 1
75*b^3 + 3*(27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cos
h(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2
+ 10*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh^{2}\left(c + dx\right)\right)^{3} \sinh^{2}\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(110) = 220$.
time = 0.57, size = 393, normalized size = 3.22

$$\frac{15\,a^3e^{(2\,dx+2\,c)} + 45\,a^2be^{(2\,dx+2\,c)} + 45\,ab^2e^{(2\,dx+2\,c)} + 15\,b^3e^{(2\,dx+2\,c)} - 60\,(a^3 + 9\,a^2b + 15\,ab^2 + 7\,b^3)(dx + c) + 15\,\left(2\,a^3e^{(2\,dx+2\,c)} + 18\,a^2be^{(2\,dx+2\,c)} + 30\,ab^2e^{(2\,dx+2\,c)} + 14\,b^3e^{(2\,dx+2\,c)} - a^3 - 3\,a^2b - 3\,ab^2 - b^3\right)e^{(-2\,dx-2\,c)} - \frac{16\,\left(45\,a^2be^{(8\,dx+8\,c)} + 135\,ab^2e^{(8\,dx+8\,c)} + 90\,b^3e^{(8\,dx+8\,c)} + 180\,a^2be^{(6\,dx+6\,c)} + 450\,ab^2e^{(6\,dx+6\,c)} + 240\,b^3e^{(6\,dx+6\,c)} + 270\,a^2be^{(4\,dx+4\,c)} + 600\,ab^2e^{(4\,dx+4\,c)} + 340\,b^3e^{(4\,dx+4\,c)} + 180\,a^2be^{(2\,dx+2\,c)} + 390\,ab^2e^{(2\,dx+2\,c)} + 200\,b^3e^{(2\,dx+2\,c)} + 45\,a^2b + 105\,ab^2 + 58\,b^3\right)}{\left(e^{(2\,dx+2\,c)} + 1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*a^3*e^(2*d*x + 2*c) + 45*a^2*b*e^(2*d*x + 2*c) + 45*a*b^2*e^(2*d*
x + 2*c) + 15*b^3*e^(2*d*x + 2*c) - 60*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*(
d*x + c) + 15*(2*a^3*e^(2*d*x + 2*c) + 18*a^2*b*e^(2*d*x + 2*c) + 30*a*b^2*
e^(2*d*x + 2*c) + 14*b^3*e^(2*d*x + 2*c) - a^3 - 3*a^2*b - 3*a*b^2 - b^3)*e
^(-2*d*x - 2*c) - 16*(45*a^2*b*e^(8*d*x + 8*c) + 135*a*b^2*e^(8*d*x + 8*c)
+ 90*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 450*a*b^2*e^(6*d*x +
 6*c) + 240*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 600*a*b^2*e^(
4*d*x + 4*c) + 340*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) + 390*a*
b^2*e^(2*d*x + 2*c) + 200*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 105*a*b^2 + 58*b
^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 0.32, size = 668, normalized size = 5.48

$$\frac{e^{2c+2dx}(a+b)^3}{8d}-\frac{\frac{2(3a^2b+6ab^2+2b^3)}{5d}+\frac{6e^{2c+2dx}(a^2b+3ab^2+2b^3)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}-\frac{\frac{2(3a^2b+6ab^2+2b^3)}{5d}+\frac{6e^{6c+6dx}(a^2b+3ab^2+2b^3)}{5d}+\frac{6e^{4c+4dx}(3a^2b+6ab^2+2b^3)}{5d}+\frac{2e^{2c+2dx}(9a^2b+15ab^2+10b^3)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}-\frac{\frac{2(9a^2b+15ab^2+10b^3)}{15d}+\frac{6e^{4c+4dx}(a^2b+3ab^2+2b^3)}{5d}+\frac{4e^{2c+2dx}(3a^2b+6ab^2+2b^3)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{6(a^2b+3ab^2+2b^3)}{5d(e^{2c+2dx}+1)}-\frac{e^{-2c-2dx}(a+b)^3}{8d}-\frac{\frac{6(a^2b+3ab^2+2b^3)}{5d}+\frac{8e^{2c+2dx}(3a^2b+6ab^2+2b^3)}{5d}+\frac{6e^{8c+8dx}(a^2b+3ab^2+2b^3)}{5d}+\frac{8e^{6c+6dx}(3a^2b+6ab^2+2b^3)}{5d}+\frac{4e^{4c+4dx}(9a^2b+15ab^2+10b^3)}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}-\frac{x(a+b)^2(a+7b)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d)
 + (6*exp(2*c + 2*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d))/(2*exp(2*c + 2*d*x
) + exp(4*c + 4*d*x) + 1) - ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (6*exp
(6*c + 6*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(6*a*b
^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(15*a*b^2 + 9*a^2*b + 10
*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x)
 + exp(8*c + 8*d*x) + 1) - ((2*(15*a*b^2 + 9*a^2*b + 10*b^3))/(15*d) + (6*e
xp(4*c + 4*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (4*exp(2*c + 2*d*x)*(6*a
*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) +
exp(6*c + 6*d*x) + 1) - (6*(3*a*b^2 + a^2*b + 2*b^3))/(5*d*(exp(2*c + 2*d*x
) + 1)) - (exp(- 2*c - 2*d*x)*(a + b)^3)/(8*d) - ((6*(3*a*b^2 + a^2*b + 2*b
^3))/(5*d) + (8*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (6*ex
p(8*c + 8*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(6*a*
b^2 + 3*a^2*b + 2*b^3))/(5*d) + (4*exp(4*c + 4*d*x)*(15*a*b^2 + 9*a^2*b + 1
0*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d
*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (x*(a + b)^2*(a + 7*b)
)/2
```

3.20 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=70

$$\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{3b(a+b)^2\text{sech}(c+dx)}{d}-\frac{b^2(a+b)\text{sech}^3(c+dx)}{d}+\frac{b^3\text{sech}^5(c+dx)}{5d}$$

```
[Out] (a+b)^3*cosh(d*x+c)/d+3*b*(a+b)^2*sech(d*x+c)/d-b^2*(a+b)*sech(d*x+c)^3/d+1
/5*b^3*sech(d*x+c)^5/d
```

Rubi [A]

time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3745, 276}

$$-\frac{b^2(a+b)\text{sech}^3(c+dx)}{d}+\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{3b(a+b)^2\text{sech}(c+dx)}{d}+\frac{b^3\text{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Cosh[c + d*x])/d + (3*b*(a + b)^2*Sech[c + d*x])/d - (b^2*(a + b
)*Sech[c + d*x]^3)/d + (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{\text{Subst}\left(\int\frac{\left(a+b-bx^2\right)^3}{x^2}\,dx,x,\text{sech}(c+dx)\right)}{d}$$

$$=-\frac{\text{Subst}\left(\int\left(-3b(a+b)^2+\frac{(a+b)^3}{x^2}+3b^2(a+b)x^2-b^3x^4\right)dx,x\right.}{d}$$

$$=\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{3b(a+b)^2\text{sech}(c+dx)}{d}-\frac{b^2(a+b)\text{sec}}{d}$$

Mathematica [A]
time = 0.61, size = 63, normalized size = 0.90

$$\frac{5(a+b)^3\cosh(c+dx)+b\operatorname{sech}(c+dx)\left(15(a+b)^2-5b(a+b)\operatorname{sech}^2(c+dx)+b^2\operatorname{sech}^4(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (5*(a + b)^3*Cosh[c + d*x] + b*Sech[c + d*x]*(15*(a + b)^2 - 5*b*(a + b)*Se
ch[c + d*x]^2 + b^2*Sech[c + d*x]^4))/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(68) = 136.
time = 1.46, size = 170, normalized size = 2.43

method	result
derivativedivides	$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+3a\,b^2\left(\frac{\sinh^4(dx+c)}{\cosh(dx+c)^3}+\frac{4\left(\sinh^2(dx+c)\right)}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)+b^3\left(\frac{\sinh^6(dx+c)}{\cosh(dx+c)^5}\right.}{d}$
default	$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+3a\,b^2\left(\frac{\sinh^4(dx+c)}{\cosh(dx+c)^3}+\frac{4\left(\sinh^2(dx+c)\right)}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)+b^3\left(\frac{\sinh^6(dx+c)}{\cosh(dx+c)^5}\right.}{d}$
risch	$\frac{e^{dx+c}a^3}{2d}+\frac{3\,e^{dx+c}a^2b}{2d}+\frac{3a\,e^{dx+c}b^2}{2d}+\frac{b^3e^{dx+c}}{2d}+\frac{e^{-dx-c}a^3}{2d}+\frac{3\,e^{-dx-c}a^2b}{2d}+\frac{3a\,e^{-dx-c}b^2}{2d}+\frac{e^{-dx-c}b^3}{2d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*cosh(d*x+c)+3*a^2*b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c))+3*a*
b^2*(sinh(d*x+c)^4/cosh(d*x+c)^3+4*sinh(d*x+c)^2/cosh(d*x+c)^3+8/3/cosh(d*x
+c)^3)+b^3*(sinh(d*x+c)^6/cosh(d*x+c)^5+6*sinh(d*x+c)^4/cosh(d*x+c)^5+8*sin
h(d*x+c)^2/cosh(d*x+c)^5+16/5/cosh(d*x+c)^5))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(68) = 136.
time = 0.28, size = 321, normalized size = 4.59

$$\frac{1}{10}b^3\left(\frac{5\,e^{(-dx-c)}}{d}+\frac{85\,e^{(-2\,dx-2\,c)}+210\,e^{(-4\,dx-4\,c)}+314\,e^{(-6\,dx-6\,c)}+185\,e^{(-8\,dx-8\,c)}+65\,e^{(-10\,dx-10\,c)}+5}{d\left(e^{(-dx-c)}+5\,e^{(-3\,dx-3\,c)}+10\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+5\,e^{(-9\,dx-9\,c)}+e^{(-11\,dx-11\,c)}\right)}\right)+\frac{1}{2}ab^2\left(\frac{3\,e^{(-dx-c)}}{d}+\frac{33\,e^{(-2\,dx-2\,c)}+41\,e^{(-4\,dx-4\,c)}+27\,e^{(-6\,dx-6\,c)}+3}{d\left(e^{(-dx-c)}+3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)}\right)}\right)+\frac{3}{2}a^2b\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}\right)+\frac{a^3\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x - 4*c) +
314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c) + 5)/(d
*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^(-7*d*x -
```

```
7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + 1/2*a*b^2*(3*e^(-d*x -
c)/d + (33*e^(-2*d*x - 2*c) + 41*e^(-4*d*x - 4*c) + 27*e^(-6*d*x - 6*c) + 3
)/(d*(e^(-d*x - c) + 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) + e^(-7*d*x -
7*c)))) + 3/2*a^2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x
- c) + e^(-3*d*x - 3*c)))) + a^3*cosh(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(68) = 136$.
time = 0.34, size = 383, normalized size = 5.47

$$\frac{5\,(a^3+3\,a^2b+3\,ab^2+b^3)\cosh\left(dx+c\right)^6+5\,(a^3+3\,a^2b+3\,ab^2+b^3)\sinh\left(dx+c\right)^6+30\,(a^3+5\,a^2b+7\,ab^2+3\,b^3)\cosh\left(dx+c\right)^4+15\left(2\,a^3+10\,a^2b+14\,ab^2+6\,b^3+5\,(a^3+3\,a^2b+3\,ab^2+b^3)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)^4+50\,a^3+330\,a^2b+430\,ab^2+182\,b^3+5\,(15\,a^3+93\,a^2b+125\,ab^2+47\,b^3)\cosh\left(dx+c\right)^2+5\left(15\,(a^3+3\,a^2b+3\,ab^2+b^3)\cosh\left(dx+c\right)^4+15\,a^3+93\,a^2b+125\,ab^2+47\,b^3+36\,(a^3+5\,a^2b+7\,ab^2+3\,b^3)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)^2}{10\left(d\cosh\left(dx+c\right)^5+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+5\,d\cosh\left(dx+c\right)^3+5\left(2\,d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2+10\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/10*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 5*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*sinh(d*x + c)^6 + 30*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)*cos
h(d*x + c)^4 + 15*(2*a^3 + 10*a^2*b + 14*a*b^2 + 6*b^3 + 5*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 50*a^3 + 330*a^2*b + 430
*a*b^2 + 182*b^3 + 5*(15*a^3 + 93*a^2*b + 125*a*b^2 + 47*b^3)*cosh(d*x + c)
^2 + 5*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*a^3 + 93*a^
2*b + 125*a*b^2 + 47*b^3 + 36*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)
^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\sinh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(68) = 136$.
time = 0.52, size = 236, normalized size = 3.37

$$\frac{5\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+15\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+15\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+5\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{4\left(15\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+30\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+15\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4-20\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-20\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+16\,b^3\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10*(5*a^3*(e^(d*x + c) + e^(-d*x - c)) + 15*a^2*b*(e^(d*x + c) + e^(-d*x
- c)) + 15*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 5*b^3*(e^(d*x + c) + e^(-d*
x - c)) + 4*(15*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 30*a*b^2*(e^(d*x + c
) + e^(-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4 - 20*a*b^2*(e^(
d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 16*b^3
)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B]
time = 1.25, size = 308, normalized size = 4.40

$$\frac{e^{c+dx}\,(a+b)^3}{2\,d}+\frac{e^{-c-dx}\,(a+b)^3}{2\,d}+\frac{6\,e^{c+dx}\,(a^2\,b+2\,a\,b^2+b^3)}{d\,(e^{2\,c+2\,d\,x}+1)}-\frac{64\,b^3\,e^{c+dx}}{5\,d\,(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1)}+\frac{8\,e^{c+dx}\,(9\,b^3+5\,a\,b^2)}{5\,d\,(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1)}+\frac{32\,b^3\,e^{c+dx}}{5\,d\,(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1)}-\frac{8\,e^{c+dx}\,(b^3+a\,b^2)}{d\,(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (exp(c + d*x)*(a + b)^3)/(2*d) + (exp(- c - d*x)*(a + b)^3)/(2*d) + (6*exp(
c + d*x)*(2*a*b^2 + a^2*b + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (64*b^3*exp(
c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x
) + exp(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(5*a*b^2 + 9*b^3))/(5*d*(3*exp
(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*exp(c
 + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (8*exp(c + d*x)*(a*b^2
 + b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.21 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=84

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+2b)\operatorname{sech}^3(c+dx)}{3d}-\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

```
[Out] -a^3*arctanh(cosh(d*x+c))/d-b*(3*a^2+3*a*b+b^2)*sech(d*x+c)/d+1/3*b^2*(3*a+
2*b)*sech(d*x+c)^3/d-1/5*b^3*sech(d*x+c)^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3745, 398, 213}

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+2b)\operatorname{sech}^3(c+dx)}{3d}-\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) - (b*(3*a^2 + 3*a*b + b^2)*Sech[c + d*x])
/d + (b^2*(3*a + 2*b)*Sech[c + d*x]^3)/(3*d) - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^3}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(-b(3a^2+3ab+b^2)+b^2(3a+2b)x^2-b^3x^4+\frac{a^3}{-1+x^2}\right)\right.}{d}$$

$$= -\frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+2b)\operatorname{sech}^3(c+dx)}{3d}-$$

$$= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}+$$

Mathematica [A]
time = 0.22, size = 79, normalized size = 0.94

$$\frac{15a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-15b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)+5b^2(3a+2b)\operatorname{sech}^3(c+dx)-3b^3\operatorname{sech}^5(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (15*a^3*Log[Tanh[(c + d*x)/2]] - 15*b*(3*a^2 + 3*a*b + b^2)*Sech[c + d*x] +
 5*b^2*(3*a + 2*b)*Sech[c + d*x]^3 - 3*b^3*Sech[c + d*x]^5)/(15*d)
```

Maple [A]
time = 1.58, size = 118, normalized size = 1.40

method	result
derivativedivides	$\frac{-2a^3\operatorname{arctanh}(e^{dx+c})-\frac{3a^2b}{\cosh(dx+c)}+3ab^2\left(-\frac{\sinh^2(dx+c)}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)+b^3\left(-\frac{\sinh^4(dx+c)}{\cosh(dx+c)^5}-\frac{4\left(\sinh^2(dx+c)\right)}{3\cosh(dx+c)^5}-\frac{8}{15\cosh(d}\right.}{d}$
default	$\frac{-2a^3\operatorname{arctanh}(e^{dx+c})-\frac{3a^2b}{\cosh(dx+c)}+3ab^2\left(-\frac{\sinh^2(dx+c)}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)+b^3\left(-\frac{\sinh^4(dx+c)}{\cosh(dx+c)^5}-\frac{4\left(\sinh^2(dx+c)\right)}{3\cosh(dx+c)^5}-\frac{8}{15\cosh(d}\right.}{d}$
risch	$-\frac{2b\,e^{dx+c}\left(45a^2e^{8dx+8c}+45ab\,e^{8dx+8c}+15b^2e^{8dx+8c}+180a^2e^{6dx+6c}+120ab\,e^{6dx+6c}+20b^2e^{6dx+6c}+270a^2e^{4dx+4c}+150\right.}{15d\left(1+e^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))-3*a^2*b/cosh(d*x+c)+3*a*b^2*(-sinh(d*x+c)^2
/cosh(d*x+c)^3-2/3/cosh(d*x+c)^3)+b^3*(-sinh(d*x+c)^4/cosh(d*x+c)^5-4/3*sin
h(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(80) = 160$.
time = 0.29, size = 560, normalized size = 6.67

$$-\frac{2}{15}b^3\left(\frac{15e^{(-dx-c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{20e^{(-3dx-3c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{58e^{(-5dx-5c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{20e^{(-7dx-7c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{15e^{(-9dx-9c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)-2ab^2\left(\frac{3e^{(-dx-c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{2e^{(-3dx-3c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{3e^{(-5dx-5c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+\frac{a^3\log\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}-\frac{6a^2b}{d(e^{(dx+c)}+e^{(-dx-c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -2/15*b^3*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(
-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 58*e^(-5*d*x - 5*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x - 2*c) +
10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1))) - 2*a*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(-2*d*x -
2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x - 5*c)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^3*lo
g(tanh(1/2*d*x + 1/2*c))/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2277 vs. $2(80) = 160$.
time = 0.39, size = 2277, normalized size = 27.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 270*(3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(3*a^2*b + 3*a*b^2 + b^3)*sinh(
d*x + c)^9 + 40*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^7 + 40*(9*a^2*b + 6
*a*b^2 + b^3 + 27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
7 + 280*(9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^2*b + 6*a*b^2 +
 b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(135*a^2*b + 75*a*b^2 + 29*b^3)*co
sh(d*x + c)^5 + 4*(945*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 135*a^2*
b + 75*a*b^2 + 29*b^3 + 210*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^5 + 20*(189*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*(9*a^2
*b + 6*a*b^2 + b^3)*cosh(d*x + c)^3 + (135*a^2*b + 75*a*b^2 + 29*b^3)*cosh(
d*x + c))*sinh(d*x + c)^4 + 40*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^3 +
40*(63*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(9*a^2*b + 6*a*b^2 +
```

```
b^3)*cosh(d*x + c)^4 + 9*a^2*b + 6*a*b^2 + b^3 + (135*a^2*b + 75*a*b^2 + 29
*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 40*(27*(3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^7 + 21*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^5 + (135*a^2*b
+ 75*a*b^2 + 29*b^3)*cosh(d*x + c)^3 + 3*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x
 + c))*sinh(d*x + c)^2 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c) + 15*(a
^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sinh(d*x +
 c)^10 + 5*a^3*cosh(d*x + c)^8 + 10*a^3*cosh(d*x + c)^6 + 5*(9*a^3*cosh(d*x
 + c)^2 + a^3)*sinh(d*x + c)^8 + 40*(3*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x +
 c))*sinh(d*x + c)^7 + 10*a^3*cosh(d*x + c)^4 + 10*(21*a^3*cosh(d*x + c)^4
+ 14*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 4*(63*a^3*cosh(d*x + c)^5
 + 70*a^3*cosh(d*x + c)^3 + 15*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 5*a^3*c
osh(d*x + c)^2 + 10*(21*a^3*cosh(d*x + c)^6 + 35*a^3*cosh(d*x + c)^4 + 15*a
^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 40*(3*a^3*cosh(d*x + c)^7 + 7*a
^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x +
c)^3 + a^3 + 5*(9*a^3*cosh(d*x + c)^8 + 28*a^3*cosh(d*x + c)^6 + 30*a^3*cos
h(d*x + c)^4 + 12*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 10*(a^3*cosh
(d*x + c)^9 + 4*a^3*cosh(d*x + c)^7 + 6*a^3*cosh(d*x + c)^5 + 4*a^3*cosh(d*
x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x +
 c) + 1) - 15*(a^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9
+ a^3*sinh(d*x + c)^10 + 5*a^3*cosh(d*x + c)^8 + 10*a^3*cosh(d*x + c)^6 + 5
*(9*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^8 + 40*(3*a^3*cosh(d*x + c)^3
+ a^3*cosh(d*x + c))*sinh(d*x + c)^7 + 10*a^3*cosh(d*x + c)^4 + 10*(21*a^3*
cosh(d*x + c)^4 + 14*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 4*(63*a^3
*cosh(d*x + c)^5 + 70*a^3*cosh(d*x + c)^3 + 15*a^3*cosh(d*x + c))*sinh(d*x
+ c)^5 + 5*a^3*cosh(d*x + c)^2 + 10*(21*a^3*cosh(d*x + c)^6 + 35*a^3*cosh(d
*x + c)^4 + 15*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 40*(3*a^3*cosh(
d*x + c)^7 + 7*a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x +
 c))*sinh(d*x + c)^3 + a^3 + 5*(9*a^3*cosh(d*x + c)^8 + 28*a^3*cosh(d*x + c
)^6 + 30*a^3*cosh(d*x + c)^4 + 12*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^
2 + 10*(a^3*cosh(d*x + c)^9 + 4*a^3*cosh(d*x + c)^7 + 6*a^3*cosh(d*x + c)^5
 + 4*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x +
 c) + sinh(d*x + c) - 1) + 10*(27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8
 + 28*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^6 + 2*(135*a^2*b + 75*a*b^2 +
 29*b^3)*cosh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 12*(9*a^2*b + 6*a*b^
2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*
x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 5*d*cosh(d*x + c)^8 + 5*(9*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 + d*cosh(d*x
 + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*(21*d*cosh(d*x + c)^4 +
14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 + 70*d*
cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*d*cosh(d*x + c)^
4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2
+ d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 + 5*d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)^2 +
5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 12*d
*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*cosh(d*x + c)^9 + 4*d*cosh(d*
```

```
x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sin
h(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^3 \operatorname{csch}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(80) = 160.
time = 0.52, size = 196, normalized size = 2.33

$$-\frac{15\,a^3\log\left(e^{(dx+c)} + e^{(-dx-c)} + 2\right) - 15\,a^3\log\left(e^{(dx+c)} + e^{(-dx-c)} - 2\right) + \frac{4\left(45\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+45\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+15\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4-60\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-40\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+48\,b^3\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/30*(15*a^3*log(e^(d*x + c) + e^(-d*x - c) + 2) - 15*a^3*log(e^(d*x + c)
+ e^(-d*x - c) - 2) + 4*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 45*a*b^2
*(e^(d*x + c) + e^(-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4 - 6
0*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 40*b^3*(e^(d*x + c) + e^(-d*x - c)
)^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B]
time = 0.25, size = 317, normalized size = 3.77

$$\frac{8\,e^{c+d\,x}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^c\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}-\frac{2\,e^{c+d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{64\,b^3\,e^{c+d\,x}}{5\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}-\frac{8\,e^{c+d\,x}\,\left(22\,b^3+15\,a\,b^2\right)}{15\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}-\frac{32\,b^3\,e^{c+d\,x}}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x),x)
```

```
[Out] (8*exp(c + d*x)*(3*a*b^2 + 2*b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d
*x) + 1)) - (2*atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^
6)^(1/2))/(-d^2)^(1/2) - (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(d*(exp
(2*c + 2*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*ex
p(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*exp(c + d
*x)*(15*a*b^2 + 22*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10
*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 1
0*d*x) + 1))
```

3.22 $\int \text{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=64

$$-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] -a^3*coth(d*x+c)/d+3*a^2*b*tanh(d*x+c)/d+a*b^2*tanh(d*x+c)^3/d+1/5*b^3*tanh
(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3744, 276}

$$-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -((a^3*Coth[c + d*x])/d) + (3*a^2*b*Tanh[c + d*x])/d + (a*b^2*Tanh[c + d*x]
^3)/d + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \text{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x^2}+3ab^2x^2+b^3x^4\right)\,dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+
\end{aligned}$$

Mathematica [A]
time = 0.50, size = 70, normalized size = 1.09

$$\frac{-5a^3\coth(c+dx)+b\left(15a^2+5ab+b^2-b(5a+2b)\operatorname{sech}^2(c+dx)+b^2\operatorname{sech}^4(c+dx)\right)\tanh(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-5*a^3*Coth[c + d*x] + b*(15*a^2 + 5*a*b + b^2 - b*(5*a + 2*b)*Sech[c + d*
x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(62) = 124.
time = 2.52, size = 344, normalized size = 5.38

method	result
risch	$-\frac{2(5a^3e^{10dx+10c}+15a^2b\,e^{10dx+10c}+15a\,b^2e^{10dx+10c}+5b^3e^{10dx+10c}+25a^3e^{8dx+8c}+45a^2b\,e^{8dx+8c}+15a\,b^2e^{8dx+8c}-5b^3e^{8dx+8c}+5}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(5*a^3*exp(10*d*x+10*c)+15*a^2*b*exp(10*d*x+10*c)+15*a*b^2*exp(10*d*x+
10*c)+5*b^3*exp(10*d*x+10*c)+25*a^3*exp(8*d*x+8*c)+45*a^2*b*exp(8*d*x+8*c)+
15*a*b^2*exp(8*d*x+8*c)-5*b^3*exp(8*d*x+8*c)+50*a^3*exp(6*d*x+6*c)+30*a^2*b
*exp(6*d*x+6*c)-10*a*b^2*exp(6*d*x+6*c)+10*b^3*exp(6*d*x+6*c)+50*a^3*exp(4*
d*x+4*c)-30*a^2*b*exp(4*d*x+4*c)-10*a*b^2*exp(4*d*x+4*c)-10*b^3*exp(4*d*x+4
*c)+25*a^3*exp(2*d*x+2*c)-45*a^2*b*exp(2*d*x+2*c)-5*a*b^2*exp(2*d*x+2*c)+b^
3*exp(2*d*x+2*c)+5*a^3-15*a^2*b-5*a*b^2-b^3)/d/(1+exp(2*d*x+2*c))^5/(exp(2*
d*x+2*c)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(62) = 124.
time = 0.28, size = 348, normalized size = 5.44

$$\frac{2}{5}b^3\left(\frac{10e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{5e^{(-8dx-8c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{1}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+2ab^2\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+\frac{6a^2b}{d(e^{(-2dx-2c)}+1)}+\frac{2a^3}{d(e^{(-2dx-2c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 2/5*b^3*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^
(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
```

```
^(-10*d*x - 10*c) + 1))) + 2*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2
*c) + 1)) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(62) = 124$.
time = 0.38, size = 572, normalized size = 8.94

$$-\frac{4\left(\left(5a^3+5ab^2+2b^3\right)\cosh\left(dx+c\right)^5+5\left(5a^3+5ab^2+2b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+\left(15a^2b+10ab^2+3b^3\right)\sinh\left(dx+c\right)^5+\left(25a^3+5ab^2-2b^3\right)\cosh\left(dx+c\right)^3+\left(45a^2b+10ab^2-3b^3+10\left(15a^2b+10ab^2+3b^3\right)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)^3+\left(10\left(5a^3+5ab^2+2b^3\right)\cosh\left(dx+c\right)^3+3\left(25a^3+5ab^2-2b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2+10\left(5a^3-ab^2\right)\cosh\left(dx+c\right)+\left(5\left(15a^2b+10ab^2+3b^3\right)\cosh\left(dx+c\right)^4+30a^2b+10b^3+3\left(45a^2b+10ab^2-3b^3\right)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)\right)}{5\left(d\cosh\left(dx+c\right)^7+7d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+d\sinh\left(dx+c\right)^7+3d\cosh\left(dx+c\right)^5+\left(21d\cosh\left(dx+c\right)^2+5d\right)\sinh\left(dx+c\right)^5+5\left(7d\cosh\left(dx+c\right)^3+3d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^4+d\cosh\left(dx+c\right)^3+\left(35d\cosh\left(dx+c\right)^4+50d\cosh\left(dx+c\right)^2+9d\right)\sinh\left(dx+c\right)^3+3\left(7d\cosh\left(dx+c\right)^5+10d\cosh\left(dx+c\right)^3+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2-5d\cosh\left(dx+c\right)+\left(7d\cosh\left(dx+c\right)^6+25d\cosh\left(dx+c\right)^4+27d\cosh\left(dx+c\right)^2+5d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/5*((5*a^3 + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 5*a*b^2 + 2*b^
3)*cosh(d*x + c)*sinh(d*x + c)^4 + (15*a^2*b + 10*a*b^2 + 3*b^3)*sinh(d*x +
 c)^5 + (25*a^3 + 5*a*b^2 - 2*b^3)*cosh(d*x + c)^3 + (45*a^2*b + 10*a*b^2 -
 3*b^3 + 10*(15*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ (10*(5*a^3 + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^3 + 3*(25*a^3 + 5*a*b^2 - 2*b
^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 - a*b^2)*cosh(d*x + c) + (5*
(15*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 30*a^2*b + 10*b^3 + 3*(45*a
^2*b + 10*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7
 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 3*d*cosh(d*x + c
)^5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3
 + 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x
+ c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + 3*(7*d*cosh(d*x + c)
^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 - 5*d*cosh(d*x
 + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2
+ 5*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh^{2}{\left(c + dx \right)}\right)^{3} \operatorname{csch}^{2}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(62) = 124$.
time = 0.54, size = 202, normalized size = 3.16

$$-\frac{2\left(\frac{5a^3}{e^{(2dx+2c)}-1}+\frac{15a^2be^{(8dx+8c)}+15ab^2e^{(8dx+8c)}+5b^3e^{(8dx+8c)}+60a^2be^{(6dx+6c)}+30ab^2e^{(6dx+6c)}+90a^2be^{(4dx+4c)}+20ab^2e^{(4dx+4c)}+10b^3e^{(4dx+4c)}+60a^2be^{(2dx+2c)}+10ab^2e^{(2dx+2c)}+15a^2b+5ab^2+b^3}{\left(e^{(2dx+2c)}+1\right)^5}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^3/(e^(2*d*x + 2*c) - 1) + (15*a^2*b*e^(8*d*x + 8*c) + 15*a*b^2*e^
(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 30*a*b^2
*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 20*a*b^2*e^(4*d*x + 4*c) + 10
*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 10*a*b^2*e^(2*d*x + 2*c)
+ 15*a^2*b + 5*a*b^2 + b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.23, size = 590, normalized size = 9.22

$$-\frac{\frac{2\,\left(3\,a^2\,b-b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,\left(3\,a^2\,b-a\,b^2+b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b-b^3\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{\frac{2\,\left(3\,a^2\,b-b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b-a\,b^2+b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b-b^3\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{12\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b-a\,b^2+b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b-b^3\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(3\,a^2\,b-b^3\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{2\,a^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^2,x)
```

```
[Out] - ((2*(3*a^2*b - b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b + b^3
))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(3*a^2*b - a*b^
2 + b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) + (4
*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c +
 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a^2*b - b^3))/(5*d) + (6*exp(2*c +
 2*d*x)*(3*a^2*b - a*b^2 + b^3))/(5*d) + (2*exp(6*c + 6*d*x)*(3*a*b^2 + 3*a
^2*b + b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a^2*b - b^3))/(5*d))/(4*exp(2*c
 + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)
 - ((2*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) + (12*exp(4*c + 4*d*x)*(3*a^2*b - a
*b^2 + b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) +
 (8*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(3*a^2*b
- b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d
*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (2*a^3)/(d*(exp(2*c +
2*d*x) - 1)) - (2*(3*a*b^2 + 3*a^2*b + b^3))/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.23 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=152

$$\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b(81a^2-28ab-4b^2)\operatorname{sech}(c+dx)}{30d}+\frac{(33a-2b)b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sec}\right.}{30d}$$

```
[Out] 1/2*a^2*(a-6*b)*arctanh(cosh(d*x+c))/d+1/30*b*(81*a^2-28*a*b-4*b^2)*sech(d*
x+c)/d+1/30*(33*a-2*b)*b*sech(d*x+c)*(a+b-b*sech(d*x+c)^2)/d+7/10*b*sech(d*
x+c)*(a+b-b*sech(d*x+c)^2)^2/d-1/2*coth(d*x+c)*csch(d*x+c)*(a+b-b*sech(d*x+
c)^2)^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {3745, 478, 542, 396, 213}

$$\frac{b(81a^2-28ab-4b^2)\operatorname{sech}(c+dx)}{30d}+\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{7b\operatorname{sech}(c+dx)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}{10d}+\frac{b(33a-2b)\operatorname{sech}(c+dx)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}{30d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^3}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(81*a^2 - 28*a*b - 4*b^2)
*Sech[c + d*x])/(30*d) + ((33*a - 2*b)*b*Sech[c + d*x]*(a + b - b*Sech[c +
d*x]^2))/(30*d) + (7*b*Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2)^2)/(10*d)
- (Coth[c + d*x]*Csch[c + d*x]*(a + b - b*Sech[c + d*x]^2)^3)/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{\text{Subst}\left(\int\frac{x^2(a+b-bx^2)^3}{(-1+x^2)^2}\,dx,x,\operatorname{sech}(c+dx)\right)}{d}$$

$$=-\frac{\coth(c+dx)\operatorname{csch}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^3}{2d}-\frac{\text{Subs}}{}$$

$$=\frac{7b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2}{10d}-\frac{\coth(c+dx)\operatorname{csch}}{}$$

$$=\frac{(33a-2b)b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)}{30d}+\frac{7b\operatorname{sech}(c}{}$$

$$=\frac{b(81a^2-28ab-4b^2)\operatorname{sech}(c+dx)}{30d}+\frac{(33a-2b)b\operatorname{sech}(c+dx)}{3}$$

$$=\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b(81a^2-28ab-4b^2)\operatorname{sech}}{30d}$$

Mathematica [A]
time = 6.15, size = 127, normalized size = 0.84

$$-\frac{a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^2(a-6b)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{3a^2b\operatorname{sech}(c+dx)}{d}-\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}+\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*(a^3*Csch[(c + d*x)/2]^2)/d - (a^2*(a - 6*b)*Log[Tanh[(c + d*x)/2]])/(
2*d) - (a^3*Sech[(c + d*x)/2]^2)/(8*d) + (3*a^2*b*Sech[c + d*x])/d - (b^2*(
3*a + b)*Sech[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]^5)/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(142) = 284$.
time = 2.92, size = 410, normalized size = 2.70

method	result
risch	$-\frac{e^{dx+c}\left(15a^3e^{12dx+12c}-90a^2b\,e^{12dx+12c}+90a^3e^{10dx+10c}-180a^2b\,e^{10dx+10c}+120a\,b^2e^{10dx+10c}+40b^3e^{10dx+10c}+225a^3e^{8dx+8c}+90a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*exp(d*x+c)*(15*a^3*exp(12*d*x+12*c)-90*a^2*b*exp(12*d*x+12*c)+90*a^3*
exp(10*d*x+10*c)-180*a^2*b*exp(10*d*x+10*c)+120*a*b^2*exp(10*d*x+10*c)+40*b
^3*exp(10*d*x+10*c)+225*a^3*exp(8*d*x+8*c)+90*a^2*b*exp(8*d*x+8*c)-96*b^3*e
xp(8*d*x+8*c)+300*a^3*exp(6*d*x+6*c)+360*a^2*b*exp(6*d*x+6*c)-240*a*b^2*exp
(6*d*x+6*c)+112*b^3*exp(6*d*x+6*c)+225*a^3*exp(4*d*x+4*c)+90*a^2*b*exp(4*d*
x+4*c)-96*b^3*exp(4*d*x+4*c)+90*a^3*exp(2*d*x+2*c)-180*a^2*b*exp(2*d*x+2*c)
+120*a*b^2*exp(2*d*x+2*c)+40*b^3*exp(2*d*x+2*c)+15*a^3-90*a^2*b)/d/(1+exp(2
*d*x+2*c))^5/(exp(2*d*x+2*c)-1)^2-1/2*a^3/d*ln(exp(d*x+c)-1)+3*a^2*b/d*ln(e
xp(d*x+c)-1)+1/2*a^3/d*ln(exp(d*x+c)+1)-3*a^2*b/d*ln(exp(d*x+c)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(151) = 302$.
time = 0.30, size = 403, normalized size = 2.65

$$\frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3dx-3c)}\right)}{d\left(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1\right)}\right)-3a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2e^{(-dx-c)}}{d\left(e^{(-2dx-2c)}+1\right)}\right)-\frac{8}{15}b^3\left(\frac{5e^{(-3dx-3c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}-\frac{2e^{(-5dx-5c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{5e^{(-7dx-7c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)-\frac{8ab^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c
) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 3*
a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(
d*(e^(-2*d*x - 2*c) + 1))) - 8/15*b^3*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
```

```
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) - 8*
a*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5037 vs. $2(151) = 302$.
time = 0.37, size = 5037, normalized size = 33.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(30*(a^3 - 6*a^2*b)*cosh(d*x + c)^13 + 390*(a^3 - 6*a^2*b)*cosh(d*x +
 c)*sinh(d*x + c)^12 + 30*(a^3 - 6*a^2*b)*sinh(d*x + c)^13 + 20*(9*a^3 - 18
*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^11 + 20*(9*a^3 - 18*a^2*b + 12*a*b
^2 + 4*b^3 + 117*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 220*(3
9*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*c
osh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x +
c)^9 + 2*(10725*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 225*a^3 + 90*a^2*b - 96*b
^3 + 550*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^9 + 6*(6435*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 + 550*(9*a^3 - 18*a^2*b + 12
*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 9*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^8 + 8*(75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x +
 c)^7 + 8*(6435*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 + 825*(9*a^3 - 18*a^2*b + 1
2*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3 +
27*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 8*(6435*
(a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 1155*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3
)*cosh(d*x + c)^5 + 63*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^3 + 7*(75
*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(75
*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^5 + 6*(6435*(a^3 - 6*a^2*b)*cosh(d*
x + c)^8 + 1540*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 126
*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^4 + 75*a^3 + 30*a^2*b - 32*b^3
+ 28*(75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^5 + 2*(10725*(a^3 - 6*a^2*b)*cosh(d*x + c)^9 + 3300*(9*a^3 - 18*a^2*b + 12
*a*b^2 + 4*b^3)*cosh(d*x + c)^7 + 378*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x
 + c)^5 + 140*(75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x + c)^3 + 15*
(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 20*(9*a^3 - 1
8*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 4*(2145*(a^3 - 6*a^2*b)*cosh(
d*x + c)^10 + 825*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^8 + 1
26*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^6 + 70*(75*a^3 + 90*a^2*b - 6
0*a*b^2 + 28*b^3)*cosh(d*x + c)^4 + 45*a^3 - 90*a^2*b + 60*a*b^2 + 20*b^3 +
 15*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(585*
(a^3 - 6*a^2*b)*cosh(d*x + c)^11 + 275*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3
)*cosh(d*x + c)^9 + 54*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^7 + 42*(7
5*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*cosh(d*x + c)^5 + 15*(75*a^3 + 30*a^2
```

```
*b - 32*b^3)*cosh(d*x + c)^3 + 15*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cos
h(d*x + c))*sinh(d*x + c)^2 + 30*(a^3 - 6*a^2*b)*cosh(d*x + c) - 15*((a^3 -
 6*a^2*b)*cosh(d*x + c)^14 + 14*(a^3 - 6*a^2*b)*cosh(d*x + c)*sinh(d*x + c)
^13 + (a^3 - 6*a^2*b)*sinh(d*x + c)^14 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c)^12
 + (3*a^3 - 18*a^2*b + 91*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^12
 + 4*(91*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 + 9*(a^3 - 6*a^2*b)*cosh(d*x + c))
*sinh(d*x + c)^11 + (a^3 - 6*a^2*b)*cosh(d*x + c)^10 + (1001*(a^3 - 6*a^2*b
)*cosh(d*x + c)^4 + a^3 - 6*a^2*b + 198*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*si
nh(d*x + c)^10 + 2*(1001*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 + 330*(a^3 - 6*a^2
*b)*cosh(d*x + c)^3 + 5*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*
(a^3 - 6*a^2*b)*cosh(d*x + c)^8 + (3003*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 + 1
485*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 - 5*a^3 + 30*a^2*b + 45*(a^3 - 6*a^2*b)
*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(429*(a^3 - 6*a^2*b)*cosh(d*x + c)^7
+ 297*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 + 15*(a^3 - 6*a^2*b)*cosh(d*x + c)^3
- 5*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 5*(a^3 - 6*a^2*b)*cosh
(d*x + c)^6 + (3003*(a^3 - 6*a^2*b)*cosh(d*x + c)^8 + 2772*(a^3 - 6*a^2*b)*
cosh(d*x + c)^6 + 210*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 - 5*a^3 + 30*a^2*b -
140*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(a^3 - 6*a^2
*b)*cosh(d*x + c)^9 + 1188*(a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 126*(a^3 - 6*a
^2*b)*cosh(d*x + c)^5 - 140*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 15*(a^3 - 6*a
^2*b)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^3 - 6*a^2*b)*cosh(d*x + c)^4 + (1
001*(a^3 - 6*a^2*b)*cosh(d*x + c)^10 + 1485*(a^3 - 6*a^2*b)*cosh(d*x + c)^8
 + 210*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 - 350*(a^3 - 6*a^2*b)*cosh(d*x + c)^
4 + a^3 - 6*a^2*b - 75*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4
*(91*(a^3 - 6*a^2*b)*cosh(d*x + c)^11 + 165*(a^3 - 6*a^2*b)*cosh(d*x + c)^9
 + 30*(a^3 - 6*a^2*b)*cosh(d*x + c)^7 - 70*(a^3 - 6*a^2*b)*cosh(d*x + c)^5
- 25*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(
d*x + c)^3 + a^3 - 6*a^2*b + 3*(a^3 - 6*a^2*b)*cosh(d*x + c)^2 + (91*(a^3 -
 6*a^2*b)*cosh(d*x + c)^12 + 198*(a^3 - 6*a^2*b)*cosh(d*x + c)^10 + 45*(a^3
 - 6*a^2*b)*cosh(d*x + c)^8 - 140*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 - 75*(a^3
 - 6*a^2*b)*cosh(d*x + c)^4 + 3*a^3 - 18*a^2*b + 6*(a^3 - 6*a^2*b)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 2*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^13 + 18*(a^3
- 6*a^2*b)*cosh(d*x + c)^11 + 5*(a^3 - 6*a^2*b)*cosh(d*x + c)^9 - 20*(a^3 -
 6*a^2*b)*cosh(d*x + c)^7 - 15*(a^3 - 6*a^2*b)*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{csch}^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**3, x)
```

Giac [A]
time = 0.55, size = 206, normalized size = 1.36

$$-\frac{\frac{60\,a^{3}\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}-4}-15\,(a^{3}-6\,a^{2}b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)+15\,(a^{3}-6\,a^{2}b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{8\left(45\,a^{2}b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{4}-60\,ab^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}-20\,b^{3}\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}+48\,b^{3}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c) + e^(-d*x - c))^2
- 4) - 15*(a^3 - 6*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 15*(a^3 - 6
*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 8*(45*a^2*b*(e^(d*x + c) + e^
(-d*x - c))^4 - 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x +
c) + e^(-d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B]
time = 1.26, size = 412, normalized size = 2.71

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^{3}\,\sqrt{-d^{2}}-6\,a^{2}\,b\,\sqrt{-d^{2}}\right)}{d\,\sqrt{a^{6}-12\,a^{5}\,b+36\,a^{4}\,b^{2}}}\right)\,\sqrt{a^{6}-12\,a^{5}\,b+36\,a^{4}\,b^{2}}}{\sqrt{-d^{2}}}-\frac{64\,b^{3}\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{8\,\mathrm{e}^{c+d\,x}\,\left(17\,b^{3}+15\,a\,b^{2}\right)}{15\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}+\frac{32\,b^{3}\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}-\frac{a^{3}\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{8\,\mathrm{e}^{c+d\,x}\,\left(b^{3}+3\,a\,b^{2}\right)}{3\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{2\,a^{3}\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{6\,a^{2}\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(a^6 -
 12*a^5*b + 36*a^4*b^2)^(1/2)))*(a^6 - 12*a^5*b + 36*a^4*b^2)^(1/2))/(-d^2)
^(1/2) - (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x
) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(15*a*b^2
 + 17*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*
x) + 1)) + (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*
d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1))
- (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (8*exp(c + d*x)*(3*a*b^2
+ b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*a^3*exp(c +
d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (6*a^2*b*exp(c + d*
x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.24 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=98

$$\frac{a^2(a-3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{3a(a-b)b\tanh(c+dx)}{d}-\frac{(3a-b)b^2\tanh^3(c+dx)}{3d}-\frac{b^3\tanh^5(c}{5d}$$

```
[Out] a^2*(a-3*b)*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d-3*a*(a-b)*b*tanh(d*x+c)/d
-1/3*(3*a-b)*b^2*tanh(d*x+c)^3/d-1/5*b^3*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3744, 459}

$$-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^2(a-3b)\coth(c+dx)}{d}-\frac{b^2(3a-b)\tanh^3(c+dx)}{3d}-\frac{3ab(a-b)\tanh(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a - 3*b)*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) - (3*a*(a - b
)*b*Tanh[c + d*x])/d - ((3*a - b)*b^2*Tanh[c + d*x]^3)/(3*d) - (b^3*Tanh[c
+ d*x]^5)/(5*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)(a+bx^2)^3}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a(a-b)b + \frac{a^3}{x^4} - \frac{a^2(a-3b)}{x^2} - (3a-b)b^2x^2 - b^3x^4\right.\right.}{d}$$
$$= \frac{a^2(a-3b)\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{3a(a-b)b\tan}{d}$$

Mathematica [A]
time = 0.85, size = 87, normalized size = 0.89

$$\frac{-5a^2\coth(c+dx)\left(-2a+9b+a\operatorname{csch}^2(c+dx)\right)+b\left(-45a^2+30ab+2b^2+b(15a+b)\operatorname{sech}^2(c+dx)-3b^2\operatorname{sech}^4(c+dx)\right)\tanh(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-5*a^2*Coth[c + d*x]*(-2*a + 9*b + a*Csch[c + d*x]^2) + b*(-45*a^2 + 30*a*
b + 2*b^2 + b*(15*a + b)*Sech[c + d*x]^2 - 3*b^2*Sech[c + d*x]^4)*Tanh[c +
d*x])/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(92) = 184.
time = 2.62, size = 403, normalized size = 4.11

method	result
risch	$-\frac{4(-15a\,b^2+45a^2b\,e^{12dx+12c}+45a\,b^2e^{12dx+12c}+90a^2b\,e^{10dx+10c}-30a\,b^2e^{10dx+10c}-45a^2b\,e^{4dx+4c}+90a^2b\,e^{2dx+2c}+45a^2b-5a^3-b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/15*(-15*a*b^2+45*a^2*b*exp(12*d*x+12*c)+45*a*b^2*exp(12*d*x+12*c)+90*a^2
*b*exp(10*d*x+10*c)-30*a*b^2*exp(10*d*x+10*c)-45*a^2*b*exp(4*d*x+4*c)+90*a^
2*b*exp(2*d*x+2*c)+45*a^2*b-5*a^3-b^3-105*a*b^2*exp(8*d*x+8*c)+60*a*b^2*exp
(6*d*x+6*c)+75*a*b^2*exp(4*d*x+4*c)-45*a^2*b*exp(8*d*x+8*c)-30*a*b^2*exp(2*
d*x+2*c)-180*a^2*b*exp(6*d*x+6*c)-10*a^3*exp(2*d*x+2*c)+15*a^3*exp(12*d*x+1
2*c)+15*b^3*exp(12*d*x+12*c)+70*a^3*exp(10*d*x+10*c)-2*b^3*exp(2*d*x+2*c)+6
5*b^3*exp(8*d*x+8*c)+25*a^3*exp(4*d*x+4*c)+17*b^3*exp(4*d*x+4*c)-50*b^3*exp
(10*d*x+10*c)+125*a^3*exp(8*d*x+8*c)+100*a^3*exp(6*d*x+6*c)-44*b^3*exp(6*d*
x+6*c))/d/(1+exp(2*d*x+2*c))^5/(exp(2*d*x+2*c)-1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(92) = 184$.
time = 0.30, size = 493, normalized size = 5.03

$$\frac{4}{15}b^3\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}-\frac{5e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{15e^{(-6dx-6c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{1}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+4ab^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+\frac{4}{3}a^3\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)+\frac{12a^2b}{d(e^{(-4dx-4c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c
)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^
(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e
^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 1
0*c) + 1))) + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-
2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*
d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 12*a^2*b/(d*(e^
(-4*d*x - 4*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(92) = 184$.
time = 0.35, size = 925, normalized size = 9.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/15*((5*a^3 + 45*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 12*(5*a^3 +
15*a*b^2 + 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^3 + 45*a^2*b + 15*a*
b^2 + 7*b^3)*sinh(d*x + c)^6 + 2*(15*a^3 + 45*a^2*b - 15*a*b^2 - 13*b^3)*co
sh(d*x + c)^4 + (30*a^3 + 90*a^2*b - 30*a*b^2 - 26*b^3 + 15*(5*a^3 + 45*a^2
*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^3 + 15*
a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 4*(5*a^3 - 3*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 50*a^3 - 90*a^2*b + 30*a*b^2 - 22*b^3 + (75*a^3 - 45*a^2*b - 15*a*
b^2 + 41*b^3)*cosh(d*x + c)^2 + (15*(5*a^3 + 45*a^2*b + 15*a*b^2 + 7*b^3)*c
osh(d*x + c)^4 + 75*a^3 - 45*a^2*b - 15*a*b^2 + 41*b^3 + 12*(15*a^3 + 45*a^
2*b - 15*a*b^2 - 13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(5*a^3 + 1
5*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + 8*(5*a^3 - 3*b^3)*cosh(d*x + c)^3 + (25*
a^3 - 45*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10
+ 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 2*d*cosh(d*x +
```

```
c)^8 + (45*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)
^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^7 - 3*d*cosh(d*x + c)^6 + (210*d*cosh
(d*x + c)^4 + 56*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d
*x + c)^5 + 56*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 8*d
*cosh(d*x + c)^4 + (210*d*cosh(d*x + c)^6 + 140*d*cosh(d*x + c)^4 - 45*d*co
sh(d*x + c)^2 - 8*d)*sinh(d*x + c)^4 + 4*(30*d*cosh(d*x + c)^7 + 28*d*cosh(
d*x + c)^5 - 5*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 2*d
*cosh(d*x + c)^2 + (45*d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^6 - 45*d*cosh
(d*x + c)^4 - 48*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x
 + c)^9 + 8*d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 - 8*d*cosh(d*x + c)^3 -
 2*d*cosh(d*x + c))*sinh(d*x + c) + 6*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^3 \operatorname{csch}^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(92) = 184.
time = 0.54, size = 257, normalized size = 2.62

$$-\frac{2\left(\frac{5\left(9a^2be^{(4dx+4c)}+6a^3e^{(2dx+2c)}-18a^2be^{(2dx+2c)}-2a^3+9a^2b\right)}{\left(e^{(2dx+2c)}-1\right)^3}-\frac{45a^2be^{(8dx+8c)}+180a^2be^{(6dx+6c)}-90ab^2e^{(6dx+6c)}-30b^3e^{(6dx+6c)}+270a^2be^{(4dx+4c)}-210ab^2e^{(4dx+4c)}+10b^3e^{(4dx+4c)}+180a^2be^{(2dx+2c)}-150ab^2e^{(2dx+2c)}-10b^3e^{(2dx+2c)}+45a^2b-30ab^2-2b^3}{\left(e^{(2dx+2c)}+1\right)^5}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/15*(5*(9*a^2*b*e^(4*d*x + 4*c) + 6*a^3*e^(2*d*x + 2*c) - 18*a^2*b*e^(2*d
*x + 2*c) - 2*a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3 - (45*a^2*b*e^(8*d*x +
 8*c) + 180*a^2*b*e^(6*d*x + 6*c) - 90*a*b^2*e^(6*d*x + 6*c) - 30*b^3*e^(6*
d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) - 210*a*b^2*e^(4*d*x + 4*c) + 10*b^3
*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) - 150*a*b^2*e^(2*d*x + 2*c) -
10*b^3*e^(2*d*x + 2*c) + 45*a^2*b - 30*a*b^2 - 2*b^3)/(e^(2*d*x + 2*c) + 1)
^5)/d
```

Mupad [**B**]
time = 0.28, size = 622, normalized size = 6.35

$$\frac{\frac{2\left(9a^2b-12ab^2+4b^3\right)}{15d}-\frac{4e^{2c+2dx}\left(-3a^2b+3ab^2+b^3\right)}{5d}+\frac{6a^2be^{4c+4dx}}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{\frac{2\left(-3a^2b+3ab^2+b^3\right)}{5d}+\frac{6e^{4c+4dx}\left(-3a^2b+3ab^2+b^3\right)}{5d}-\frac{2e^{2c+2dx}\left(9a^2b-12ab^2+4b^3\right)}{5d}-\frac{6a^2be^{6c+6dx}}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}+\frac{\frac{6a^2b}{5d}-\frac{8e^{6c+6dx}\left(-3a^2b+3ab^2+b^3\right)}{5d}-\frac{8e^{2c+2dx}\left(-3a^2b+3ab^2+b^3\right)}{5d}+\frac{4e^{4c+4dx}\left(9a^2b-12ab^2+4b^3\right)}{5d}+\frac{6a^2be^{8c+8dx}}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}-\frac{\frac{2\left(-3a^2b+3ab^2+b^3\right)}{5d}-\frac{6a^2be^{2c+2dx}}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}-\frac{4a^3}{d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}-\frac{8a^3}{3d\left(3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1\right)}-\frac{6a^2b}{d\left(e^{2c+2dx}-1\right)}+\frac{6a^2b}{5d\left(e^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^4,x)
```

```
[Out] ((2*(9*a^2*b - 12*a*b^2 + 4*b^3))/(15*d) - (4*exp(2*c + 2*d*x)*(3*a*b^2 - 3
*a^2*b + b^3))/(5*d) + (6*a^2*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a*b^2 - 3*a^2*b + b
^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (2*exp(
2*c + 2*d*x)*(9*a^2*b - 12*a*b^2 + 4*b^3))/(5*d) - (6*a^2*b*exp(6*c + 6*d*x
))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + e
xp(8*c + 8*d*x) + 1) + ((6*a^2*b)/(5*d) - (8*exp(6*c + 6*d*x)*(3*a*b^2 - 3*
a^2*b + b^3))/(5*d) - (8*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d)
+ (4*exp(4*c + 4*d*x)*(9*a^2*b - 12*a*b^2 + 4*b^3))/(5*d) + (6*a^2*b*exp(8*
c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c +
 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(3*a*b^2 - 3*a
^2*b + b^3))/(5*d) - (6*a^2*b*exp(2*c + 2*d*x))/(5*d))/(2*exp(2*c + 2*d*x)
+ exp(4*c + 4*d*x) + 1) - (4*a^3)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
 + 1)) - (8*a^3)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
6*d*x) - 1)) - (6*a^2*b)/(d*(exp(2*c + 2*d*x) - 1)) + (6*a^2*b)/(5*d*(exp(2
*c + 2*d*x) + 1))
```

3.25 $\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{(3a^2-6ab-b^2)\,x}{8(a+b)^3}+\frac{a^{3/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^3d}-\frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d}+\frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}$$

```
[Out] 1/8*(3*a^2-6*a*b-b^2)*x/(a+b)^3-1/8*(5*a+b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2
/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d+a^(3/2)*arctan(b^(1/2)*tanh(d*x+c)
/a^(1/2))*b^(1/2)/(a+b)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.261$, Rules used = {3744, 481, 541, 536, 212, 211}

$$\frac{a^{3/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^3}+\frac{x(3a^2-6ab-b^2)}{8(a+b)^3}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)}-\frac{(5a+b)\sinh(c+dx)\cosh(c+dx)}{8d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 6*a*b - b^2)*x)/(8*(a + b)^3) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tanh[c + d*x])/Sqrt[a]])/((a + b)^3*d) - ((5*a + b)*Cosh[c + d*x]*Sinh[c +
d*x])/(8*(a + b)^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} - \frac{\text{Subst}\left(\int \frac{a+(4a+b)x^2}{(1-x^2)^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= -\frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} + \frac{(a^2b)\,\text{Subs}}{}$$

$$= \frac{(3a^2-6ab-b^2)\,x}{8(a+b)^3} + \frac{a^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^3d} - \frac{(5a+b)\cosh(c+dx)\,\text{si}}{8(a+b)^2d}$$

Mathematica [A]
time = 0.20, size = 93, normalized size = 0.79

$$\frac{4(3a^2-6ab-b^2)(c+dx)+32a^{3/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)-8a(a+b)\sinh(2(c+dx))+(a+b)^2\sinh(4(c+dx))}{32(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (4*(3*a^2 - 6*a*b - b^2)*(c + d*x) + 32*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan
h[c + d*x])/Sqrt[a]] - 8*a*(a + b)*Sinh[2*(c + d*x)] + (a + b)^2*Sinh[4*(c
+ d*x)])/(32*(a + b)^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(104) = 208.
time = 2.47, size = 434, normalized size = 3.68

method	result
risch	$\frac{3x\,a^2}{8(a+b)^3}-\frac{3xab}{4(a+b)^3}-\frac{x\,b^2}{8(a+b)^3}+\frac{e^{4dx+4c}}{64(a+b)d}-\frac{a\,e^{2dx+2c}}{8(a+b)^2d}+\frac{a\,e^{-2dx-2c}}{8(a^2+2ab+b^2)d}-\frac{e^{-4dx-4c}}{64(a+b)d}+\frac{\sqrt{-ab}\,a\ln\Big(e}{}$
derivativedivides	$\frac{-\frac{8}{(32a+32b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{32}{(64a+64b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-a+3b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{3a-b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+}{}$
default	$\frac{-\frac{8}{(32a+32b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{32}{(64a+64b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-a+3b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{3a-b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8/(32*a+32*b)/(tanh(1/2*d*x+1/2*c)+1)^4+32/(64*a+64*b)/(tanh(1/2*d*x+
1/2*c)+1)^3-1/8*(-a+3*b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/8*(3*a-b)/(a+b
)^2/(tanh(1/2*d*x+1/2*c)+1)+1/8*(3*a^2-6*a*b-b^2)/(a+b)^3*ln(tanh(1/2*d*x+1
/2*c)+1)-2*a^3*b/(a+b)^3*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2
*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b)
)^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b
*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1
/2)+a+2*b)*a)^(1/2)))+8/(32*a+32*b)/(tanh(1/2*d*x+1/2*c)-1)^4+32/(64*a+64*b
)/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(a-3*b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1
/8*(3*a-b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)+1/8/(a+b)^3*(-3*a^2+6*a*b+b^2)*l
n(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(104) = 208$.

time = 0.54, size = 514, normalized size = 4.36

$$-\frac{(ab-b^2)(dx+c)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}+\frac{\left(8\,be^{(-2\,dx-2\,c)}+a+b\right)e^{(4\,dx+4\,c)}}{64\,(a^2+2\,ab+b^2)d}-\frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d}+\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d}+\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}\,d}-\frac{(a^2b-6\,ab^2+b^3)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}\,d}-\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}\,d}-\frac{3\,b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}\,(a+b)d}-\frac{8\,be^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}}{64\,(a^2+2\,ab+b^2)d}+\frac{3\,(dx+c)}{8\,(a+b)d}-\frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d}+\frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*b - b^2)*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*(8*b*
e^(-2*d*x - 2*c) + a + b)*e^(4*d*x + 4*c)/((a^2 + 2*a*b + b^2)*d) - 1/4*b*l
og((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a
*b + b^2)*d) + 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4
*c) + a + b)/((a^2 + 2*a*b + b^2)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*
e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 1/8
*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt
(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*d) - 1/4*(a*b - b^2)*arct
an(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*s
qrt(a*b)*d) - 3/8*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/(sqrt(a*b)*(a + b)*d) - 1/64*(8*b*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
4*c))/((a^2 + 2*a*b + b^2)*d) + 3/8*(d*x + c)/((a + b)*d) - 1/8*e^(2*d*x +
2*c)/((a + b)*d) + 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(104) = 208$.

time = 0.40, size = 2024, normalized size = 17.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 - 6*
```

```
a*b - b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c
)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 - 6*a*b - b^2)*d
*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c) - 20*(a^2
+ a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x + c)^2 + 4
*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 - 6*a*b - b^2)*d*x*cosh
(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c)
^2 + 32*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)^3*sinh(d*x + c) + 6*a*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x
+ c)^4)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*co
sh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x +
c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2
*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x
+ c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - a^2 - 2*a*b - b^2
 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cos
h(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x + c))
*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*d*sinh(d*x + c)^4), 1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(
d*x + c)^8 + 8*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*co
sh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*s
inh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 6*(a^2 + a*b)*c
osh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
 4*(3*a^2 - 6*a*b - b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 6*a*b - b^2)*d*
x*cosh(d*x + c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2
+ a*b)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a
^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*
a^2 + 2*a*b)*sinh(d*x + c)^2 + 64*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)^3*
sinh(d*x + c) + 6*a*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a*cosh(d*x + c)*sin
h(d*x + c)^3 + a*sinh(d*x + c)^4)*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
- b)*sqrt(a*b)/(a*b)) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^7 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(
d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b +
```

```
3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh
(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(104) = 208.
time = 1.71, size = 276, normalized size = 2.34

$$\frac{\frac{64\,a^2b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}}+\frac{8\,(3\,a^2-6\,ab-b^2)(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{(18\,a^2e^{(4\,dx+4\,c)}-36\,abe^{(4\,dx+4\,c)}-6\,b^2e^{(4\,dx+4\,c)}-8\,a^2e^{(2\,dx+2\,c)}-8\,abe^{(2\,dx+2\,c)}+a^2+2\,ab+b^2)e^{(-4\,dx-4\,c)}}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}}{a^2+2\,ab+b^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(64*a^2*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) + 8*(3*a^2 - 6*a*b -
b^2)*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (18*a^2*e^(4*d*x + 4*c) -
36*a*b*e^(4*d*x + 4*c) - 6*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 8*
a*b*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c)/(a^3 + 3*a^2*b +
3*a*b^2 + b^3) + (a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a*e^(2*d*x + 2*
c))/(a^2 + 2*a*b + b^2))/d
```

Mupad [B]
time = 1.68, size = 250, normalized size = 2.12

$$\frac{e^{4c+4dx}}{64\,d\,(a+b)}-\frac{e^{-4c-4dx}}{64\,d\,(a+b)}-\frac{x\,(-3\,a^2+6\,a\,b+b^2)}{8\,(a+b)^3}+\frac{a\,e^{-2c-2dx}}{8\,d\,(a+b)^2}-\frac{a\,e^{2c+2dx}}{8\,d\,(a+b)^2}+\frac{(-a)^{3/2}\sqrt{b}\,\ln\left((-a)^{3/2}\,b^{3/2}\,(e^{2c+2dx}-1)-2\,a^2\,b\,e^{2c+2dx}+(-a)^{5/2}\sqrt{b}\,(e^{2c+2dx}+1)\right)}{2\,d\,(a+b)^3}-\frac{(-a)^{3/2}\sqrt{b}\,\ln\left(2\,a^2\,b\,e^{2c+2dx}+(-a)^{3/2}\,b^{3/2}\,(e^{2c+2dx}-1)+(-a)^{5/2}\sqrt{b}\,(e^{2c+2dx}+1)\right)}{2\,d\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] exp(4*c + 4*d*x)/(64*d*(a + b)) - exp(- 4*c - 4*d*x)/(64*d*(a + b)) - (x*(6
*a*b - 3*a^2 + b^2))/(8*(a + b)^3) + (a*exp(- 2*c - 2*d*x))/(8*d*(a + b)^2)
 - (a*exp(2*c + 2*d*x))/(8*d*(a + b)^2) + ((-a)^(3/2)*b^(1/2)*log((-a)^(3/2
)*b^(3/2)*(exp(2*c + 2*d*x) - 1) - 2*a^2*b*exp(2*c + 2*d*x) + (-a)^(5/2)*b^
(1/2)*(exp(2*c + 2*d*x) + 1)))/(2*d*(a + b)^3) - ((-a)^(3/2)*b^(1/2)*log(2*
a^2*b*exp(2*c + 2*d*x) + (-a)^(3/2)*b^(3/2)*(exp(2*c + 2*d*x) - 1) + (-a)^(
5/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1)))/(2*d*(a + b)^3)
```

3.26 $\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d}-\frac{a\cosh(c+dx)}{(a+b)^2d}+\frac{\cosh^3(c+dx)}{3(a+b)d}$$

```
[Out] -a*cosh(d*x+c)/(a+b)^2/d+1/3*cosh(d*x+c)^3/(a+b)/d+a*arctanh(sech(d*x+c)*b^
(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3745, 464, 331, 214}

$$\frac{\cosh^3(c+dx)}{3d(a+b)}-\frac{a\cosh(c+dx)}{d(a+b)^2}+\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/((a + b)^(5/2)*d)
- (a*Cosh[c + d*x])/((a + b)^2*d) + Cosh[c + d*x]^3/(3*(a + b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= \frac{\cosh^3(c+dx)}{3(a+b)d} + \frac{a\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{(a+b)d}\\ &= -\frac{a\cosh(c+dx)}{(a+b)^2d} + \frac{\cosh^3(c+dx)}{3(a+b)d} + \frac{(ab)\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{(a+b)^2d}\\ &= \frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} - \frac{a\cosh(c+dx)}{(a+b)^2d} + \frac{\cosh^3(c+dx)}{3(a+b)d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.40, size = 135, normalized size = 1.80

$$\frac{12ia\sqrt{b}\left(\text{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\text{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)-3(3a-b)\sqrt{a+b}\cosh(c+dx)+(a+b)^{3/2}\cosh(3(c+dx))}{12(a+b)^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((12*I)*a*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2]])/Sq
rt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2]])/Sqrt[b]]) -
3*(3*a - b)*Sqrt[a + b]*Cosh[c + d*x] + (a + b)^(3/2)*Cosh[3*(c + d*x)])/(1
2*(a + b)^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(65) = 130.
time = 2.32, size = 202, normalized size = 2.69

method	result
derivativedivides	$\frac{-\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{16}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(16a+16b)}-\frac{a-b}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{ab\,\mathrm{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{4\sqrt{ab+b}}\right)}{(a+b)^2\sqrt{ab+b^2}}}{d}$
default	$\frac{-\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{16}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(16a+16b)}-\frac{a-b}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{ab\,\mathrm{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{4\sqrt{ab+b}}\right)}{(a+b)^2\sqrt{ab+b^2}}}{d}$
risch	$\frac{e^{3dx+3c}}{24(a+b)d}-\frac{3\,e^{dx+c}a}{8(a+b)^2d}+\frac{e^{dx+c}b}{8(a+b)^2d}-\frac{3\,e^{-dx-c}a}{8(a+b)^2d}+\frac{e^{-dx-c}b}{8(a+b)^2d}+\frac{e^{-3dx-3c}}{24(a+b)d}+\frac{\sqrt{b(a+b)}\,a\ln\left(e^{2dx+2c}+\right.}{2(a+b)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/d*(-8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)+1)^2+16/3/(tanh(1/2*d*x+1/2*c)+1)^3/(16*a+16*b)-1/2*(a-b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)+a*b/(a+b)^2/(a*b+b^2)^(1/2)*arctanh(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+2*a+4*b)/(a*b+b^2)^(1/2))-16/3/(tanh(1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^2*(-a+b)/(tanh(1/2*d*x+1/2*c)-1))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/24*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) - 3*(3*a*e^(4*c) - b*e^(4*c))*e^(4*d*x) - 3*(3*a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-3*d*x)/(a^2*d*e^(3*c) + 2*a*b*d*e^(3*c) + b^2*d*e^(3*c)) - 1/8*integrate(16*(a*b*e^(3*d*x + 3*c) - a*b*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(65) = 130.
time = 0.36, size = 1367, normalized size = 18.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 +
(a + b)*sinh(d*x + c)^6 - 3*(3*a - b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d
*x + c)^2 - 3*a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 - 3*(3*
a - b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a - b)*cosh(d*x + c)^2 + 3*(5*
(a + b)*cosh(d*x + c)^4 - 6*(3*a - b)*cosh(d*x + c)^2 - 3*a + b)*sinh(d*x +
 c)^2 + 12*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*cos
h(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqrt(b/(a + b))*log(((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a
 + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*s
inh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a +
b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
 b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b)) + 6*((a + b)*cosh(d*x + c)^5 - 2*(3*a - b)*cosh(d*x
+ c)^3 - (3*a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^
2)*d*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2*sinh(d*x + c
) + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 + 2*a*b +
b^2)*d*sinh(d*x + c)^3), 1/24*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x
 + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(3*a - b)*cosh(d*x + c)
^4 + 3*(5*(a + b)*cosh(d*x + c)^2 - 3*a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)
*cosh(d*x + c)^3 - 3*(3*a - b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a - b)
*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(3*a - b)*cosh(d*x + c)
^2 - 3*a + b)*sinh(d*x + c)^2 + 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2
*sinh(d*x + c) + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqr
t(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)
*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(
a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - 24*(
a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*cosh(d*x + c)*s
inh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqrt(-b/(a + b))*arctan(1/2*((a + b)*co
sh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) + 6*((a + b)*cosh(
d*x + c)^5 - 2*(3*a - b)*cosh(d*x + c)^3 - (3*a - b)*cosh(d*x + c))*sinh(d*
x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b
^2)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)
*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 2.65, size = 955, normalized size = 12.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] exp(- 3*c - 3*d*x)/(24*d*(a + b)) + exp(3*c + 3*d*x)/(24*d*(a + b)) - ((a^2
*b)^(1/2)*(2*atan(((exp(d*x)*exp(c)*((4*(2*a^2*b^3*d*(a^2*b)^(1/2) + 4*a^3*
b^2*d*(a^2*b)^(1/2) + 2*a^4*b*d*(a^2*b)^(1/2)))/(a*(a + b)*(-d^2*(a + b)^5)
^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(- a^5*d^2 - b^5
*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2))
+ (2*a^3*b)/(d*(a + b)^3*(a^2*b)^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2
*b + a^3 + b^3))) + (2*a^3*b*exp(3*c)*exp(3*d*x))/(d*(a + b)^3*(a^2*b)^(1/2
)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))*(a^6*(- a^5*d^2 - b
^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2)
 + b^6*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 -
10*a^3*b^2*d^2)^(1/2) + 15*a^2*b^4*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a
^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 20*a^3*b^3*(- a^5*d^2 -
 b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/
2) + 15*a^4*b^2*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b
^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 6*a*b^5*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2
 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 6*a^5*b*(- a^5*d^
2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^
(1/2)))/(4*a^2*b)) - 2*atan((a*exp(d*x)*exp(c)*(-d^2*(a + b)^5)^(1/2))/(2*d
*(a + b)^2*(a^2*b)^(1/2)))))/(2*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*
b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2)) - (exp(- c - d*x)*(3*a - b)
)/(8*d*(a + b)^2) - (exp(c + d*x)*(3*a - b))/(8*d*(a + b)^2)
```

3.27 $\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=78

$$-\frac{(a-b)x}{2(a+b)^2}-\frac{\sqrt{a}\,\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

```
[Out] -1/2*(a-b)*x/(a+b)^2+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d-arctan(b^(1/2)*tan
h(d*x+c)/a^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {3744, 482, 536, 212, 211}

$$-\frac{\sqrt{a}\,\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)}-\frac{x(a-b)}{2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((a - b)*x)/(a + b)^2 - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a]])/((a + b)^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} - \frac{\text{Subst}\left(\int \frac{a-bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)^2d} - \frac{(ab)\text{S}}{}$$

$$= -\frac{(a-b)x}{2(a+b)^2} - \frac{\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

Mathematica [A]
time = 0.12, size = 67, normalized size = 0.86

$$\frac{-2(a-b)(c+dx) - 4\sqrt{a}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right) + (a+b)\sinh(2(c+dx))}{4(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-2*(a - b)*(c + d*x) - 4*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sq
rt[a]] + (a + b)*Sinh[2*(c + d*x)])/(4*(a + b)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(66) = 132$.

time = 2.38, size = 310, normalized size = 3.97

method	result
risch	$-\frac{ax}{2(a+b)^2} + \frac{xb}{2(a+b)^2} + \frac{e^{2dx+2c}}{8(a+b)d} - \frac{e^{-2dx-2c}}{8(a+b)d} + \frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c} - \frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2(a+b)^2 d} - \frac{\sqrt{-ab}\,\ln\left(e^{2dx}\right.}{2(}$
derivativedivides	$\frac{2a^2b\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}} + \frac{\left(a+\sqrt{b(a+b)}+b\right)\arctan\left(\right.}{2a\sqrt{b(a+b)}\sqrt{(}}\right.}{(a+b)^2}$
default	$\frac{2a^2b\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}} + \frac{\left(a+\sqrt{b(a+b)}+b\right)\arctan\left(\right.}{2a\sqrt{b(a+b)}\sqrt{(}}\right.}{(a+b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2*b/(a+b)^2*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*
(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1
/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+
b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+
a+2*b)*a)^(1/2)))-4/(8*a+8*b)/(tanh(1/2*d*x+1/2*c)+1)^2+8/(16*a+16*b)/(tanh
(1/2*d*x+1/2*c)+1)+1/2/(a+b)^2*(-a+b)*ln(tanh(1/2*d*x+1/2*c)+1)+4/(8*a+8*b)
/(tanh(1/2*d*x+1/2*c)-1)^2+8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)-1)+1/2*(a-b)/
(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(66) = 132$.

time = 0.51, size = 316, normalized size = 4.05

$$\frac{b\log\left((a+b)e^{(4\,dx+4\,c)} + 2\,(a-b)e^{(2\,dx+2\,c)} + a + b\right)}{4\,(a^2+2\,ab+b^2)d} - \frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)} + (a+b)e^{(-4\,dx-4\,c)} + a + b\right)}{4\,(a^2+2\,ab+b^2)d} - \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}\,d} + \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}\,d} + \frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}\,(a+b)d} - \frac{dx+c}{2\,(a+b)d} + \frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d} - \frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
2 + 2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*
d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2*((
a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d
) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) + 1/2*b*arctan(1/2*((a + b)*e^(-2*d*x -
 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) - 1/2*(d*x + c)/((a + b)*d)
 + 1/8*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(66) = 132$.
time = 0.41, size = 916, normalized size = 11.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh(d*x + c)^4 - 4*(a + b)*
cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x + c)^4 + 2*(2*(a - b)*d*x
- 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*sqrt(-a*b)*(cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*log(((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*
a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
 c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*s
inh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d
*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)
^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh
(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x
+ c) + a + b)) + 4*(2*(a - b)*d*x*cosh(d*x + c) - (a + b)*cosh(d*x + c)^3)*
sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*
a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x +
 c)^2), -1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh(d*x + c)^4 - 4*(
a + b)*cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x + c)^4 + 2*(2*(a -
b)*d*x - 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*sqrt(a*b)*(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*arctan(1/2*((a +
 b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(
d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 4*(2*(a - b)*d*x*cosh(d*x + c) - (a
+ b)*cosh(d*x + c)^3)*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*
x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a
*b + b^2)*d*sinh(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(66) = 132.
time = 0.96, size = 159, normalized size = 2.04

$$-\frac{\frac{8\,ab\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}}+\frac{4\,(dx+c)(a-b)}{a^2+2\,ab+b^2}-\frac{\left(2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}-a-b\right)e^{(-2\,dx-2\,c)}}{a^2+2\,ab+b^2}-\frac{e^{(2\,dx+2\,c)}}{a+b}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*a*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt
(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 4*(d*x + c)*(a - b)/(a^2 + 2*a*b +
 b^2) - (2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) - a - b)*e^(-2*d*x - 2*c
)/(a^2 + 2*a*b + b^2) - e^(2*d*x + 2*c)/(a + b))/d
```

Mupad [B]
time = 1.51, size = 198, normalized size = 2.54

$$\frac{e^{2c+2dx}}{8\,d\,(a+b)}-\frac{e^{-2c-2dx}}{8\,d\,(a+b)}-\frac{x\,(a-b)}{2\,(a+b)^2}-\frac{\sqrt{-a}\,\sqrt{b}\,\ln\left(\sqrt{-a}\,b^{3/2}\,\left(e^{2c+2dx}-1\right)+(-a)^{3/2}\,\sqrt{b}\,\left(e^{2c+2dx}+1\right)-2\,a\,b\,e^{2c+2dx}\right)}{2\,d\,(a+b)^2}+\frac{\sqrt{-a}\,\sqrt{b}\,\ln\left(\sqrt{-a}\,b^{3/2}\,\left(e^{2c+2dx}-1\right)+(-a)^{3/2}\,\sqrt{b}\,\left(e^{2c+2dx}+1\right)+2\,a\,b\,e^{2c+2dx}\right)}{2\,d\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/(8*d*(a + b)) - (x*(a -
 b))/(2*(a + b)^2) - ((-a)^(1/2)*b^(1/2)*log((-a)^(1/2)*b^(3/2)*(exp(2*c +
2*d*x) - 1) + (-a)^(3/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1) - 2*a*b*exp(2*c + 2
*d*x)))/(2*d*(a + b)^2) + ((-a)^(1/2)*b^(1/2)*log((-a)^(1/2)*b^(3/2)*(exp(2
*c + 2*d*x) - 1) + (-a)^(3/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1) + 2*a*b*exp(2*
c + 2*d*x)))/(2*d*(a + b)^2)
```

3.28 $\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d}+\frac{\cosh(c+dx)}{(a+b)d}$$

```
[Out] cosh(d*x+c)/(a+b)/d-arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^
(3/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3745, 331, 214}

$$\frac{\cosh(c+dx)}{d(a+b)}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/((a + b)^(3/2)*d))
 + Cosh[c + d*x]/((a + b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
```

```
- 1)/2]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{d}$$
$$= \frac{\cosh(c+dx)}{(a+b)d} - \frac{b\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{(a+b)d}$$
$$= -\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{\cosh(c+dx)}{(a+b)d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 107, normalized size = 2.02

$$\frac{-i\sqrt{b}\left(\text{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\text{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)+\sqrt{a+b}\,\cosh(c+dx)}{(a+b)^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
[Out] ((-I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b
]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]) + Sqrt
[a + b]*Cosh[c + d*x])/((a + b)^(3/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(45) = 90$.
time = 1.54, size = 104, normalized size = 1.96

method	result
derivativedivides	$\frac{\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\,\text{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a+4b}{4\sqrt{ab+b^2}}\right)}{(a+b)\sqrt{ab+b^2}}-\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$
default	$\frac{\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\,\text{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a+4b}{4\sqrt{ab+b^2}}\right)}{(a+b)\sqrt{ab+b^2}}-\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$

risch	$\frac{e^{dx+c}}{2(a+b)d}+\frac{e^{-dx-c}}{2(a+b)d}+\frac{\sqrt{b(a+b)}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{b(a+b)}\,e^{dx+c}}{a+b}+1\right)}{2(a+b)^2d}-\frac{\sqrt{b(a+b)}\,\ln\left(e^{2dx+2c}\right.}{2(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)+1)-b/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/
4*(2*a*tanh(1/2*d*x+1/2*c)^2+2*a+4*b)/(a*b+b^2)^(1/2))-4/(4*a+4*b)/(tanh(1/
2*d*x+1/2*c)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(b
*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a*b
*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)
), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(45) = 90$.
time = 0.42, size = 666, normalized size = 12.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*log(((a + b)*cosh(d*x
 + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*si
nh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(
d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x +
c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*
x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x
 + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d
*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
c) + a + b)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c)), -1/2*(2*sqr
```

```
t(-b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x
+ c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3
+ (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x
+ c))*sqrt(-b/(a + b))/b) - 2*sqrt(-b/(a + b))*(cosh(d*x + c) + sinh(d*x +
c))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a +
 b))/b) - cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) - sinh(d*x + c)^2
 - 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 2.00, size = 520, normalized size = 9.81

$$\frac{e^{c+dx}}{2d(a+b)}+\frac{e^{-c-dx}}{2d(a+b)}-\frac{\sqrt{b}\left(2\,\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\sqrt{-d^2(a+b)^3}}{2\sqrt{b}\,d\,(a+b)}\right)-2\,\mathrm{atan}\left(\frac{\left(e^{dx}\,e^{c}\left(\frac{2a\sqrt{b}}{d(a+b)^2(a^3+3a^2b+3ab^2+b^3)}+\frac{4(2a^2b^{3/2}d+2ab^{5/2}d)}{(a+b)\sqrt{-d^2(a+b)^3}\sqrt{-a^3d^2-3a^2bd^2-3ab^2d^2-b^3d^2}\,(a^3+3a^2b+3ab^2+b^3)}\right)+\frac{2a\sqrt{b}\,e^{3c}\,e^{3dx}}{d(a+b)^2(a^3+3a^2b+3ab^2+b^3)}\right)\left(a^4\sqrt{-a^3d^2-3a^2bd^2-3ab^2d^2-b^3d^2}+b^4\sqrt{-a^3d^2-3a^2bd^2-3ab^2d^2-b^3d^2}+4ab^3\sqrt{-a^3d^2-3a^2bd^2-3ab^2d^2-b^3d^2}+4a^3b\sqrt{-a^3d^2-3a^2bd^2-3ab^2d^2-b^3d^2}+6a^2b^2\sqrt{-a^3d^2-3a^2bd^2-3ab^2d^2-b^3d^2}\right)}{4ab}\right)\right)}{2\sqrt{-a^3d^2-3a^2bd^2-3ab^2d^2-b^3d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] exp(c + d*x)/(2*d*(a + b)) + exp(- c - d*x)/(2*d*(a + b)) - (b^(1/2)*(2*ata
n((exp(d*x)*exp(c)*(-d^2*(a + b)^3)^(1/2))/(2*b^(1/2)*d*(a + b))) - 2*atan(
((exp(d*x)*exp(c)*((2*a*b^(1/2))/(d*(a + b)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^
3)) + (4*(2*a^2*b^(3/2)*d + 2*a*b^(5/2)*d))/((a + b)*(-d^2*(a + b)^3)^(1/2)
*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)*(3*a*b^2 + 3*a^2*b
```

```
 + a^3 + b^3))) + (2*a*b^(1/2)*exp(3*c)*exp(3*d*x))/(d*(a + b)^2*(3*a*b^2 +
 3*a^2*b + a^3 + b^3)))*(a^4*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d
^2)^(1/2) + b^4*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 4
*a*b^3*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 4*a^3*b*(-
 a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 6*a^2*b^2*(- a^3*d^
2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)))/(4*a*b))))/(2*(- a^3*d^2 -
 b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))
```

3.29 $\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}\,d}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d+arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/
a/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3745, 400, 213, 214}

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/
Sqrt[a + b]])/(a*Sqrt[a + b]*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
```

```
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a+b-bx^2)}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(c+dx)\right)}{ad}+\frac{b\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \text{sech}(c+dx)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 123, normalized size = 2.24

$$\frac{i\sqrt{b}\,\text{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+i\sqrt{b}\,\text{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\sqrt{a+b}\,\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a\sqrt{a+b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
[Out] (I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] +
 I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] +
 Sqrt[a + b]*Log[Tanh[(c + d*x)/2]])/(a*Sqrt[a + b]*d)
```

Maple [A]
time = 2.40, size = 67, normalized size = 1.22

method	result
derivativedivides	$\frac{\frac{b\,\text{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a+4b}{4\sqrt{ab+b^2}}\right)}{a\sqrt{ab+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{\frac{b\,\text{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a+4b}{4\sqrt{ab+b^2}}\right)}{a\sqrt{ab+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$

risch	$\frac{\ln(e^{dx+c}-1)}{da} - \frac{\ln(e^{dx+c}+1)}{da} + \frac{\sqrt{b(a+b)}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{b(a+b)}\,e^{dx+c}}{a+b}+1\right)}{2(a+b)da} - \frac{\sqrt{b(a+b)}\,\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*b/(a*b+b^2)^(1/2)*arctanh(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+2*a+4*b)/
(a*b+b^2)^(1/2))+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*c)
 + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(47) = 94$.
time = 0.40, size = 587, normalized size = 10.67

$$\left[\frac{\sqrt{\frac{b}{a+b}}\log\left(\frac{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2(a+3b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2+a+3b\right)\sinh(dx+c)^2+4\left((a+b)\cosh(dx+c)^3+(a+3b)\cosh(dx+c)\right)\sinh(dx+c)+4\left((a+b)\cosh(dx+c)^3+3(a+b)\cosh(dx+c)\sinh(dx+c)^2+(a+b)\sinh(dx+c)^3+(a+b)\cosh(dx+c)+\left(3(a+b)\cosh(dx+c)^2+a+b\right)\sinh(dx+c)\right)\sqrt{\frac{b}{a+b}}+a+b}{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2(a-b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2+a-b\right)\sinh(dx+c)^2+4\left((a+b)\cosh(dx+c)^3+(a-b)\cosh(dx+c)\right)\sinh(dx+c)+a+b}\right)-2\log\left(\cosh(dx+c)+\sinh(dx+c)+1\right)+2\log\left(\cosh(dx+c)+\sinh(dx+c)-1\right)}{2ad}, \frac{\sqrt{-\frac{b}{a+b}}\arctan\left(\frac{\left((a+b)\cosh(dx+c)^3+3(a+b)\cosh(dx+c)\sinh(dx+c)^2+(a+b)\sinh(dx+c)^3+(a-3b)\cosh(dx+c)+\left(3(a+b)\cosh(dx+c)^2+a-3b\right)\sinh(dx+c)\right)\sqrt{-\frac{b}{a+b}}}{2b}\right)-\sqrt{-\frac{b}{a+b}}\arctan\left(\frac{\left((a+b)\cosh(dx+c)+(a+b)\sinh(dx+c)\right)\sqrt{-\frac{b}{a+b}}}{2b}\right)-\log\left(\cosh(dx+c)+\sinh(dx+c)+1\right)+\log\left(\cosh(dx+c)+\sinh(dx+c)-1\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(
d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x +
 c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 +
 (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))
*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2
*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x
+ c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 2*log(cosh(d*x +
c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(a*d),
(sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) +
(3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) -
```

```
sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))
*sqrt(-b/(a + b))/b) - log(cosh(d*x + c) + sinh(d*x + c) + 1) + log(cosh(d*
x + c) + sinh(d*x + c) - 1))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.92, size = 284, normalized size = 5.16

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\left(9\,b^4\,\sqrt{-a^2\,d^2}+16\,a^2\,b^2\,\sqrt{-a^2\,d^2}+24\,a\,b^3\,\sqrt{-a^2\,d^2}\right)}{16\,d\,a^3\,b^2+24\,d\,a^2\,b^3+9\,d\,a\,b^4}\right)}{\sqrt{-a^2\,d^2}}-\frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-a^3\,d^2-b\,a^2\,d^2}\,\sqrt{-a^2\,d^2\,\left(a+b\right)}+\mathrm{e}^{3\,c}\,\mathrm{e}^{3\,d\,x}\,\sqrt{-a^3\,d^2-b\,a^2\,d^2}\,\sqrt{-a^2\,d^2\,\left(a+b\right)}+4\,a^2\,b\,d^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c}{2\,a\,\sqrt{b}\,d\,\sqrt{-a^2\,d^2\,\left(a+b\right)}}\right)-2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-a^2\,d^2\,\left(a+b\right)}}{2\,a\,\sqrt{b}\,d}\right)\right)}{2\,\sqrt{-a^3\,d^2-b\,a^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] - (2*atan((exp(d*x)*exp(c)*(9*b^4*(-a^2*d^2)^(1/2) + 16*a^2*b^2*(-a^2*d^2)^
(1/2) + 24*a*b^3*(-a^2*d^2)^(1/2)))/(24*a^2*b^3*d + 16*a^3*b^2*d + 9*a*b^4*
d)))/(-a^2*d^2)^(1/2) - (b^(1/2)*(2*atan((exp(d*x)*exp(c)*(- a^3*d^2 - a^2*
b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2) + exp(3*c)*exp(3*d*x)*(- a^3*d^2 - a^
2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2) + 4*a^2*b*d^2*exp(d*x)*exp(c))/(2*a
*b^(1/2)*d*(-a^2*d^2*(a + b))^(1/2))) - 2*atan((exp(d*x)*exp(c)*(-a^2*d^2*(
a + b))^(1/2))/(2*a*b^(1/2)*d))))/(2*(- a^3*d^2 - a^2*b*d^2)^(1/2))
```

3.30 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{b}\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\coth(c+dx)}{ad}$$

```
[Out] -coth(d*x+c)/a/d-arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(3/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3744, 331, 211}

$$-\frac{\sqrt{b}\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - Coth[c +
 d*x]/(a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{\coth(c+dx)}{ad} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{ad}\\ &= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 1.00

$$-\frac{\sqrt{b}\,\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - Coth[c +
 d*x]/(a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(40) = 80$.
time = 2.53, size = 188, normalized size = 3.92

method	result
risch	$-\frac{2}{da\left(e^{2dx+2c}-1\right)} + \frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2a^2d} - \frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a^2d}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+2b\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(a+\sqrt{b(a+\right.}{2a\sqrt{b}}\right.}{d}$

default	$$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+2b\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(a+\sqrt{b(a+\right.}{2a\sqrt{b(}}\right.}{d}$$	

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)+2*b*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b)
)^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/(
(2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(
1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
b*(a+b))^(1/2)+a+2*b)*a)^(1/2)))-1/2/a/tanh(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.51, size = 62, normalized size = 1.29

$$\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,ad}+\frac{2}{(ae^{(-2\,dx-2\,c)}-a)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*d)
+ 2/((a*e^(-2*d*x - 2*c) - a)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(40) = 80$.
time = 0.39, size = 618, normalized size = 12.88

$$\left[\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+2ab+b^2)\cosh(dx+c)^4+4(a^2+2ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2ab+b^2)\sinh(dx+c)^4+2(a^2-b^2)\cosh(dx+c)^2+2\left(3(a^2+2ab+b^2)\cosh(dx+c)^2+a^2-b^2\right)\sinh(dx+c)^2+a^2-6ab+b^2+4\left((a^2+2ab+b^2)\cosh(dx+c)^3+(a^2-b^2)\cosh(dx+c)\right)\sinh(dx+c)-4\left((a^2+ab)\cosh(dx+c)^2+2(a^2+ab)\cosh(dx+c)\sinh(dx+c)+(a^2+ab)\sinh(dx+c)^2+a^2-ab\right)\sqrt{-\frac{b}{a}}}{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2(a-b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2+a-b\right)\sinh(dx+c)^2+4\left((a+b)\cosh(dx+c)^3+(a-b)\cosh(dx+c)\right)\sinh(dx+c)+a+b}\right)-4}{2\left(ad\cosh(dx+c)^2+2ad\cosh(dx+c)\sinh(dx+c)+ad\sinh(dx+c)^2-ad\right)}, -\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cosh(dx+c)^2+2(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a-b\right)\sqrt{\frac{b}{a}}}{2b}\right)+2}{ad\cosh(dx+c)^2+2ad\cosh(dx+c)\sinh(dx+c)+ad\sinh(dx+c)^2-ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2
*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a
```

```
^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cos
h(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh
(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*c
osh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b
)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4)/(a*
d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2
 - a*d), -((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
+ c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 2)/(a*
d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2
 - a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [A]
time = 0.65, size = 69, normalized size = 1.44

$$-\frac{\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] -(b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(
sqrt(a*b)*a) + 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B]
time = 1.31, size = 136, normalized size = 2.83

$$\frac{2}{a\,d-a\,d\,\mathrm{e}^{2\,c+2\,d\,x}}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a^3\,d^2}}{2\,a\,\sqrt{b}\,d}-\frac{\sqrt{b}\,\sqrt{a^3\,d^2}}{2\,a^2\,d}+\frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{a^3\,d^2}}{2\,a\,\sqrt{b}\,d}+\frac{\sqrt{b}\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{a^3\,d^2}}{2\,a^2\,d}\right)}{\sqrt{a^3\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] 2/(a*d - a*d*exp(2*c + 2*d*x)) - (b^(1/2)*atan((a^3*d^2)^(1/2)/(2*a*b^(1/2)
*d) - (b^(1/2)*(a^3*d^2)^(1/2))/(2*a^2*d) + (exp(2*c)*exp(2*d*x)*(a^3*d^2)^
(1/2))/(2*a*b^(1/2)*d) + (b^(1/2)*exp(2*c)*exp(2*d*x)*(a^3*d^2)^(1/2))/(2*a
^2*d)))/(a^3*d^2)^(1/2)
```

3.31 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\sqrt{b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

```
[Out] 1/2*(a+2*b)*arctanh(cosh(d*x+c))/a^2/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d-arct
anh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 482, 536, 213, 214}

$$\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\sqrt{b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Sqrt[b]*Sqrt[a + b]*ArcTanh
[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*
x])/(2*a*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+b-bx^2)}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}\\
&= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\operatorname{Subst}\left(\int \frac{a+b+bx^2}{(-1+x^2)(a+b-bx^2)}\,dx, x, \operatorname{sech}(c+dx)\right)}{2ad}\\
&= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{(b(a+b))\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{a^2d} -\\
&= \frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d} - \frac{\sqrt{b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \coth(
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.47, size = 170, normalized size = 2.00

$$-\frac{8i\sqrt{b}\,\sqrt{a+b}\,\operatorname{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+8i\sqrt{b}\,\sqrt{a+b}\,\operatorname{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+4a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+8b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/8*((8*I)*Sqrt[b]*Sqrt[a + b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c
+ d*x)/2])/Sqrt[b]] + (8*I)*Sqrt[b]*Sqrt[a + b]*ArcTan[((-I)*Sqrt[a + b] +
Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] + a*Csch[(c + d*x)/2]^2 + 4*a*Log[Tanh[
(c + d*x)/2]] + 8*b*Log[Tanh[(c + d*x)/2]] + a*Sech[(c + d*x)/2]^2)/(a^2*d)
```

Maple [A]
time = 2.84, size = 111, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}-\frac{(a+b)b\,\mathrm{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a+4b}{4\sqrt{ab+b^2}}\right)}{a^2\sqrt{ab+b^2}}}{d}$
default	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}-\frac{(a+b)b\,\mathrm{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a+4b}{4\sqrt{ab+b^2}}\right)}{a^2\sqrt{ab+b^2}}}{d}$
risch	$-\frac{e^{dx+c}\left(1+e^{2dx+2c}\right)}{da\left(e^{2dx+2c}-1\right)^2}-\frac{b\ln\left(e^{dx+c}-1\right)}{a^2d}-\frac{\ln\left(e^{dx+c}-1\right)}{2da}+\frac{b\ln\left(e^{dx+c}+1\right)}{a^2d}+\frac{\ln\left(e^{dx+c}+1\right)}{2da}+\frac{\sqrt{ab+b^2}\ln\left(e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^2*(-2*a-
4*b)*ln(tanh(1/2*d*x+1/2*c))-(a+b)*b/a^2/(a*b+b^2)^(1/2)*arctanh(1/4*(2*a*t
anh(1/2*d*x+1/2*c)^2+2*a+4*b)/(a*b+b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*
c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a +
2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*((a*b*e^(3*c)
+ b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3 + a^2*b + (a^3
*e^(4*c) + a^2*b*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*
d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 858 vs. 2(73) = 146.
time = 0.43, size = 1790, normalized size = 21.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d
*x + c)^3 - (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x +
 c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(
cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(a*b + b^2)*log(((a
 + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*s
inh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2
 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d
*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)
^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c
))*sqrt(a*b + b^2) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 2*a*cosh(d*x +
c) - ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)
*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3
- (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sin
h(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2
*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cos
h(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d
*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(
a^2*d*cosh(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(
d*x + c)^4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 -
 a^2*d)*sinh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*s
inh(d*x + c)), -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^
2 + 2*a*sinh(d*x + c)^3 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c
)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(
d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-a
*b - b^2)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a +
b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-a*b - b^2)/(a*b + b^2))
- 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 +
2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x
 + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-a*b - b^2)*arctan(1/2*sqr
t(-a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/b) + 2*a*cosh(d*x + c) - ((a
+ 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2
*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x
 + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3 - (a + 2*
b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sinh(d*x + c
) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a +
2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c
)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(a^2*d*cos
```

```
h(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(d*x + c)^
4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 - a^2*d)*s
inh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*sinh(d*x +
 c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 1.80, size = 787, normalized size = 9.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(18*b^7*(-a^4*d^2)^(1/2) + 48*a^2*b^5*(-a^4*d^2)^(1/
2) + 27*a^3*b^4*(-a^4*d^2)^(1/2) + 8*a^4*b^3*(-a^4*d^2)^(1/2) + a^5*b^2*(-a
^4*d^2)^(1/2) + 45*a*b^6*(-a^4*d^2)^(1/2)))/(9*a^2*b^6*d*(4*a*b + a^2 + 4*b
^2)^(1/2) + 18*a^3*b^5*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 15*a^4*b^4*d*(4*a*b
+ a^2 + 4*b^2)^(1/2) + 6*a^5*b^3*d*(4*a*b + a^2 + 4*b^2)^(1/2) + a^6*b^2*d*
(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 + 4*b^2)^(1/2))/(-a^4*d^2)^(1/2)
 - ((2*atan((exp(d*x)*exp(c)*(a + b)*(-a^4*d^2)^(1/2))/(2*a^2*d*(b*(a + b))
^(1/2))) + 2*atan(((exp(d*x)*exp(c)*((64*(2*a^4*b*d*(a*b + b^2)^(1/2) + 6*a
^2*b^3*d*(a*b + b^2)^(1/2) + 6*a^3*b^2*d*(a*b + b^2)^(1/2)))/(a^9*d^2*(a +
```

```
b)^2*(2*a*b + a^2 + b^2)) - (32*(3*b^4*(-a^4*d^2)^(1/2) + 4*a^2*b^2*(-a^4*d
^2)^(1/2) + 6*a*b^3*(-a^4*d^2)^(1/2) + a^3*b*(-a^4*d^2)^(1/2)))/(a^7*d*(a +
 b)*(-a^4*d^2)^(1/2)*(b*(a + b))^(1/2)*(2*a*b + a^2 + b^2))) - (32*exp(3*c)
*exp(3*d*x)*(3*b^4*(-a^4*d^2)^(1/2) + 4*a^2*b^2*(-a^4*d^2)^(1/2) + 6*a*b^3*
(-a^4*d^2)^(1/2) + a^3*b*(-a^4*d^2)^(1/2)))/(a^7*d*(a + b)*(-a^4*d^2)^(1/2)
*(b*(a + b))^(1/2)*(2*a*b + a^2 + b^2)))*(a^8*(-a^4*d^2)^(1/2) + a^5*b^3*(-
a^4*d^2)^(1/2) + 3*a^6*b^2*(-a^4*d^2)^(1/2) + 3*a^7*b*(-a^4*d^2)^(1/2)))/(1
92*a*b^2 + 64*a^2*b + 192*b^3)))*(a*b + b^2)^(1/2))/(2*(-a^4*d^2)^(1/2)) -
exp(c + d*x)/(a*d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x))/(a*d*(exp(4*c
+ 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.32 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b}\,(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

```
[Out] (a+b)*coth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d+(a+b)*arctan(b^(1/2)*tanh(d*x
+c)/a^(1/2))*b^(1/2)/a^(5/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3744, 464, 331, 211}

$$\frac{\sqrt{b}\,(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*d) + ((a
 + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{\coth^3(c+dx)}{3ad} - \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{ad}\\
&= \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} + \frac{(b(a+b))\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+
\end{aligned}$$

$$= \frac{\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}$$

Mathematica [A]
time = 0.23, size = 71, normalized size = 1.01

$$\frac{3\sqrt{b}\,(a+b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + \sqrt{a}\coth(c+dx)\left(2a+3b-a\operatorname{csch}^2(c+dx)\right)}{3a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
[Out] (3*Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Coth[c
 + d*x]*(2*a + 3*b - a*Csch[c + d*x]^2))/(3*a^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(60) = 120.
time = 2.80, size = 247, normalized size = 3.53

method	result

risch	$-\frac{2(-3b\,e^{4dx+4c}+6a\,e^{2dx+2c}+6b\,e^{2dx+2c}-2a-3b)}{3d\,a^2(e^{2dx+2c}-1)^3}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a^2d}+\frac{\sqrt{-ab}\,\ln\left(e^{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{2b(a+b)\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\cdots}{\sqrt{\left(2\sqrt{b(a+\cdots}\right.}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-\cdots\right.}}\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{2b(a+b)\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\cdots}{\sqrt{\left(2\sqrt{b(a+\cdots}\right.}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-\cdots\right.}}\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a^2*(1/3*a*tanh(1/2*d*x+1/2*c)^3-3*a*tanh(1/2*d*x+1/2*c)-4*b*tanh
(1/2*d*x+1/2*c))-2*b/a*(a+b)*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)
/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(
a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((
2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2)))-1/24/a/tanh(1/2*d*x+1/2*c)^3-1/8/a^2*(-3*a-4*b)/t
anh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(60) = 120$.
time = 0.51, size = 134, normalized size = 1.91

$$\frac{2\left(6(a+b)e^{(-2dx-2c)}-3be^{(-4dx-4c)}-2a-3b\right)}{3\left(3a^2e^{(-2dx-2c)}-3a^2e^{(-4dx-4c)}+a^2e^{(-6dx-6c)}-a^2\right)d}-\frac{(ab+b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2/3*(6*(a + b)*e^(-2*d*x - 2*c) - 3*b*e^(-4*d*x - 4*c) - 2*a - 3*b)/((3*a^2
*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c) + a^2*e^(-6*d*x - 6*c) - a^2)*d)
 - (a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sq
rt(a*b)*a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(60) = 120$.
time = 0.39, size = 1628, normalized size = 23.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*sinh
(d*x + c)^4 - 24*(a + b)*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 - a - b)
*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(a + b)*cosh(d*x + c)^4 + 3*(5*(
a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c
)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2
+ 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*
x + c)^2 + 6*((a + b)*cosh(d*x + c)^5 - 2*(a + b)*cosh(d*x + c)^3 + (a + b)
*cosh(d*x + c))*sinh(d*x + c) - a - b)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b
 + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c)
)*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x +
 c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((
a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*
sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2
+ a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x +
c))*sinh(d*x + c) + a + b)) + 48*(b*cosh(d*x + c)^3 - (a + b)*cosh(d*x + c)
)*sinh(d*x + c) + 8*a + 12*b)/(a^2*d*cosh(d*x + c)^6 + 6*a^2*d*cosh(d*x + c
)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3*a^2
*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)^4 +
4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*d
 + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2*d)*sinh(d*x +
 c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 + a^2*d*cosh(d*x
 + c))*sinh(d*x + c)), 1/3*(6*b*cosh(d*x + c)^4 + 24*b*cosh(d*x + c)*sinh(d
*x + c)^3 + 6*b*sinh(d*x + c)^4 - 12*(a + b)*cosh(d*x + c)^2 + 12*(3*b*cosh
(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^6 + 6*(a +
b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(a + b)*cosh
(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^4 + 4*(5*
```

```
(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a +
 b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(a + b)*cosh(d*x + c
)^2 + a + b)*sinh(d*x + c)^2 + 6*((a + b)*cosh(d*x + c)^5 - 2*(a + b)*cosh(
d*x + c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) - a - b)*sqrt(b/a)*arctan
(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a
+ b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 24*(b*cosh(d*x + c)^3 - (a + b
)*cosh(d*x + c))*sinh(d*x + c) + 4*a + 6*b)/(a^2*d*cosh(d*x + c)^6 + 6*a^2*
d*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x
+ c)^4 + 3*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh
(d*x + c)^4 + 4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x
+ c)^3 - a^2*d + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2
*d)*sinh(d*x + c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 +
a^2*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Giac [A]
time = 0.64, size = 119, normalized size = 1.70

$$\frac{\frac{3\left(ab+b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\left(3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+2\,a+3\,b\right)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a -
b)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c
) - 6*b*e^(2*d*x + 2*c) + 2*a + 3*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B]
time = 1.44, size = 254, normalized size = 3.63

$$\frac{2\,b}{a^2\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{8}{3\,a\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{4}{a\,d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{\sqrt{-b}\,\ln\left(-\frac{4\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{a^2}-\frac{2\,\sqrt{-b}\,\left(a\,d+b\,d+a\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{a^{5/2}\,d}\right)\,\left(a+b\right)}{2\,a^{5/2}\,d}-\frac{\sqrt{-b}\,\ln\left(\frac{2\,\sqrt{-b}\,\left(a\,d+b\,d+a\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{a^{5/2}\,d}-\frac{4\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{a^2}\right)\,\left(a+b\right)}{2\,a^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] (2*b)/(a^2*d*(exp(2*c + 2*d*x) - 1)) - 8/(3*a*d*(3*exp(2*c + 2*d*x) - 3*exp
(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - 4/(a*d*(exp(4*c + 4*d*x) - 2*exp(2
*c + 2*d*x) + 1)) + ((-b)^(1/2)*log(- (4*b*exp(2*c + 2*d*x))/a^2 - (2*(-b)^
(1/2)*(a*d + b*d + a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(5/2)*d
))*(a + b))/(2*a^(5/2)*d) - ((-b)^(1/2)*log((2*(-b)^(1/2)*(a*d + b*d + a*d*
exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(5/2)*d) - (4*b*exp(2*c + 2*d*
x))/a^2)*(a + b))/(2*a^(5/2)*d)
```

3.33 $\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=192

$$\frac{3(a^2-6ab+b^2)\,x}{8(a+b)^4}+\frac{3\sqrt{a}\,(a-b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2(a+b)^4d}-\frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}+\frac{\cosh}{4(a+}$$

```
[Out] 3/8*(a^2-6*a*b+b^2)*x/(a+b)^4+3/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))
*a^(1/2)*b^(1/2)/(a+b)^4/d-1/8*(5*a-b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d/(a
+b*tanh(d*x+c)^2)+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)
+3/8*(3*a-b)*b*tanh(d*x+c)/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.18, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3744, 481, 541, 536, 212, 211}

$$\frac{3x(a^2-6ab+b^2)}{8(a+b)^4}+\frac{3\sqrt{a}\,\sqrt{b}\,(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2d(a+b)^4}+\frac{3b(3a-b)\tanh(c+dx)}{8d(a+b)^3\left(a+b\tanh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{(5a-b)\sinh(c+dx)\cosh(c+dx)}{8d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (3*(a^2 - 6*a*b + b^2)*x)/(8*(a + b)^4) + (3*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan
[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*(a + b)^4*d) - ((5*a - b)*Cosh[c + d*
x]*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)) + (Cosh[c + d*x]^
3*Sinh[c + d*x])/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)) + (3*(3*a - b)*b*Tan
h[c + d*x])/(8*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(4a-b)x^2}{(1-x^2)^2(a+bx^2)^2}\,dx, x, \tanh(c+d\right.}{4(a+b)d}$$

$$= -\frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} -$$

$$= -\frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} +$$

$$= -\frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} +$$

$$= \frac{3(a^2-6ab+b^2)\,x}{8(a+b)^4} + \frac{3\sqrt{a}\,(a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2(a+b)^4d} - \frac{(5a-b)\,\text{co}}{8(a+b)^2}$$

Mathematica [A]
time = 0.78, size = 132, normalized size = 0.69

$$\frac{12(a^2-6ab+b^2)(c+dx) + 48\sqrt{a}\,(a-b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right) - 8(a-b)(a+b)\sinh(2(c+dx)) + \frac{16ab(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))} + (a+b)^2\sinh(4(c+dx))}{32(a+b)^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (12*(a^2 - 6*a*b + b^2)*(c + d*x) + 48*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt
[b]*Tanh[c + d*x])/Sqrt[a]] - 8*(a - b)*(a + b)*Sinh[2*(c + d*x)] + (16*a*b
*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + (a + b)^2
*Sinh[4*(c + d*x)])/(32*(a + b)^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(174) = 348$.
time = 2.57, size = 512, normalized size = 2.67 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a*b/(a+b)^4*(((-1/2*a-1/2*b)*tanh(1/2*d*x+1/2*c)^3+(-1/2*a-1/2*b)*t
anh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*
tanh(1/2*d*x+1/2*c)^2+a)+1/2*(3*a-3*b)*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*
```

```
(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+
b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))+1/4/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^
4+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(a-7*b)/(a+b)^3/(tanh(1/2*d*x+1
/2*c)-1)^2-1/8*(3*a-5*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+1/8/(a+b)^4*(-3*a^
2+18*a*b-3*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/4/(a+b)^2/(tanh(1/2*d*x+1/2*c)+
1)^4+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(-a+7*b)/(a+b)^3/(tanh(1/2*d
*x+1/2*c)+1)^2-1/8*(3*a-5*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)+1/8/(a+b)^4*(3
*a^2-18*a*b+3*b^2)*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(174) = 348$.
time = 0.71, size = 1690, normalized size = 8.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*b - 2*b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
+ a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/2*b*log((a + b
)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d) + 1/4*(a*b - 2*b^2)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b
)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)
 + 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)
/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/32*(3*a^3*b - 33*a^2*b^2 + 13*a*b^
3 + b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 4*
a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)*d) + 1/8*(3*a^2*b - 6*a*b^
2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/32*(3*a^3*b - 33*a^2*b^2 + 13*a
*b^3 + b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5
+ 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)*d) - 1/8*(3*a^2*b - 6*
a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 3/16*(3*a*b + b^2)*arctan(1/
2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sq
rt(a*b)*d) - 1/16*(a^3*b - 5*a^2*b^2 - 5*a*b^3 + b^4 + (a^3*b - 15*a^2*b^2
+ 15*a*b^3 - b^4)*e^(2*d*x + 2*c))/((a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^
3 + 5*a^2*b^4 + a*b^5 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^
4 + a*b^5)*e^(4*d*x + 4*c) + 2*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a
^2*b^4 - a*b^5)*e^(2*d*x + 2*c))*d) + 1/16*(a^3*b - 5*a^2*b^2 - 5*a*b^3 + b
^4 + (a^3*b - 15*a^2*b^2 + 15*a*b^3 - b^4)*e^(-2*d*x - 2*c))/((a^6 + 5*a^5*
b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5 + 2*(a^6 + 3*a^5*b + 2*a^4*
b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*e^(-2*d*x - 2*c) + (a^6 + 5*a^5*b + 10
*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*e^(-4*d*x - 4*c))*d) - 1/4*(a^2*
```

```
b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x + 2*c))/((a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e
^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*e^(2*d*x + 2*c))*d)
+ 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(-2*d*x - 2*c))/((a^5 + 4*a^
4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)
*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(-4*d
*x - 4*c))*d) + 3/8*(a*b + b^2 + (a*b - b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^
3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c
) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*c))*d) + 3/8*(d*x + c
)/((a^2 + 2*a*b + b^2)*d) + 1/64*((a + b)*e^(4*d*x + 4*c) + 16*b*e^(2*d*x +
 2*c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/64*(16*b*e^(-2*d*x - 2*c) +
(a + b)*e^(-4*d*x - 4*c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/8*e^(2*d*
x + 2*c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b + b^2
)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3531 vs. $2(174) = 348$.
time = 0.46, size = 7366, normalized size = 38.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/64*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 12*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*sinh(d*x + c)^12 - 6*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^10 -
 6*(a^3 + a^2*b - a*b^2 - b^3 - 11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^3 - 3*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - (1
5*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d
*x)*cosh(d*x + c)^8 + (495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4
- 15*a^3 + 19*a^2*b + 19*a*b^2 - 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3
)*d*x - 270*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
8*(99*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 - 90*(a^3 + a^2*b - a
*b^2 - b^3)*cosh(d*x + c)^3 - (15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(
a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 16*(4*
a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^6 +
4*(231*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 315*(a^3 + a^2*b -
 a*b^2 - b^3)*cosh(d*x + c)^4 - 16*a^2*b + 16*a*b^2 + 12*(a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*d*x - 7*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*
a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 189*(a^3 + a^2*b - a*b^2 - b^3)
*cosh(d*x + c)^5 - 7*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a
^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 12*(4*a^2*b - 4*a*b^2 - 3*(a^3
```

- 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + (15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cos h(d*x + c)^4 + (495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 - 1260*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^6 - 70*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x - 240*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 - 180*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^7 - 14*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 80*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^3 + (15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 6*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 - 135*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^8 - 14*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 120*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^4 + 3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + 3*(15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 48*((a^2 - b^2)*cos h(d*x + c)^8 + 8*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - b^2)*si nh(d*x + c)^8 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^6 + 2*(14*(a^2 - b^2)*c osh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^6 + 4*(14*(a^2 - b^2)*cos h(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^2 - b^2)*cosh(d*x + c)^4 + (70*(a^2 - b^2)*cosh(d*x + c)^4 + 30*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^4 + 4*(14*(a^2 - b^2)*cos h(d*x + c)^5 + 10*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*(a^2 - b^2)*cosh(d*x + c)^6 + 15*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(a^2 - b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(a^2 - b^2)*cosh(d*x + c)^7 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^5 + (a^2 - b^2)*cosh(d*x + c)^3)*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*co sh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^11 - 15*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^9 - 2*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 24*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*cosh(d*x + c)^5 + (15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^

```
2 + b^3)*d*x)*cosh(d*x + c)^3 + 3*(a^3 + a^2*b ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sinh(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(174) = 348.
time = 2.06, size = 476, normalized size = 2.48

$$\frac{\frac{24\left(a^2-6ab+b^2\right)(dx+c)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{\left(18a^2e^{(4dx+4c)}-108abe^{(4dx+4c)}+18b^2e^{(4dx+4c)}-8a^2e^{(2dx+2c)}+8b^2e^{(2dx+2c)}+a^2+2ab+b^2\right)e^{(-4dx-4c)}}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{96\left(a^2b-ab^2\right)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\sqrt{ab}}+\frac{a^2e^{(4dx+4c)}+2abe^{(4dx+4c)}+b^2e^{(4dx+4c)}-8a^2e^{(2dx+2c)}+8b^2e^{(2dx+2c)}}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{64\left(a^2be^{(2dx+2c)}-ab^2e^{(2dx+2c)}+a^2b+ab^2\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 - 6*a*b + b^2)*(d*x + c)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
 + b^4) - (18*a^2*e^(4*d*x + 4*c) - 108*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d
*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 8*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b
^2)*e^(-4*d*x - 4*c)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 96*(a^2*
b - a*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) + (a^2*e^(4*d
*x + 4*c) + 2*a*b*e^(4*d*x + 4*c) + b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x +
2*c) + 8*b^2*e^(2*d*x + 2*c))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) -
 64*(a^2*b*e^(2*d*x + 2*c) - a*b^2*e^(2*d*x + 2*c) + a^2*b + a*b^2)/((a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c
) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sinh(c+d\,x)}^4}{{\left(b\,{\tanh\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)
```

3.34 $\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=124

$$\frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d}-\frac{(a-b)\cosh(c+dx)}{(a+b)^3d}+\frac{\cosh^3(c+dx)}{3(a+b)^2d}+\frac{ab\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\text{sech}^2(c+\right.}$$

```
[Out] -(a-b)*cosh(d*x+c)/(a+b)^3/d+1/3*cosh(d*x+c)^3/(a+b)^2/d+1/2*a*b*sech(d*x+c
)/(a+b)^3/d/(a+b-b*sech(d*x+c)^2)+1/2*(3*a-2*b)*arctanh(sech(d*x+c)*b^(1/2)
/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3745, 467, 1275, 214}

$$\frac{\cosh^3(c+dx)}{3d(a+b)^2}-\frac{(a-b)\cosh(c+dx)}{d(a+b)^3}+\frac{ab\text{sech}(c+dx)}{2d(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)}+\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*(a +
b)^(7/2)*d) - ((a - b)*Cosh[c + d*x])/((a + b)^3*d) + Cosh[c + d*x]^3/(3*(a
 + b)^2*d) + (a*b*Sech[c + d*x])/(2*(a + b)^3*d*(a + b - b*Sech[c + d*x]^2)
)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+b-bx^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= \frac{ab\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{b\text{Subst}\left(\int \frac{-\frac{2}{b(a+b)}+\frac{2ax^2}{b(a+b)^2}+\frac{ax^4}{(a+b)^3}}{x^4(a+b-bx^2)}\,dx, x\right.}{2d}$$

$$= \frac{ab\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)} + \frac{b\text{Subst}\left(\int \left(-\frac{2}{b(a+b)^2x^4}+\frac{2(a-b)}{b(a+b)^3x^2}+\right.\right.}{2d}$$

$$= -\frac{(a-b)\cosh(c+dx)}{(a+b)^3d} + \frac{\cosh^3(c+dx)}{3(a+b)^2d} + \frac{ab\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\text{sech}^2(c+d\right.}$$

$$= \frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d} - \frac{(a-b)\cosh(c+dx)}{(a+b)^3d} + \frac{\cosh^3(c+}{3(a+b)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.95, size = 160, normalized size = 1.29

$$\frac{\frac{6i(3a-2b)\sqrt{b}\left(\text{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\text{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{7/2}}+\frac{3\cosh(c+dx)\left(5b+a\left(-3+\frac{4b}{a-b+(a+b)\cosh(2(c+dx))}\right)\right)}{(a+b)^3}+\frac{\cosh(3(c+dx))}{(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (((6*I)*(3*a - 2*b)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d
*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sq
```

```
rt[b]]))/(a + b)^(7/2) + (3*Cosh[c + d*x]*(5*b + a*(-3 + (4*b)/(a - b + (a
+ b)*Cosh[2*(c + d*x)]))))/(a + b)^3 + Cosh[3*(c + d*x)]/(a + b)^2)/(12*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(110) = 220$.
time = 2.61, size = 267, normalized size = 2.15

method	result
derivativedivides	$\frac{-\frac{4b\left(\frac{\left(-\frac{b}{2}-\frac{a}{4}\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a}{4}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{(3a-2b)\operatorname{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a+4b}{4\sqrt{ab+b^2}}\right)}{8\sqrt{ab+b^2}}\right)}{(a+b)^3}+\frac{}{3(a+b}}{}$
default	$\frac{-\frac{4b\left(\frac{\left(-\frac{b}{2}-\frac{a}{4}\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a}{4}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{(3a-2b)\operatorname{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a+4b}{4\sqrt{ab+b^2}}\right)}{8\sqrt{ab+b^2}}\right)}{(a+b)^3}+\frac{}{3(a+b}}{}$
risch	$\frac{e^{3dx+3c}}{24(a^2+2ab+b^2)d}-\frac{3\,e^{dx+c}a}{8(a+b)(a^2+2ab+b^2)d}+\frac{5\,e^{dx+c}b}{8(a+b)(a^2+2ab+b^2)d}-\frac{3\,e^{-dx-c}a}{8(a^3+3a^2b+3a\,b^2+b^3)d}+\frac{5\,e^{-dx-c}b}{8(a^3+3a^2b+3a\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4*b/(a+b)^3*(((-1/2*b-1/4*a)*tanh(1/2*d*x+1/2*c)^2-1/4*a)/(a*tanh(1/2
*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)-1/8*(3
*a-2*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+2*a+4*b)/(a*
b+b^2)^(1/2)))+1/3/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/(a+b)^2/(tanh(1/2*
d*x+1/2*c)+1)^2-1/2*(a-3*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)-1/3/(a+b)^2/(ta
nh(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^3*(-
a+3*b)/(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(a^2 + 2*a*b + b^2 + (a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c))*e^
(10*d*x) - (7*a^2*e^(8*c) - 6*a*b*e^(8*c) - 13*b^2*e^(8*c))*e^(8*d*x) - 2*(
13*a^2*e^(6*c) - 40*a*b*e^(6*c) + 7*b^2*e^(6*c))*e^(6*d*x) - 2*(13*a^2*e^(4
```

```
*c) - 40*a*b*e^(4*c) + 7*b^2*e^(4*c))*e^(4*d*x) - (7*a^2*e^(2*c) - 6*a*b*e^
(2*c) - 13*b^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(7*c) + 4*a^3*b*d*e^(7*c) + 6*
a^2*b^2*d*e^(7*c) + 4*a*b^3*d*e^(7*c) + b^4*d*e^(7*c))*e^(7*d*x) + 2*(a^4*d
*e^(5*c) + 2*a^3*b*d*e^(5*c) - 2*a*b^3*d*e^(5*c) - b^4*d*e^(5*c))*e^(5*d*x)
 + (a^4*d*e^(3*c) + 4*a^3*b*d*e^(3*c) + 6*a^2*b^2*d*e^(3*c) + 4*a*b^3*d*e^(
3*c) + b^4*d*e^(3*c))*e^(3*d*x)) - 1/8*integrate(8*((3*a*b*e^(3*c) - 2*b^2*
e^(3*c))*e^(3*d*x) - (3*a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4 + (a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c)
+ 4*a*b^3*e^(4*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*
c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2650 vs. $2(113) = 226$.
time = 0.41, size = 5025, normalized size = 40.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 10*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)*sinh(d*x + c)^9 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 - (7*a^2 - 6
*a*b - 13*b^2)*cosh(d*x + c)^8 + (45*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 -
7*a^2 + 6*a*b + 13*b^2)*sinh(d*x + c)^8 + 8*(15*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*
a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^6 + 2*(105*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^4 - 14*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^2 - 13*a^2 + 40*a*b - 7
*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 14*(7*a
^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^3 - 3*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^5 - 2*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^4 + 2*(1
05*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 35*(7*a^2 - 6*a*b - 13*b^2)*cosh(d
*x + c)^4 - 15*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^2 - 13*a^2 + 40*a*b
- 7*b^2)*sinh(d*x + c)^4 + 8*(15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 7*(7
*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^5 - 5*(13*a^2 - 40*a*b + 7*b^2)*cosh(d
*x + c)^3 - (13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (7*a
^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^2 + (45*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^8 - 28*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^6 - 30*(13*a^2 - 40*a*b + 7
*b^2)*cosh(d*x + c)^4 - 12*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^2 - 7*a^
2 + 6*a*b + 13*b^2)*sinh(d*x + c)^2 - 6*((3*a^2 + a*b - 2*b^2)*cosh(d*x + c
)^7 + 7*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + a*b
- 2*b^2)*sinh(d*x + c)^7 + 2*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^5 + (21*
(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^2 + 6*a^2 - 10*a*b + 4*b^2)*sinh(d*x +
c)^5 + 5*(7*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^3 + 2*(3*a^2 - 5*a*b + 2*b^
2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^3 +
 (35*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^4 + 20*(3*a^2 - 5*a*b + 2*b^2)*cos
```

h(d*x + c)^2 + 3*a^2 + a*b - 2*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^5 + 20*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^6 + 10*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + a^2 + 2*a*b + b^2 + 2*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 - 4*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^7 - 6*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^5 - 4*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^7 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^6 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*sinh(d*x + c)^7 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^5 + (21*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d)*sinh(d*x + c)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 5*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 20*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)*sinh(d*x + c)^3 + (21*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^5 + 20*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 10*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^8 + (45*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - 7*a^2 + 6*a*b + 13*b^2)*sinh(d*x + c)^8 + 8*(15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^6 + 2*(105*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 14*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^2 - 13*a^2 + 40*a*b - 7*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 14*(7*a^2 - 6*a*b - 1...

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sinh\left(c+d\,x\right)}^3}{{\left(b\,{\tanh\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

3.35 $\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=132

$$-\frac{(a-3b)x}{2(a+b)^3}-\frac{(3a-b)\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a+b)^3d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}-\frac{b\tanh(c+dx}{(a+b)^2d\left(a+b\tanh^2\right.}$$

```
[Out] -1/2*(a-3*b)*x/(a+b)^3-1/2*(3*a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1
/2)/(a+b)^3/d/a^(1/2)+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^
2)-b*tanh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3744, 482, 541, 536, 212, 211}

$$-\frac{\sqrt{b}\,(3a-b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,d(a+b)^3}-\frac{b\tanh(c+dx)}{d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{x(a-3b)}{2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a - 3*b)*x)/(a + b)^3 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(2*Sqrt[a]*(a + b)^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*
(a + b)*d*(a + b*Tanh[c + d*x]^2)) - (b*Tanh[c + d*x])/((a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a-3bx^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right.}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\tanh(c+dx)}{(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\tanh(c+dx)}{(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-3}{}$$

$$= -\frac{(a-3b)x}{2(a+b)^3} - \frac{(3a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a+b)^3d} + \frac{\cosh(c+dx)\sinh(c}{2(a+b)d\left(a+b\tanh^2(\right.}$$

Mathematica [A]
time = 0.49, size = 105, normalized size = 0.80

$$\frac{-2(a-3b)(c+dx) + \frac{2\sqrt{b}(-3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} + (a+b)\sinh(2(c+dx)) - \frac{2b(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{4(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-2*(a - 3*b)*(c + d*x) + (2*Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*
x])/Sqrt[a]])/Sqrt[a] + (a + b)*Sinh[2*(c + d*x)] - (2*b*(a + b)*Sinh[2*(c
+ d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(4*(a + b)^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(118) = 236$.
time = 2.55, size = 387, normalized size = 2.93

method	result
risch	$-\frac{xa}{2(a+b)(a^2+2ab+b^2)} + \frac{3xb}{2(a+b)(a^2+2ab+b^2)} + \frac{e^{2dx+2c}}{8(a^2+2ab+b^2)d} - \frac{e^{-2dx-2c}}{8(a^2+2ab+b^2)d} + \frac{b(a\,e^{2dx+2}}{d(a+b)^3(a\,e^{4dx+4c}+b\,e^{4dx}}$

derivativedivides	$\frac{-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-a+3b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2(a+b)^3}+\frac{4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{4}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\right.}\right.}{}}{}$
default	$\frac{-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-a+3b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2(a+b)^3}+\frac{4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{4}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\right.}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c
)+1)+1/2/(a+b)^3*(-a+3*b)*ln(tanh(1/2*d*x+1/2*c)+1)+4*b/(a+b)^3*(((-1/4*a-1
/4*b)*tanh(1/2*d*x+1/2*c)^3+(-1/4*a-1/4*b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2
*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/4*(3
*a-b)*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-
```

```
a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)
^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/
2))))+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c
)-1)+1/2*(a-3*b)/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(118) = 236$.
time = 0.64, size = 840, normalized size = 6.36

$$\frac{b\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{2(a^3+3a^2b+3ab^2+b^3)d}-\frac{b\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a^3+3a^2b+3ab^2+b^3)d}-\frac{(3a^2b-6ab^2-b^3)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab}\,d}+\frac{(3a^2b-6ab^2-b^3)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab}\,d}+\frac{(3ab+b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{4(a^3+2a^2b+ab^2)\sqrt{ab}\,d}+\frac{a^2b-b^3+(a^2b-6ab^2+b^3)e^{(2dx+2c)}}{4(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4+(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)e^{(4dx+4c)}+2(a^5+2a^4b-2a^2b^3-ab^4)e^{(2dx+2c)})d}-\frac{a^2b-b^3+(a^2b-6ab^2+b^3)e^{(-2dx-2c)}}{4(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4+2(a^5+2a^4b-2a^2b^3-ab^4)e^{(-2dx-2c)}+(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)e^{(-4dx-4c)})d}-\frac{ab+b^2+(ab-b^2)e^{(-2dx-2c)}}{2(a^4+3a^3b+3a^2b^2+ab^3+2(a^4+a^3b-a^2b^2-ab^3)e^{(-2dx-2c)}+(a^4+3a^3b+3a^2b^2+ab^3)e^{(-4dx-4c)})d}-\frac{dx+c}{2(a^2+2ab+b^2)d}+\frac{e^{(2dx+2c)}}{8(a^2+2ab+b^2)d}-\frac{e^{(-2dx-2c)}}{8(a^2+2ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a
 + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/8*(
3*a^2*b - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/8*(3*a^2*b - 6*
a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/4*(3*a*b + b^2)*arctan(1/2
*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqr
t(a*b)*d) + 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x + 2*c))/((a
^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 +
 4*a^2*b^3 + a*b^4)*e^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)
*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(-2*d*x
 - 2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(a^5 + 2*a^4*b
 - 2*a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
*b^3 + a*b^4)*e^(-4*d*x - 4*c))*d) - 1/2*(a*b + b^2 + (a*b - b^2)*e^(-2*d*x
 - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a
*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*
c))*d) - 1/2*(d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(2*d*x + 2*c)/((a^2
+ 2*a*b + b^2)*d) - 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(118) = 236$.
time = 0.40, size = 3918, normalized size = 29.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 - 2*(2*(a^2 - 2*
```

a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 - 2*(2*(a^2 - 2*a*b - 3*b^2)*
d*x - 14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^6 +
4*(14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x
- a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x
- a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 -
4*(a^2 - 4*a*b + 3*b^2)*d*x - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)
*cosh(d*x + c)^2 + 4*a*b - 4*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^5 - 5*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x +
c)^3 - 4*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^3 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c)^
2 + 2*(14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 15*(2*(a^2 - 2*a*b - 3*b^2)
*d*x - a^2 + b^2)*cosh(d*x + c)^4 - 2*(a^2 - 2*a*b - 3*b^2)*d*x - 24*((a^2
- 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c)^2 - a^2 + 4*a*b + 5*b^2)*si
nh(d*x + c)^2 - 2*((3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^6 + 6*(3*a^2 + 2*a*b
- b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^2 + 2*a*b - b^2)*sinh(d*x + c)
^6 + 2*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c)^4 + (15*(3*a^2 + 2*a*b - b^2)*co
sh(d*x + c)^2 + 6*a^2 - 8*a*b + 2*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 2*a*
b - b^2)*cosh(d*x + c)^3 + 2*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^2 + (15*(3*a^2 + 2*a*b - b^2)*
cosh(d*x + c)^4 + 12*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b
- b^2)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^5 + 4*(3*
a^2 - 4*a*b + b^2)*cosh(d*x + c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x
+ c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2
+ a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 +
4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)) - a^2 - 2*a*b - b^2 + 4*(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 3*(2*(a
^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*
b^2)*d*x - a*b + b^2)*cosh(d*x + c)^3 - (2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2
- 4*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
+ b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4)*d*sinh(d*x + c)^6 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x
+ c)^4 + (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2
+ 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d)*sinh(d*x + c)^4 + (a^4 + 4*a^3*b +
6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
+ b^4)*d*cosh(d*x + c)^4 + 12*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x +

```
c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + 2*
(3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^5 + 4*(a^4 +
 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^2 + 2*a*b + b^2)*co
sh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2
+ 2*a*b + b^2)*sinh(d*x + c)^8 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2
)*cosh(d*x + c)^6 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x - 14*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^6 + 4*(14*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c))
*sinh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c)^
4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 4*(a^2 - 4*a*b + 3*b^2)*d*x
 - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^2 + 4*a*b - 4
*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 5*(2*(a^
2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^3 - 4*((a^2 - 4*a*b + 3*b
^2)*d*x - a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*(a^2 - 2*a*b - 3
*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c)^2 + 2*(14*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^6 - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x +
 c)^4 - 2*(a^2 - 2*a*b - 3*b^2)*d*x - 24*((a^2 - 4*a*b + 3*b^2)*d*x - a*b +
 b^2)*cosh(d*x + c)^2 - a^2 + 4*a*b + 5*b^2)*sinh(d*x + c)^2 - 4*((3*a^2 +
2*a*b - b^2)*cosh(d*x + c)^6 + 6*(3*a^2 + 2*a*b...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sinh(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(118) = 236.
time = 1.21, size = 367, normalized size = 2.78

$$-\frac{\frac{12\,(dx+c)(a-3\,b)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{12\,(3\,ab-b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}}-\frac{3\,e^{(2\,dx+2\,c)}}{a^2+2\,ab+b^2}-\frac{2\,a^2e^{(6\,dx+6\,c)}-4\,abe^{(6\,dx+6\,c)}-6\,b^2e^{(6\,dx+6\,c)}+a^2e^{(4\,dx+4\,c)}+2\,abe^{(4\,dx+4\,c)}-15\,b^2e^{(4\,dx+4\,c)}-4\,a^2e^{(2\,dx+2\,c)}+20\,abe^{(2\,dx+2\,c)}+24\,b^2e^{(2\,dx+2\,c)}-3\,a^2-6\,ab-3\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+2\,ae^{(4\,dx+4\,c)}-2\,be^{(4\,dx+4\,c)}+ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}\right)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(d*x + c)*(a - 3*b)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 12*(3*a*b -
 b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))
/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) - 3*e^(2*d*x + 2*c)/(a^2 + 2*a
*b + b^2) - (2*a^2*e^(6*d*x + 6*c) - 4*a*b*e^(6*d*x + 6*c) - 6*b^2*e^(6*d*x
```

```
 + 6*c) + a^2*e^(4*d*x + 4*c) + 2*a*b*e^(4*d*x + 4*c) - 15*b^2*e^(4*d*x + 4
*c) - 4*a^2*e^(2*d*x + 2*c) + 20*a*b*e^(2*d*x + 2*c) + 24*b^2*e^(2*d*x + 2*
c) - 3*a^2 - 6*a*b - 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a*e^(6*d*x +
6*c) + b*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x + 4*c) + a*e^
(2*d*x + 2*c) + b*e^(2*d*x + 2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

3.36 $\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\mathrm{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d}+\frac{3\cosh(c+dx)}{2(a+b)^2d}-\frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\mathrm{sech}^2(c+dx)\right)}$$

```
[Out] 3/2*cosh(d*x+c)/(a+b)^2/d-1/2*cosh(d*x+c)/(a+b)/d/(a+b-b*sech(d*x+c)^2)-3/2
*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3745, 296, 331, 214}

$$\frac{3\cosh(c+dx)}{2d(a+b)^2}-\frac{\cosh(c+dx)}{2d(a+b)\left(a-b\mathrm{sech}^2(c+dx)+b\right)}-\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\mathrm{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*
d) + (3*Cosh[c + d*x])/(2*(a + b)^2*d) - Cosh[c + d*x]/(2*(a + b)*d*(a + b
- b*Sech[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^2}\,dx, x, \text{sech}(c+dx)\right)}{d}\\ &= -\frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{3\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \text{sech}(c+\right.}{2(a+b)d}\\ &= \frac{3\cosh(c+dx)}{2(a+b)^2d} - \frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)} - \frac{(3b)\text{Subst}\left(\int \frac{1}{a+b-bx}\right.}{2(a}\\ &= -\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} + \frac{3\cosh(c+dx)}{2(a+b)^2d} - \frac{\cosh(c+d}{2(a+b)d\left(a+b-b\text{se}\right.}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.54, size = 133, normalized size = 1.45

$$\frac{-\frac{3i\sqrt{b}\left(\text{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\text{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{5/2}}+\frac{2\cosh(c+dx)\left(1-\frac{b}{a-b+(a+b)\cosh(2(c+dx))}\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (((-3*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2]])/Sqr
t[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2]])/Sqrt[b]]))/(a
 + b)^(5/2) + (2*Cosh[c + d*x]*(1 - b/(a - b + (a + b)*Cosh[2*(c + d*x)])))
/(a + b)^2)/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.
time = 2.16, size = 167, normalized size = 1.82

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{(2b+a)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{1}{2}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{3\,\mathrm{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{4\sqrt{ab+b}}\right.}{4\sqrt{ab+b^2}}\right.}{(a+b)^2}}{d}$
default	$\frac{-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{(2b+a)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{1}{2}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{3\,\mathrm{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{4\sqrt{ab+b}}\right.}{4\sqrt{ab+b^2}}\right.}{(a+b)^2}}{d}$
risch	$\frac{e^{dx+c}}{2(a^2+2ab+b^2)d}+\frac{e^{-dx-c}}{2(a^2+2ab+b^2)d}-\frac{b\,e^{dx+c}(1+e^{2dx+2c})}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{3\sqrt{b\,(a+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)+2*b/(a+b)^2*((-1/2*(2*b+a)/a*tanh(1
/2*d*x+1/2*c)^2-1/2)/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b
*tanh(1/2*d*x+1/2*c)^2+a)-3/4/(a*b+b^2)^(1/2)*arctanh(1/4*(2*a*tanh(1/2*d*x
+1/2*c)^2+2*a+4*b)/(a*b+b^2)^(1/2)))+1/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) + 3*(a*e^(4*c) - b*e^(4*c))*e^(4*d*x
) + 3*(a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)/((a^3*d*e^(5*c) + 3*a^2*b*
d*e^(5*c) + 3*a*b^2*d*e^(5*c) + b^3*d*e^(5*c))*e^(5*d*x) + 2*(a^3*d*e^(3*c)
 + a^2*b*d*e^(3*c) - a*b^2*d*e^(3*c) - b^3*d*e^(3*c))*e^(3*d*x) + (a^3*d*e^
c + 3*a^2*b*d*e^c + 3*a*b^2*d*e^c + b^3*d*e^c)*e^(d*x)) + 1/2*integrate(6*(
b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^
(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3
*e^(2*c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(81) = 162.
time = 0.39, size = 2252, normalized size = 24.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(2*(a + b)*cosh(d*x + c)^6 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a + b)*sinh(d*x + c)^6 + 6*(a - b)*cosh(d*x + c)^4 + 6*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^4 + 8*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a - b)*cosh(d*x + c)^2 + 6*(5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^5 + 5*(a + b)*cosh(d*x + c)*sinh(d*x + c)^4 + (a + b)*sinh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + (a + b)*cosh(d*x + c) + (5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 12*((a + b)*cosh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 2*b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^5 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^3 + 2*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c) + 2*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 6*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)*sinh(d*x + c)), 1/2*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 + 3*(a - b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 - 3*((a + b)*cosh(d*x + c)^5 + 5*(a + b)*cosh(d*x + c)*sinh(d*x + c)^4 + (a + b)*sinh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + (a + b)*cosh(d*x + c) + (5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 +

```
(a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x +
c))*sqrt(-b/(a + b))/b) + 3*((a + b)*cosh(d*x + c)^5 + 5*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^4 + (a + b)*sinh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 +
 2*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 2*(5*(a + b)*cosh(
d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + (a + b)*cosh(d*x +
c) + (5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d
*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(
d*x + c))*sqrt(-b/(a + b))/b) + 6*((a + b)*cosh(d*x + c)^5 + 2*(a - b)*cosh
(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*c
osh(d*x + c)*sinh(d*x + c)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x +
 c)^5 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^3 + 2*(5*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh
(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c) + 2*(5*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3)
*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*co
sh(d*x + c)^4 + 6*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d)*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

3.37 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=103

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d}+\frac{b\operatorname{sech}(c+dx)}{2a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

```
[Out] -arctanh(cosh(d*x+c))/a^2/d+1/2*b*sech(d*x+c)/a/(a+b)/d/(a+b-b*sech(d*x+c)^
2)+1/2*(3*a+2*b)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b)
^(3/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3745, 425, 536, 213, 214}

$$\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{b\operatorname{sech}(c+dx)}{2ad(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a^2*d)) + (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*S
ech[c + d*x])/Sqrt[a + b]])/(2*a^2*(a + b)^(3/2)*d) + (b*Sech[c + d*x])/(2*
a*(a + b)*d*(a + b - b*Sech[c + d*x]^2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x^2)(a+b-bx^2)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}(c+dx)}{2a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{2a+b+bx^2}{(-1+x^2)(a+b-bx^2)}\,dx, x, \operatorname{sech}(\right.}{2a(a+b)d}$$

$$= \frac{b\operatorname{sech}(c+dx)}{2a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{a^2d}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} + \frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} + \frac{}{2a(a+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.54, size = 175, normalized size = 1.70

$$\frac{\frac{i\sqrt{b}\,(3a+2b)\operatorname{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{i\sqrt{b}\,(3a+2b)\operatorname{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{2ab\cosh(c+dx)}{(a+b)(a-b+(a+b)\cosh(2(c+dx)))} + 2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2
])/Sqrt[b]])/(a + b)^(3/2) + (I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a + b
] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (2*a*b*Cosh[c + d*
x])/((a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])) + 2*Log[Tanh[(c + d*x)/2]
])/(2*a^2*d)
```

Maple [A]
time = 3.21, size = 161, normalized size = 1.56

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{4b\left(\frac{-\frac{(2b+a)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)}-\frac{a}{4(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{(3a+2b)\operatorname{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2}{4\sqrt{ab+b^2}}\right)}{8(a+b)\sqrt{ab+b^2}}\right)}{a^2}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{4b\left(\frac{-\frac{(2b+a)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)}-\frac{a}{4(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{(3a+2b)\operatorname{arctanh}\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2}{4\sqrt{ab+b^2}}\right)}{8(a+b)\sqrt{ab+b^2}}\right)}{a^2}}{d}$
risch	$\frac{b\,e^{dx+c}\left(1+e^{2dx+2c}\right)}{a(a+b)d\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{\ln\left(e^{dx+c}-1\right)}{a^2d}-\frac{\ln\left(e^{dx+c}+1\right)}{a^2d}+\frac{3\sqrt{b(a+b)}\ 1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c))-4*b/a^2*((-1/4*(2*b+a)/(a+b)*tanh(1/2*d*
x+1/2*c)^2-1/4/(a+b)*a)/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+
4*b*tanh(1/2*d*x+1/2*c)^2+a)-1/8*(3*a+2*b)/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/
4*(2*a*tanh(1/2*d*x+1/2*c)^2+2*a+4*b)/(a*b+b^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e
^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c)
- a*b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log
((e^(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) + 2*b^
2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 2*b^2*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^
```

```
2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^
4*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(94) = 188$.
time = 0.40, size = 2614, normalized size = 25.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b*
sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x +
 c)^4 + 4*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 +
5*a*b + 2*b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 +
2*(3*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*
x + c)^2 + 3*a^2 + 5*a*b + 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)
^3 + (3*a^2 - a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*lo
g(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
 c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*c
osh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d
*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) +
 (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*
x + c))*sinh(d*x + c) + a + b)) - 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*s
inh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 +
 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) + 1) + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c
)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b +
b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*(
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x +
 c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*a*b*cosh(d*x + c)^2 + a*
b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2
*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^
2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*
a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 + (
a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3
 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*cosh(d*x + c
```

```
)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 + 2*a*b*c
osh(d*x + c) + ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 5*a*b
+ 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 5*a*b + 2*b^2)*sinh(d*x +
 c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 5*a*b + 2*b
^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + 5*a*b
+ 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 + (3*a^2 - a*b - 2*b^2
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)
^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d
*x + c))*sqrt(-b/(a + b))/b) - ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 4
*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 5*a*b + 2
*b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a
^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*x + c)^2
+ 3*a^2 + 5*a*b + 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 + (3*a
^2 - a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2
*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - 2*((
a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*
x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 -
 b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
+ 2*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*c
osh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh
(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 +
(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
 - 1) + 2*(3*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^
2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh
(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^
2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (
a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 +
 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*si
nh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c+d\,x)\,\left(b\tanh(c+d\,x)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)
```

3.38 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{3\coth(c+dx)}{2a^2d}+\frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] -3/2*coth(d*x+c)/a^2/d-3/2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(5
/2)/d+1/2*coth(d*x+c)/a/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3744, 296, 331, 211}

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d}-\frac{3\coth(c+dx)}{2a^2d}+\frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(5/2)*d) - (3*Cot
h[c + d*x])/(2*a^2*d) + Coth[c + d*x]/(2*a*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)} + \frac{3\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= -\frac{3\coth(c+dx)}{2a^2d} + \frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \operatorname{ta}\right.}{2a^2d}$$

$$= -\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d} - \frac{3\coth(c+dx)}{2a^2d} + \frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx\right.}$$

Mathematica [A]
time = 0.35, size = 86, normalized size = 1.05

$$\frac{-3\sqrt{b}\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right) - 2\sqrt{a}\,\coth(c+dx) - \frac{\sqrt{a}\,b\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 2*Sqrt[a]*Coth[c + d*
x] - (Sqrt[a]*b*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*
a^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(68) = 136.
time = 2.81, size = 266, normalized size = 3.24

method	result
risch	$-\frac{2a^2e^{4dx+4c}+3ab\,e^{4dx+4c}+3b^2e^{4dx+4c}+4a^2e^{2dx+2c}-6b^2e^{2dx+2c}+2a^2+5ab+3b^2}{d(a+b)a^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)\left(e^{2dx+2c}-1\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-a}}{a}\right.}{4a^3d}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}+\frac{4b\left(\frac{-\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{3a\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}}{2a\sqrt{b(a+b)}\sqrt{(}}\right.}{}\right.}{}}{}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}+\frac{4b\left(\frac{-\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{3a\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}}{2a\sqrt{b(a+b)}\sqrt{(}}\right.}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2*tanh(1/2*d*x+1/2*c)+4*b/a^2*((-1/4*tanh(1/2*d*x+1/2*c)^3-1/4*
tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b
*tanh(1/2*d*x+1/2*c)^2+a)+3/4*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1
/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(
b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)
/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2))))-1/2/a^2/tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(68) = 136$.
time = 0.60, size = 212, normalized size = 2.59

$$-\frac{2\,a^2+5\,ab+3\,b^2+2\,(2\,a^2-3\,b^2)e^{(-2\,dx-2\,c)}+(2\,a^2+3\,ab+3\,b^2)e^{(-4\,dx-4\,c)}}{(a^4+2\,a^3b+a^2b^2+(a^4-2\,a^3b-3\,a^2b^2)e^{(-2\,dx-2\,c)}-(a^4-2\,a^3b-3\,a^2b^2)e^{(-4\,dx-4\,c)}-(a^4+2\,a^3b+a^2b^2)e^{(-6\,dx-6\,c)})d}+\frac{3\,b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(2*a^2 + 5*a*b + 3*b^2 + 2*(2*a^2 - 3*b^2)*e^(-2*d*x - 2*c) + (2*a^2 + 3*a
*b + 3*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b -
3*a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 - 2*a^3*b - 3*a^2*b^2)*e^(-4*d*x - 4*c)
- (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 3/2*b*arctan(1/2*((a + b
)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(68) = 136$.
time = 0.41, size = 2562, normalized size = 31.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + 3*a*b + 3*b^
2)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + 3*a*b + 3*b^2)*sinh(d*x + c)^
4 + 8*(2*a^2 - 3*b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + 3*a*b + 3*b^2)*cosh(d
*x + c)^2 + 2*a^2 - 3*b^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*
x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a
*b + b^2)*sinh(d*x + c)^6 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^4 + (15*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^4 + 4
*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 + (15*(a^2 + 2*
a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 - a^2
+ 2*a*b + 3*b^2)*sinh(d*x + c)^2 - a^2 - 2*a*b - b^2 + 2*(3*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^5 + 2*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 - 2*a
*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^
```

2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*
a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))
/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)
^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x
+ c))*sinh(d*x + c) + a + b)) + 8*a^2 + 20*a*b + 12*b^2 + 16*((2*a^2 + 3*a
*b + 3*b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))
/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)
*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c
)^6 + (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^4 + (15*(a^4 + 2*a^3*b +
a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh(d*x + c)^4
- (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^
2*b^2)*d*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c))*sin
h(d*x + c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 - 2
*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh
(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*
d*cosh(d*x + c)^5 + 2*(a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^3 - (a^4
- 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(2*a^2 + 3*
a*b + 3*b^2)*cosh(d*x + c)^4 + 8*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)*sinh
(d*x + c)^3 + 2*(2*a^2 + 3*a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(2*a^2 - 3*b^2)
*cosh(d*x + c)^2 + 4*(3*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a^2 - 3
*b^2)*sinh(d*x + c)^2 + 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x +
c)^6 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^4 + (15*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 + a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^
2)*cosh(d*x + c)^3 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 -
(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^4 + 6*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 - a^2 + 2*a*b + 3*b^2)*sinh
(d*x + c)^2 - a^2 - 2*a*b - b^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5
+ 2*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x
+ c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a +
b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a
)/b) + 4*a^2 + 10*a*b + 6*b^2 + 8*((2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^3
+ (2*a^2 - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*
d*cosh(d*x + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x +
c)^5 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 - 2*a^3*b - 3*a^2
*b^2)*d*cosh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 +
(a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh(d*x + c)^4 - (a^4 - 2*a^3*b - 3*a^2*b^
2)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (
a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^4 + 2*
a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 - 2*a^3*b - 3*a^2*b^2)*d*cosh(d

```
*x + c)^2 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b
 + a^2*b^2)*d + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 -
 2*a^3*b - 3*a^2*b^2)*d*cosh(d*x + c)^3 - (a^4 - 2*a^3*b - 3*a^2*b^2)*d*cos
h(d*x + c))*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(68) = 136.
time = 0.72, size = 227, normalized size = 2.77

$$-\frac{\frac{3\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,\left(2\,a^2e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}+3\,b^2e^{(4\,dx+4\,c)}+4\,a^2e^{(2\,dx+2\,c)}-6\,b^2e^{(2\,dx+2\,c)}+2\,a^2+5\,ab+3\,b^2\right)}{(a^3+a^2b)\left(ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}-a-b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a
*b))/(sqrt(a*b)*a^2) + 2*(2*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) + 3
*b^2*e^(4*d*x + 4*c) + 4*a^2*e^(2*d*x + 2*c) - 6*b^2*e^(2*d*x + 2*c) + 2*a^
2 + 5*a*b + 3*b^2)/((a^3 + a^2*b)*(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) +
a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x
+ 2*c) - a - b)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\sinh\left(c+d\,x\right)}^2\,{\left(b\,{\tanh\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2), x)
```

3.39 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=141

$$\frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}\,d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)}-\frac{\quad}{a^2d}$$

```
[Out] 1/2*(a+4*b)*arctanh(cosh(d*x+c))/a^3/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a+b
-b*sech(d*x+c)^2)-b*sech(d*x+c)/a^2/d/(a+b-b*sech(d*x+c)^2)-1/2*(3*a+4*b)*a
rctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^3/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3745, 482, 541, 536, 213, 214}

$$\frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}}-\frac{b\operatorname{sech}(c+dx)}{a^2d\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b\operatorname{sech}^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + 4*b)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) - (Sqrt[b]*(3*a + 4*b)*ArcTanh
[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*d) - (Coth[c + d*
x]*Csch[c + d*x])/(2*a*d*(a + b - b*Sech[c + d*x]^2)) - (b*Sech[c + d*x])/(
a^2*d*(a + b - b*Sech[c + d*x]^2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+b-bx^2)^2}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+b+3bx^2}{(-1+x^2)(a+b-bx^2)^2}\,dx, x, \operatorname{sech}(c+d\right.}{2ad}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{b\operatorname{sech}(c+dx)}{a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{\text{Subst}\left(\int 2\right.}{}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{b\operatorname{sech}(c+dx)}{a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{(a+4b)\text{Su}}{}$$

$$= \frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d} - \frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}\,d} - \frac{}{2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.84, size = 203, normalized size = 1.44

$$-\frac{\frac{4i\sqrt{b}\,(3a+4b)\text{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}} + \frac{4i\sqrt{b}\,(3a+4b)\text{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}} + \frac{8ab\cosh(c+dx)}{a-b+(a+b)\cosh(2(c+dx))} + a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) + 4(a+4b)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/8*(((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c
 + d*x)/2])/Sqrt[b]])/Sqrt[a + b] + ((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[((-I)
*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/Sqrt[a + b] + (8*a*b*Co
sh[c + d*x])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + a*Csch[(c + d*x)/2]^2 +
4*(a + 4*b)*Log[Tanh[(c + d*x)/2]] + a*Sech[(c + d*x)/2]^2)/(a^3*d)
```

Maple [A]
time = 3.20, size = 187, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2} - \frac{1}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2} + \frac{(-8b-2a)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3} + \frac{2b\left(\frac{\left(-b-\frac{a}{2}\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a}{2}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\right.\right.}\right.}{a^3}}{d}$

default	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{1}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-8b-2a)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{2b\left(\frac{\left(-b-\frac{a}{2}\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a}{2}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}\cdots\right.\right.}\right.}{\cdots}}{d}$
risch	$-\frac{e^{dx+c}\left(a\,e^{6dx+6c}+2b\,e^{6dx+6c}+3a\,e^{4dx+4c}-2b\,e^{4dx+4c}+3a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+2b\right)}{d\,a^2\left(e^{2dx+2c}-1\right)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{dx+c}-1\right)}{2a^2d}-\frac{2\ln\left(e^{d\cdots}\right.}{a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^2-1/8/a^2/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-
8*b-2*a)*ln(tanh(1/2*d*x+1/2*c))+2/a^3*b*(((-b-1/2*a)*tanh(1/2*d*x+1/2*c)^2
-1/2*a)/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x
+1/2*c)^2+a)-1/4*(3*a+4*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*a*tanh(1/2*d*x+1/
2*c)^2+2*a+4*b)/(a*b+b^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a*e^(7*c) + 2*b*e^(7*c))*e^(7*d*x) + (3*a*e^(5*c) - 2*b*e^(5*c))*e^(5*d*x
) + (3*a*e^(3*c) - 2*b*e^(3*c))*e^(3*d*x) + (a*e^c + 2*b*e^c)*e^(d*x))/(4*a
^2*b*d*e^(6*d*x + 6*c) + 4*a^2*b*d*e^(2*d*x + 2*c) - a^3*d - a^2*b*d - (a^3
*d*e^(8*c) + a^2*b*d*e^(8*c))*e^(8*d*x) + 2*(a^3*d*e^(4*c) - 3*a^2*b*d*e^(4
*c))*e^(4*d*x)) + 1/2*(a + 4*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^3*d) - 1/2
*(a + 4*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) + 8*integrate(1/8*((3*a*b*
e^(3*c) + 4*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 4*b^2*e^c)*e^(d*x))/(a^4
+ a^3*b + (a^4*e^(4*c) + a^3*b*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) - a^3*b*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3368 vs. $2(132) = 264$.
time = 0.46, size = 6335, normalized size = 44.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + 2*a*b)*cosh(d*x + c)^7 + 28*(a^2 + 2*a*b)*cosh(d*x + c)*sinh
(d*x + c)^6 + 4*(a^2 + 2*a*b)*sinh(d*x + c)^7 + 4*(3*a^2 - 2*a*b)*cosh(d*x
+ c)^5 + 4*(21*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b)*sinh(d*x + c)
^5 + 20*(7*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b)*cosh(d*x + c))*s
inh(d*x + c)^4 + 4*(3*a^2 - 2*a*b)*cosh(d*x + c)^3 + 4*(35*(a^2 + 2*a*b)*co
sh(d*x + c)^4 + 10*(3*a^2 - 2*a*b)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b)*sinh(d*
x + c)^3 + 4*(21*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 10*(3*a^2 - 2*a*b)*cosh(d*
x + c)^3 + 3*(3*a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 - ((3*a^2 + 7*a
*b + 4*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)*sinh(
d*x + c)^7 + (3*a^2 + 7*a*b + 4*b^2)*sinh(d*x + c)^8 - 4*(3*a*b + 4*b^2)*co
sh(d*x + c)^6 + 4*(7*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^2 - 3*a*b - 4*b^
2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^3 - 3*(3*a*
b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2 - 5*a*b - 12*b^2)*cosh
(d*x + c)^4 + 2*(35*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^4 - 30*(3*a*b + 4
*b^2)*cosh(d*x + c)^2 - 3*a^2 + 5*a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a
^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^5 - 10*(3*a*b + 4*b^2)*cosh(d*x + c)^3 -
(3*a^2 - 5*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a*b + 4*b^2)
*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 7*a*b + 4*b^2)*cosh(d*x + c)^6 - 15*(3*a*b
 + 4*b^2)*cosh(d*x + c)^4 - 3*(3*a^2 - 5*a*b - 12*b^2)*cosh(d*x + c)^2 - 3*
a*b - 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 7*a*b + 4*b^2 + 8*((3*a^2 + 7*a*b +
4*b^2)*cosh(d*x + c)^7 - 3*(3*a*b + 4*b^2)*cosh(d*x + c)^5 - (3*a^2 - 5*a*b
 - 12*b^2)*cosh(d*x + c)^3 - (3*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*
(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x +
c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3
+ 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a +
b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(
b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3
 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(a^2 + 2*a*b)*cosh(d*
x + c) - 2*((a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^8 + 8*(a^2 + 5*a*b + 4*b^2)
*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 5*a*b + 4*b^2)*sinh(d*x + c)^8 - 4*
(a*b + 4*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^2
- a*b - 4*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^3
 - 3*(a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*
cosh(d*x + c)^4 + 2*(35*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^4 - 30*(a*b + 4
*b^2)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 5
*a*b + 4*b^2)*cosh(d*x + c)^5 - 10*(a*b + 4*b^2)*cosh(d*x + c)^3 - (a^2 + a
*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b + 4*b^2)*cosh(d*x + c)
^2 + 4*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^6 - 15*(a*b + 4*b^2)*cosh(d*x
 + c)^4 - 3*(a^2 + a*b - 12*b^2)*cosh(d*x + c)^2 - a*b - 4*b^2)*sinh(d*x +
c)^2 + a^2 + 5*a*b + 4*b^2 + 8*((a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^7 - 3*(
a*b + 4*b^2)*cosh(d*x + c)^5 - (a^2 + a*b - 12*b^2)*cosh(d*x + c)^3 - (a*b
```

```
+ 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
1) + 2*((a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^8 + 8*(a^2 + 5*a*b + 4*b^2)*cos
h(d*x + c)*sinh(d*x + c)^7 + (a^2 + 5*a*b + 4*b^2)*sinh(d*x + c)^8 - 4*(a*b
 + 4*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^2 - a*
b - 4*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^3 - 3
*(a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*cosh
(d*x + c)^4 + 2*(35*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^4 - 30*(a*b + 4*b^2
)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 5*a*b
 + 4*b^2)*cosh(d*x + c)^5 - 10*(a*b + 4*b^2)*cosh(d*x + c)^3 - (a^2 + a*b -
 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b + 4*b^2)*cosh(d*x + c)^2 +
 4*(7*(a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^6 - 15*(a*b + 4*b^2)*cosh(d*x + c
)^4 - 3*(a^2 + a*b - 12*b^2)*cosh(d*x + c)^2 - a*b - 4*b^2)*sinh(d*x + c)^2
 + a^2 + 5*a*b + 4*b^2 + 8*((a^2 + 5*a*b + 4*b^2)*cosh(d*x + c)^7 - 3*(a*b
+ 4*b^2)*cosh(d*x + c)^5 - (a^2 + a*b - 12*b^2)*cosh(d*x + c)^3 - (a*b + 4*
b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) +
 4*(7*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 5*(3*a^2 - 2*a*b)*cosh(d*x + c)^4 + 3
*(3*a^2 - 2*a*b)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c))/(4*a^3*b*d*c
osh(d*x + c)^6 - (a^4 + a^3*b)*d*cosh(d*x + c)^8 - 8*(a^4 + a^3*b)*d*cosh(d
*x + c)*sinh(d*x + c)^7 - (a^4 + a^3*b)*d*sinh(d*x + c)^8 + 4*a^3*b*d*cosh(
d*x + c)^2 + 4*(a^3*b*d - 7*(a^4 + a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 2*(a^4 - 3*a^3*b)*d*cosh(d*x + c)^4 + 8*(3*a^3*b*d*cosh(d*x + c) - 7*(a
^4 + a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^5 ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(c+d\,x\right)^3\,{\left(b\,\tanh\left(c+d\,x\right)^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)
```

3.40 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{b}\,(3a+5b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d}+\frac{(a+2b)\coth(c+dx)}{a^3d}-\frac{\coth^3(c+dx)}{3a^2d}+\frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] (a+2*b)*coth(d*x+c)/a^3/d-1/3*coth(d*x+c)^3/a^2/d+1/2*(3*a+5*b)*arctan(b^(1
/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(7/2)/d+1/2*b*(a+b)*tanh(d*x+c)/a^3/d/(a
+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3744, 467, 1275, 211}

$$\frac{\sqrt{b}\,(3a+5b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d}+\frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)}+\frac{(a+2b)\coth(c+dx)}{a^3d}-\frac{\coth^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*d)
 + ((a + 2*b)*Coth[c + d*x])/(a^3*d) - Coth[c + d*x]^3/(3*a^2*d) + (b*(a +
b)*Tanh[c + d*x])/(2*a^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\operatorname{Subst}\left(\int \frac{-\frac{2}{ab}+\frac{2(a+b)x^2}{a^2b}-\frac{(a+b)x^4}{a^3}}{x^4(a+bx^2)}\,dx, x, \tanh(c+$$

$$= \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\operatorname{Subst}\left(\int \left(-\frac{2}{a^2bx^4}+\frac{2(a+2b)}{a^3bx^2}+\frac{-3a-5b}{a^3(a+bx^2)}\right)dx, x$$

$$= \frac{(a+2b)\coth(c+dx)}{a^3d} - \frac{\coth^3(c+dx)}{3a^2d} + \frac{b(a+b)\tanh(c+dx)}{2a^3d\left(a+b\tanh^2(c+dx)\right)} + \frac{(b(3$$

$$= \frac{\sqrt{b}\,(3a+5b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d} + \frac{(a+2b)\coth(c+dx)}{a^3d} - \frac{\coth^3(c+d}{3a^2d}$$

Mathematica [A]
time = 0.68, size = 114, normalized size = 1.01

$$\frac{3\sqrt{b}\,(3a+5b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)+2\sqrt{a}\,\coth(c+dx)\left(2a+6b-a\operatorname{csch}^2(c+dx)\right)+\frac{3\sqrt{a}\,b(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{6a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + 2*Sqrt[a]*
Coth[c + d*x]*(2*a + 6*b - a*Csch[c + d*x]^2) + (3*Sqrt[a]*b*(a + b)*Sinh[2
*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(6*a^(7/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(99) = 198$.

time = 3.18, size = 338, normalized size = 2.99

method	result
derivativedivides	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{2b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}\right.}{}$
default	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{2b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}\right.}{}$
risch	$-\frac{-9ab\,e^{8dx+8c}-15b^2e^{8dx+8c}+12a^2e^{6dx+6c}+6ab\,e^{6dx+6c}+60b^2e^{6dx+6c}+20a^2e^{4dx+4c}-4ab\,e^{4dx+4c}-90b^2e^{4dx+4c}+4a}{3a^3d\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)\left(e^{2dx+2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a^3*(1/3*a*tanh(1/2*d*x+1/2*c)^3-3*a*tanh(1/2*d*x+1/2*c)-8*b*tanh
(1/2*d*x+1/2*c))-2/a^3*b*(((-1/2*a-1/2*b)*tanh(1/2*d*x+1/2*c)^3+(-1/2*a-1/2
*b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2
+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(3*a+5*b)*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/
a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(
b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))-1/24/a^2/tanh(1/2*d*x+1/2*c)^3-
1/8/a^3*(-8*b-3*a)/tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(99) = 198$.
time = 0.60, size = 282, normalized size = 2.50

$$\frac{4a^2 + 19ab + 15b^2 - 2(2a^2 + 13ab + 30b^2)e^{(-2dx-2c)} - 2(10a^2 - 2ab - 45b^2)e^{(-4dx-4c)} - 6(2a^2 + ab + 10b^2)e^{(-6dx-6c)} + 3(3ab + 5b^2)e^{(-8dx-8c)}}{3(a^4 + a^3b - (a^4 + 5a^3b)e^{(-2dx-2c)} - 2(a^4 - 5a^3b)e^{(-4dx-4c)} + 2(a^4 - 5a^3b)e^{(-6dx-6c)} + (a^4 + 5a^3b)e^{(-8dx-8c)} - (a^4 + a^3b)e^{(-10dx-10c)})d} - \frac{(3ab + 5b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a^2 + 19*a*b + 15*b^2 - 2*(2*a^2 + 13*a*b + 30*b^2)*e^(-2*d*x - 2*c)
 - 2*(10*a^2 - 2*a*b - 45*b^2)*e^(-4*d*x - 4*c) - 6*(2*a^2 + a*b + 10*b^2)*
e^(-6*d*x - 6*c) + 3*(3*a*b + 5*b^2)*e^(-8*d*x - 8*c))/((a^4 + a^3*b - (a^4
 + 5*a^3*b)*e^(-2*d*x - 2*c) - 2*(a^4 - 5*a^3*b)*e^(-4*d*x - 4*c) + 2*(a^4
- 5*a^3*b)*e^(-6*d*x - 6*c) + (a^4 + 5*a^3*b)*e^(-8*d*x - 8*c) - (a^4 + a^3
*b)*e^(-10*d*x - 10*c))*d) - 1/2*(3*a*b + 5*b^2)*arctan(1/2*((a + b)*e^(-2*
d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. $2(99) = 198$.
time = 0.41, size = 5062, normalized size = 44.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(3*a*b + 5*b^2)*cosh(d*x + c)^8 + 96*(3*a*b + 5*b^2)*cosh(d*x + c
)*sinh(d*x + c)^7 + 12*(3*a*b + 5*b^2)*sinh(d*x + c)^8 - 24*(2*a^2 + a*b +
10*b^2)*cosh(d*x + c)^6 + 24*(14*(3*a*b + 5*b^2)*cosh(d*x + c)^2 - 2*a^2 -
a*b - 10*b^2)*sinh(d*x + c)^6 + 48*(14*(3*a*b + 5*b^2)*cosh(d*x + c)^3 - 3*
```

(2*a^2 + a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(10*a^2 - 2*a*b - 45*b^2)*cosh(d*x + c)^4 + 8*(105*(3*a*b + 5*b^2)*cosh(d*x + c)^4 - 45*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^2 - 10*a^2 + 2*a*b + 45*b^2)*sinh(d*x + c)^4 + 32*(21*(3*a*b + 5*b^2)*cosh(d*x + c)^5 - 15*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^3 - (10*a^2 - 2*a*b - 45*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(2*a^2 + 13*a*b + 30*b^2)*cosh(d*x + c)^2 + 8*(42*(3*a*b + 5*b^2)*cosh(d*x + c)^6 - 45*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^4 - 6*(10*a^2 - 2*a*b - 45*b^2)*cosh(d*x + c)^2 - 2*a^2 - 13*a*b - 30*b^2)*sinh(d*x + c)^2 + 3*((3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^10 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (3*a^2 + 8*a*b + 5*b^2)*sinh(d*x + c)^10 - (3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^8 + (45*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^2 - 3*a^2 - 20*a*b - 25*b^2)*sinh(d*x + c)^8 + 8*(15*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^3 - (3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^6 + 2*(105*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^4 - 14*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^2 - 3*a^2 + 10*a*b + 25*b^2)*sinh(d*x + c)^6 + 4*(63*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^5 - 14*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^3 - 3*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^4 + 2*(105*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^6 - 35*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^4 - 15*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^2 + 3*a^2 - 10*a*b - 25*b^2)*sinh(d*x + c)^4 + 8*(15*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^7 - 7*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^5 - 5*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^3 + (3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^2 + (45*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^8 - 28*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^6 - 30*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^2 + 3*a^2 + 20*a*b + 25*b^2)*sinh(d*x + c)^2 - 3*a^2 - 8*a*b - 5*b^2 + 2*(5*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^9 - 4*(3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c)^7 - 6*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 10*a*b - 25*b^2)*cosh(d*x + c)^3 + (3*a^2 + 20*a*b + 25*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 16*a^2 + 76*a*b + 60*b^2 + 16*(6*(3*a*b + 5*b^2)*cosh(d*x + c)^7 - 9*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^5 - 2*(10*a^2 - 2*a*b - 45*b^2)*cosh(d*x + c)^3 - (2*a^2 + 13*a*b + 30*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^10 + 10*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + a^3*b)*d*sinh(d*x + c)^10 - (a^4 + 5

```
*a^3*b)*d*cosh(d*x + c)^8 + (45*(a^4 + a^3*b)*d*cosh(d*x + c)^2 - (a^4 + 5*
a^3*b)*d)*sinh(d*x + c)^8 - 2*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^6 + 8*(15*(a^
4 + a^3*b)*d*cosh(d*x + c)^3 - (a^4 + 5*a^3*b)*d*cosh(d*x + c))*sinh(d*x +
c)^7 + 2*(105*(a^4 + a^3*b)*d*cosh(d*x + c)^4 - 14*(a^4 + 5*a^3*b)*d*cosh(d
*x + c)^2 - (a^4 - 5*a^3*b)*d)*sinh(d*x + c)^6 + 2*(a^4 - 5*a^3*b)*d*cosh(d
*x + c)^4 + 4*(63*(a^4 + a^3*b)*d*cosh(d*x + c)^5 - 14*(a^4 + 5*a^3*b)*d*co
sh(d*x + c)^3 - 3*(a^4 - 5*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105
*(a^4 + a^3*b)*d*cosh(d*x + c)^6 - 35*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^4 - 1
5*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^2 + (a^4 - 5*a^3*b)*d)*sinh(d*x + c)^4 +
(a^4 + 5*a^3*b)*d*cosh(d*x + c)^2 + 8*(15*(a^4 + a^3*b)*d*cosh(d*x + c)^7 -
 7*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^5 - 5*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^3
+ (a^4 - 5*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^4 + a^3*b)*d*co
sh(d*x + c)^8 - 28*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^6 - 30*(a^4 - 5*a^3*b)*d
*cosh(d*x + c)^4 + 12*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^2 + (a^4 + 5*a^3*b)*d
)*sinh(d*x + c)^2 - (a^4 + a^3*b)*d + 2*(5*(a^4 + a^3*b)*d*cosh(d*x + c)^9
- 4*(a^4 + 5*a^3*b)*d*cosh(d*x + c)^7 - 6*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^5
 + 4*(a^4 - 5*a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 5*a^3*b)*d*cosh(d*x + c))*s
inh(d*x + c)), 1/6*(6*(3*a*b + 5*b^2)*cosh(d*x ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(99) = 198.
time = 0.67, size = 209, normalized size = 1.85

$$\frac{\frac{3\,(3\,ab+5\,b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{6\,\left(abe^{(2\,dx+2\,c)}-b^2e^{(2\,dx+2\,c)}+ab+b^2\right)}{\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)a^3}+\frac{8\,\left(3\,be^{(4\,dx+4\,c)}-3\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+a+3\,b\right)}{a^3\left(e^{(2\,dx+2\,c)}-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(3*a*b + 5*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) +
a - b)/sqrt(a*b))/(sqrt(a*b)*a^3) - 6*(a*b*e^(2*d*x + 2*c) - b^2*e^(2*d*x +
 2*c) + a*b + b^2)/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*a^3) + 8*(3*b*e^(4*d*x + 4*c) - 3*a*e^
(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + a + 3*b)/(a^3*(e^(2*d*x + 2*c) - 1)^3
))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(c+d\,x\right)^4\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)
```

3.41 $\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=240

$$\frac{3(a^2-10ab+5b^2)\,x}{8(a+b)^5}+\frac{3\sqrt{b}\,(5a^2-10ab+b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a+b)^5d}-\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2}$$

```
[Out] 3/8*(a^2-10*a*b+5*b^2)*x/(a+b)^5+3/8*(5*a^2-10*a*b+b^2)*arctan(b^(1/2)*tanh
(d*x+c)/a^(1/2))*b^(1/2)/(a+b)^5/d/a^(1/2)-1/8*(5*a-3*b)*cosh(d*x+c)*sinh(d
*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d
/(a+b*tanh(d*x+c)^2)^2+1/8*(7*a-5*b)*b*tanh(d*x+c)/(a+b)^3/d/(a+b*tanh(d*x+
c)^2)^2+3/2*(a-b)*b*tanh(d*x+c)/(a+b)^4/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.25, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3744, 481, 541, 536, 212, 211}

$$\frac{3\sqrt{b}\,(5a^2-10ab+b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,d(a+b)^5}+\frac{3x(a^2-10ab+5b^2)}{8(a+b)^5}+\frac{3b(a-b)\tanh(c+dx)}{2d(a+b)^4\left(a+b\tanh^2(c+dx)\right)}+\frac{b(7a-5b)\tanh(c+dx)}{8d(a+b)^3\left(a+b\tanh^2(c+dx)\right)^2}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{(5a-3b)\sinh(c+dx)\cosh(c+dx)}{8d(a+b)^2\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a^2 - 10*a*b + 5*b^2)*x)/(8*(a + b)^5) + (3*Sqrt[b]*(5*a^2 - 10*a*b + b
^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a + b)^5*d) - ((5*
a - 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b*Tanh[c + d*x]^2
)^2) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)
^2) + ((7*a - 5*b)*b*Tanh[c + d*x])/(8*(a + b)^3*d*(a + b*Tanh[c + d*x]^2)^
2) + (3*(a - b)*b*Tanh[c + d*x])/(2*(a + b)^4*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
```

```
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+(4a-3b)x^2}{(1-x^2)^2(a+bx^2)^3}\,dx, x, \tanh(c+dx\right.}{4(a+b)d} \\
&= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \\
&= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \\
&= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \\
&= -\frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \\
&= \frac{3(a^2-10ab+5b^2)\,x}{8(a+b)^5} + \frac{3\sqrt{b}\,(5a^2-10ab+b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a+b)^5d} - \frac{(5a}{8(}
\end{aligned}
$$

Mathematica [A]
time = 0.62, size = 184, normalized size = 0.77

$$
\frac{12(a^2-10ab+5b^2)(c+dx) + \frac{12\sqrt{b}\,(5a^2-10ab+b^2)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} - 8(a-2b)(a+b)\sinh(2(c+dx)) + \frac{16ab^2(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{4(9a-5b)b(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))} + (a+b)^2\sinh(4(c+dx))}{32(a+b)^5d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (12*(a^2 - 10*a*b + 5*b^2)*(c + d*x) + (12*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*A
rcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - 8*(a - 2*b)*(a + b)*Sinh[
2*(c + d*x)] + (16*a*b^2*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2
*(c + d*x)])^2 + (4*(9*a - 5*b)*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a +
b)*Cosh[2*(c + d*x)]) + (a + b)^2*Sinh[4*(c + d*x)])/(32*(a + b)^5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(220) = 440$.
time = 2.81, size = 610, normalized size = 2.54 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b/(a+b)^5*((-3/8*a*(3*a^2+2*a*b-b^2)*tanh(1/2*d*x+1/2*c)^7+(-27/8*a
^3-23/4*a^2*b+1/8*a*b^2+5/2*b^3)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a^3-23/4*a^2*
b+1/8*a*b^2+5/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(-9/8*a^3-3/4*a^2*b+3/8*a*b^2)*t
anh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*
tanh(1/2*d*x+1/2*c)^2+a)^2+1/8*(15*a^2-30*a*b+3*b^2)*a*(-1/2*(-a+(b*(a+b))^
(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/
2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))+1/4/(a+b)^3/(tanh(1/2*
d*x+1/2*c)-1)^4+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(3*a-9*b)/(a+b)^4
/(tanh(1/2*d*x+1/2*c)-1)-1/8*(-11*b+a)/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)^2+1/
8/(a+b)^5*(-3*a^2+30*a*b-15*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/4/(a+b)^3/(tan
h(1/2*d*x+1/2*c)+1)^4+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(3*a-9*b)/(
a+b)^4/(tanh(1/2*d*x+1/2*c)+1)-1/8*(11*b-a)/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)
^2+1/8/(a+b)^5*(3*a^2-30*a*b+15*b^2)*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3392 vs. $2(220) = 440$.
time = 1.02, size = 3392, normalized size = 14.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -3/8*(a*b - 3*b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
+ a + b)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d) - 3/
4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + 3/8*(a*b - 3*b^2)*log(2*(a - b)
*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^5 + 5*a^4*b + 10*
a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d) + 3/4*b*log(2*(a - b)*e^(-2*d*x -
2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4)*d) + 3/128*(5*a^4*b - 80*a^3*b^2 + 50*a^2*b^3 + 8*a*b^4 + b^5)*a
rctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 5*a^6*b + 10
*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) + 3/32*(5*a^3*b -
 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) -
 3/128*(5*a^4*b - 80*a^3*b^2 + 50*a^2*b^3 + 8*a*b^4 + b^5)*arctan(1/2*((a +
 b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*
a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 -
5*a*b^3 - b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a
^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 3/64*(15*a^2
*b + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a
*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) - 1/64*(9*a^5*b -
```

65*a^4*b^2 - 134*a^3*b^3 - 34*a^2*b^4 + 29*a*b^5 + 3*b^6 + (9*a^5*b - 183*a
^4*b^2 + 98*a^3*b^3 + 266*a^2*b^4 - 27*a*b^5 - 3*b^6)*e^(6*d*x + 6*c) + (27
*a^5*b - 459*a^4*b^2 + 710*a^3*b^3 - 542*a^2*b^4 + 63*a*b^5 + 9*b^6)*e^(4*d
*x + 4*c) + (27*a^5*b - 341*a^4*b^2 + 86*a^3*b^3 + 398*a^2*b^4 - 65*a*b^5 -
 9*b^6)*e^(2*d*x + 2*c))/((a^9 + 7*a^8*b + 21*a^7*b^2 + 35*a^6*b^3 + 35*a^5
*b^4 + 21*a^4*b^5 + 7*a^3*b^6 + a^2*b^7 + (a^9 + 7*a^8*b + 21*a^7*b^2 + 35*
a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^3*b^6 + a^2*b^7)*e^(8*d*x + 8*c) +
4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 5*a^6*b^3 - 5*a^5*b^4 - 9*a^4*b^5 - 5*a^3*b^
6 - a^2*b^7)*e^(6*d*x + 6*c) + 2*(3*a^9 + 13*a^8*b + 23*a^7*b^2 + 25*a^6*b^
3 + 25*a^5*b^4 + 23*a^4*b^5 + 13*a^3*b^6 + 3*a^2*b^7)*e^(4*d*x + 4*c) + 4*(
a^9 + 5*a^8*b + 9*a^7*b^2 + 5*a^6*b^3 - 5*a^5*b^4 - 9*a^4*b^5 - 5*a^3*b^6 -
 a^2*b^7)*e^(2*d*x + 2*c))*d) + 1/64*(9*a^5*b - 65*a^4*b^2 - 134*a^3*b^3 -
34*a^2*b^4 + 29*a*b^5 + 3*b^6 + (27*a^5*b - 341*a^4*b^2 + 86*a^3*b^3 + 398*
a^2*b^4 - 65*a*b^5 - 9*b^6)*e^(-2*d*x - 2*c) + (27*a^5*b - 459*a^4*b^2 + 71
0*a^3*b^3 - 542*a^2*b^4 + 63*a*b^5 + 9*b^6)*e^(-4*d*x - 4*c) + (9*a^5*b - 1
83*a^4*b^2 + 98*a^3*b^3 + 266*a^2*b^4 - 27*a*b^5 - 3*b^6)*e^(-6*d*x - 6*c))
/((a^9 + 7*a^8*b + 21*a^7*b^2 + 35*a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^
3*b^6 + a^2*b^7 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 5*a^6*b^3 - 5*a^5*b^4 - 9*
a^4*b^5 - 5*a^3*b^6 - a^2*b^7)*e^(-2*d*x - 2*c) + 2*(3*a^9 + 13*a^8*b + 23*
a^7*b^2 + 25*a^6*b^3 + 25*a^5*b^4 + 23*a^4*b^5 + 13*a^3*b^6 + 3*a^2*b^7)*e^
(-4*d*x - 4*c) + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 5*a^6*b^3 - 5*a^5*b^4 - 9*a
^4*b^5 - 5*a^3*b^6 - a^2*b^7)*e^(-6*d*x - 6*c) + (a^9 + 7*a^8*b + 21*a^7*b^
2 + 35*a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^3*b^6 + a^2*b^7)*e^(-8*d*x -
 8*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(
3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^
4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a
^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 +
 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^
8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e
^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2
*b^6)*e^(6*d*x + 6*c) + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*
a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^
6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) + 1/16*(9*a^4*
b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + (27*a^4*b - 86*a^3*b^2 - 84
*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^(-2*d*x - 2*c) + (27*a^4*b - 156*a^3*b^2 + 1
10*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b - 22*a^3*b^2 -
 20*a^2*b^3 + 6*a*b^4 + b^5)*e^(-6*d*x - 6*c))/((a^8 + 6*a^7*b + 15*a^6*b^2
 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + 4*(a^8 + 4*a^7*b + 5*a^6
*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 10*a^
7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(-4*
d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6
)*e^(-6*d*x - 6*c) + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4
+ 6*a^3*b^5 + a^2*b^6)*e^(-8*d*x - 8*c))*d) + 3/32*(9*a^3*b + 21*a^2*b^2 +
15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4)*e^(-2*d*x - 2
c) + 3(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b

```
 - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b
^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^
4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) +...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9248 vs. $2(220) = 440$.
time = 0.55, size = 18818, normalized size = 78.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/64*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^16 + 16*(a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^15 +
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x + c)^16 - 4*(a^4 - 6*a
^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^14 - 4*(a^4 - 6*a^2*b^2 - 8*a*b^3 -
 3*b^4 - 30*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^2)*si
nh(d*x + c)^14 + 56*(10*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*
x + c)^3 - (a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)
^13 - 2*(13*a^4 - 20*a^3*b - 50*a^2*b^2 + 12*a*b^3 + 29*b^4 - 12*(a^4 - 8*a
^3*b - 14*a^2*b^2 + 5*b^4)*d*x)*cosh(d*x + c)^12 + 2*(910*(a^4 + 4*a^3*b +
6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 - 13*a^4 + 20*a^3*b + 50*a^2*b^2
 - 12*a*b^3 - 29*b^4 + 12*(a^4 - 8*a^3*b - 14*a^2*b^2 + 5*b^4)*d*x - 182*(a
^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 8*(54
6*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^5 - 182*(a^4 -
6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^3 - 3*(13*a^4 - 20*a^3*b - 50*a^
2*b^2 + 12*a*b^3 + 29*b^4 - 12*(a^4 - 8*a^3*b - 14*a^2*b^2 + 5*b^4)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^11 - 4*(9*a^4 + 12*a^3*b - 34*a^2*b^2 - 36*a*b^3
 + b^4 - 24*(a^4 - 10*a^3*b + 4*a^2*b^2 + 10*a*b^3 - 5*b^4)*d*x)*cosh(d*x +
 c)^10 + 4*(2002*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^
6 - 1001*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^4 - 9*a^4 - 12*a
^3*b + 34*a^2*b^2 + 36*a*b^3 - b^4 + 24*(a^4 - 10*a^3*b + 4*a^2*b^2 + 10*a*
b^3 - 5*b^4)*d*x - 33*(13*a^4 - 20*a^3*b - 50*a^2*b^2 + 12*a*b^3 + 29*b^4 -
 12*(a^4 - 8*a^3*b - 14*a^2*b^2 + 5*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c
)^10 + 8*(1430*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^7
- 1001*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^5 - 55*(13*a^4 - 2
0*a^3*b - 50*a^2*b^2 + 12*a*b^3 + 29*b^4 - 12*(a^4 - 8*a^3*b - 14*a^2*b^2 +
 5*b^4)*d*x)*cosh(d*x + c)^3 - 5*(9*a^4 + 12*a^3*b - 34*a^2*b^2 - 36*a*b^3
+ b^4 - 24*(a^4 - 10*a^3*b + 4*a^2*b^2 + 10*a*b^3 - 5*b^4)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^9 - 16*(27*a^3*b - 33*a^2*b^2 + 37*a*b^3 - 15*b^4 - 3*(3*
a^4 - 32*a^3*b + 38*a^2*b^2 - 40*a*b^3 + 15*b^4)*d*x)*cosh(d*x + c)^8 + 2*(
6435*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^8 - 6006*(a^
4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^6 - 495*(13*a^4 - 20*a^3*b -
 50*a^2*b^2 + 12*a*b^3 + 29*b^4 - 12*(a^4 - 8*a^3*b - 14*a^2*b^2 + 5*b^4)*d
```

```
*x)*cosh(d*x + c)^4 - 216*a^3*b + 264*a^2*b^2 - 296*a*b^3 + 120*b^4 + 24*(3
*a^4 - 32*a^3*b + 38*a^2*b^2 - 40*a*b^3 + 15*b^4)*d*x - 90*(9*a^4 + 12*a^3*
b - 34*a^2*b^2 - 36*a*b^3 + b^4 - 24*(a^4 - 10*a^3*b + 4*a^2*b^2 + 10*a*b^3
 - 5*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(715*(a^4 + 4*a^3*b +
6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^9 - 858*(a^4 - 6*a^2*b^2 - 8*a*b^3
 - 3*b^4)*cosh(d*x + c)^7 - 99*(13*a^4 - 20*a^3*b - 50*a^2*b^2 + 12*a*b^3 +
 29*b^4 - 12*(a^4 - 8*a^3*b - 14*a^2*b^2 + 5*b^4)*d*x)*cosh(d*x + c)^5 - 30
*(9*a^4 + 12*a^3*b - 34*a^2*b^2 - 36*a*b^3 + b^4 - 24*(a^4 - 10*a^3*b + 4*a
^2*b^2 + 10*a*b^3 - 5*b^4)*d*x)*cosh(d*x + c)^3 - 8*(27*a^3*b - 33*a^2*b^2
+ 37*a*b^3 - 15*b^4 - 3*(3*a^4 - 32*a^3*b + 38*a^2*b^2 - 40*a*b^3 + 15*b^4)
*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(9*a^4 - 132*a^3*b + 46*a^2*b^2 +
108*a*b^3 - 79*b^4 + 24*(a^4 - 10*a^3*b + 4*a^2*b^2 + 10*a*b^3 - 5*b^4)*d*x
)*cosh(d*x + c)^6 + 4*(2002*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cos
h(d*x + c)^10 - 3003*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^8 -
462*(13*a^4 - 20*a^3*b - 50*a^2*b^2 + 12*a*b^3 + 29*b^4 - 12*(a^4 - 8*a^3*b
 - 14*a^2*b^2 + 5*b^4)*d*x)*cosh(d*x + c)^6 - 210*(9*a^4 + 12*a^3*b - 34*a^
2*b^2 - 36*a*b^3 + b^4 - 24*(a^4 - 10*a^3*b + 4*a^2*b^2 + 10*a*b^3 - 5*b^4)
*d*x)*cosh(d*x + c)^4 + 9*a^4 - 132*a^3*b + 46*a^2*b^2 + 108*a*b^3 - 79*b^4
 + 24*(a^4 - 10*a^3*b + 4*a^2*b^2 + 10*a*b^3 - 5*b^4)*d*x - 112*(27*a^3*b -
 33*a^2*b^2 + 37*a*b^3 - 15*b^4 - 3*(3*a^4 - 32*a^3*b + 38*a^2*b^2 - 40*a*b
^3 + 15*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(546*(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^11 - 1001*(a^4 - 6*a^2*b^2 - 8*a
*b^3 - 3*b^4)*cosh(d*x + c)^9 - 198*(13*a^4 - 20*a^3*b - 50*a^2*b^2 + 12*a*
b^3 + 29*b^4 - 12*(a^4 - 8*a^3*b - 14*a^2*b^2 + 5*b^4)*d*x)*cosh(d*x + c)^7
 - 126*(9*a^4 + 12*a^3*b - 34*a^2*b^2 - 36*a*b^3 + b^4 - 24*(a^4 - 10*a^3*b
 + 4*a^2*b^2 + 10*a*b^3 - 5*b^4)*d*x)*cosh(d*x + c)^5 - 112*(27*a^3*b - 33*
a^2*b^2 + 37*a*b^3 - 15*b^4 - 3*(3*a^4 - 32*a^3*b + 38*a^2*b^2 - 40*a*b^3 +
 15*b^4)*d*x)*cosh(d*x + c)^3 + 3*(9*a^4 - 132*a^3*b + 46*a^2*b^2 + 108*a*b
^3 - 79*b^4 + 24*(a^4 - 10*a^3*b + 4*a^2*b^2 + 10*a*b^3 - 5*b^4)*d*x)*cosh(
d*x + c))*sinh(d*x + c)^5 + 2*(13*a^4 - 92*a^3*b - 154*a^2*b^2 + 20*a*b^3 +
 69*b^4 + 12*(a^4 - 8*a^3*b - 14*a^2*b^2 + 5*b^4)*d*x)*cosh(d*x + c)^4 + 2*
(910*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^12 - 2002*(a
^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^10 - 495*(13*a^4 - 20*a^3*b
 - 50*a^2*b^2 + 12*a*b^3 + 29*b^4 - 12*(a^4 - 8*a^3*b - 14*a^2*b^2 + 5*b^4)
*d*x)*cosh(d*x + c)^8 - 420*(9*a^4 + 12*a^3*b -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(220) = 440.
time = 3.18, size = 861, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 - 10*a*b + 5*b^2)*(d*x + c)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*
a^2*b^3 + 5*a*b^4 + b^5) + 24*(5*a^2*b - 10*a*b^2 + b^3)*arctan(1/2*(a*e^(2
*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 5*a^4*b + 10*a^
3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b)) + (a^3*e^(4*d*x + 4*c) + 3*a
^2*b*e^(4*d*x + 4*c) + 3*a*b^2*e^(4*d*x + 4*c) + b^3*e^(4*d*x + 4*c) - 8*a^
3*e^(2*d*x + 2*c) + 24*a*b^2*e^(2*d*x + 2*c) + 16*b^3*e^(2*d*x + 2*c))/(a^6
 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6) - (6*a^4
*e^(12*d*x + 12*c) - 48*a^3*b*e^(12*d*x + 12*c) - 84*a^2*b^2*e^(12*d*x + 12
*c) + 30*b^4*e^(12*d*x + 12*c) + 16*a^4*e^(10*d*x + 10*c) - 104*a^3*b*e^(10
*d*x + 10*c) - 24*a^2*b^2*e^(10*d*x + 10*c) + 72*a*b^3*e^(10*d*x + 10*c) -
24*b^4*e^(10*d*x + 10*c) + 5*a^4*e^(8*d*x + 8*c) + 84*a^3*b*e^(8*d*x + 8*c)
 + 30*a^2*b^2*e^(8*d*x + 8*c) + 84*a*b^3*e^(8*d*x + 8*c) - 123*b^4*e^(8*d*x
 + 8*c) - 20*a^4*e^(6*d*x + 6*c) + 280*a^3*b*e^(6*d*x + 6*c) - 64*a^2*b^2*e
^(6*d*x + 6*c) - 152*a*b^3*e^(6*d*x + 6*c) + 212*b^4*e^(6*d*x + 6*c) - 20*a
^4*e^(4*d*x + 4*c) + 136*a^3*b*e^(4*d*x + 4*c) + 224*a^2*b^2*e^(4*d*x + 4*c
) - 40*a*b^3*e^(4*d*x + 4*c) - 108*b^4*e^(4*d*x + 4*c) - 4*a^4*e^(2*d*x + 2
*c) + 24*a^2*b^2*e^(2*d*x + 2*c) + 32*a*b^3*e^(2*d*x + 2*c) + 12*b^4*e^(2*d
*x + 2*c) + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/((a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*
c) + 2*a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x + 4*c) + a*e^(2*d*x + 2*c) + b*e^(2
*d*x + 2*c))^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)
```

3.42 $\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=166

$$\frac{5(3a-4b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d}-\frac{(a-2b)\cosh(c+dx)}{(a+b)^4d}+\frac{\cosh^3(c+dx)}{3(a+b)^3d}+\frac{ab\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\text{sech}^2(\right.}$$

```
[Out] -(a-2*b)*cosh(d*x+c)/(a+b)^4/d+1/3*cosh(d*x+c)^3/(a+b)^3/d+1/4*a*b*sech(d*x
+c)/(a+b)^3/d/(a+b-b*sech(d*x+c)^2)^2+1/8*(7*a-4*b)*b*sech(d*x+c)/(a+b)^4/d
/(a+b-b*sech(d*x+c)^2)+5/8*(3*a-4*b)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2
))*b^(1/2)/(a+b)^(9/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 467, 1273, 1275, 214}

$$\frac{\cosh^3(c+dx)}{3d(a+b)^3}-\frac{(a-2b)\cosh(c+dx)}{d(a+b)^4}+\frac{b(7a-4b)\text{sech}(c+dx)}{8d(a+b)^4\left(a-b\text{sech}^2(c+dx)+b\right)}+\frac{ab\text{sech}(c+dx)}{4d(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)^2}+\frac{5\sqrt{b}\,(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (5*(3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*(a
+ b)^(9/2)*d) - ((a - 2*b)*Cosh[c + d*x])/((a + b)^4*d) + Cosh[c + d*x]^3/(
3*(a + b)^3*d) + (a*b*Sech[c + d*x])/(4*(a + b)^3*d*(a + b - b*Sech[c + d*x
]^2)^2) + ((7*a - 4*b)*b*Sech[c + d*x])/(8*(a + b)^4*d*(a + b - b*Sech[c +
d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+b-bx^2)^3}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= \frac{ab\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{b\text{Subst}\left(\int \frac{-\frac{4}{b(a+b)}+\frac{4ax^2}{b(a+b)^2}+\frac{3ax^4}{(a+b)^3}}{x^4(a+b-bx^2)^2}\,dx, x, \ldots\right.}{4d}$$

$$= \frac{ab\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\text{sech}(c+dx)}{8(a+b)^4d\left(a+b-b\text{sech}^2(c+dx)\right)}$$

$$= \frac{ab\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\text{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\text{sech}(c+dx)}{8(a+b)^4d\left(a+b-b\text{sech}^2(c+dx)\right)}$$

$$= -\frac{(a-2b)\cosh(c+dx)}{(a+b)^4d} + \frac{\cosh^3(c+dx)}{3(a+b)^3d} + \frac{ab\text{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\text{sech}^2(c+d\right.}$$

$$= \frac{5(3a-4b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d} - \frac{(a-2b)\cosh(c+dx)}{(a+b)^4d} + \frac{\cosh^3(c-}{3(a+b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.37, size = 227, normalized size = 1.37

$$\frac{\frac{15i(3a-4b)\sqrt{b}\left(\text{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\text{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{9/2}}-\frac{6\cosh(c+dx)\left(3a^3-24a^2b+30ab^2-13b^3+(6a^3-27a^2b-11ab^2+22b^3)\cosh(2(c+dx))+3(a-3b)(a+b)^2\cosh^2(2(c+dx))\right)}{(a+b)^4(a-b+(a+b)\cosh(2(c+dx)))^2}+\frac{2\cosh(3(c+dx))}{(a+b)^3}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (((15*I)*(3*a - 4*b)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c +
d*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/S
qrt[b]]))/(a + b)^(9/2) - (6*Cosh[c + d*x]*(3*a^3 - 24*a^2*b + 30*a*b^2 - 1
3*b^3 + (6*a^3 - 27*a^2*b - 11*a*b^2 + 22*b^3)*Cosh[2*(c + d*x)] + 3*(a - 3
*b)*(a + b)^2*Cosh[2*(c + d*x)]^2))/((a + b)^4*(a - b + (a + b)*Cosh[2*(c +
 d*x)])^2) + (2*Cosh[3*(c + d*x)])/(a + b)^3)/(24*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(150) = 300$.
time = 2.66, size = 341, normalized size = 2.05

method	result

derivativedivides	$-\frac{2b\left(\frac{-\frac{(9a+20b)a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{\left(27a^3+66a^2b+56a\,b^2-16b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\left(-\frac{27}{8}a^2-\frac{11}{2}ab+2b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{9a}{8}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}\right.}{(a+b)^4}$
default	$-\frac{2b\left(\frac{-\frac{(9a+20b)a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{\left(27a^3+66a^2b+56a\,b^2-16b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\left(-\frac{27}{8}a^2-\frac{11}{2}ab+2b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{9a}{8}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}\right.}{(a+b)^4}$
risch	$\frac{e^{3dx+3c}}{24(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{3\,e^{dx+c}a}{8(a+b)(a^3+3a^2b+3a\,b^2+b^3)d}+\frac{9\,e^{dx+c}b}{8(a+b)(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{3\,e^{-dx-c}a}{8(a^4+4a^3b+6a^2b^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b/(a+b)^4*((-1/8*(9*a+20*b)*a*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+66*
a^2*b+56*a*b^2-16*b^3)/a*tanh(1/2*d*x+1/2*c)^4+(-27/8*a^2-11/2*a*b+2*b^2)*t
anh(1/2*d*x+1/2*c)^2-9/8*a^2+1/4*a*b)/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2
*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-5/16*(3*a-4*b)/(a*b+b^2)^(1/2)
*arctanh(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+2*a+4*b)/(a*b+b^2)^(1/2)))-1/3/(a+b
)^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+
b)^4*(-a+5*b)/(tanh(1/2*d*x+1/2*c)-1)+1/3/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^3
-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2-1/2*(a-5*b)/(a+b)^4/(tanh(1/2*d*x+1/
2*c)+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7129 vs. $2(156) = 312$.
time = 0.51, size = 13095, normalized size = 78.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/48*(2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^14 + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^13 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^14 - 2*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c)^12 - 2*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3 - 91*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 8*(91*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 - 3*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 2*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^3)*cosh(d*x + c)^10 + 2*(1001*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 39*a^3 + 173*a^2*b + 113*a*b^2 - 99*b^3 - 66*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(1001*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 - 110*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c)^3 - 5*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - 10*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c)^8 + 2*(3003*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 495*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c)^4 - 85*a^3 + 475*a^2*b - 205*a*b^2 + 75*b^3 - 45*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(429*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 99*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c)^5 - 15*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^3)*cosh(d*x + c)^3 - 5*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 10*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c)^6 + 2*(3003*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 - 924*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c)^6 - 210*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^3)*cosh(d*x + c)^4 - 85*a^3 + 475*a^2*b - 205*a*b^2 + 75*b^3 - 140*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(1001*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 - 396*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c)^7 - 126*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^3)*cosh(d*x + c)^5 - 140*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c)^3 - 15*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^3)*cosh(d*x + c)^4 + 2*(1001*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 - 495*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c)^8 - 210*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^3)*cosh(d*x + c)^6 - 350*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c)^4 - 39*a^3 + 173*a^2*b + 113*a*b^2 - 99*b^3 - 75*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(91*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^11 - 55*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c)^9 - 30*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^3)*cosh(d*x + c)^7 - 70*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c)^5 - 25*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c)^3 - (39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - 2*(5*a^3 - 13*a^2*b - 41*a*b^2 - 23*b^3)*cosh(d*x + c)^2 + 2*(91*(a^

```
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 - 66*(5*a^3 - 13*a^2*b - 41*a
*b^2 - 23*b^3)*cosh(d*x + c)^10 - 45*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b
^3)*cosh(d*x + c)^8 - 140*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x
+ c)^6 - 75*(17*a^3 - 95*a^2*b + 41*a*b^2 - 15*b^3)*cosh(d*x + c)^4 - 5*a^3
 + 13*a^2*b + 41*a*b^2 + 23*b^3 - 6*(39*a^3 - 173*a^2*b - 113*a*b^2 + 99*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 15*((3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^
3)*cosh(d*x + c)^11 + 11*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*cosh(d*x + c)*
sinh(d*x + c)^10 + (3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*sinh(d*x + c)^11 + 4
*(3*a^3 - 4*a^2*b - 3*a*b^2 + 4*b^3)*cosh(d*x + c)^9 + (12*a^3 - 16*a^2*b -
 12*a*b^2 + 16*b^3 + 55*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^9 + 3*(55*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*cosh(d*x + c)
^3 + 12*(3*a^3 - 4*a^2*b - 3*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^8
+ 2*(9*a^3 - 18*a^2*b + 17*a*b^2 - 12*b^3)*cosh(d*x + c)^7 + 2*(165*(3*a^3
+ 2*a^2*b - 5*a*b^2 - 4*b^3)*cosh(d*x + c)^4 + 9*a^3 - 18*a^2*b + 17*a*b^2
- 12*b^3 + 72*(3*a^3 - 4*a^2*b - 3*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^7 + 14*(33*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 24*(
3*a^3 - 4*a^2*b - 3*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + (9*a^3 - 18*a^2*b + 17
*a*b^2 - 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(3*a^3 - 4*a^2*b - 3*a*
b^2 + 4*b^3)*cosh(d*x + c)^5 + 2*(231*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*c
osh(d*x + c)^6 + 252*(3*a^3 - 4*a^2*b - 3*a*b^2 + 4*b^3)*cosh(d*x + c)^4 +
6*a^3 - 8*a^2*b - 6*a*b^2 + 8*b^3 + 21*(9*a^3 - 18*a^2*b + 17*a*b^2 - 12*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(165*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4
*b^3)*cosh(d*x + c)^7 + 252*(3*a^3 - 4*a^2*b - 3*a*b^2 + 4*b^3)*cosh(d*x +
c)^5 + 35*(9*a^3 - 18*a^2*b + 17*a*b^2 - 12*b^3)*cosh(d*x + c)^3 + 10*(3*a^
3 - 4*a^2*b - 3*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^3 + 2*
a^2*b - 5*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + (165...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^3}{{\left(b\,{\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)
```

3.43 $\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=185

$$-\frac{(a-5b)x}{2(a+b)^4}-\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^4d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}-\frac{}{4(a+b)^2}$$

```
[Out] -1/2*(a-5*b)*x/(a+b)^4-1/8*(15*a^2-10*a*b-b^2)*arctan(b^(1/2)*tanh(d*x+c)/a
^(1/2))*b^(1/2)/a^(3/2)/(a+b)^4/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*
tanh(d*x+c)^2)^2-3/4*b*tanh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2-1/8*(11*
a-b)*b*tanh(d*x+c)/a/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.18, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3744, 482, 541, 536, 212, 211}

$$-\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a+b)^4}-\frac{b(11a-b)\tanh(c+dx)}{8ad(a+b)^3\left(a+b\tanh^2(c+dx)\right)}-\frac{3b\tanh(c+dx)}{4d(a+b)^2\left(a+b\tanh^2(c+dx)\right)^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{x(a-5b)}{2(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*((a - 5*b)*x)/(a + b)^4 - (Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqr
t[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a + b)^4*d) + (Cosh[c + d*x]*Sinh
[c + d*x])/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - (3*b*Tanh[c + d*x])/(4
*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) - ((11*a - b)*b*Tanh[c + d*x])/(8*a
*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a-5bx^2}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+d\right.}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Su}}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{}{8a}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{}{8a}$$

$$= -\frac{(a-5b)x}{2(a+b)^4} - \frac{\sqrt{b}\,(15a^2-10ab-b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^4d} + \frac{\cosh(c}{2(a+b)d\,(}$$

Mathematica [A]
time = 0.89, size = 158, normalized size = 0.85

$$\frac{-4(a-5b)(c+dx) + \frac{\sqrt{b}\,(-15a^2+10ab+b^2)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + 2(a+b)\sinh(2(c+dx)) - \frac{4b^2(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2} - \frac{(9a-b)b(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-4*(a - 5*b)*(c + d*x) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[(Sqrt[b]
*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + 2*(a + b)*Sinh[2*(c + d*x)] - (4*b^2*(a
 + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2 - ((9*a - b)
*b*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(8*(
a + b)^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(167) = 334$.
time = 2.71, size = 488, normalized size = 2.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/(a+b)^4*(((-9/8*a^2-5/4*a*b-1/8*b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(27
*a^3+58*a^2*b+27*a*b^2-4*b^3)/a*tanh(1/2*d*x+1/2*c)^5-1/8*(27*a^3+58*a^2*b+
27*a*b^2-4*b^3)/a*tanh(1/2*d*x+1/2*c)^3+(-9/8*a^2-5/4*a*b-1/8*b^2)*tanh(1/2
*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/
2*d*x+1/2*c)^2+a)^2+1/8*(15*a^2-10*a*b-b^2)*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/
(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*
(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-
1)^2+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+1/2*(a-5*b)/(a+b)^4*ln(tanh(1/2*d*
x+1/2*c)-1)-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/(a+b)^3/(tanh(1/2*d*x
+1/2*c)+1)+1/2/(a+b)^4*(-a+5*b)*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. $2(167) = 334$.
time = 0.85, size = 1806, normalized size = 9.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/4*b*log(2*(a - b)*e^(-2*d*x
 - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*(
(a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4
*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b
^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*
b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 1/16*(15*a^2*b + 10*a*b
^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/16*(9*a^4*b + 4*a^3*b^2 -
 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a
*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*
b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b
^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15
*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 +
 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a
^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c) + 2*(3*a^8 + 10*a
^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(4*
d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6
)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 -
 3*b^5 + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^(-2*d*x
- 2*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^(-4*d*
x - 4*c) + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a*b^4 + b^5)*e^(-6*d*x
```

```
- 6*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5
+ a^2*b^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)
*e^(-2*d*x - 2*c) + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*
b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(-6*d*x - 6*c) + (a^8 + 6*a^7*b + 1
5*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(-8*d*x - 8*c)
)*d) - 1/8*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^
2 - 23*a*b^3 - 9*b^4)*e^(-2*d*x - 2*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 +
 3*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x
 - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 +
4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x -
 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)
*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 -
a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^
3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) - 1/2*(d*x + c)/((a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d) + 1/8*e^(2*d*x + 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) -
1/8*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6319 vs. $2(167) = 334$.
time = 0.53, size = 12965, normalized size = 70.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^12 + 24*(a^4 + 3
*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*sinh(d*x + c)^12 + 8*(a^4 + a^3*b - a^2*b^2 - a*b^3
 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2*a^4 +
 2*a^3*b - 2*a^2*b^2 - 2*a*b^3 - 2*(a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*
x + 33*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^1
0 + 40*(11*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + 2*(a^4 + a
^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*
x + c))*sinh(d*x + c)^9 + 2*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b
^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^8 + 2*(495*(
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^4 + 5*a^4 + 17*a^3*b - 11*
a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x + 1
80*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d
*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*cosh(d*x + c)^5 + 60*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b
 - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^3 + (5*a^4 + 17*a^3*b - 11*a^2*b
^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*
x + c))*sinh(d*x + c)^7 + 4*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(
```

3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(462*(a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^6 + 420*(a^4 + a^3*b - a^2*b
^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^4 + 2
7*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2
- 15*a*b^3)*d*x + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16
*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6
+ 8*(198*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^7 + 252*(a^4 + a
^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*
x + c)^5 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 -
5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^3 + 3*(27*a^3*b - 21*a^2*b
^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*c
osh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3
- 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^4 + 2*(
495*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^8 + 840*(a^4 + a^3*b
- a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c
)^6 + 70*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^
3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^4 - 5*a^4 + 55*a^3*b + 3*a^2*b^
2 - 51*a*b^3 + 6*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x + 30*(27*
a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 -
15*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 2*a^4 - 6*a^3*b - 6*a^2*b
^2 - 2*a*b^3 + 8*(55*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(d*x + c)^9 +
120*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*
d*x)*cosh(d*x + c)^7 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4
- 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^5 + 10*(27*a^3
*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*
a*b^3)*d*x)*cosh(d*x + c)^3 - (5*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*
b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x +
c)^3 - 4*(2*a^4 - 7*a^3*b - 19*a^2*b^2 - 9*a*b^3 + b^4 + 2*(a^4 - 3*a^3*b
- 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^4 + 3*a^3*b + 3*a^2*
b^2 + a*b^3)*cosh(d*x + c)^10 + 90*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 -
3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*cosh(d*x + c)^8 + 14*(5*a^4 + 17*a^3*b
- 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*
x)*cosh(d*x + c)^6 + 15*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^
4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*cosh(d*x + c)^4 - 2*a^4 + 7*a^3*
b + 19*a^2*b^2 + 9*a*b^3 - b^4 - 2*(a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*
x - 3*(5*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b
- a^2*b^2 + 5*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((15*a^4 + 20*
a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^10 + 10*(15*a^4 + 20*a^3*
b - 6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (15*a^4 + 2
0*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*sinh(d*x + c)^10 + 4*(15*a^4 - 10*a^3
*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*cosh(d*x + c)^8 + (60*a^4 - 40*a^3*b - 64
*a^2*b^2 + 40*a*b^3 + 4*b^4 + 45*(15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3
- b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(15*a^4 + 20*a^3*b - 6*a^2*
b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^3 + 4*(15*a^4 - 10*a^3*b - 16*a^2*b^2 +
10*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(45*a^4 - 60*a^3*b + 62

```
*a^2*b^2 - 28*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 2*(105*(15*a^4 + 20*a^3*b -
6*a^2*b^2 - 12*a*b^3 - b^4)*cosh(d*x + c)^4 + 45*a^4 - 60*a^3*b + 62*a^2*b^
2 - 28*a*b^3 - 3*b^4 + 56*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)
*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(15*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(167) = 334.
time = 2.13, size = 535, normalized size = 2.89

$$-\frac{\frac{4\,(dx+c)(a-5\,b)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{\left(2\,ae^{(2\,dx+2\,c)}-10\,be^{(2\,dx+2\,c)}-a-b\right)e^{(-2\,dx-2\,c)}}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{\left(15\,a^2b-10\,ab^2-b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\sqrt{ab}}-\frac{e^{(2\,dx+2\,c)}}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(9\,a^3be^{(6\,dx+6\,c)}-5\,a^2b^2e^{(6\,dx+6\,c)}-13\,ab^3e^{(6\,dx+6\,c)}+b^4e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}-21\,a^2b^2e^{(4\,dx+4\,c)}+29\,ab^3e^{(4\,dx+4\,c)}-3\,b^4e^{(4\,dx+4\,c)}+27\,a^3be^{(2\,dx+2\,c)}+a^2b^2e^{(2\,dx+2\,c)}-23\,ab^3e^{(2\,dx+2\,c)}+3\,b^4e^{(2\,dx+2\,c)}+9\,a^3b+17\,a^2b^2+7\,ab^3-b^4\right)}{\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*(a - 5*b)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (
2*a*e^(2*d*x + 2*c) - 10*b*e^(2*d*x + 2*c) - a - b)*e^(-2*d*x - 2*c)/(a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (15*a^2*b - 10*a*b^2 - b^3)*arctan(
1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 4*a^
4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - e^(2*d*x + 2*c)/(a^3 + 3*
a^2*b + 3*a*b^2 + b^3) - 2*(9*a^3*b*e^(6*d*x + 6*c) - 5*a^2*b^2*e^(6*d*x +
6*c) - 13*a*b^3*e^(6*d*x + 6*c) + b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x +
 4*c) - 21*a^2*b^2*e^(4*d*x + 4*c) + 29*a*b^3*e^(4*d*x + 4*c) - 3*b^4*e^(4*
d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + a^2*b^2*e^(2*d*x + 2*c) - 23*a*b^3*
e^(2*d*x + 2*c) + 3*b^4*e^(2*d*x + 2*c) + 9*a^3*b + 17*a^2*b^2 + 7*a*b^3 -
b^4)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*(a*e^(4*d*x + 4*c) +
b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/
d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{sinh}\left(c+d\,x\right)}^2}{{\left(b\,{\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

3.44 $\int \frac{\sinh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$-\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d}+\frac{15\cosh(c+dx)}{8(a+b)^3d}-\frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2}-\frac{5\cosh(c+dx)}{8(a+b)^2d\left(a+b\ldots\right.}$$

```
[Out] 15/8*cosh(d*x+c)/(a+b)^3/d-1/4*cosh(d*x+c)/(a+b)/d/(a+b-b*sech(d*x+c)^2)^2-
5/8*cosh(d*x+c)/(a+b)^2/d/(a+b-b*sech(d*x+c)^2)-15/8*arctanh(sech(d*x+c)*b^
(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3745, 296, 331, 214}

$$\frac{15\cosh(c+dx)}{8d(a+b)^3}-\frac{5\cosh(c+dx)}{8d(a+b)^2\left(a-b\text{sech}^2(c+dx)+b\right)}-\frac{\cosh(c+dx)}{4d(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)
*d) + (15*Cosh[c + d*x])/(8*(a + b)^3*d) - Cosh[c + d*x]/(4*(a + b)*d*(a +
b - b*Sech[c + d*x]^2)^2) - (5*Cosh[c + d*x])/(8*(a + b)^2*d*(a + b - b*Sec
h[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^3}\,dx, x, \text{sech}(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} - \frac{5\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^2}\,dx, x, \text{sech}(c\right.}{4(a+b)d}$$

$$= -\frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} - \frac{5\cosh(c+dx)}{8(a+b)^2d\left(a+b-b\text{sech}^2(c+dx\right.}$$

$$= \frac{15\cosh(c+dx)}{8(a+b)^3d} - \frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\text{sech}^2(c+dx)\right)^2} - \frac{5\cosh(}{8(a+b)^2d\left(a+b\right.}$$

$$= -\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d} + \frac{15\cosh(c+dx)}{8(a+b)^3d} - \frac{\cosh(c-}{4(a+b)d\left(a+b-\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.27, size = 157, normalized size = 1.25

$$\frac{-\frac{15i\sqrt{b}\left(\text{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\text{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{7/2}}+\frac{2\cosh(c+dx)\left(4-\frac{4b^2}{(a-b+(a+b)\cosh(2(c+dx)))^2}-\frac{9b}{a-b+(a+b)\cosh(2(c+dx))}\right)}{(a+b)^3}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (((-15*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sq
rt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]))/(
```

```
a + b)^(7/2) + (2*Cosh[c + d*x]*(4 - (4*b^2)/(a - b + (a + b)*Cosh[2*(c + d
*x)])^2 - (9*b)/(a - b + (a + b)*Cosh[2*(c + d*x)])))/(a + b)^3)/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(110) = 220$.

time = 2.20, size = 252, normalized size = 2.00

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)^3(\tanh(\frac{dx}{2}+\frac{c}{2})-1)}+\frac{2b\left(\frac{-\frac{(9a^2+24ab+8b^2)(\tanh^6(\frac{dx}{2}+\frac{c}{2}))}{8a}-\frac{(27a^3+78a^2b+88ab^2+16b^3)(\tanh^4(\frac{dx}{2}+\frac{c}{2}))}{8a^2}-\frac{(27a^2+5}{}}{(a(\tanh^4(\frac{dx}{2}+\frac{c}{2}))+2a(\tanh^2(\frac{dx}{2}+\frac{c}{2}))+4b(\tanh^2(\frac{dx}{2}+\frac{c}{2}))+}\right.}{(a+b)^3}}{d}$
default	$\frac{-\frac{1}{(a+b)^3(\tanh(\frac{dx}{2}+\frac{c}{2})-1)}+\frac{2b\left(\frac{-\frac{(9a^2+24ab+8b^2)(\tanh^6(\frac{dx}{2}+\frac{c}{2}))}{8a}-\frac{(27a^3+78a^2b+88ab^2+16b^3)(\tanh^4(\frac{dx}{2}+\frac{c}{2}))}{8a^2}-\frac{(27a^2+5}{}}{(a(\tanh^4(\frac{dx}{2}+\frac{c}{2}))+2a(\tanh^2(\frac{dx}{2}+\frac{c}{2}))+4b(\tanh^2(\frac{dx}{2}+\frac{c}{2}))+}\right.}{(a+b)^3}}{d}$
risch	$\frac{e^{dx+c}}{2(a^3+3a^2b+3ab^2+b^3)d}+\frac{e^{-dx-c}}{2(a^3+3a^2b+3ab^2+b^3)d}-\frac{(9a\,e^{6dx+6c}+9b\,e^{6dx+6c}+27a\,e^{4dx+4c}-b\,e^{4dx+4c}+27a\,e^{2dx+2c}-}{4(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b)^2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+2*b/(a+b)^3*((-1/8*(9*a^2+24*a*b+8*
b^2)/a*tanh(1/2*d*x+1/2*c)^6-1/8/a^2*(27*a^3+78*a^2*b+88*a*b^2+16*b^3)*tanh
(1/2*d*x+1/2*c)^4-1/8*(27*a^2+56*a*b+8*b^2)/a*tanh(1/2*d*x+1/2*c)^2-9/8*a-1
/4*b)/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1
/2*c)^2+a)^2-15/16/(a*b+b^2)^(1/2)*arctanh(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+2
*a+4*b)/(a*b+b^2)^(1/2)))+1/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2 + 4*a*b + 2*b^2 + 2*(a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c
))*e^(10*d*x) + 5*(2*a^2*e^(8*c) - a*b*e^(8*c) - 3*b^2*e^(8*c))*e^(8*d*x) +
 5*(4*a^2*e^(6*c) - 7*a*b*e^(6*c) + b^2*e^(6*c))*e^(6*d*x) + 5*(4*a^2*e^(4*
c) - 7*a*b*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 5*(2*a^2*e^(2*c) - a*b*e^(2*c
```

```
) - 3*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*e^(9*c) + 5*a^4*b*d*e^(9*c) + 10*a^3*
b^2*d*e^(9*c) + 10*a^2*b^3*d*e^(9*c) + 5*a*b^4*d*e^(9*c) + b^5*d*e^(9*c))*e
^(9*d*x) + 4*(a^5*d*e^(7*c) + 3*a^4*b*d*e^(7*c) + 2*a^3*b^2*d*e^(7*c) - 2*a
^2*b^3*d*e^(7*c) - 3*a*b^4*d*e^(7*c) - b^5*d*e^(7*c))*e^(7*d*x) + 2*(3*a^5*
d*e^(5*c) + 7*a^4*b*d*e^(5*c) + 6*a^3*b^2*d*e^(5*c) + 6*a^2*b^3*d*e^(5*c) +
 7*a*b^4*d*e^(5*c) + 3*b^5*d*e^(5*c))*e^(5*d*x) + 4*(a^5*d*e^(3*c) + 3*a^4*
b*d*e^(3*c) + 2*a^3*b^2*d*e^(3*c) - 2*a^2*b^3*d*e^(3*c) - 3*a*b^4*d*e^(3*c)
 - b^5*d*e^(3*c))*e^(3*d*x) + (a^5*d*e^c + 5*a^4*b*d*e^c + 10*a^3*b^2*d*e^c
 + 10*a^2*b^3*d*e^c + 5*a*b^4*d*e^c + b^5*d*e^c)*e^(d*x)) + 1/2*integrate(1
5/2*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4 + (a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4
*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c) - 2*a*b^3*e
^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3792 vs. $2(116) = 232$.
time = 0.46, size = 7119, normalized size = 56.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 80*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^9 + 8*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 + 20*(2*
a^2 - a*b - 3*b^2)*cosh(d*x + c)^8 + 20*(18*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^2 + 2*a^2 - a*b - 3*b^2)*sinh(d*x + c)^8 + 160*(6*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^3 + (2*a^2 - a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(
4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^6 + 20*(84*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^4 + 28*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a^2 - 7*a*b + b^2)*si
nh(d*x + c)^6 + 8*(252*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 140*(2*a^2 - a
*b - 3*b^2)*cosh(d*x + c)^3 + 15*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c))*sinh(
d*x + c)^5 + 20*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^4 + 20*(84*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^6 + 70*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^4 + 15*(4*
a^2 - 7*a*b + b^2)*cosh(d*x + c)^2 + 4*a^2 - 7*a*b + b^2)*sinh(d*x + c)^4 +
 80*(12*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 14*(2*a^2 - a*b - 3*b^2)*cosh
(d*x + c)^5 + 5*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^3 + (4*a^2 - 7*a*b + b^
2)*cosh(d*x + c))*sinh(d*x + c)^3 + 20*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^
2 + 20*(18*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 28*(2*a^2 - a*b - 3*b^2)*c
osh(d*x + c)^6 + 15*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^4 + 6*(4*a^2 - 7*a*
b + b^2)*cosh(d*x + c)^2 + 2*a^2 - a*b - 3*b^2)*sinh(d*x + c)^2 + 15*((a^2
+ 2*a*b + b^2)*cosh(d*x + c)^9 + 9*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d
*x + c)^8 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^9 + 4*(a^2 - b^2)*cosh(d*x +
c)^7 + 4*(9*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^
7 + 28*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*
```

```
sinh(d*x + c)^6 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 2*(63*(a^2 +
2*a*b + b^2)*cosh(d*x + c)^4 + 42*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a
*b + 3*b^2)*sinh(d*x + c)^5 + 2*(63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 7
0*(a^2 - b^2)*cosh(d*x + c)^3 + 5*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*si
nh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)^3 + 4*(21*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^6 + 35*(a^2 - b^2)*cosh(d*x + c)^4 + 5*(3*a^2 - 2*a*b + 3*b^2)
*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^3 + 4*(9*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^7 + 21*(a^2 - b^2)*cosh(d*x + c)^5 + 5*(3*a^2 - 2*a*b + 3*b^2)*
cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^2 + 2*a
*b + b^2)*cosh(d*x + c) + (9*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 28*(a^2
- b^2)*cosh(d*x + c)^6 + 10*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 12*(a
^2 - b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c))*sqrt(b/(a + b
))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(
d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3
*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*c
osh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x +
 c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) +
a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*
x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*co
sh(d*x + c))*sinh(d*x + c) + a + b)) + 8*a^2 + 16*a*b + 8*b^2 + 40*(2*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^9 + 4*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^7 +
3*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^5 + 2*(4*a^2 - 7*a*b + b^2)*cosh(d*x
+ c)^3 + (2*a^2 - a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^9 + 9*(a^5 + 5
*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x
+ c)^8 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d
*x + c)^9 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cos
h(d*x + c)^7 + 4*(9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^
5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b
^5)*d)*sinh(d*x + c)^7 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b
^4 + 3*b^5)*d*cosh(d*x + c)^5 + 28*(3*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*
b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2
*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 42*(a^5 +
 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^
5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^5 +
 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^
3 + 2*(63*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(
d*x + c)^5 + 70*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*c
osh(d*x + c)^3 + 5*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b
^5)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a^5 + 5*a^4*b + 10*a^3*b^2 +
10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^6 + 35*(a^5 + 3*a^4*b + 2*a^3*b
^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)}{{\left(b\,{\tanh\left(c+d\,x\right)}^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

3.45 $\int \frac{\text{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal. Leaf size=156

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}+\frac{\sqrt{b}\,(15a^2+20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d}+\frac{b\text{sech}(c+dx)}{4a(a+b)d\left(a+b-b\text{sech}^2(c+\right.}$$

[Out] -arctanh(cosh(d*x+c))/a^3/d+1/4*b*sech(d*x+c)/a/(a+b)/d/(a+b-b*sech(d*x+c)^2)^2+1/8*b*(7*a+4*b)*sech(d*x+c)/a^2/(a+b)^2/d/(a+b-b*sech(d*x+c)^2)+1/8*(15*a^2+20*a*b+8*b^2)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^3/(a+b)^(5/2)/d

Rubi [A]
time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3745, 425, 541, 536, 213, 214}

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}+\frac{b(7a+4b)\text{sech}(c+dx)}{8a^2d(a+b)^2\left(a-b\text{sech}^2(c+dx)+b\right)}+\frac{\sqrt{b}\,(15a^2+20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}}+\frac{b\text{sech}(c+dx)}{4ad(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] -(ArcTanh[Cosh[c + d*x]]/(a^3*d)) + (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*a^3*(a + b)^(5/2)*d) + (b*Sech[c + d*x])/(4*a*(a + b)*d*(a + b - b*Sech[c + d*x]^2)^2) + (b*(7*a + 4*b)*Sech[c + d*x])/(8*a^2*(a + b)^2*d*(a + b - b*Sech[c + d*x]^2))

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c - a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x^2)(a+b-bx^2)^3}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}(c+dx)}{4a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{4a+b+3bx^2}{(-1+x^2)(a+b-bx^2)^2}\,dx, x, \operatorname{sech}\right.}{4a(a+b)d}$$

$$= \frac{b\operatorname{sech}(c+dx)}{4a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{b(7a+4b)\operatorname{sech}(c+dx)}{8a^2(a+b)^2d\left(a+b-b\operatorname{sech}^2(c+dx\right.}$$

$$= \frac{b\operatorname{sech}(c+dx)}{4a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{b(7a+4b)\operatorname{sech}(c+dx)}{8a^2(a+b)^2d\left(a+b-b\operatorname{sech}^2(c+dx\right.}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{\sqrt{b}\,(15a^2+20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.05, size = 236, normalized size = 1.51

$$\frac{\frac{i\sqrt{b}\,(15a^2+20ab+8b^2)\operatorname{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{5/2}} + \frac{i\sqrt{b}\,(15a^2+20ab+8b^2)\operatorname{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{5/2}} + \frac{8a^2b^2\cosh(c+dx)}{(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{2ab(9a+4b)\cosh(c+dx)}{(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))} + 8\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((I*Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Ta
nh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(5/2) + (I*Sqrt[b]*(15*a^2 + 20*a*b + 8*
b^2)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b
)^(5/2) + (8*a^2*b^2*Cosh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*(c +
 d*x)])^2) + (2*a*b*(9*a + 4*b)*Cosh[c + d*x])/((a + b)^2*(a - b + (a + b)*
Cosh[2*(c + d*x)])) + 8*Log[Tanh[(c + d*x)/2]])/(8*a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(142) = 284$.
time = 3.20, size = 304, normalized size = 1.95

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3} - \frac{2b\left(\frac{-\frac{(9a^2+28ab+16b^2)a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a^2+2ab+b^2)} - \frac{3(9a^3+30a^2b+40a\,b^2+16b^3)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a^2+2ab+b^2)} - \frac{a(27a^2+68ab+3}{8(a^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}\right.}{a^3}}{d}$

default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2b\left(\frac{-\frac{(9a^2+28ab+16b^2)a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a^2+2ab+b^2)}-\frac{3(9a^3+30a^2b+40a\,b^2+16b^3)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a^2+2ab+b^2)}-\frac{a(27a^2+68ab+}{8(}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right.}\right.}{a^3}}{d}$
risch	$\frac{(9a^2\mathrm{e}^{6dx+6c}+13ab\,\mathrm{e}^{6dx+6c}+4b^2\mathrm{e}^{6dx+6c}+27a^2\mathrm{e}^{4dx+4c}+11ab\,\mathrm{e}^{4dx+4c}-4b^2\mathrm{e}^{4dx+4c}+27a^2\mathrm{e}^{2dx+2c}+11ab\,\mathrm{e}^{2dx+2c}-4b^2\mathrm{e}^{}}{4(a^2+2ab+b^2)(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}+2a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+a+b)^2a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^3*ln(tanh(1/2*d*x+1/2*c))-2/a^3*b*((-1/8*(9*a^2+28*a*b+16*b^2)*a/(
a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-3/8*(9*a^3+30*a^2*b+40*a*b^2+16*b^3)/(
a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-1/8*a*(27*a^2+68*a*b+32*b^2)/(a^2+2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^2-3/8*a^2*(3*a+2*b)/(a^2+2*a*b+b^2))/(a*tanh(1/2
*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1/16
*(15*a^2+20*a*b+8*b^2)/(a^2+2*a*b+b^2)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*a*tan
h(1/2*d*x+1/2*c)^2+2*a+4*b)/(a*b+b^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((9*a^2*b*e^(7*c) + 13*a*b^2*e^(7*c) + 4*b^3*e^(7*c))*e^(7*d*x) + (27*a
^2*b*e^(5*c) + 11*a*b^2*e^(5*c) - 4*b^3*e^(5*c))*e^(5*d*x) + (27*a^2*b*e^(3
*c) + 11*a*b^2*e^(3*c) - 4*b^3*e^(3*c))*e^(3*d*x) + (9*a^2*b*e^c + 13*a*b^2
*e^c + 4*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3*d +
 a^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c) + 4*a
^3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 2*a^5*
b*d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6
*d*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*e^(4*c)
+ 3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^(2*c) - 2
*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*
e^(-c))/(a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/8*(
(15*a^2*b*e^(3*c) + 20*a*b^2*e^(3*c) + 8*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b
*e^c + 20*a*b^2*e^c + 8*b^3*e^c)*e^(d*x))/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3 + (a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))
*e^(4*d*x) + 2*(a^6*e^(2*c) + a^5*b*e^(2*c) - a^4*b^2*e^(2*c) - a^3*b^3*e^(
2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5783 vs. $2(148) = 296$.
time = 0.56, size = 10716, normalized size = 68.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^7 + 28*(9*a^3*b + 1
3*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(9*a^3*b + 13*a^2*b^
2 + 4*a*b^3)*sinh(d*x + c)^7 + 4*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x
 + c)^5 + 4*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3 + 21*(9*a^3*b + 13*a^2*b^2 + 4
*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(9*a^3*b + 13*a^2*b^2 + 4*
a*b^3)*cosh(d*x + c)^3 + (27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*s
inh(d*x + c)^4 + 4*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 4*(3
5*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^4 + 27*a^3*b + 11*a^2*b^2
- 4*a*b^3 + 10*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^3 + 4*(21*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^5 + 10*(27*a^
3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 3*(27*a^3*b + 11*a^2*b^2 - 4*
a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((15*a^4 + 50*a^3*b + 63*a^2*b^2 +
36*a*b^3 + 8*b^4)*cosh(d*x + c)^8 + 8*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*
a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 50*a^3*b + 63*a^2*
b^2 + 36*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2
- 20*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20
*a*b^3 - 8*b^4 + 7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*
b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3
- 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 30*a^3*b + 29*a^2*b^2
 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 50*a^3*b + 63*a^2*b
^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 45*a^4 + 30*a^3*b + 29*a^2*b^2 + 4
4*a*b^3 + 24*b^4 + 30*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3
+ 8*b^4 + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x
 + c)^5 + 10*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x +
c)^3 + (45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh
(d*x + c)^2 + 4*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh
(d*x + c)^6 + 15*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*
x + c)^4 + 15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4 + 3*(45*a^4 + 3
0*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 8*((15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^7 +
3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (45*
a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a^4
+ 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sq
```

```
rt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)
^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 +
3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)
*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/
(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a +
 b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 +
 (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(9*a^3*b + 13*a^2*b^2 +
 4*a*b^3)*cosh(d*x + c) - 16*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*c
osh(d*x + c)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c
)*sinh(d*x + c)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x +
c)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(a^4 + 2*a^3*b
 - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x +
 c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*c
osh(d*x + c)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4
+ 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a^4
 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^5 + 1
0*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^
2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b
- 2*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4)*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)
^4 + a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b
^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh
(d*x + c)^5 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)
^3 + (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)
```

3.46 $\int \frac{\text{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3} \, dx$

Optimal. Leaf size=112

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d}-\frac{15\coth(c+dx)}{8a^3d}+\frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2}+\frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] -15/8*coth(d*x+c)/a^3/d-15/8*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^
(7/2)/d+1/4*coth(d*x+c)/a/d/(a+b*tanh(d*x+c)^2)^2+5/8*coth(d*x+c)/a^2/d/(a+
b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3744, 296, 331, 211}

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d}-\frac{15\coth(c+dx)}{8a^3d}+\frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}+\frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(7/2)*d) - (15*C
oth[c + d*x])/(8*a^3*d) + Coth[c + d*x]/(4*a*d*(a + b*Tanh[c + d*x]^2)^2) +
 (5*Coth[c + d*x])/(8*a^2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{15\operatorname{Subst}\left(\int \frac{1}{x^2(a+\ldots}\right.}{\ldots}$$

$$= -\frac{15\coth(c+dx)}{8a^3d} + \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right.}$$

$$= -\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d} - \frac{15\coth(c+dx)}{8a^3d} + \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+d\right.}$$

Mathematica [A]
time = 0.74, size = 109, normalized size = 0.97

$$\frac{-15\sqrt{b}\,\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right) - 8\sqrt{a}\,\coth(c+dx) - \frac{\sqrt{a}\,b(9a-7b+(9a+7b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*Sqrt[a]*Coth[c + d
*x] - (Sqrt[a]*b*(9*a - 7*b + (9*a + 7*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*
x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(8*a^(7/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(96) = 192$.

time = 2.92, size = 306, normalized size = 2.73

method	result
derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}+\frac{2b\left(\frac{-\frac{9a\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{9a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}+\right.}{}$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}+\frac{2b\left(\frac{-\frac{9a\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{9a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}+\right.}{}$
risch	$-\frac{8a^4e^{8dx+8c}+23a^3b\,e^{8dx+8c}+45a^2b^2e^{8dx+8c}+45a\,b^3e^{8dx+8c}+15b^4e^{8dx+8c}+32a^4e^{6dx+6c}+46a^3b\,e^{6dx+6c}-90a\,b^3e^{6dx}}{4(e^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^3*tanh(1/2*d*x+1/2*c)+2/a^3*b*((-9/8*a*tanh(1/2*d*x+1/2*c)^7+(-
27/8*a-7/2*b)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a-7/2*b)*tanh(1/2*d*x+1/2*c)^3-9
/8*a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^
2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+15/8*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(
a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b
))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/(
(2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))-1/2/a^3/tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(96) = 192$.
time = 0.66, size = 478, normalized size = 4.27

$$-\frac{8\,a^4+41\,a^3b+73\,a^2b^2+55\,ab^3+15\,b^4+2\,(16\,a^4+41\,a^3b-55\,ab^3-30\,b^4)e^{(-2\,dx-2\,c)}+2\,(24\,a^4+32\,a^3b+5\,a^2b^2+50\,ab^3+45\,b^4)e^{(-4\,dx-4\,c)}+2\,(16\,a^4+23\,a^3b-45\,ab^3-30\,b^4)e^{(-6\,dx-6\,c)}+(8\,a^4+23\,a^3b+45\,a^2b^2+45\,ab^3+15\,b^4)e^{(-8\,dx-8\,c)}}{4\,(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+(3\,a^7+4\,a^6b-6\,a^5b^2-12\,a^4b^3-5\,a^3b^4)e^{(-2\,dx-2\,c)}+2\,(a^7+2\,a^5b^2+8\,a^4b^3+5\,a^3b^4)e^{(-4\,dx-4\,c)}-2\,(a^7+2\,a^5b^2+8\,a^4b^3+5\,a^3b^4)e^{(-6\,dx-6\,c)}-(3\,a^7+4\,a^6b-6\,a^5b^2-12\,a^4b^3-5\,a^3b^4)e^{(-8\,dx-8\,c)}-(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)e^{(-10\,dx-10\,c)})d}+\frac{15\,b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(8*a^4 + 41*a^3*b + 73*a^2*b^2 + 55*a*b^3 + 15*b^4 + 2*(16*a^4 + 41*a^
3*b - 55*a*b^3 - 30*b^4)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 32*a^3*b + 5*a^2*b^
2 + 50*a*b^3 + 45*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 23*a^3*b - 45*a*b^3 -
 30*b^4)*e^(-6*d*x - 6*c) + (8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*
b^4)*e^(-8*d*x - 8*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 +
(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-2*d*x - 2*c) + 2
*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-4*d*x - 4*c) - 2*(a^7 + 2*a^
5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-6*d*x - 6*c) - (3*a^7 + 4*a^6*b - 6*a^5*
b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-8*d*x - 8*c) - (a^7 + 4*a^6*b + 6*a^5*b^2
 + 4*a^4*b^3 + a^3*b^4)*e^(-10*d*x - 10*c))*d) + 15/8*b*arctan(1/2*((a + b)
*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3995 vs. $2(96) = 192$.
time = 0.43, size = 8312, normalized size = 74.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)
^8 + 32*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)*s
inh(d*x + c)^7 + 4*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*sinh
```

(d*x + c)^8 + 8*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c)^6 + 8
*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4 + 14*(8*a^4 + 23*a^3*b + 45*a^2*b^2
 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(14*(8*a^4 + 23
*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^3 + 3*(16*a^4 + 23*a
^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(24*a^4 + 32*a
^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*cosh(d*x + c)^4 + 8*(35*(8*a^4 + 23*a
^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^4 + 24*a^4 + 32*a^3*b
+ 5*a^2*b^2 + 50*a*b^3 + 45*b^4 + 15*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*a^4 + 164*a^3*b + 292*a^2*b^2 + 220
*a*b^3 + 60*b^4 + 32*(7*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)
*cosh(d*x + c)^5 + 5*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c)^
3 + (24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*cosh(d*x + c))*sinh
(d*x + c)^3 + 8*(16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b^4)*cosh(d*x + c)^2 + 8
(14(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^6 +
15*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c)^4 + 16*a^4 + 41*a^
3*b - 55*a*b^3 - 30*b^4 + 6*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 15*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4)*cosh(d*x + c)^10 + 10*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*sinh(d*x + c)^10 + (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*co
sh(d*x + c)^8 + (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4 + 45*(a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*
(15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^3 + (3*a^4 +
4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*
(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^6 + 2*(105*(a^4 + 4*a^3*b
 + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 + a^4 + 2*a^2*b^2 + 8*a*b^3 +
 5*b^4 + 14*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 4*(63*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh
(d*x + c)^5 + 14*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x
+ c)^3 + 3*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c))*sinh(d*x + c)
^5 - 2*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^4 + 2*(105*(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^6 + 35*(3*a^4 + 4*a^3*b
- 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^4 - a^4 - 2*a^2*b^2 - 8*a*b^3
 - 5*b^4 + 15*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 - a^4 - 4*a^3*b - 6*a^2*b^2 - 4*a*b^3 - b^4 + 8*(15*(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^7 + 7*(3*a^4 + 4*a^3*b - 6*a^2*b
^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 5*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b
^4)*cosh(d*x + c)^3 - (a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c))*si
nh(d*x + c)^3 - (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x +
 c)^2 + (45*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^8 + 2
8*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 30*(a^
4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^4 - 3*a^4 - 4*a^3*b + 6*a^2*
b^2 + 12*a*b^3 + 5*b^4 - 12*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 2*(5*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*co
sh(d*x + c)^9 + 4*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*cosh(d*x

```
 + c)^7 + 6*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^5 - 4*(a^4 +
2*a^2*b^2 + 8*a*b^3 + 5*b^4)*cosh(d*x + c)^3 - (3*a^4 + 4*a^3*b - 6*a^2*b^2
 - 12*a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2
*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^
2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 +
 a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*c
osh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b
)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sq
rt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)
*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 16*(2*(8*a^4 + 23*a^3*b + 45*a^2*
b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^7 + 3*(16*a^4 + 23*a^3*b - 45*a*b^3
- 30*b^4)*cosh(d*x + c)^5 + 2*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 4
5*b^4)*cosh(d*x + c)^3 + (16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b^4)*cosh(d*x +
 c))*sinh(d*x + c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*co
sh(d*x + c)^10 + 10*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cos
h(d*x + c)*sinh(d*x + c)^9 + (a^7 + 4*a^6*b + 6...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(96) = 192.
time = 1.04, size = 351, normalized size = 3.13

$$-\frac{\frac{15\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{2\,\left(9\,a^3be^{(6\,dx+6\,c)}+3\,a^2b^2e^{(6\,dx+6\,c)}-13\,ab^3e^{(6\,dx+6\,c)}-7\,b^4e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+3\,a^2b^2e^{(4\,dx+4\,c)}+13\,ab^3e^{(4\,dx+4\,c)}+21\,b^4e^{(4\,dx+4\,c)}+27\,a^3be^{(2\,dx+2\,c)}+25\,a^2b^2e^{(2\,dx+2\,c)}-23\,ab^3e^{(2\,dx+2\,c)}-21\,b^4e^{(2\,dx+2\,c)}+9\,a^3b+25\,a^2b^2+23\,ab^3+7\,b^4\right)}{\left(a^5+2\,a^4b+a^3b^2\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)^2}+\frac{16}{a^3\left(e^{(2\,dx+2\,c)}-1\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/(sqrt(a*b)*a^3) - 2*(9*a^3*b*e^(6*d*x + 6*c) + 3*a^2*b^2*e^(6*d*x + 6
*c) - 13*a*b^3*e^(6*d*x + 6*c) - 7*b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x
+ 4*c) + 3*a^2*b^2*e^(4*d*x + 4*c) + 13*a*b^3*e^(4*d*x + 4*c) + 21*b^4*e^(4
*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 25*a^2*b^2*e^(2*d*x + 2*c) - 23*a*
```

```
b^3*e^(2*d*x + 2*c) - 21*b^4*e^(2*d*x + 2*c) + 9*a^3*b + 25*a^2*b^2 + 23*a*
b^3 + 7*b^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4
*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) + 16/(a^3*(e^(2
*d*x + 2*c) - 1)))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c+dx)^2\left(b\tanh(c+dx)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

3.47 $\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal. Leaf size=196

$$\frac{(a+6b)\tanh^{-1}(\cosh(c+dx))}{2a^4d}-\frac{\sqrt{b}\,(15a^2+40ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d}-\frac{\coth(c+dx)\operatorname{csch}(c+}{2ad\left(a+b-b\operatorname{sech}^2(c+\right.}$$

```
[Out] 1/2*(a+6*b)*arctanh(cosh(d*x+c))/a^4/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a+b
-b*sech(d*x+c)^2)^2-3/4*b*sech(d*x+c)/a^2/d/(a+b-b*sech(d*x+c)^2)^2-1/8*b*(
11*a+12*b)*sech(d*x+c)/a^3/(a+b)/d/(a+b-b*sech(d*x+c)^2)-1/8*(15*a^2+40*a*b
+24*b^2)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(3/2)/d
```

Rubi [A]
time = 0.23, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {3745, 482, 541, 536, 213, 214}

$$\frac{(a+6b)\tanh^{-1}(\cosh(c+dx))}{2a^4d}-\frac{b(11a+12b)\operatorname{sech}(c+dx)}{8a^3d(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{\sqrt{b}\,(15a^2+40ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{3/2}}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d) - (Sqrt[b]*(15*a^2 + 40*a*b +
24*b^2)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*
d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a + b - b*Sech[c + d*x]^2)^2) -
(3*b*Sech[c + d*x])/(4*a^2*d*(a + b - b*Sech[c + d*x]^2)^2) - (b*(11*a + 12
*b)*Sech[c + d*x])/(8*a^3*(a + b)*d*(a + b - b*Sech[c + d*x]^2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+b-bx^2)^3}\,dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{a+b+5bx^2}{(-1+x^2)(a+b-bx^2)^3}\,dx, x, \operatorname{sech}(c+\right.}{2ad}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{b}{8a^3(a+}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{b}{8a^3(a+}$$

$$= \frac{(a+6b)\tanh^{-1}(\cosh(c+dx))}{2a^4d} - \frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}}{\sqrt{a+}}\right.}{8a^4(a+b)^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.13, size = 269, normalized size = 1.37

$$-\frac{\frac{i\sqrt{b}\left(15a^2+40ab+24b^2\right)\operatorname{ArcTan}\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}}+\frac{i\sqrt{b}\left(15a^2+40ab+24b^2\right)\operatorname{ArcTan}\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}}+\frac{8a^2b^2\cosh(c+dx)}{(a+b)(a-b+(a+b)\cosh(2(c+dx)))^2}+\frac{2ab(9a+8b)\cosh(c+dx)}{(a+b)(a-b+(a+b)\cosh(2(c+dx)))}+a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+4(a+6b)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*((I*Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[((-I)*Sqrt[a + b] - Sqrt
[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (I*Sqrt[b]*(15*a^2 + 40*a*
b + 24*b^2)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])
/(a + b)^(3/2) + (8*a^2*b^2*Cosh[c + d*x])/((a + b)*(a - b + (a + b)*Cosh[2
*(c + d*x)])^2) + (2*a*b*(9*a + 8*b)*Cosh[c + d*x])/((a + b)*(a - b + (a +
b)*Cosh[2*(c + d*x)])) + a*Csch[(c + d*x)/2]^2 + 4*(a + 6*b)*Log[Tanh[(c +
d*x)/2]] + a*Sech[(c + d*x)/2]^2)/(a^4*d)
```

Maple [A]
time = 3.41, size = 304, normalized size = 1.55

method	result

derivativedivides	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}+\frac{2b\left(\frac{-\frac{(9a^2+32ab+24b^2)a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{(27a^3+102a^2b+152a\,b^2+80b^3)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{a(27a^2+80ab+\cdots}{\cdots}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}\cdots\right.}{a^4}\cdots}{\cdots}$
default	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}+\frac{2b\left(\frac{-\frac{(9a^2+32ab+24b^2)a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{(27a^3+102a^2b+152a\,b^2+80b^3)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{a(27a^2+80ab+\cdots}{\cdots}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}\cdots\right.}{a^4}\cdots}{\cdots}$
risch	$-\frac{(4a^3e^{10dx+10c}+21a^2b\,e^{10dx+10c}+29a\,b^2e^{10dx+10c}+12b^3e^{10dx+10c}+20a^3e^{8dx+8c}+37a^2b\,e^{8dx+8c}-15a\,b^2e^{8dx+8c}-3\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^3+2*b/a^4*((-1/8*(9*a^2+32*a*b+24*b^2)*a/(
a+b)*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+102*a^2*b+152*a*b^2+80*b^3)/(a+b)*ta
nh(1/2*d*x+1/2*c)^4-1/8*a*(27*a^2+80*a*b+56*b^2)/(a+b)*tanh(1/2*d*x+1/2*c)^
2-1/8*a^2*(9*a+10*b)/(a+b))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c
)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1/16*(15*a^2+40*a*b+24*b^2)/(a+b)/(a*b+b
^2)^(1/2)*arctanh(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+2*a+4*b)/(a*b+b^2)^(1/2)))
-1/8/a^3/tanh(1/2*d*x+1/2*c)^2+1/4/a^4*(-12*b-2*a)*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((4*a^3*e^(11*c) + 21*a^2*b*e^(11*c) + 29*a*b^2*e^(11*c) + 12*b^3*e^(1
1*c))*e^(11*d*x) + (20*a^3*e^(9*c) + 37*a^2*b*e^(9*c) - 15*a*b^2*e^(9*c) -
36*b^3*e^(9*c))*e^(9*d*x) + 2*(20*a^3*e^(7*c) + 3*a^2*b*e^(7*c) - 7*a*b^2*e
^(7*c) + 12*b^3*e^(7*c))*e^(7*d*x) + 2*(20*a^3*e^(5*c) + 3*a^2*b*e^(5*c) -
7*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) + (20*a^3*e^(3*c) + 37*a^2*b*e^
(3*c) - 15*a*b^2*e^(3*c) - 36*b^3*e^(3*c))*e^(3*d*x) + (4*a^3*e^c + 21*a^2*
b*e^c + 29*a*b^2*e^c + 12*b^3*e^c)*e^(d*x))/(a^6*d + 3*a^5*b*d + 3*a^4*b^2*
d + a^3*b^3*d + (a^6*d*e^(12*c) + 3*a^5*b*d*e^(12*c) + 3*a^4*b^2*d*e^(12*c)
 + a^3*b^3*d*e^(12*c))*e^(12*d*x) + 2*(a^6*d*e^(10*c) - a^5*b*d*e^(10*c) -
5*a^4*b^2*d*e^(10*c) - 3*a^3*b^3*d*e^(10*c))*e^(10*d*x) - (a^6*d*e^(8*c) +
```

```
3*a^5*b*d*e^(8*c) - 13*a^4*b^2*d*e^(8*c) - 15*a^3*b^3*d*e^(8*c))*e^(8*d*x)
- 4*(a^6*d*e^(6*c) - a^5*b*d*e^(6*c) + 3*a^4*b^2*d*e^(6*c) + 5*a^3*b^3*d*e^
(6*c))*e^(6*d*x) - (a^6*d*e^(4*c) + 3*a^5*b*d*e^(4*c) - 13*a^4*b^2*d*e^(4*c
) - 15*a^3*b^3*d*e^(4*c))*e^(4*d*x) + 2*(a^6*d*e^(2*c) - a^5*b*d*e^(2*c) -
5*a^4*b^2*d*e^(2*c) - 3*a^3*b^3*d*e^(2*c))*e^(2*d*x)) + 1/2*(a + 6*b)*log((
e^(d*x + c) + 1)*e^(-c))/(a^4*d) - 1/2*(a + 6*b)*log((e^(d*x + c) - 1)*e^(-
c))/(a^4*d) + 8*integrate(1/32*((15*a^2*b*e^(3*c) + 40*a*b^2*e^(3*c) + 24*b
^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c + 40*a*b^2*e^c + 24*b^3*e^c)*e^(d*x))
/(a^6 + 2*a^5*b + a^4*b^2 + (a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(4*c
))*e^(4*d*x) + 2*(a^6*e^(2*c) - a^4*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11576 vs. $2(187) = 374$.
time = 0.60, size = 21301, normalized size = 108.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^4 + 21*a^3*b + 29*a^2*b^2 + 12*a*b^3)*cosh(d*x + c)^11 + 44*
(4*a^4 + 21*a^3*b + 29*a^2*b^2 + 12*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 +
 4*(4*a^4 + 21*a^3*b + 29*a^2*b^2 + 12*a*b^3)*sinh(d*x + c)^11 + 4*(20*a^4
+ 37*a^3*b - 15*a^2*b^2 - 36*a*b^3)*cosh(d*x + c)^9 + 4*(20*a^4 + 37*a^3*b
- 15*a^2*b^2 - 36*a*b^3 + 55*(4*a^4 + 21*a^3*b + 29*a^2*b^2 + 12*a*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^9 + 12*(55*(4*a^4 + 21*a^3*b + 29*a^2*b^2 + 12*
a*b^3)*cosh(d*x + c)^3 + 3*(20*a^4 + 37*a^3*b - 15*a^2*b^2 - 36*a*b^3)*cosh
(d*x + c))*sinh(d*x + c)^8 + 8*(20*a^4 + 3*a^3*b - 7*a^2*b^2 + 12*a*b^3)*co
sh(d*x + c)^7 + 8*(165*(4*a^4 + 21*a^3*b + 29*a^2*b^2 + 12*a*b^3)*cosh(d*x
+ c)^4 + 20*a^4 + 3*a^3*b - 7*a^2*b^2 + 12*a*b^3 + 18*(20*a^4 + 37*a^3*b -
15*a^2*b^2 - 36*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 56*(33*(4*a^4 + 2
1*a^3*b + 29*a^2*b^2 + 12*a*b^3)*cosh(d*x + c)^5 + 6*(20*a^4 + 37*a^3*b - 1
5*a^2*b^2 - 36*a*b^3)*cosh(d*x + c)^3 + (20*a^4 + 3*a^3*b - 7*a^2*b^2 + 12*
a*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 8*(20*a^4 + 3*a^3*b - 7*a^2*b^2 + 1
2*a*b^3)*cosh(d*x + c)^5 + 8*(231*(4*a^4 + 21*a^3*b + 29*a^2*b^2 + 12*a*b^3
)*cosh(d*x + c)^6 + 63*(20*a^4 + 37*a^3*b - 15*a^2*b^2 - 36*a*b^3)*cosh(d*x
 + c)^4 + 20*a^4 + 3*a^3*b - 7*a^2*b^2 + 12*a*b^3 + 21*(20*a^4 + 3*a^3*b -
7*a^2*b^2 + 12*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 8*(165*(4*a^4 + 21
*a^3*b + 29*a^2*b^2 + 12*a*b^3)*cosh(d*x + c)^7 + 63*(20*a^4 + 37*a^3*b - 1
5*a^2*b^2 - 36*a*b^3)*cosh(d*x + c)^5 + 35*(20*a^4 + 3*a^3*b - 7*a^2*b^2 +
12*a*b^3)*cosh(d*x + c)^3 + 5*(20*a^4 + 3*a^3*b - 7*a^2*b^2 + 12*a*b^3)*cos
h(d*x + c))*sinh(d*x + c)^4 + 4*(20*a^4 + 37*a^3*b - 15*a^2*b^2 - 36*a*b^3)
*cosh(d*x + c)^3 + 4*(165*(4*a^4 + 21*a^3*b + 29*a^2*b^2 + 12*a*b^3)*cosh(d
*x + c)^8 + 84*(20*a^4 + 37*a^3*b - 15*a^2*b^2 - 36*a*b^3)*cosh(d*x + c)^6
+ 70*(20*a^4 + 3*a^3*b - 7*a^2*b^2 + 12*a*b^3)*cosh(d*x + c)^4 + 20*a^4 + 3
```

```
7*a^3*b - 15*a^2*b^2 - 36*a*b^3 + 20*(20*a^4 + 3*a^3*b - 7*a^2*b^2 + 12*a*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(55*(4*a^4 + 21*a^3*b + 29*a^2*b^2
 + 12*a*b^3)*cosh(d*x + c)^9 + 36*(20*a^4 + 37*a^3*b - 15*a^2*b^2 - 36*a*b^
3)*cosh(d*x + c)^7 + 42*(20*a^4 + 3*a^3*b - 7*a^2*b^2 + 12*a*b^3)*cosh(d*x
+ c)^5 + 20*(20*a^4 + 3*a^3*b - 7*a^2*b^2 + 12*a*b^3)*cosh(d*x + c)^3 + 3*(
20*a^4 + 37*a^3*b - 15*a^2*b^2 - 36*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 -
 ((15*a^4 + 70*a^3*b + 119*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^12 +
12*(15*a^4 + 70*a^3*b + 119*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)*sinh
(d*x + c)^11 + (15*a^4 + 70*a^3*b + 119*a^2*b^2 + 88*a*b^3 + 24*b^4)*sinh(d
*x + c)^12 + 2*(15*a^4 + 10*a^3*b - 101*a^2*b^2 - 168*a*b^3 - 72*b^4)*cosh(
d*x + c)^10 + 2*(15*a^4 + 10*a^3*b - 101*a^2*b^2 - 168*a*b^3 - 72*b^4 + 33*
(15*a^4 + 70*a^3*b + 119*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^10 + 20*(11*(15*a^4 + 70*a^3*b + 119*a^2*b^2 + 88*a*b^3 + 24*b^4)
*cosh(d*x + c)^3 + (15*a^4 + 10*a^3*b - 101*a^2*b^2 - 168*a*b^3 - 72*b^4)*c
osh(d*x + c))*sinh(d*x + c)^9 - (15*a^4 + 70*a^3*b - 121*a^2*b^2 - 552*a*b^
3 - 360*b^4)*cosh(d*x + c)^8 + (495*(15*a^4 + 70*a^3*b + 119*a^2*b^2 + 88*a
*b^3 + 24*b^4)*cosh(d*x + c)^4 - 15*a^4 - 70*a^3*b + 121*a^2*b^2 + 552*a*b^
3 + 360*b^4 + 90*(15*a^4 + 10*a^3*b - 101*a^2*b^2 - 168*a*b^3 - 72*b^4)*cos
h(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(15*a^4 + 70*a^3*b + 119*a^2*b^2 + 88
*a*b^3 + 24*b^4)*cosh(d*x + c)^5 + 30*(15*a^4 + 10*a^3*b - 101*a^2*b^2 - 16
8*a*b^3 - 72*b^4)*cosh(d*x + c)^3 - (15*a^4 + 70*a^3*b - 121*a^2*b^2 - 552*
a*b^3 - 360*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^4 + 10*a^3*b + 19
*a^2*b^2 + 152*a*b^3 + 120*b^4)*cosh(d*x + c)^6 + 4*(231*(15*a^4 + 70*a^3*b
 + 119*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^6 + 105*(15*a^4 + 10*a^3*
b - 101*a^2*b^2 - 168*a*b^3 - 72*b^4)*cosh(d*x + c)^4 - 15*a^4 - 10*a^3*b -
 19*a^2*b^2 - 152*a*b^3 - 120*b^4 - 7*(15*a^4 + 70*a^3*b - 121*a^2*b^2 - 55
2*a*b^3 - 360*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*(15*a^4 + 70*a^
3*b + 119*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^7 + 63*(15*a^4 + 10*a^
3*b - 101*a^2*b^2 - 168*a*b^3 - 72*b^4)*cosh(d*x + c)^5 - 7*(15*a^4 + 70*a^
3*b - 121*a^2*b^2 - 552*a*b^3 - 360*b^4)*cosh(d*x + c)^3 - 3*(15*a^4 + 10*a
^3*b + 19*a^2*b^2 + 152*a*b^3 + 120*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - (
15*a^4 + 70*a^3*b - 121*a^2*b^2 - 552*a*b^3 - 360*b^4)*cosh(d*x + c)^4 + (4
95*(15*a^4 + 70*a^3*b + 119*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^8 +
420*(15*a^4 + 10*a^3*b - 101*a^2*b^2 - 168*a*b^3 - 72*b^4)*cosh(d*x + c)^6
- 70*(15*a^4 + 70*a^3*b - 121*a^2*b^2 - 552*a*b^3 - 360*b^4)*cosh(d*x + c)^
4 - 15*a^4 - 70*a^3*b + 121*a^2*b^2 + 552*a*b^3 + 360*b^4 - 60*(15*a^4 + 10
*a^3*b + 19*a^2*b^2 + 152*a*b^3 + 120*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4
 + 15*a^4 + 70*a^3*b + 119*a^2*b^2 + 88*a*b^3 + 24*b^4 + 4*(55*(15*a^4 + 70
*a^3*b + 119*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^9 + 60*(15*a^4 + 10
*a^3*b - 101*a^2*b^2 - 168*a*b^3 - 72*b^4)*cosh(d*x + c)^7 - 14*(15*a^4 + 7
0*a^3*b - 121*a^2*b^2 - 552*a*b^3 - 360*b^4)*cosh(d*x + c)^5 - 20*(15*a^4 +
 10*a^3*b + 19*a^2*b^2 + 152*a*b^3 + 120*b^4)*c...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c+d\,x)^3\,\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)
```

3.48 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=151

$$\frac{5\sqrt{b}\,(3a+7b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d}+\frac{(a+3b)\coth(c+dx)}{a^4d}-\frac{\coth^3(c+dx)}{3a^3d}+\frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] (a+3*b)*coth(d*x+c)/a^4/d-1/3*coth(d*x+c)^3/a^3/d+5/8*(3*a+7*b)*arctan(b^(1
/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(9/2)/d+1/4*b*(a+b)*tanh(d*x+c)/a^3/d/(a
+b*tanh(d*x+c)^2)^2+1/8*b*(7*a+11*b)*tanh(d*x+c)/a^4/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$,
Rules used = {3744, 467, 1273, 1275, 211}

$$\frac{5\sqrt{b}\,(3a+7b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d}+\frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)}+\frac{(a+3b)\coth(c+dx)}{a^4d}+\frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\coth^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(9/2)*
d) + ((a + 3*b)*Coth[c + d*x])/(a^4*d) - Coth[c + d*x]^3/(3*a^3*d) + (b*(a
+ b)*Tanh[c + d*x])/(4*a^3*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(7*a + 11*b)*T
anh[c + d*x])/(8*a^4*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\text{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{b\text{Subst}\left(\int \frac{-\frac{4}{ab}+\frac{4(a+b)x^2}{a^2b}-\frac{3(a+b)x^4}{a^3}}{x^4(a+bx^2)^2}\,dx, x, \tanh(c\right.}{4d}$$

$$= \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-8ab}{}\right.}{}$$

$$= \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(-\frac{8}{x}\right.\right.}{}$$

$$= \frac{(a+3b)\coth(c+dx)}{a^4d} - \frac{\coth^3(c+dx)}{3a^3d} + \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b}{8a}$$

$$= \frac{5\sqrt{b}\,(3a+7b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d} + \frac{(a+3b)\coth(c+dx)}{a^4d} - \frac{\coth^3(c+}{3a^3d}$$

Mathematica [A]
time = 0.98, size = 149, normalized size = 0.99

$$\frac{15\sqrt{b}\,(3a+7b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right) - 8\sqrt{a}\,\coth(c+dx)\left(-2a-9b+a\,\text{csch}^2(c+dx)\right) + \frac{3\sqrt{a}\,b\left(9a^2+6ab-11b^2+(9a^2+20ab+11b^2)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2}}{24a^{9/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*Sqrt[a]
*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c + d*x]^2) + (3*Sqrt[a]*b*(9*a^2 + 6*a
*b - 11*b^2 + (9*a^2 + 20*a*b + 11*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)
])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(24*a^(9/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(135) = 270$.
time = 3.21, size = 397, normalized size = 2.63 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a^4*(1/3*a*tanh(1/2*d*x+1/2*c)^3-3*a*tanh(1/2*d*x+1/2*c)-12*b*tan
h(1/2*d*x+1/2*c))-1/24/a^3/tanh(1/2*d*x+1/2*c)^3-1/8/a^4*(-12*b-3*a)/tanh(1
```

```
/2*d*x+1/2*c)-2*b/a^4*((-1/8*a*(9*a+13*b)*tanh(1/2*d*x+1/2*c)^7+(-27/8*a^2-
67/8*a*b-11/2*b^2)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a^2-67/8*a*b-11/2*b^2)*tanh
(1/2*d*x+1/2*c)^3+(-9/8*a^2-13/8*a*b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+
1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8*(15*a
+35*b)*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)
-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a
)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1
/2)))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(135) = 270$.
time = 0.70, size = 615, normalized size = 4.07

$$\frac{16\,a^4+147\,a^3b+351\,a^2b^2+325\,ab^3+105\,b^4+2\,(8\,a^4+32\,a^3b-251\,a^2b^2-590\,ab^3-315\,b^4)e^{(-2\,dx-2\,c)}-(96\,a^4+313\,a^3b+19\,a^2b^2-1725\,ab^3-1575\,b^4)e^{(-4\,dx-4\,c)}-4\,(56\,a^4+80\,a^3b-65\,a^2b^2+400\,ab^3+525\,b^4)e^{(-6\,dx-6\,c)}-(176\,a^4+135\,a^3b+15\,a^2b^2-1375\,ab^3-1575\,b^4)e^{(-8\,dx-8\,c)}-6\,(8\,a^4+45\,a^2b^2+150\,ab^3+105\,b^4)e^{(-10\,dx-10\,c)}+15\,(3\,a^3b+13\,a^2b^2+17\,ab^3+7\,b^4)e^{(-12\,dx-12\,c)}}{12\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3+(a^7-5\,a^6b-13\,a^5b^2-7\,a^4b^3)e^{(-2\,dx-2\,c)}-(3\,a^7+a^6b-23\,a^5b^2-21\,a^4b^3)e^{(-4\,dx-4\,c)}-(3\,a^7-7\,a^6b+25\,a^5b^2+35\,a^4b^3)e^{(-6\,dx-6\,c)}+(3\,a^7-7\,a^6b+25\,a^5b^2+35\,a^4b^3)e^{(-8\,dx-8\,c)}+(3\,a^7+a^6b-23\,a^5b^2-21\,a^4b^3)e^{(-10\,dx-10\,c)}-(a^7-5\,a^6b-13\,a^5b^2-7\,a^4b^3)e^{(-12\,dx-12\,c)}-(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)e^{(-14\,dx-14\,c)})d}-\frac{5\,(3\,ab+7\,b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(16*a^4 + 147*a^3*b + 351*a^2*b^2 + 325*a*b^3 + 105*b^4 + 2*(8*a^4 + 3
2*a^3*b - 251*a^2*b^2 - 590*a*b^3 - 315*b^4)*e^(-2*d*x - 2*c) - (96*a^4 + 3
13*a^3*b + 19*a^2*b^2 - 1725*a*b^3 - 1575*b^4)*e^(-4*d*x - 4*c) - 4*(56*a^4
 + 80*a^3*b - 65*a^2*b^2 + 400*a*b^3 + 525*b^4)*e^(-6*d*x - 6*c) - (176*a^4
 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*e^(-8*d*x - 8*c) - 6*(8*
a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*e^(-10*d*x - 10*c) + 15*(3*a^3*b +
13*a^2*b^2 + 17*a*b^3 + 7*b^4)*e^(-12*d*x - 12*c))/((a^7 + 3*a^6*b + 3*a^5*
b^2 + a^4*b^3 + (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b^3)*e^(-2*d*x - 2*c) -
 (3*a^7 + a^6*b - 23*a^5*b^2 - 21*a^4*b^3)*e^(-4*d*x - 4*c) - (3*a^7 - 7*a^
6*b + 25*a^5*b^2 + 35*a^4*b^3)*e^(-6*d*x - 6*c) + (3*a^7 - 7*a^6*b + 25*a^5
*b^2 + 35*a^4*b^3)*e^(-8*d*x - 8*c) + (3*a^7 + a^6*b - 23*a^5*b^2 - 21*a^4*
b^3)*e^(-10*d*x - 10*c) - (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b^3)*e^(-12*d
*x - 12*c) - (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-14*d*x - 14*c))*d) -
 5/8*(3*a*b + 7*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b
))/(sqrt(a*b)*a^4*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7006 vs. $2(135) = 270$.
time = 0.48, size = 14334, normalized size = 94.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(60*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)^12 + 720*
(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)*sinh(d*x + c)^11 +
```

60*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*sinh(d*x + c)^12 - 24*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*cosh(d*x + c)^10 - 24*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4 - 165*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 240*(55*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)^3 - (8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*cosh(d*x + c))*sinh(d*x + c)^9 - 4*(176*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*cosh(d*x + c)^8 + 4*(7425*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)^4 - 176*a^4 - 135*a^3*b - 15*a^2*b^2 + 1375*a*b^3 + 1575*b^4 - 270*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 32*(1485*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)^5 - 90*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*cosh(d*x + c)^3 - (176*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 - 16*(56*a^4 + 80*a^3*b - 65*a^2*b^2 + 400*a*b^3 + 525*b^4)*cosh(d*x + c)^6 + 16*(3465*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)^6 - 315*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*cosh(d*x + c)^4 - 56*a^4 - 80*a^3*b + 65*a^2*b^2 - 400*a*b^3 - 525*b^4 - 7*(176*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 32*(1485*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)^7 - 189*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*cosh(d*x + c)^5 - 7*(176*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*cosh(d*x + c)^3 - 3*(56*a^4 + 80*a^3*b - 65*a^2*b^2 + 400*a*b^3 + 525*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(96*a^4 + 313*a^3*b + 19*a^2*b^2 - 1725*a*b^3 - 1575*b^4)*cosh(d*x + c)^4 + 4*(7425*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)^8 - 1260*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*cosh(d*x + c)^6 - 70*(176*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*cosh(d*x + c)^4 - 96*a^4 - 313*a^3*b - 19*a^2*b^2 + 1725*a*b^3 + 1575*b^4 - 60*(56*a^4 + 80*a^3*b - 65*a^2*b^2 + 400*a*b^3 + 525*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 64*a^4 + 588*a^3*b + 1404*a^2*b^2 + 1300*a*b^3 + 420*b^4 + 16*(825*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)^9 - 180*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*cosh(d*x + c)^7 - 14*(176*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*cosh(d*x + c)^5 - 20*(56*a^4 + 80*a^3*b - 65*a^2*b^2 + 400*a*b^3 + 525*b^4)*cosh(d*x + c)^3 - (96*a^4 + 313*a^3*b + 19*a^2*b^2 - 1725*a*b^3 - 1575*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(8*a^4 + 32*a^3*b - 251*a^2*b^2 - 590*a*b^3 - 315*b^4)*cosh(d*x + c)^2 + 8*(495*(3*a^3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*cosh(d*x + c)^10 - 135*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*cosh(d*x + c)^8 - 14*(176*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*cosh(d*x + c)^6 - 30*(56*a^4 + 80*a^3*b - 65*a^2*b^2 + 400*a*b^3 + 525*b^4)*cosh(d*x + c)^4 + 8*a^4 + 32*a^3*b - 251*a^2*b^2 - 590*a*b^3 - 315*b^4 - 3*(96*a^4 + 313*a^3*b + 19*a^2*b^2 - 1725*a*b^3 - 1575*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 15*((3*a^4 + 16*a^3*b + 30*a^2*b^2 + 24*a*b^3 + 7*b^4)*cosh(d*x + c)^14 + 14*(3*a^4 + 16*a^3*b + 30*a^2*b^2 + 24*a*b^3 + 7*b^4)*cosh(d*x + c)*sinh(d*x + c)^13 + (3*a^4 + 16*a^3*b + 30*a^2*b^2 + 24*a*b^3 + 7*b^4)*sinh(d*x + c)^14 + (3*a^4 - 8*a^3*b - 74*a^2*b^2 - 112*a*b^3 - 49*b^4)*cosh(d*x + c)^12 + (3*a^4 - 8*a^3*b - 74*a^2*b^2 - 112*a*b^3 - 49*b^4 + 91*(3*a^4 + 16*a^3*b + 30*a^2*b^2 + 24*a*b^3

```
+ 7*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 4*(91*(3*a^4 + 16*a^3*b + 30*a
^2*b^2 + 24*a*b^3 + 7*b^4)*cosh(d*x + c)^3 + 3*(3*a^4 - 8*a^3*b - 74*a^2*b^
2 - 112*a*b^3 - 49*b^4)*cosh(d*x + c))*sinh(d*x + c)^11 - (9*a^4 + 24*a^3*b
 - 62*a^2*b^2 - 224*a*b^3 - 147*b^4)*cosh(d*x + c)^10 + (1001*(3*a^4 + 16*a
^3*b + 30*a^2*b^2 + 24*a*b^3 + 7*b^4)*cosh(d*x + c)^4 - 9*a^4 - 24*a^3*b +
62*a^2*b^2 + 224*a*b^3 + 147*b^4 + 66*(3*a^4 - 8*a^3*b - 74*a^2*b^2 - 112*a
*b^3 - 49*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(3*a^4 + 16*a^3*
b + 30*a^2*b^2 + 24*a*b^3 + 7*b^4)*cosh(d*x + c)^5 + 110*(3*a^4 - 8*a^3*b -
 74*a^2*b^2 - 112*a*b^3 - 49*b^4)*cosh(d*x + c)^3 - 5*(9*a^4 + 24*a^3*b - 6
2*a^2*b^2 - 224*a*b^3 - 147*b^4)*cosh(d*x + c))*sinh(d*x + c)^9 - (9*a^4 +
26*a^2*b^2 + 280*a*b^3 + 245*b^4)*cosh(d*x + c)^8 + (3003*(3*a^4 + 16*a^3*b
 + 30*a^2*b^2 + 24*a*b^3 + 7*b^4)*cosh(d*x + c)^6 + 495*(3*a^4 - 8*a^3*b -
74*a^2*b^2 - 112*a*b^3 - 49*b^4)*cosh(d*x + c)^4 - 9*a^4 - 26*a^2*b^2 - 280
*a*b^3 - 245*b^4 - 45*(9*a^4 + 24*a^3*b - 62*a^2*b^2 - 224*a*b^3 - 147*b^4)
*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(429*(3*a^4 + 16*a^3*b + 30*a^2*b^2 +
 24*a*b^3 + 7*b^4)*cosh(d*x + c)^7 + 99*(3*a^4 - 8*a^3*b - 74*a^2*b^2 - 112
*a*b^3 - 49*b^4)*cosh(d*x + c)^5 - 15*(9*a^4 + 24*a^3*b - 62*a^2*b^2 - 224*
a*b^3 - 147*b^4)*cosh(d*x + c)^3 - (9*a^4 + 26*a^2*b^2 + 280*a*b^3 + 245*b^
4)*cosh(d*x + c))*sinh(d*x + c)^7 + (9*a^4 + 26*a^2*b^2 + 280*a*b^3 + 245*b
^4)*cosh(d*x + c)^6 + (3003*(3*a^4 + 16*a^3*b +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(135) = 270.
time = 1.02, size = 395, normalized size = 2.62

$$-\frac{\frac{6\left(9a^3be^{(6dx+6c)}+7a^2b^2e^{(6dx+6c)}-13ab^3e^{(6dx+6c)}-11b^4e^{(6dx+6c)}+27a^3be^{(4dx+4c)}+15a^2b^2e^{(4dx+4c)}+5ab^3e^{(4dx+4c)}+33b^4e^{(4dx+4c)}+27a^3be^{(2dx+2c)}+37a^2b^2e^{(2dx+2c)}-23ab^3e^{(2dx+2c)}-33b^4e^{(2dx+2c)}+9a^3b+29a^2b^2+31ab^3+11b^4\right)}{(a^5+a^4b)\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)^2}-\frac{15\left(3ab+7b^2\right)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}-\frac{16\left(9be^{(4dx+4c)}-6ae^{(2dx+2c)}-18be^{(2dx+2c)}+2a+9b\right)}{a^4\left(e^{(2dx+2c)}-1\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(6*(9*a^3*b*e^(6*d*x + 6*c) + 7*a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e^
(6*d*x + 6*c) - 11*b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x + 4*c) + 15*a^2*
b^2*e^(4*d*x + 4*c) + 5*a*b^3*e^(4*d*x + 4*c) + 33*b^4*e^(4*d*x + 4*c) + 27
*a^3*b*e^(2*d*x + 2*c) + 37*a^2*b^2*e^(2*d*x + 2*c) - 23*a*b^3*e^(2*d*x + 2
```

```
*c) - 33*b^4*e^(2*d*x + 2*c) + 9*a^3*b + 29*a^2*b^2 + 31*a*b^3 + 11*b^4)/((
a^5 + a^4*b)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
 2*b*e^(2*d*x + 2*c) + a + b)^2) - 15*(3*a*b + 7*b^2)*arctan(1/2*(a*e^(2*d*
x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^4) - 16*(9*b*
e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 18*b*e^(2*d*x + 2*c) + 2*a + 9*b)/(
a^4*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(c+d\,x\right)^4\,{\left(b\,\mathrm{tanh}\left(c+d\,x\right)^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)
```

3.49 $\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx$

Optimal. Leaf size=132

$$-\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d}+\frac{3(a-8b)\log(1+\tanh(c+dx))}{16d}-\frac{3a\tanh(c+dx)}{8d}-\frac{3b\tanh^2(c+dx)}{2d}+\text{si}$$

```
[Out] -3/16*(a+8*b)*ln(1-tanh(d*x+c))/d+3/16*(a-8*b)*ln(1+tanh(d*x+c))/d-3/8*a*ta
nh(d*x+c)/d-3/2*b*tanh(d*x+c)^2/d+1/4*sinh(d*x+c)^4*(b+a*tanh(d*x+c))/d-1/8
*sinh(d*x+c)^2*tanh(d*x+c)*(a+8*b*tanh(d*x+c))/d
```

Rubi [A]
time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3744, 1818, 815, 647, 31}

$$-\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d}+\frac{3(a-8b)\log(\tanh(c+dx)+1)}{16d}+\frac{\sinh^4(c+dx)(a\tanh(c+dx)+b)}{4d}-\frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b\tanh(c+dx))}{8d}-\frac{3a\tanh(c+dx)}{8d}-\frac{3b\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-3*(a + 8*b)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a - 8*b)*Log[1 + Tanh[c
+ d*x]])/(16*d) - (3*a*Tanh[c + d*x])/(8*d) - (3*b*Tanh[c + d*x]^2)/(2*d) +
 (Sinh[c + d*x]^4*(b + a*Tanh[c + d*x]))/(4*d) - (Sinh[c + d*x]^2*Tanh[c +
d*x]*(a + 8*b*Tanh[c + d*x]))/(8*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{x^4(a+bx^3)}{(1-x^2)^3}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d}+\frac{\text{Subst}\left(\int \frac{x^3(-4b-ax-4bx^2)}{(1-x^2)^2}\,d\right.}{4d}\\
&= \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d}-\frac{\sinh^2(c+dx)\tanh(c+d}{8d}\\
&= \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d}-\frac{\sinh^2(c+dx)\tanh(c+d}{8d}\\
&= -\frac{3a\tanh(c+dx)}{8d}-\frac{3b\tanh^2(c+dx)}{2d}+\frac{\sinh^4(c+dx)(b+a\,t}{4d}\\
&= -\frac{3a\tanh(c+dx)}{8d}-\frac{3b\tanh^2(c+dx)}{2d}+\frac{\sinh^4(c+dx)(b+a\,t}{4d}\\
&= -\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d}+\frac{3(a-8b)\log(1+\tanh(c}{16d}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 92, normalized size = 0.70

$$\frac{3a(c+dx)}{8d}+\frac{b\left(12\log(\cosh(c+dx))+2\text{sech}^2(c+dx)-4\sinh^2(c+dx)+\sinh^4(c+dx)\right)}{4d}-\frac{a\sinh(2(c+dx))}{4d}+\frac{a\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (b*(12*Log[Cosh[c + d*x]] + 2*Sech[c + d*x]^2 - 4*S
inh[c + d*x]^2 + Sinh[c + d*x]^4))/(4*d) - (a*Sinh[2*(c + d*x)])/(4*d) + (a
*Sinh[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 2.36, size = 183, normalized size = 1.39

method	result
risch	$\frac{3ax}{8} - 3bx + \frac{e^{4dx+4c}a}{64d} + \frac{e^{4dx+4c}b}{64d} - \frac{e^{2dx+2c}a}{8d} - \frac{5\,e^{2dx+2c}b}{16d} + \frac{e^{-2dx-2c}a}{8d} - \frac{5\,e^{-2dx-2c}b}{16d} - \frac{e^{-4dx-4c}a}{64d} + \frac{e^{-4dx-4c}b}{64d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a*x-3*b*x+1/64/d*exp(4*d*x+4*c)*a+1/64/d*exp(4*d*x+4*c)*b-1/8/d*exp(2*d
*x+2*c)*a-5/16/d*exp(2*d*x+2*c)*b+1/8/d*exp(-2*d*x-2*c)*a-5/16/d*exp(-2*d*x
-2*c)*b-1/64/d*exp(-4*d*x-4*c)*a+1/64/d*exp(-4*d*x-4*c)*b-6*b*c/d+2*b*exp(2
*d*x+2*c)/d/(1+exp(2*d*x+2*c))^2+3*b/d*ln(1+exp(2*d*x+2*c))
```

Maxima [A]
time = 0.50, size = 194, normalized size = 1.47

$$\frac{1}{64}a\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + \frac{1}{64}b\left(\frac{192(dx+c)}{d} - \frac{20e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} + \frac{192\log\left(e^{(-2dx-2c)}+1\right)}{d} - \frac{18e^{(-2dx-2c)} + 39e^{(-4dx-4c)} - 108e^{(-6dx-6c)} - 1}{d(e^{(-4dx-4c)} + 2e^{(-6dx-6c)} + e^{(-8dx-8c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*c) -
 e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x - 2*
c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*c) +
 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. 2(120) = 240.
time = 0.36, size = 1530, normalized size = 11.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/64*((a + b)*cosh(d*x + c)^12 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^11
+ (a + b)*sinh(d*x + c)^12 - 6*(a + 3*b)*cosh(d*x + c)^10 + 6*(11*(a + b)*c
osh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^10 + 20*(11*(a + b)*cosh(d*x + c)^3
 - 3*(a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(8*(a - 8*b)*d*x - 5*a -
```

```
13*b)*cosh(d*x + c)^8 + 3*(165*(a + b)*cosh(d*x + c)^4 + 8*(a - 8*b)*d*x -
90*(a + 3*b)*cosh(d*x + c)^2 - 5*a - 13*b)*sinh(d*x + c)^8 + 24*(33*(a + b)
*cosh(d*x + c)^5 - 30*(a + 3*b)*cosh(d*x + c)^3 + (8*(a - 8*b)*d*x - 5*a -
13*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x
+ c)^6 + 4*(231*(a + b)*cosh(d*x + c)^6 - 315*(a + 3*b)*cosh(d*x + c)^4 + 1
2*(a - 8*b)*d*x + 21*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^2 + 22*b)
*sinh(d*x + c)^6 + 24*(33*(a + b)*cosh(d*x + c)^7 - 63*(a + 3*b)*cosh(d*x +
 c)^5 + 7*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^3 + 2*(6*(a - 8*b)*d
*x + 11*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(8*(a - 8*b)*d*x + 5*a - 13*b
)*cosh(d*x + c)^4 + 3*(165*(a + b)*cosh(d*x + c)^8 - 420*(a + 3*b)*cosh(d*x
 + c)^6 + 70*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^4 + 8*(a - 8*b)*d
*x + 40*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^2 + 5*a - 13*b)*sinh(d*x + c
)^4 + 4*(55*(a + b)*cosh(d*x + c)^9 - 180*(a + 3*b)*cosh(d*x + c)^7 + 42*(8
*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^5 + 40*(6*(a - 8*b)*d*x + 11*b)*
cosh(d*x + c)^3 + 3*(8*(a - 8*b)*d*x + 5*a - 13*b)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 6*(a - 3*b)*cosh(d*x + c)^2 + 6*(11*(a + b)*cosh(d*x + c)^10 - 45*
(a + 3*b)*cosh(d*x + c)^8 + 14*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)
^6 + 20*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^4 + 3*(8*(a - 8*b)*d*x + 5*a
 - 13*b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c)^2 + 192*(b*cosh(d*x + c)^
8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 + 2*b*cosh(d*x +
c)^6 + 2*(14*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^6 + 4*(14*b*cosh(d*x + c)
^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + b*cosh(d*x + c)^4 + (70*b*cosh(d*
x + c)^4 + 30*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 4*(14*b*cosh(d*x + c
)^5 + 10*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*b*cos
h(d*x + c)^6 + 15*b*cosh(d*x + c)^4 + 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 4*(2*b*cosh(d*x + c)^7 + 3*b*cosh(d*x + c)^5 + b*cosh(d*x + c)^3)*sinh(d*
x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*((a + b)*
cosh(d*x + c)^11 - 5*(a + 3*b)*cosh(d*x + c)^9 + 2*(8*(a - 8*b)*d*x - 5*a -
 13*b)*cosh(d*x + c)^7 + 4*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^5 + (8*(a
 - 8*b)*d*x + 5*a - 13*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c))*sinh(d
*x + c) - a + b)/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d
*sinh(d*x + c)^8 + 2*d*cosh(d*x + c)^6 + 2*(14*d*cosh(d*x + c)^2 + d)*sinh(
d*x + c)^6 + 4*(14*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 +
 d*cosh(d*x + c)^4 + (70*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*sinh
(d*x + c)^4 + 4*(14*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 2*(14*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 3*d
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 3*d*cosh(d*x +
 c)^5 + d*cosh(d*x + c)^3)*sinh(d*x + c))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^3\left(c+dx\right)\right)\sinh^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**4, x)
```

Giac [A]
time = 0.44, size = 203, normalized size = 1.54

$$\frac{24\,(dx+c)(a-8\,b)+ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-20\,be^{(2\,dx+2\,c)}+192\,b\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{9\,ae^{(8\,dx+8\,c)}+72\,be^{(8\,dx+8\,c)}+10\,ae^{(6\,dx+6\,c)}+36\,be^{(6\,dx+6\,c)}-6\,ae^{(4\,dx+4\,c)}+111\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+18\,be^{(2\,dx+2\,c)}+a-b}{\left(e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}\right)^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*(a - 8*b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a*
e^(2*d*x + 2*c) - 20*b*e^(2*d*x + 2*c) + 192*b*log(e^(2*d*x + 2*c) + 1) - (
9*a*e^(8*d*x + 8*c) + 72*b*e^(8*d*x + 8*c) + 10*a*e^(6*d*x + 6*c) + 36*b*e^
(6*d*x + 6*c) - 6*a*e^(4*d*x + 4*c) + 111*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x
+ 2*c) + 18*b*e^(2*d*x + 2*c) + a - b)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c))^
2)/d
```

Mupad [B]
time = 0.27, size = 156, normalized size = 1.18

$$x\left(\frac{3\,a}{8}-3\,b\right)+\frac{2\,b}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a+b\right)}{64\,d}-\frac{2\,b}{d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{{\mathrm{e}}^{-4\,c-4\,d\,x}\,\left(a-b\right)}{64\,d}+\frac{3\,b\,\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+1\right)}{d}+\frac{{\mathrm{e}}^{-2\,c-2\,d\,x}\,\left(2\,a-5\,b\right)}{16\,d}-\frac{{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a+5\,b\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3),x)
```

```
[Out] x*((3*a)/8 - 3*b) + (2*b)/(d*(exp(2*c + 2*d*x) + 1)) + (exp(4*c + 4*d*x)*(a
 + b))/(64*d) - (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (ex
p(- 4*c - 4*d*x)*(a - b))/(64*d) + (3*b*log(exp(2*c)*exp(2*d*x) + 1))/d + (
exp(- 2*c - 2*d*x)*(2*a - 5*b))/(16*d) - (exp(2*c + 2*d*x)*(2*a + 5*b))/(16
*d)
```

3.50 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal. Leaf size=98

$$\frac{5b\text{ArcTan}(\sinh(c+dx))}{2d}-\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{5b\sinh(c+dx)}{2d}+\frac{5b\sinh^3(c+dx)}{6d}-\frac{b\sinh^3(c+$$

```
[Out] 5/2*b*arctan(sinh(d*x+c))/d-a*cosh(d*x+c)/d+1/3*a*cosh(d*x+c)^3/d-5/2*b*sin
h(d*x+c)/d+5/6*b*sinh(d*x+c)^3/d-1/2*b*sinh(d*x+c)^3*tanh(d*x+c)^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3747, 2713, 2672, 294, 308, 209}

$$\frac{a\cosh^3(c+dx)}{3d}-\frac{a\cosh(c+dx)}{d}+\frac{5b\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{5b\sinh^3(c+dx)}{6d}-\frac{5b\sinh(c+dx)}{2d}-\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (5*b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Cosh[c + d*x])/d + (a*Cosh[c + d*x]^
3)/(3*d) - (5*b*Sinh[c + d*x])/(2*d) + (5*b*Sinh[c + d*x]^3)/(6*d) - (b*Sin
h[c + d*x]^3*Tanh[c + d*x]^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = i\int\left(-ia\sinh^3(c+dx)-ib\sinh^3(c+dx)\tanh^3(c+dx)\right)\,dx$$
$$= a\int\sinh^3(c+dx)\,dx+b\int\sinh^3(c+dx)\tanh^3(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cosh(c+dx)\right)}{d}+\frac{b\text{Subst}\left(\int\frac{x^6}{(1+x^2)^2}\,dx\right.}{d}$$
$$= -\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{b\sinh^3(c+dx)\tanh^2(c+}{2d}$$
$$= -\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{b\sinh^3(c+dx)\tanh^2(c+}{2d}$$
$$= -\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{5b\sinh(c+dx)}{2d}+\frac{5b\sinh}{}$$
$$= \frac{5b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{5}{}$$

Mathematica [A]
time = 0.23, size = 104, normalized size = 1.06

$$-\frac{3a\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}+\frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{3d}-\frac{5b(2\sinh(c+dx)\tanh^2(c+dx)-3(\text{ArcTan}(\sinh(c+dx))-\text{sech}(c+dx)\tanh(c+dx)))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) + (a*Cosh[3*(c + d*x)])/(12*d) + (b*Sinh[c + d*x
]^3*Tanh[c + d*x]^2)/(3*d) - (5*b*(2*Sinh[c + d*x]*Tanh[c + d*x]^2 - 3*(Arc
Tan[Sinh[c + d*x]] - Sech[c + d*x]*Tanh[c + d*x])))/(6*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.21, size = 186, normalized size = 1.90

method	result
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}-\frac{3a\,e^{dx+c}}{8d}-\frac{9b\,e^{dx+c}}{8d}-\frac{3\,e^{-dx-c}a}{8d}+\frac{9\,e^{-dx-c}b}{8d}+\frac{e^{-3dx-3c}a}{24d}-\frac{e^{-3dx-3c}b}{24d}-\frac{b\,e^{dx+c}(e^{2dx+2c}}{d(1+e^{2dx+2c})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*a+1/24/d*exp(3*d*x+3*c)*b-3/8*a/d*exp(d*x+c)-9/8*b/d*
exp(d*x+c)-3/8/d*exp(-d*x-c)*a+9/8/d*exp(-d*x-c)*b+1/24/d*exp(-3*d*x-3*c)*a
-1/24/d*exp(-3*d*x-3*c)*b-b*exp(d*x+c)*(exp(2*d*x+2*c)-1)/d/(1+exp(2*d*x+2*
c))^2+5/2*I*b/d*ln(exp(d*x+c)+I)-5/2*I*b/d*ln(exp(d*x+c)-I)
```

Maxima [A]
time = 0.49, size = 174, normalized size = 1.78

$$\frac{1}{24}\,b\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-1}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right)+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/24*b*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d
 - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*
(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a*(e^(3
*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1070 vs. 2(88) = 176.
time = 0.36, size = 1070, normalized size = 10.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/24*((a + b)*cosh(d*x + c)^10 + 10*(a + b)*cosh(d*x + c)*sinh(d*x + c)^9 +
 (a + b)*sinh(d*x + c)^10 - (7*a + 25*b)*cosh(d*x + c)^8 + (45*(a + b)*cosh
(d*x + c)^2 - 7*a - 25*b)*sinh(d*x + c)^8 + 8*(15*(a + b)*cosh(d*x + c)^3 -
```

```
 (7*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a + 25*b)*cosh(d*x + c
)^6 + 2*(105*(a + b)*cosh(d*x + c)^4 - 14*(7*a + 25*b)*cosh(d*x + c)^2 - 13
*a - 25*b)*sinh(d*x + c)^6 + 4*(63*(a + b)*cosh(d*x + c)^5 - 14*(7*a + 25*b
)*cosh(d*x + c)^3 - 3*(13*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(13*
a - 25*b)*cosh(d*x + c)^4 + 2*(105*(a + b)*cosh(d*x + c)^6 - 35*(7*a + 25*b
)*cosh(d*x + c)^4 - 15*(13*a + 25*b)*cosh(d*x + c)^2 - 13*a + 25*b)*sinh(d*
x + c)^4 + 8*(15*(a + b)*cosh(d*x + c)^7 - 7*(7*a + 25*b)*cosh(d*x + c)^5 -
 5*(13*a + 25*b)*cosh(d*x + c)^3 - (13*a - 25*b)*cosh(d*x + c))*sinh(d*x +
c)^3 - (7*a - 25*b)*cosh(d*x + c)^2 + (45*(a + b)*cosh(d*x + c)^8 - 28*(7*a
 + 25*b)*cosh(d*x + c)^6 - 30*(13*a + 25*b)*cosh(d*x + c)^4 - 12*(13*a - 25
*b)*cosh(d*x + c)^2 - 7*a + 25*b)*sinh(d*x + c)^2 + 120*(b*cosh(d*x + c)^7
+ 7*b*cosh(d*x + c)*sinh(d*x + c)^6 + b*sinh(d*x + c)^7 + 2*b*cosh(d*x + c)
^5 + (21*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x + c)^5 + 5*(7*b*cosh(d*x + c)^3
+ 2*b*cosh(d*x + c))*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (35*b*cosh(d*x +
 c)^4 + 20*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^3 + (21*b*cosh(d*x + c)^5 +
 20*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^2 + (7*b*cosh(d*x
+ c)^6 + 10*b*cosh(d*x + c)^4 + 3*b*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(
cosh(d*x + c) + sinh(d*x + c)) + 2*(5*(a + b)*cosh(d*x + c)^9 - 4*(7*a + 25
*b)*cosh(d*x + c)^7 - 6*(13*a + 25*b)*cosh(d*x + c)^5 - 4*(13*a - 25*b)*cos
h(d*x + c)^3 - (7*a - 25*b)*cosh(d*x + c))*sinh(d*x + c) + a - b)/(d*cosh(d
*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 2*d*cos
h(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d
*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*
cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*
x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d
*cosh(d*x + c)^6 + 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^{3}\left(c + dx\right)\right)\sinh^{3}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**3, x)
```

Giac [A]
time = 0.44, size = 132, normalized size = 1.35

$$\frac{120\,b\arctan\left(e^{(dx+c)}\right) + ae^{(3\,dx+3\,c)} + be^{(3\,dx+3\,c)} - 9\,ae^{(dx+c)} - 27\,be^{(dx+c)} - \left(9\,ae^{(2\,dx+2\,c)} - 27\,be^{(2\,dx+2\,c)} - a + b\right)e^{(-3\,dx-3\,c)} - \frac{24\,\left(be^{(3\,dx+3\,c)} - be^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^{2}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/24*(120*b*arctan(e^(d*x + c)) + a*e^(3*d*x + 3*c) + b*e^(3*d*x + 3*c) - 9
*a*e^(d*x + c) - 27*b*e^(d*x + c) - (9*a*e^(2*d*x + 2*c) - 27*b*e^(2*d*x +
2*c) - a + b)*e^(-3*d*x - 3*c) - 24*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^
(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B]
time = 0.23, size = 171, normalized size = 1.74

$$\frac{e^{3c+3dx}(a+b)}{24d}+\frac{5\,\mathrm{atan}\left(\frac{b\,e^{dx}\,e^{c}\sqrt{d^2}}{d\sqrt{b^2}}\right)\sqrt{b^2}}{\sqrt{d^2}}+\frac{e^{-3c-3dx}(a-b)}{24d}-\frac{e^{c+dx}(3a+9b)}{8d}-\frac{e^{-c-dx}(3a-9b)}{8d}-\frac{b\,e^{c+dx}}{d\,(e^{2c+2dx}+1)}+\frac{2\,b\,e^{c+dx}}{d\,(2\,e^{2c+2dx}+e^{4c+4dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3),x)
```

```
[Out] (exp(3*c + 3*d*x)*(a + b))/(24*d) + (5*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))
/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) + (exp(- 3*c - 3*d*x)*(a - b))/(
24*d) - (exp(c + d*x)*(3*a + 9*b))/(8*d) - (exp(- c - d*x)*(3*a - 9*b))/(8*
d) - (b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) + (2*b*exp(c + d*x))/(d*(2
*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.51 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal. Leaf size=100

$$\frac{(a+4b)\log(1-\tanh(c+dx))}{4d}-\frac{(a-4b)\log(1+\tanh(c+dx))}{4d}+\frac{a\tanh(c+dx)}{2d}+\frac{b\tanh^2(c+dx)}{2d}+\sinh^2(c$$

```
[Out] 1/4*(a+4*b)*ln(1-tanh(d*x+c))/d-1/4*(a-4*b)*ln(1+tanh(d*x+c))/d+1/2*a*tanh(
d*x+c)/d+1/2*b*tanh(d*x+c)^2/d+1/2*sinh(d*x+c)^2*(b+a*tanh(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3744, 1818, 1816, 647, 31}

$$\frac{(a+4b)\log(1-\tanh(c+dx))}{4d}-\frac{(a-4b)\log(\tanh(c+dx)+1)}{4d}+\frac{\sinh^2(c+dx)(a\tanh(c+dx)+b)}{2d}+\frac{a\tanh(c+dx)}{2d}+\frac{b\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] ((a + 4*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 4*b)*Log[1 + Tanh[c + d*x]
])/(4*d) + (a*Tanh[c + d*x])/(2*d) + (b*Tanh[c + d*x]^2)/(2*d) + (Sinh[c +
d*x]^2*(b + a*Tanh[c + d*x]))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^3)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d} + \frac{\text{Subst}\left(\int \frac{x(-2b-ax-2bx^2)}{1-x^2}\,d\right.}{2d}$$

$$= \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d} + \frac{\text{Subst}\left(\int \left(a+2bx-\frac{a+4b}{1-x^2}\right.\right.}{2d}$$

$$= \frac{a\tanh(c+dx)}{2d} + \frac{b\tanh^2(c+dx)}{2d} + \frac{\sinh^2(c+dx)(b+a\tanh(}{2d}$$

$$= \frac{a\tanh(c+dx)}{2d} + \frac{b\tanh^2(c+dx)}{2d} + \frac{\sinh^2(c+dx)(b+a\tanh(}{2d}$$

$$= \frac{(a+4b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-4b)\log(1+\tanh(c+d}{4d}$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 0.69

$$\frac{a(-c-dx)}{2d} - \frac{b\left(4\log(\cosh(c+dx)) + \text{sech}^2(c+dx) - \sinh^2(c+dx)\right)}{2d} + \frac{a\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (a*(-c - d*x))/(2*d) - (b*(4*Log[Cosh[c + d*x]] + Sech[c + d*x]^2 - Sinh[c
+ d*x]^2))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 1.53, size = 68, normalized size = 0.68

method	result
derivativedivides	$\frac{a\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh^4(dx+c)}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh^2(dx+c)\right)}{d}$
default	$\frac{a\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh^4(dx+c)}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh^2(dx+c)\right)}{d}$
risch	$-\frac{ax}{2}+2bx+\frac{e^{2dx+2c}a}{8d}+\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}a}{8d}+\frac{e^{-2dx-2c}b}{8d}+\frac{4bc}{d}-\frac{2b\,e^{2dx+2c}}{d\left(1+e^{2dx+2c}\right)^2}-\frac{2b\ln\left(1+e^{2dx+2c}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sinh(d*x+c)*cosh(d*x+c)-1/2*d*x-1/2*c)+b*(1/2*sinh(d*x+c)^4/cos
h(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2))
```

Maxima [A]
time = 0.49, size = 141, normalized size = 1.41

$$-\frac{1}{8}a\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{8}b\left(\frac{16(dx+c)}{d}-\frac{e^{(-2dx-2c)}}{d}+\frac{16\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{2e^{(-2dx-2c)}-15e^{(-4dx-4c)}+1}{d\left(e^{(-2dx-2c)}+2e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b*(16*(d*x + c)
/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d - (2*e^(-2*d*x - 2
*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(90) = 180.
time = 0.36, size = 924, normalized size = 9.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/8*((a + b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
 + b)*sinh(d*x + c)^8 - 2*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^6 - 2*(2*
(a - 4*b)*d*x - 14*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^6 + 4*(14
*(a + b)*cosh(d*x + c)^3 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c))*sinh(
d*x + c)^5 - 2*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh
(d*x + c)^4 - 4*(a - 4*b)*d*x - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^
2 - 7*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x + c)^5 - 5*(2*(a - 4*b)*d*
```

```
x - a - b)*cosh(d*x + c)^3 - (4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c))*sinh(d*
x + c)^3 - 2*(2*(a - 4*b)*d*x + a - b)*cosh(d*x + c)^2 + 2*(14*(a + b)*cosh
(d*x + c)^6 - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^4 - 2*(a - 4*b)*d*
x - 6*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 - 16
*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6
 + 2*b*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x + c)^4 + 4*(
5*b*cosh(d*x + c)^3 + 2*b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^
2 + (15*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(
3*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c))
*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*cosh(d
*x + c)^7 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^5 - 2*(4*(a - 4*b)*d*
x + 7*b)*cosh(d*x + c)^3 - (2*(a - 4*b)*d*x + a - b)*cosh(d*x + c))*sinh(d*
x + c) - a + b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*
sinh(d*x + c)^6 + 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 + 2*d)*sinh(d
*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^3 + d
*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d
*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))
*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^3\left(c + dx\right)\right)\sinh^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3),x)
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**2, x)
```

Giac [A]
time = 0.44, size = 140, normalized size = 1.40

$$-\frac{4\,(dx+c)(a-4\,b) - ae^{(2\,dx+2\,c)} - be^{(2\,dx+2\,c)} - \left(2\,ae^{(2\,dx+2\,c)} - 8\,be^{(2\,dx+2\,c)} - a + b\right)e^{(-2\,dx-2\,c)} + 16\,b\log\left(e^{(2\,dx+2\,c)} + 1\right) - \frac{8\,\left(3\,be^{(4\,dx+4\,c)} + 4\,be^{(2\,dx+2\,c)} + 3\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
[Out] -1/8*(4*(d*x + c)*(a - 4*b) - a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - (2*a*
e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c) - a + b)*e^(-2*d*x - 2*c) + 16*b*log(
e^(2*d*x + 2*c) + 1) - 8*(3*b*e^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + 3*b)/
(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B]
time = 1.15, size = 115, normalized size = 1.15

$$\frac{e^{2\,c+2\,d\,x}\,(a+b)}{8\,d} - \frac{2\,b}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - x\,\left(\frac{a}{2} - 2\,b\right) + \frac{2\,b}{d\,\left(2\,e^{2\,c+2\,d\,x} + e^{4\,c+4\,d\,x} + 1\right)} - \frac{e^{-2\,c-2\,d\,x}\,(a-b)}{8\,d} - \frac{2\,b\,\ln\left(e^{2\,c}\,e^{2\,d\,x}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3),x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b))/(8*d) - (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - x*(a/
2 - 2*b) + (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (exp(- 2
*c - 2*d*x)*(a - b))/(8*d) - (2*b*log(exp(2*c)*exp(2*d*x) + 1))/d
```

3.52 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal. Leaf size=63

$$-\frac{3b\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{a\cosh(c+dx)}{d}+\frac{3b\sinh(c+dx)}{2d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}$$

```
[Out] -3/2*b*arctan(sinh(d*x+c))/d+a*cosh(d*x+c)/d+3/2*b*sinh(d*x+c)/d-1/2*b*sinh
(d*x+c)*tanh(d*x+c)^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3747, 2718, 2672, 294, 327, 209}

$$\frac{a\cosh(c+dx)}{d}-\frac{3b\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{3b\sinh(c+dx)}{2d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-3*b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Cosh[c + d*x])/d + (3*b*Sinh[c + d*
x])/(2*d) - (b*Sinh[c + d*x]*Tanh[c + d*x]^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx &= -\left(i\int \left(ia\sinh(c+dx)+ib\sinh(c+dx)\tanh^3(c+dx)\right)\,dx\right)\\ &= a\int \sinh(c+dx)\,dx+b\int \sinh(c+dx)\tanh^3(c+dx)\,dx\\ &= \frac{a\cosh(c+dx)}{d}+\frac{b\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a\cosh(c+dx)}{d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}+\frac{(3b)\text{Subst}\left(\int \frac{}{1-}\right.}{}\\ &= \frac{a\cosh(c+dx)}{d}+\frac{3b\sinh(c+dx)}{2d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}\\ &= -\frac{3b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a\cosh(c+dx)}{d}+\frac{3b\sinh(c+dx)}{2d}-\end{aligned}$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 1.14

$$\frac{a\cosh(c)\cosh(dx)}{d}+\frac{a\sinh(c)\sinh(dx)}{d}+\frac{b\sinh(c+dx)\tanh^2(c+dx)}{d}-\frac{3b(\text{ArcTan}(\sinh(c+dx))-\text{sech}(c+dx)\tanh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

[Out] (a*Cosh[c]*Cosh[d*x])/d + (a*Sinh[c]*Sinh[d*x])/d + (b*Sinh[c + d*x]*Tanh[c + d*x]^2)/d - (3*b*(ArcTan[Sinh[c + d*x]] - Sech[c + d*x]*Tanh[c + d*x]))/(2*d)

Maple [C] Result contains complex when optimal does not.
time = 2.19, size = 125, normalized size = 1.98

method	result	size
risch	$\frac{a\,{\mathrm e}^{d x +c}}{2 d}+\frac{b\,{\mathrm e}^{d x +c}}{2 d}+\frac{{\mathrm e}^{-d x -c} a}{2 d}-\frac{{\mathrm e}^{-d x -c} b}{2 d}+\frac{b\,{\mathrm e}^{d x +c} \left({\mathrm e}^{2 d x +2 c}-1\right)}{d \left(1+{\mathrm e}^{2 d x +2 c}\right)^{2}}+\frac{3 i b \ln \left({\mathrm e}^{d x +c}-i\right)}{2 d}-\frac{3 i b \ln \left({\mathrm e}^{d x +c}+i\right)}{2 d}$	125

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)

[Out] 1/2*a/d*exp(d*x+c)+1/2*b/d*exp(d*x+c)+1/2/d*exp(-d*x-c)*a-1/2/d*exp(-d*x-c)*b+b*exp(d*x+c)*(exp(2*d*x+2*c)-1)/d/(1+exp(2*d*x+2*c))^2+3/2*I*b/d*ln(exp(d*x+c)-I)-3/2*I*b/d*ln(exp(d*x+c)+I)

Maxima [A]
time = 0.48, size = 105, normalized size = 1.67

$$\frac{1}{2}\,b\left(\frac{6\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d\left(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)}\right)}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] 1/2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + a*cosh(d*x + c)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(57) = 114$.
time = 0.35, size = 528, normalized size = 8.38

$$\frac{(a+b)\cosh(dx+c)^6+6(a+b)\cosh(dx+c)\sinh(dx+c)^5+(a+b)\sinh(dx+c)^6+3(a+b)\cosh(dx+c)^4+3\left(5(a+b)\cosh(dx+c)^2+a+b\right)\sinh(dx+c)^4+4\left(5(a+b)\cosh(dx+c)^3+3(a+b)\cosh(dx+c)\right)\sinh(dx+c)^3+3(a-b)\cosh(dx+c)^2+3\left(5(a+b)\cosh(dx+c)^4+6(a+b)\cosh(dx+c)^2+a-b\right)\sinh(dx+c)^2-6\left(b\cosh(dx+c)^5+5b\cosh(dx+c)\sinh(dx+c)^4+b\sinh(dx+c)^5+2b\cosh(dx+c)^3+2\left(5b\cosh(dx+c)^2+b\right)\sinh(dx+c)^3+2\left(5b\cosh(dx+c)^3+3b\cosh(dx+c)\right)\sinh(dx+c)^2+b\cosh(dx+c)+\left(5b\cosh(dx+c)^4+6b\cosh(dx+c)^2+b\right)\sinh(dx+c)\right)\arctan\left(\cosh(dx+c)+\sinh(dx+c)\right)+6\left((a+b)\cosh(dx+c)^5+2(a+b)\cosh(dx+c)^3+(a-b)\cosh(dx+c)\right)\sinh(dx+c)+a-b}{2\left(d\cosh(dx+c)^5+5d\cosh(dx+c)\sinh(dx+c)^4+d\sinh(dx+c)^5+2d\cosh(dx+c)^3+2\left(5d\cosh(dx+c)^2+d\right)\sinh(dx+c)^3+2\left(5d\cosh(dx+c)^3+3d\cosh(dx+c)\right)\sinh(dx+c)^2+d\cosh(dx+c)+\left(5d\cosh(dx+c)^4+6d\cosh(dx+c)^2+d\right)\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] 1/2*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 + 3*(a + b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 + 6*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 - 6*(b*cosh(d*x + c)^5 + 5*b*cosh(d*x + c)*sinh(d*x + c)^4 + b*sinh(d*x + c)^5 + 2*

```
b*cosh(d*x + c)^3 + 2*(5*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^3 + 2*(5*b*co
sh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^2 + b*cosh(d*x + c) + (5*b
*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c))*arctan(cosh(d*x
+ c) + sinh(d*x + c)) + 6*((a + b)*cosh(d*x + c)^5 + 2*(a + b)*cosh(d*x + c
)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)/(d*cosh(d*x + c)^5 + 5*
d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 +
 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3*d
*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 +
6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^3\left(c + dx\right)\right)\sinh\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x), x)
```

Giac [A]
time = 0.42, size = 86, normalized size = 1.37

$$-\frac{6\,b\arctan\left(e^{(dx+c)}\right) - ae^{(dx+c)} - be^{(dx+c)} - (a-b)e^{(-dx-c)} - \frac{2\,\left(be^{(3\,dx+3\,c)} - be^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/2*(6*b*arctan(e^(d*x + c)) - a*e^(d*x + c) - b*e^(d*x + c) - (a - b)*e^(
-d*x - c) - 2*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/
d
```

Mupad [B]
time = 0.14, size = 128, normalized size = 2.03

$$\frac{\mathrm{e}^{-c-d\,x}\,\left(a-b\right)}{2\,d} - \frac{3\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}} + \frac{\mathrm{e}^{c+d\,x}\,\left(a+b\right)}{2\,d} + \frac{b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3),x)
```

```
[Out] (exp(- c - d*x)*(a - b))/(2*d) - (3*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d
*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) + (exp(c + d*x)*(a + b))/(2*d) + (b
*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(2*exp(2*
c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.53 $\int \mathrm{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal. Leaf size=49

$$\frac{b\mathrm{ArcTan}(\sinh(c+dx))}{2d}-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\mathrm{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*b*arctan(sinh(d*x+c))/d-a*arctanh(cosh(d*x+c))/d-1/2*b*sech(d*x+c)*tanh
(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3747, 3855, 2691}

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\mathrm{ArcTan}(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*ArcTanh[Cosh[c + d*x]])/d - (b*Sech[c
+ d*x]*Tanh[c + d*x])/(2*d)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = i\int\left(-ia\operatorname{csch}(c+dx)-ib\operatorname{sech}(c+dx)\tanh^2(c+dx)\right)\,dx$$
$$= a\int \operatorname{csch}(c+dx)\,dx + b\int \operatorname{sech}(c+dx)\tanh^2(c+dx)\,dx$$
$$= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{1}{2}b\int$$
$$= \frac{b\tan^{-1}(\sinh(c+dx))}{2d} - \frac{a\tanh^{-1}(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}(c+dx}{2}$$

Mathematica [A]
time = 0.03, size = 75, normalized size = 1.53

$$\frac{b\text{ArcTan}(\sinh(c+dx))}{2d} - \frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[S
inh[c/2 + (d*x)/2]])/d - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.89, size = 101, normalized size = 2.06

method	result	size
risch	$-\frac{b\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}-1\right)}{d\left(1+\mathrm{e}^{2dx+2c}\right)^{2}}+\frac{ib\ln\left(\mathrm{e}^{dx+c}+i\right)}{2d}-\frac{ib\ln\left(\mathrm{e}^{dx+c}-i\right)}{2d}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -b*exp(d*x+c)*(exp(2*d*x+2*c)-1)/d/(1+exp(2*d*x+2*c))^2+1/2*I*b/d*ln(exp(d*
x+c)+I)-1/2*I*b/d*ln(exp(d*x+c)-I)+a/d*ln(exp(d*x+c)-1)-a/d*ln(exp(d*x+c)+1
)
```

Maxima [A]
time = 0.48, size = 83, normalized size = 1.69

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(45) = 90$.
time = 0.37, size = 522, normalized size = 10.65

$$-\frac{b \cosh(dx+c)^3 + 3 b \cosh(dx+c) \sinh(dx+c)^2 + b \sinh(dx+c)^3 - \left(b \cosh(dx+c)^4 + 4 b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 + 2 b \cosh(dx+c)^2 + 2 \left(3 b \cosh(dx+c)^2 + b\right) \sinh(dx+c)^2 + 4 \left(b \cosh(dx+c)^3 + b \cosh(dx+c)\right) \sinh(dx+c) + b\right) \arctan\left(\cosh(dx+c) + \sinh(dx+c)\right) - b \cosh(dx+c) + \left(a \cosh(dx+c)^4 + 4 a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 + 2 a \cosh(dx+c)^2 + 2 \left(3 a \cosh(dx+c)^2 + a\right) \sinh(dx+c)^2 + 4 \left(a \cosh(dx+c)^3 + a \cosh(dx+c)\right) \sinh(dx+c) + a\right) \log\left(\cosh(dx+c) + \sinh(dx+c) + 1\right) - \left(a \cosh(dx+c)^4 + 4 a \cosh(dx+c) \sinh(dx+c)^3 + a \sinh(dx+c)^4 + 2 a \cosh(dx+c)^2 + 2 \left(3 a \cosh(dx+c)^2 + a\right) \sinh(dx+c)^2 + 4 \left(a \cosh(dx+c)^3 + a \cosh(dx+c)\right) \sinh(dx+c) + a\right) \log\left(\cosh(dx+c) + \sinh(dx+c) - 1\right) + \left(3 b \cosh(dx+c)^2 - b\right) \sinh(dx+c)}{d \cosh(dx+c)^4 + 4 d \cosh(dx+c) \sinh(dx+c)^3 + d \sinh(dx+c)^4 + 2 d \cosh(dx+c)^2 + 2 \left(3 d \cosh(dx+c)^2 + d\right) \sinh(dx+c)^2 + 4 \left(d \cosh(dx+c)^3 + d \cosh(dx+c)\right) \sinh(dx+c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)
^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*
(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*arctan(cosh(d*x +
c) + sinh(d*x + c)) - b*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
 c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh
(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*
sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x
+ c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3
 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) -
1) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh
(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x
+ c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh^3(c + dx)\right) \operatorname{csch}(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x), x)
```

Giac [A]
time = 0.44, size = 74, normalized size = 1.51

$$\frac{b \arctan\left(e^{(dx+c)}\right) - a \log\left(e^{(dx+c)} + 1\right) + a \log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{b e^{(3\,dx+3\,c)} - b e^{(dx+c)}}{\left(e^{(2\,dx+2\,c)} + 1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] (b*arctan(e^(d*x + c)) - a*log(e^(d*x + c) + 1) + a*log(abs(e^(d*x + c) - 1
)) - (b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B]
time = 2.47, size = 233, normalized size = 4.76

$$\frac{2\,b\,{\mathrm{e}}^{c+d\,x}}{d+2\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}+d\,{\mathrm{e}}^{4\,c+4\,d\,x}}-\frac{a\,\ln\left(-8\,a\,b^2-32\,a^3-32\,a^3\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}-8\,a\,b^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\right)}{d}+\frac{a\,\ln\left(8\,a\,b^2+32\,a^3-32\,a^3\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}-8\,a\,b^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\right)}{d}-\frac{b\,\left(\ln\left(4\,b^3\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}+16\,a^2\,b\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}-a^2\,b\,16{}\mathrm{i}-b^3\,4{}\mathrm{i}\right)\,1{}\mathrm{i}-\ln\left(4\,b^3\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}+16\,a^2\,b\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}+a^2\,b\,16{}\mathrm{i}+b^3\,4{}\mathrm{i}\right)\,1{}\mathrm{i}\right)}{2\,d}-\frac{b\,{\mathrm{e}}^{c+d\,x}}{d+d\,{\mathrm{e}}^{2\,c+2\,d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x),x)
```

```
[Out] (2*b*exp(c + d*x))/(d + 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*log
(- 8*a*b^2 - 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 8*a*b^2*exp(d*x)*exp(c)))/d
+ (a*log(8*a*b^2 + 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 8*a*b^2*exp(d*x)*exp(c
)))/d - (b*(log(4*b^3*exp(d*x)*exp(c) - b^3*4i - a^2*b*16i + 16*a^2*b*exp(d
*x)*exp(c))*1i - log(a^2*b*16i + b^3*4i + 4*b^3*exp(d*x)*exp(c) + 16*a^2*b*
exp(d*x)*exp(c))*1i))/(2*d) - (b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x))
```

3.54 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal. Leaf size=29

$$-\frac{a\coth(c+dx)}{d}+\frac{b\tanh^2(c+dx)}{2d}$$

```
[Out] -a*coth(d*x+c)/d+1/2*b*tanh(d*x+c)^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3744, 14}

$$\frac{b\tanh^2(c+dx)}{2d}-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^3}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^2}+bx\right)\,dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{a\coth(c+dx)}{d}+\frac{b\tanh^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 1.00

$$-\frac{a\coth(c+dx)}{d}-\frac{b\mathrm{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -((a*Coth[c + d*x])/d) - (b*Sech[c + d*x]^2)/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(27) = 54.
time = 2.87, size = 80, normalized size = 2.76

method	result	size
risch	$-\frac{2\left(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}+2a\,\mathrm{e}^{2dx+2c}-b\,\mathrm{e}^{2dx+2c}+a\right)}{d\left(1+\mathrm{e}^{2dx+2c}\right)^2\left(\mathrm{e}^{2dx+2c}-1\right)}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(a*exp(4*d*x+4*c)+b*exp(4*d*x+4*c)+2*a*exp(2*d*x+2*c)-b*exp(2*d*x+2*c)+a
)/d/(1+exp(2*d*x+2*c))^2/(exp(2*d*x+2*c)-1)
```

Maxima [A]
time = 0.27, size = 44, normalized size = 1.52

$$\frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)}-\frac{2\,b}{d(e^{(dx+c)}+e^{(-dx-c)})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 2*a/(d*(e^(-2*d*x - 2*c) - 1)) - 2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(27) = 54.
time = 0.33, size = 141, normalized size = 4.86

$$-\frac{2\left((2\,a+b)\cosh\left(dx+c\right)^2+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(2\,a+b)\sinh\left(dx+c\right)^2+2\,a-b\right)}{d\cosh\left(dx+c\right)^4+6\,d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^2+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4+4\left(d\cosh\left(dx+c\right)^3+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -2*((2*a + b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (2*a + b)
*sinh(d*x + c)^2 + 2*a - b)/(d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2*sinh(d
*x + c)^2 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*(d*co
sh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^{3}\left(c+dx\right)\right)\operatorname{csch}^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**2, x)
```

Giac [A]
time = 0.44, size = 45, normalized size = 1.55

$$-\frac{2\left(\frac{a}{e^{(2\,dx+2\,c)}-1}+\frac{be^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^{2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -2*(a/(e^(2*d*x + 2*c) - 1) + b*e^(2*d*x + 2*c)/(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B]
time = 0.17, size = 79, normalized size = 2.72

$$-\frac{2\left(a+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}+a\,\mathrm{e}^{4\,c+4\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x}+b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^2,x)
```

```
[Out] -(2*(a + 2*a*exp(2*c + 2*d*x) + a*exp(4*c + 4*d*x) - b*exp(2*c + 2*d*x) + b
*exp(4*c + 4*d*x)))/(d*(exp(2*c + 2*d*x) - 1)*(exp(2*c + 2*d*x) + 1)^2)
```

3.55 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal. Leaf size=71

$$\frac{b\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{b\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*b*arctan(sinh(d*x+c))/d+1/2*a*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*
csch(d*x+c)/d+1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3747, 3853, 3855}

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{b\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Cot
h[c + d*x]*Csch[c + d*x])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\left(i\int\left(ia\operatorname{csch}^3(c+dx)+ib\operatorname{sech}^3(c+dx)\right)\,dx\right)$$
$$= a\int \operatorname{csch}^3(c+dx)\,dx + b\int \operatorname{sech}^3(c+dx)\,dx$$
$$= -\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} -$$
$$= \frac{b\tan^{-1}(\sinh(c+dx))}{2d} + \frac{a\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a\coth(c+}{}$$

Mathematica [A]
time = 0.02, size = 95, normalized size = 1.34

$$\frac{b\text{ArcTan}(\sinh(c+dx))}{2d} - \frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Ta
nh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sech[c + d*x]*
Tanh[c + d*x])/(2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 3.20, size = 177, normalized size = 2.49

method	result
risch	$-\frac{e^{dx+c}\left(a\,e^{6dx+6c}-b\,e^{6dx+6c}+3a\,e^{4dx+4c}+3b\,e^{4dx+4c}+3a\,e^{2dx+2c}-3b\,e^{2dx+2c}+a+b\right)}{d\left(e^{2dx+2c}-1\right)^2\left(1+e^{2dx+2c}\right)^2} + \frac{a\ln\left(e^{dx+c}+1\right)}{2d} - \frac{a\ln\left(e^{dx+c}-1\right)}{2d} + \frac{ib}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -exp(d*x+c)*(a*exp(6*d*x+6*c)-b*exp(6*d*x+6*c)+3*a*exp(4*d*x+4*c)+3*b*exp(4
*d*x+4*c)+3*a*exp(2*d*x+2*c)-3*b*exp(2*d*x+2*c)+a+b)/d/(exp(2*d*x+2*c)-1)^2
/(1+exp(2*d*x+2*c))^2+1/2*a/d*ln(exp(d*x+c)+1)-1/2*a/d*ln(exp(d*x+c)-1)+1/2
*I*b/d*ln(exp(d*x+c)+I)-1/2*I*b/d*ln(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(63) = 126.
time = 0.49, size = 156, normalized size = 2.20

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)} + e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a*(log(e^(-d*x - c) + 1)/d - log
(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. $2(63) = 126$.
time = 0.38, size = 1188, normalized size = 16.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a - b)*cosh(d*x + c)^7 + 14*(a - b)*cosh(d*x + c)*sinh(d*x + c)^6
+ 2*(a - b)*sinh(d*x + c)^7 + 6*(a + b)*cosh(d*x + c)^5 + 6*(7*(a - b)*cosh
(d*x + c)^2 + a + b)*sinh(d*x + c)^5 + 10*(7*(a - b)*cosh(d*x + c)^3 + 3*(a
 + b)*cosh(d*x + c))*sinh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^3 + 2*(35*(a
 - b)*cosh(d*x + c)^4 + 30*(a + b)*cosh(d*x + c)^2 + 3*a - 3*b)*sinh(d*x +
c)^3 + 6*(7*(a - b)*cosh(d*x + c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*(a - b
)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c)^8 + 56*b*cosh(d*x + c
)^3*sinh(d*x + c)^5 + 28*b*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*b*cosh(d*x +
 c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 2*b*cosh(d*x + c)^4 + 2*(35*b*cos
h(d*x + c)^4 - b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - b*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(7*b*cosh(d*x + c)^6 - 3*b*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*(b*cosh(d*x + c)^7 - b*cosh(d*x + c)^3)*sinh(d*x + c) + b)*arct
an(cosh(d*x + c) + sinh(d*x + c)) + 2*(a + b)*cosh(d*x + c) - (a*cosh(d*x +
 c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*cosh(d*x + c)^2*sinh(d*
x + c)^6 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 2*a*cosh
(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*
x + c)^5 - a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a*cosh(d*x + c)^6 - 3*a*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - a*cosh(d*x + c)^3
)*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a*cosh(d*x +
 c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*cosh(d*x + c)^2*sinh(d*
x + c)^6 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 2*a*cosh
(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*
x + c)^5 - a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a*cosh(d*x + c)^6 - 3*a*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - a*cosh(d*x + c)^3
)*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(a - b)*
cosh(d*x + c)^6 + 15*(a + b)*cosh(d*x + c)^4 + 9*(a - b)*cosh(d*x + c)^2 +
a + b)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x +
c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x +
```

```
c)^7 + d*sinh(d*x + c)^8 - 2*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 -
d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(7*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*
cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^3\left(c + dx\right)\right)\operatorname{csch}^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(63) = 126.
time = 0.45, size = 143, normalized size = 2.01

$$\frac{2\,b\arctan\left(e^{(dx+c)}\right) + a\log\left(e^{(dx+c)}+1\right) - a\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\left(ae^{(7\,dx+7\,c)} - be^{(7\,dx+7\,c)} + 3\,ae^{(5\,dx+5\,c)} + 3\,be^{(5\,dx+5\,c)} + 3\,ae^{(3\,dx+3\,c)} - 3\,be^{(3\,dx+3\,c)} + ae^{(dx+c)} + be^{(dx+c)}\right)}{\left(e^{(4\,dx+4\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*arctan(e^(d*x + c)) + a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x +
c) - 1)) - 2*(a*e^(7*d*x + 7*c) - b*e^(7*d*x + 7*c) + 3*a*e^(5*d*x + 5*c) +
 3*b*e^(5*d*x + 5*c) + 3*a*e^(3*d*x + 3*c) - 3*b*e^(3*d*x + 3*c) + a*e^(d*x
 + c) + b*e^(d*x + c))/(e^(4*d*x + 4*c) - 1)^2)/d
```

Mupad [B]
time = 2.48, size = 173, normalized size = 2.44

$$\frac{a\,\ln\left({\mathrm{e}}^{c+d\,x}+1\right)}{2\,d}-\frac{\frac{4\,{\mathrm{e}}^{3\,c+3\,d\,x}\,\left(a-b\right)}{d}+\frac{4\,{\mathrm{e}}^{c+d\,x}\,\left(a+b\right)}{d}}{{\mathrm{e}}^{8\,c+8\,d\,x}-2\,{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{a\,\ln\left({\mathrm{e}}^{c+d\,x}-1\right)}{2\,d}-\frac{\frac{{\mathrm{e}}^{3\,c+3\,d\,x}\,\left(a-b\right)}{d}+\frac{3\,{\mathrm{e}}^{c+d\,x}\,\left(a+b\right)}{d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-1}-\frac{b\,\ln\left({\mathrm{e}}^{c+d\,x}-\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d}+\frac{b\,\ln\left({\mathrm{e}}^{c+d\,x}+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^3,x)
```

```
[Out] (a*log(exp(c + d*x) + 1))/(2*d) - ((4*exp(3*c + 3*d*x)*(a - b))/d + (4*exp(
c + d*x)*(a + b))/d)/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (a*log(e
xp(c + d*x) - 1))/(2*d) - ((exp(3*c + 3*d*x)*(a - b))/d + (3*exp(c + d*x)*(
a + b))/d)/(exp(4*c + 4*d*x) - 1) - (b*log(exp(c + d*x) - 1i)*1i)/(2*d) + (
b*log(exp(c + d*x) + 1i)*1i)/(2*d)
```

3.56 $\int \mathbf{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal. Leaf size=56

$$\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}+\frac{b\log(\tanh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

```
[Out] a*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d+b*ln(tanh(d*x+c))/d-1/2*b*tanh(d*x+c)
^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3744, 1816}

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}-\frac{b\tanh^2(c+dx)}{2d}+\frac{b\log(\tanh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d) + (b*Log[Tanh[c + d*x]])/d
- (b*Tanh[c + d*x]^2)/(2*d)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+bx^3)}{x^4}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^4}-\frac{a}{x^2}+\frac{b}{x}-bx\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}+\frac{b\log(\tanh(c+dx))}{d}-\frac{b\tan}{}\end{aligned}$$

Mathematica [**A**]
time = 0.16, size = 74, normalized size = 1.32

$$\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\text{csch}^2(c+dx)}{3d}-\frac{b\left(2\log(\cosh(c+dx))-2\log(\sinh(c+dx))-\text{sech}^2(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b*(2
*Log[Cosh[c + d*x]] - 2*Log[Sinh[c + d*x]] - Sech[c + d*x]^2))/(2*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(52) = 104.
time = 3.09, size = 156, normalized size = 2.79

method	result
risch	$-\frac{2\left(-3b\,e^{8dx+8c}+6a\,e^{6dx+6c}+9b\,e^{6dx+6c}+10a\,e^{4dx+4c}-9b\,e^{4dx+4c}+2a\,e^{2dx+2c}+3b\,e^{2dx+2c}-2a\right)}{3d\left(e^{2dx+2c}-1\right)^3\left(1+e^{2dx+2c}\right)^2}+\frac{b\ln\left(e^{2dx+2c}-1\right)}{d}-\frac{b\ln(1+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-3*b*exp(8*d*x+8*c)+6*a*exp(6*d*x+6*c)+9*b*exp(6*d*x+6*c)+10*a*exp(4*
d*x+4*c)-9*b*exp(4*d*x+4*c)+2*a*exp(2*d*x+2*c)+3*b*exp(2*d*x+2*c)-2*a)/d/(e
xp(2*d*x+2*c)-1)^3/(1+exp(2*d*x+2*c))^2+b/d*ln(exp(2*d*x+2*c)-1)-b/d*ln(1+e
xp(2*d*x+2*c))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(52) = 104.
time = 0.48, size = 184, normalized size = 3.29

$$b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}\right)+\frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}-\frac{1}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c)
 + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)
)) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
+ e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1739 vs. 2(52) = 104.
time = 0.34, size = 1739, normalized size = 31.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] 1/3*(6*b*cosh(d*x + c)^8 + 48*b*cosh(d*x + c)*sinh(d*x + c)^7 + 6*b*sinh(d*x + c)^8 - 6*(2*a + 3*b)*cosh(d*x + c)^6 + 6*(28*b*cosh(d*x + c)^2 - 2*a - 3*b)*sinh(d*x + c)^6 + 12*(28*b*cosh(d*x + c)^3 - 3*(2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(10*a - 9*b)*cosh(d*x + c)^4 + 2*(210*b*cosh(d*x + c)^4 - 45*(2*a + 3*b)*cosh(d*x + c)^2 - 10*a + 9*b)*sinh(d*x + c)^4 + 8*(42*b*cosh(d*x + c)^5 - 15*(2*a + 3*b)*cosh(d*x + c)^3 - (10*a - 9*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*a + 3*b)*cosh(d*x + c)^2 + 2*(84*b*cosh(d*x + c)^6 - 45*(2*a + 3*b)*cosh(d*x + c)^4 - 6*(10*a - 9*b)*cosh(d*x + c)^2 - 2*a - 3*b)*sinh(d*x + c)^2 - 3*(b*cosh(d*x + c)^10 + 10*b*cosh(d*x + c)*sinh(d*x + c)^9 + b*sinh(d*x + c)^10 - b*cosh(d*x + c)^8 + (45*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^8 + 8*(15*b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)^7 - 2*b*cosh(d*x + c)^6 + 2*(105*b*cosh(d*x + c)^4 - 14*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 4*(63*b*cosh(d*x + c)^5 - 14*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*b*cosh(d*x + c)^4 + 2*(105*b*cosh(d*x + c)^6 - 35*b*cosh(d*x + c)^4 - 15*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^7 - 7*b*cosh(d*x + c)^5 - 5*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (45*b*cosh(d*x + c)^8 - 28*b*cosh(d*x + c)^6 - 30*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^9 - 4*b*cosh(d*x + c)^7 - 6*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(b*cosh(d*x + c)^10 + 10*b*cosh(d*x + c)*sinh(d*x + c)^9 + b*sinh(d*x + c)^10 - b*cosh(d*x + c)^8 + (45*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^8 + 8*(15*b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)^7 - 2*b*cosh(d*x + c)^6 + 2*(105*b*cosh(d*x + c)^4 - 14*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 4*(63*b*cosh(d*x + c)^5 - 14*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*b*cosh(d*x + c)^4 + 2*(105*b*cosh(d*x + c)^6 - 35*b*cosh(d*x + c)^4 - 15*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^7 - 7*b*cosh(d*x + c)^5 - 5*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (45*b*cosh(d*x + c)^8 - 28*b*cosh(d*x + c)^6 - 30*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^9 - 4*b*cosh(d*x + c)^7 - 6*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(12*b*cosh(d*x + c)^7 - 9*(2*a + 3*b)*cosh(d*x + c)^5 - 2*(10*a - 9*b)*cosh(d*x + c)^3 - (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*a)/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 - d*cosh(d*x + c)^8 + (45*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^7 - 2*d*cosh(d*x + c)^6 + 2*(105*d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 - 14*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^4 + 2*(105*d*cosh(d*x + c)^6 - 35*d*cosh(d*x + c)^4 - 15*

```
d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 8*(15*d*cosh(d*x + c)^7 - 7*d*cosh
(d*x + c)^5 - 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + d*co
sh(d*x + c)^2 + (45*d*cosh(d*x + c)^8 - 28*d*cosh(d*x + c)^6 - 30*d*cosh(d*
x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)
^9 - 4*d*cosh(d*x + c)^7 - 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*co
sh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tanh^3(c + dx))\operatorname{csch}^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(52) = 104.
time = 0.44, size = 149, normalized size = 2.66

$$-\frac{6\,b\log\left(e^{(2\,dx+2\,c)}+1\right)-6\,b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)-\frac{3\,\left(3\,be^{(4\,dx+4\,c)}+10\,be^{(2\,dx+2\,c)}+3\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}+\frac{11\,be^{(6\,dx+6\,c)}-33\,be^{(4\,dx+4\,c)}+24\,ae^{(2\,dx+2\,c)}+33\,be^{(2\,dx+2\,c)}-8\,a-11\,b}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(6*b*log(e^(2*d*x + 2*c) + 1) - 6*b*log(abs(e^(2*d*x + 2*c) - 1)) - 3*
(3*b*e^(4*d*x + 4*c) + 10*b*e^(2*d*x + 2*c) + 3*b)/(e^(2*d*x + 2*c) + 1)^2
+ (11*b*e^(6*d*x + 6*c) - 33*b*e^(4*d*x + 4*c) + 24*a*e^(2*d*x + 2*c) + 33*
b*e^(2*d*x + 2*c) - 8*a - 11*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B]
time = 1.24, size = 162, normalized size = 2.89

$$\frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{4\,a}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{8\,a}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^4,x)
```

```
[Out] (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (4*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c +
 2*d*x) + 1)) - (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (8*
a)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) -
 (2*atan((b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))
/(-d^2)^(1/2)
```

3.57 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=170

$$\frac{3}{8}\left(a^2+21b^2\right)x+\frac{6ab\log(\cosh(c+dx))}{d}-\frac{6b^2\tanh(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{d}-\frac{b^2\tanh^5(c-}{5d}$$

```
[Out] 3/8*(a^2+21*b^2)*x+6*a*b*ln(cosh(d*x+c))/d-6*b^2*tanh(d*x+c)/d-a*b*tanh(d*x
+c)^2/d-b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d+1/4*cosh(d*x+c)^3*sinh(
d*x+c)*(a^2+b^2+2*a*b*tanh(d*x+c))/d-1/8*cosh(d*x+c)*sinh(d*x+c)*(5*a^2+17*
b^2+20*a*b*tanh(d*x+c))/d
```

Rubi [A]
time = 0.21, antiderivative size = 206, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 1818, 1816, 647, 31}

$$-\frac{3(a^2+21b^2)\tanh(c+dx)}{8d}-\frac{3(a^2+16ab+21b^2)\log(1-\tanh(c+dx))}{16d}+\frac{3(a^2-16ab+21b^2)\log(\tanh(c+dx)+1)}{16d}+\frac{\sinh^4(c+dx)\left((a^2+b^2)\tanh(c+dx)+2ab\right)}{4d}-\frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+16ab\tanh(c+dx)+13b^2\right)}{8d}-\frac{3ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^3(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (-3*(a^2 + 16*a*b + 21*b^2)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a^2 - 16*a
*b + 21*b^2)*Log[1 + Tanh[c + d*x]])/(16*d) - (3*(a^2 + 21*b^2)*Tanh[c + d*
x])/(8*d) - (3*a*b*Tanh[c + d*x]^2)/d - (b^2*Tanh[c + d*x]^3)/d - (b^2*Tanh
[c + d*x]^5)/(5*d) - (Sinh[c + d*x]^2*Tanh[c + d*x]*(a^2 + 13*b^2 + 16*a*b*
Tanh[c + d*x]))/(8*d) + (Sinh[c + d*x]^4*(2*a*b + (a^2 + b^2)*Tanh[c + d*x]
))/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4\left(a+bx^3\right)^2}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\sinh^4(c+dx)\left(2ab+\left(a^2+b^2\right)\tanh(c+dx)\right)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{x^3(-\ldots}{\ldots}\right.}{\ldots} \\
&= -\frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+13b^2+16ab\tanh(c+dx)\right)\ldots}{8d} \\
&= -\frac{\sinh^2(c+dx)\tanh(c+dx)\left(a^2+13b^2+16ab\tanh(c+dx)\right)\ldots}{8d} \\
&= -\frac{3\left(a^2+21b^2\right)\tanh(c+dx)}{8d} - \frac{3ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^3(\ldots}{d} \\
&= -\frac{3\left(a^2+21b^2\right)\tanh(c+dx)}{8d} - \frac{3ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^3(\ldots}{d} \\
&= -\frac{3\left(a^2+16ab+21b^2\right)\log(1-\tanh(c+dx))}{16d} + \frac{3\left(a^2-16ab+\ldots\right.}{\ldots}
\end{aligned}
$$

Mathematica [A]
time = 1.88, size = 156, normalized size = 0.92

$$
\frac{60\left(a^2+21b^2\right)(c+dx)-200ab\cosh(2(c+dx))+10ab\cosh(4(c+dx))+960ab\log(\cosh(c+dx))+160ab\operatorname{sech}^2(c+dx)-40\left(a^2+4b^2\right)\sinh(2(c+dx))+5\left(a^2+b^2\right)\sinh(4(c+dx))-1152b^2\tanh(c+dx)+224b^2\operatorname{sech}^2(c+dx)\tanh(c+dx)-32b^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{160d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (60*(a^2 + 21*b^2)*(c + d*x) - 200*a*b*Cosh[2*(c + d*x)] + 10*a*b*Cosh[4*(c
 + d*x)] + 960*a*b*Log[Cosh[c + d*x]] + 160*a*b*Sech[c + d*x]^2 - 40*(a^2 +
 4*b^2)*Sinh[2*(c + d*x)] + 5*(a^2 + b^2)*Sinh[4*(c + d*x)] - 1152*b^2*Tanh
[c + d*x] + 224*b^2*Sech[c + d*x]^2*Tanh[c + d*x] - 32*b^2*Sech[c + d*x]^4*
Tanh[c + d*x])/(160*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(162) = 324$.
time = 2.84, size = 365, normalized size = 2.15

method	result
risch	$\frac{3a^2x}{8} - 6abx + \frac{63b^2x}{8} + \frac{e^{4dx+4c}a^2}{64d} + \frac{e^{4dx+4c}ab}{32d} + \frac{e^{4dx+4c}b^2}{64d} - \frac{e^{2dx+2c}a^2}{8d} - \frac{5\,e^{2dx+2c}ab}{8d} - \frac{e^{2dx+2c}b^2}{2d} + \frac{e^{-2dx-2c}a^2}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^2*x-6*a*b*x+63/8*b^2*x+1/64/d*exp(4*d*x+4*c)*a^2+1/32/d*exp(4*d*x+4*c
)*a*b+1/64/d*exp(4*d*x+4*c)*b^2-1/8/d*exp(2*d*x+2*c)*a^2-5/8/d*exp(2*d*x+2*
c)*a*b-1/2/d*exp(2*d*x+2*c)*b^2+1/8/d*exp(-2*d*x-2*c)*a^2-5/8/d*exp(-2*d*x-
2*c)*a*b+1/2/d*exp(-2*d*x-2*c)*b^2-1/64/d*exp(-4*d*x-4*c)*a^2+1/32/d*exp(-4
*d*x-4*c)*a*b-1/64/d*exp(-4*d*x-4*c)*b^2-12/d*a*b*c+4/5*b*(5*a*exp(8*d*x+8*
c)+25*b*exp(8*d*x+8*c)+15*a*exp(6*d*x+6*c)+75*b*exp(6*d*x+6*c)+15*a*exp(4*d
*x+4*c)+105*b*exp(4*d*x+4*c)+5*a*exp(2*d*x+2*c)+65*b*exp(2*d*x+2*c)+18*b)/d
/(1+exp(2*d*x+2*c))^5+6*a*b/d*ln(1+exp(2*d*x+2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(162) = 324$.
time = 0.51, size = 379, normalized size = 2.23

$$\frac{1}{64}a^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + \frac{1}{320}b^2\left(\frac{2520(dx+c)}{d} + \frac{5\left(32e^{(-2dx-2c)} - e^{(-4dx-4c)}\right)}{d} - \frac{135e^{(-2dx-2c)} + 5358e^{(-4dx-4c)} + 18190e^{(-6dx-6c)} + 28455e^{(-8dx-8c)} + 19995e^{(-10dx-10c)} + 6560e^{(-12dx-12c)} - 5}{d(e^{(-4dx-4c)} + 5e^{(-6dx-6c)} + 10e^{(-8dx-8c)} + 10e^{(-10dx-10c)} + 5e^{(-12dx-12c)} + e^{(-14dx-14c)})}\right) + \frac{1}{32}ab\left(\frac{192(dx+c)}{d} - \frac{20e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} + \frac{192\log\left(e^{(-2dx-2c)} + 1\right)}{d} - \frac{18e^{(-2dx-2c)} + 39e^{(-4dx-4c)} - 108e^{(-6dx-6c)} - 1}{d(e^{(-4dx-4c)} + 2e^{(-6dx-6c)} + e^{(-8dx-8c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4*c
) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x - 10
*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c)))) + 1/32*a*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*c) - e^
```

```
(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x - 2*c)
+ 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*c) + 2*
e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5034 vs. $2(162) = 324$.
time = 0.41, size = 5034, normalized size = 29.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/320*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^18 + 90*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^17 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^18 - 15*(a
^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^16 + 15*(51*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 - a^2 - 10*a*b - 9*b^2)*sinh(d*x + c)^16 + 240*(17*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 - (a^2 + 10*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^1
5 + 30*(4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x +
 c)^14 + 30*(510*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 - 16*a*b + 21
*b^2)*d*x - 60*(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^2 - 5*a^2 - 30*a*b - 25
*b^2)*sinh(d*x + c)^14 + 420*(102*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 20*
(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^3 + (4*(a^2 - 16*a*b + 21*b^2)*d*x - 5
*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c))*sinh(d*x + c)^13 + 10*(60*(a^2 - 16*
a*b + 21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^12 + 10*(9282*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 2730*(a^2 + 10*a*b + 9*b^2)*cosh(d*x
+ c)^4 + 60*(a^2 - 16*a*b + 21*b^2)*d*x + 273*(4*(a^2 - 16*a*b + 21*b^2)*d*
x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^2 - 31*a^2 - 82*a*b + 501*b^2)*s
inh(d*x + c)^12 + 120*(1326*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 546*(a^2
+ 10*a*b + 9*b^2)*cosh(d*x + c)^5 + 91*(4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a
^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^3 + (60*(a^2 - 16*a*b + 21*b^2)*d*x - 3
1*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 + 60*(20*(a^2 - 1
6*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x + c)^10 + 30*(7293
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 4004*(a^2 + 10*a*b + 9*b^2)*cosh(d*x
 + c)^6 + 1001*(4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*co
sh(d*x + c)^4 + 40*(a^2 - 16*a*b + 21*b^2)*d*x + 22*(60*(a^2 - 16*a*b + 21*
b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^2 - 6*a^2 + 30*a*b + 61
4*b^2)*sinh(d*x + c)^10 + 20*(12155*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 - 8
580*(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^7 + 3003*(4*(a^2 - 16*a*b + 21*b^2
)*d*x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^5 + 110*(60*(a^2 - 16*a*b +
21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^3 + 30*(20*(a^2 - 16
*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x + c))*sinh(d*x + c)
^9 + 60*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^2)*cosh(d*
x + c)^8 + 30*(7293*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 - 6435*(a^2 + 10*a
*b + 9*b^2)*cosh(d*x + c)^8 + 3003*(4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 -
```

```
 30*a*b - 25*b^2)*cosh(d*x + c)^6 + 165*(60*(a^2 - 16*a*b + 21*b^2)*d*x - 3
1*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^4 + 40*(a^2 - 16*a*b + 21*b^2)*d*x
+ 90*(20*(a^2 - 16*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x +
 c)^2 + 6*a^2 + 30*a*b + 922*b^2)*sinh(d*x + c)^8 + 240*(663*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^11 - 715*(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^9 + 429*(
4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^7 +
33*(60*(a^2 - 16*a*b + 21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x +
c)^5 + 30*(20*(a^2 - 16*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(
d*x + c)^3 + 2*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^2)*
cosh(d*x + c))*sinh(d*x + c)^7 + 10*(60*(a^2 - 16*a*b + 21*b^2)*d*x + 31*a^
2 - 82*a*b + 1803*b^2)*cosh(d*x + c)^6 + 10*(9282*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^12 - 12012*(a^2 + 10*a*b + 9*b^2)*cosh(d*x + c)^10 + 9009*(4*(a^2
- 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^8 + 924*(60
*(a^2 - 16*a*b + 21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^6 +
 1260*(20*(a^2 - 16*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x
+ c)^4 + 60*(a^2 - 16*a*b + 21*b^2)*d*x + 168*(20*(a^2 - 16*a*b + 21*b^2)*d
*x + 3*a^2 + 15*a*b + 461*b^2)*cosh(d*x + c)^2 + 31*a^2 - 82*a*b + 1803*b^2
)*sinh(d*x + c)^6 + 60*(714*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^13 - 1092*(a^
2 + 10*a*b + 9*b^2)*cosh(d*x + c)^11 + 1001*(4*(a^2 - 16*a*b + 21*b^2)*d*x
- 5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^9 + 132*(60*(a^2 - 16*a*b + 21*b^2
)*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^7 + 252*(20*(a^2 - 16*a*b
+ 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x + c)^5 + 56*(20*(a^2 - 1
6*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^2)*cosh(d*x + c)^3 + (60*(a^2
- 16*a*b + 21*b^2)*d*x + 31*a^2 - 82*a*b + 1803*b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + 6*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 25*a^2 - 150*a*b + 893*b^2)*
cosh(d*x + c)^4 + 6*(2550*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^14 - 4550*(a^2
+ 10*a*b + 9*b^2)*cosh(d*x + c)^12 + 5005*(4*(a^2 - 16*a*b + 21*b^2)*d*x -
5*a^2 - 30*a*b - 25*b^2)*cosh(d*x + c)^10 + 825*(60*(a^2 - 16*a*b + 21*b^2)
*d*x - 31*a^2 - 82*a*b + 501*b^2)*cosh(d*x + c)^8 + 2100*(20*(a^2 - 16*a*b
+ 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*cosh(d*x + c)^6 + 700*(20*(a^2 -
16*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^2)*cosh(d*x + c)^4 + 20*(a^2
- 16*a*b + 21*b^2)*d*x + 25*(60*(a^2 - 16*a*b + 21*b^2)*d*x + 31*a^2 - 82*a
*b + 1803*b^2)*cosh(d*x + c)^2 + 25*a^2 - 150*a*b + 893*b^2)*sinh(d*x + c)^
4 + 8*(510*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^15 - 1050*(a^2 + 10*a*b + 9*b^
2)*cosh(d*x + c)^13 + 1365*(4*(a^2 - 16*a*b + 2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^{3}\left(c+dx\right)\right)^{2}\sinh^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(162) = 324.
time = 0.58, size = 374, normalized size = 2.20

$$\frac{5\,a^{2}e^{\left(4\,dx+4\,c\right)} + 10\,abe^{\left(4\,dx+4\,c\right)} + 5\,b^{2}e^{\left(4\,dx+4\,c\right)} - 40\,a^{2}e^{\left(2\,dx+2\,c\right)} - 200\,abe^{\left(2\,dx+2\,c\right)} - 160\,b^{2}e^{\left(2\,dx+2\,c\right)} + 1920\,ab\log\left(e^{\left(2\,dx+2\,c\right)} + 1\right) + 120\,\left(a^{2} - 16\,ab + 21\,b^{2}\right)\left(dx + c\right) - 5\,\left(18\,a^{2}e^{\left(4\,dx+4\,c\right)} - 288\,abe^{\left(4\,dx+4\,c\right)} + 378\,b^{2}e^{\left(4\,dx+4\,c\right)} - 8\,a^{2}e^{\left(2\,dx+2\,c\right)} + 40\,abe^{\left(2\,dx+2\,c\right)} - 32\,b^{2}e^{\left(2\,dx+2\,c\right)} + a^{2} - 2\,ab + b^{2}\right)e^{\left(-4\,dx-4\,c\right)} - \frac{32\,\left(137\,abe^{\left(10\,dx+10\,c\right)} + 645\,abe^{\left(8\,dx+8\,c\right)} - 200\,b^{2}e^{\left(8\,dx+8\,c\right)} + 1250\,abe^{\left(6\,dx+6\,c\right)} - 600\,b^{2}e^{\left(6\,dx+6\,c\right)} + 1250\,abe^{\left(4\,dx+4\,c\right)} - 840\,b^{2}e^{\left(4\,dx+4\,c\right)} + 645\,abe^{\left(2\,dx+2\,c\right)} - 520\,b^{2}e^{\left(2\,dx+2\,c\right)} + 137\,ab - 144\,b^{2}\right)}{\left(e^{\left(2\,dx+2\,c\right)} + 1\right)^{5}}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/320*(5*a^2*e^(4*d*x + 4*c) + 10*a*b*e^(4*d*x + 4*c) + 5*b^2*e^(4*d*x + 4*
c) - 40*a^2*e^(2*d*x + 2*c) - 200*a*b*e^(2*d*x + 2*c) - 160*b^2*e^(2*d*x +
2*c) + 1920*a*b*log(e^(2*d*x + 2*c) + 1) + 120*(a^2 - 16*a*b + 21*b^2)*(d*x
 + c) - 5*(18*a^2*e^(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 378*b^2*e^(4*
d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 40*a*b*e^(2*d*x + 2*c) - 32*b^2*e^(2*d
*x + 2*c) + a^2 - 2*a*b + b^2)*e^(-4*d*x - 4*c) - 32*(137*a*b*e^(10*d*x + 1
0*c) + 645*a*b*e^(8*d*x + 8*c) - 200*b^2*e^(8*d*x + 8*c) + 1250*a*b*e^(6*d*
x + 6*c) - 600*b^2*e^(6*d*x + 6*c) + 1250*a*b*e^(4*d*x + 4*c) - 840*b^2*e^(
4*d*x + 4*c) + 645*a*b*e^(2*d*x + 2*c) - 520*b^2*e^(2*d*x + 2*c) + 137*a*b
- 144*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 0.44, size = 359, normalized size = 2.11

$$x\,\left(\frac{3\,a^2}{8}-6\,a\,b+\frac{63\,b^2}{8}\right)+\frac{4\,\left(5\,b^2+a\,b\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{{\mathrm{e}}^{-2\,c-2\,d\,x}\,\left(a^2-5\,a\,b+4\,b^2\right)}{8\,d}-\frac{{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2+5\,a\,b+4\,b^2\right)}{8\,d}+\frac{{\mathrm{e}}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^2}{64\,d}-\frac{4\,\left(5\,b^2+a\,b\right)}{d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}+\frac{24\,b^2}{d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}-\frac{{\mathrm{e}}^{-4\,c-4\,d\,x}\,{\left(a-b\right)}^2}{64\,d}-\frac{16\,b^2}{d\,\left(4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}+\frac{32\,b^2}{5\,d\,\left(5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1\right)}+\frac{6\,a\,b\,\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)^2,x)
```

```
[Out] x*((3*a^2)/8 - 6*a*b + (63*b^2)/8) + (4*(a*b + 5*b^2))/(d*(exp(2*c + 2*d*x)
 + 1)) + (exp(- 2*c - 2*d*x)*(a^2 - 5*a*b + 4*b^2))/(8*d) - (exp(2*c + 2*d*
x)*(5*a*b + a^2 + 4*b^2))/(8*d) + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d) - (4*
(a*b + 5*b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (24*b^2)/(
d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(
- 4*c - 4*d*x)*(a - b)^2)/(64*d) - (16*b^2)/(d*(4*exp(2*c + 2*d*x) + 6*exp(
4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (32*b^2)/(5*d*
(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c
 + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*a*b*log(exp(2*c)*exp(2*d*x) + 1))
/d
```

3.58 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=182

$$\frac{5ab\text{ArcTan}(\sinh(c+dx))}{d}-\frac{a^2\cosh(c+dx)}{d}-\frac{4b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{6b^2\text{sech}$$

```
[Out] 5*a*b*arctan(sinh(d*x+c))/d-a^2*cosh(d*x+c)/d-4*b^2*cosh(d*x+c)/d+1/3*a^2*c
osh(d*x+c)^3/d+1/3*b^2*cosh(d*x+c)^3/d-6*b^2*sech(d*x+c)/d+4/3*b^2*sech(d*x
+c)^3/d-1/5*b^2*sech(d*x+c)^5/d-5*a*b*sinh(d*x+c)/d+5/3*a*b*sinh(d*x+c)^3/d
-a*b*sinh(d*x+c)^3*tanh(d*x+c)^2/d
```

Rubi [A]
time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3747, 2713, 2672, 294, 308, 209, 2670, 276}

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a^2\cosh(c+dx)}{d}+\frac{5ab\text{ArcTan}(\sinh(c+dx))}{d}+\frac{5ab\sinh^3(c+dx)}{3d}-\frac{5ab\sinh(c+dx)}{d}-\frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{4b^2\cosh(c+dx)}{d}-\frac{b^2\text{sech}^5(c+dx)}{5d}+\frac{4b^2\text{sech}^3(c+dx)}{3d}-\frac{6b^2\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (5*a*b*ArcTan[Sinh[c + d*x]])/d - (a^2*Cosh[c + d*x])/d - (4*b^2*Cosh[c + d
*x])/d + (a^2*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^3)/(3*d) - (6*b^2
*Sech[c + d*x])/d + (4*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(
5*d) - (5*a*b*Sinh[c + d*x])/d + (5*a*b*Sinh[c + d*x]^3)/(3*d) - (a*b*Sinh[
c + d*x]^3*Tanh[c + d*x]^2)/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = i\int\left(-ia^2\sinh^3(c+dx)-2iab\sinh^3(c+dx)\tanh^3(c+dx)-i\right.$$

$$= a^2\int \sinh^3(c+dx)\,dx+(2ab)\int \sinh^3(c+dx)\tanh^3(c+dx)\,d$$

$$= -\frac{a^2\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cosh(c+dx)\right)}{d}+\frac{(2ab)\text{Subst}\left(\int \frac{}{(1+}\right.}{}$$

$$= -\frac{a^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}-\frac{ab\sinh^3(c+dx)\tanh^2}{d}$$

$$= -\frac{a^2\cosh(c+dx)}{d}-\frac{4b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}+\frac{b^2}{}$$

$$= -\frac{a^2\cosh(c+dx)}{d}-\frac{4b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}+\frac{b^2}{}$$

$$= \frac{5ab\tan^{-1}(\sinh(c+dx))}{d}-\frac{a^2\cosh(c+dx)}{d}-\frac{4b^2\cosh(c+dx)}{d}$$

Mathematica [A]
time = 0.52, size = 121, normalized size = 0.66

$$\frac{-45(a^2+5b^2)\cosh(c+dx)+5(a^2+b^2)\cosh(3(c+dx))-2b\left(-40b\text{sech}^3(c+dx)+6b\text{sech}^5(c+dx)-5a\left(60\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-27\sinh(c+dx)+\sinh(3(c+dx))\right)+30\text{sech}(c+dx)(6b+a\tanh(c+dx))\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (-45*(a^2 + 5*b^2)*Cosh[c + d*x] + 5*(a^2 + b^2)*Cosh[3*(c + d*x)] - 2*b*(-
40*b*Sech[c + d*x]^3 + 6*b*Sech[c + d*x]^5 - 5*a*(60*ArcTan[Tanh[(c + d*x)/
2]] - 27*Sinh[c + d*x] + Sinh[3*(c + d*x)]) + 30*Sech[c + d*x]*(6*b + a*Tan
h[c + d*x])))/(60*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.78, size = 345, normalized size = 1.90

method	result
risch	$\frac{e^{3dx+3c}a^2}{24d}+\frac{e^{3dx+3c}ab}{12d}+\frac{e^{3dx+3c}b^2}{24d}-\frac{3\,e^{dx+c}a^2}{8d}-\frac{9ab\,e^{dx+c}}{4d}-\frac{15\,e^{dx+c}b^2}{8d}-\frac{3\,e^{-dx-c}a^2}{8d}+\frac{9\,e^{-dx-c}ab}{4d}-\frac{15\,e^{-dx-c}b^2}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*a^2+1/12/d*exp(3*d*x+3*c)*a*b+1/24/d*exp(3*d*x+3*c)*b
^2-3/8/d*exp(d*x+c)*a^2-9/4*a*b/d*exp(d*x+c)-15/8/d*exp(d*x+c)*b^2-3/8/d*ex
```

```
p(-d*x-c)*a^2+9/4/d*exp(-d*x-c)*a*b-15/8/d*exp(-d*x-c)*b^2+1/24/d*exp(-3*d*
x-3*c)*a^2-1/12/d*exp(-3*d*x-3*c)*a*b+1/24/d*exp(-3*d*x-3*c)*b^2-2/15*b*exp
(d*x+c)*(15*a*exp(8*d*x+8*c)+90*b*exp(8*d*x+8*c)+30*a*exp(6*d*x+6*c)+280*b*
exp(6*d*x+6*c)+428*b*exp(4*d*x+4*c)-30*a*exp(2*d*x+2*c)+280*b*exp(2*d*x+2*c
)-15*a+90*b)/d/(1+exp(2*d*x+2*c))^5+5*I*b*a/d*ln(exp(d*x+c)+I)-5*I*b*a/d*ln
(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(172) = 344$.
time = 0.50, size = 348, normalized size = 1.91

$$-\frac{1}{120}b^2\left(\frac{5\left(45e^{(-dx-c)}-e^{(-3dx-3c)}\right)}{d}+\frac{200e^{(-2dx-2c)}+2515e^{(-4dx-4c)}+6680e^{(-6dx-6c)}+9073e^{(-8dx-8c)}+5600e^{(-10dx-10c)}+1665e^{(-12dx-12c)}-5}{d(e^{(-3dx-3c)}+5e^{(-5dx-5c)}+10e^{(-7dx-7c)}+10e^{(-9dx-9c)}+5e^{(-11dx-11c)}+e^{(-13dx-13c)})}\right)+\frac{1}{12}ab\left(\frac{27e^{(-dx-c)}-e^{(-3dx-3c)}}{d}-\frac{120\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25e^{(-2dx-2c)}+77e^{(-4dx-4c)}+3e^{(-6dx-6c)}-1}{d(e^{(-3dx-3c)}+2e^{(-5dx-5c)}+e^{(-7dx-7c)})}\right)+\frac{1}{24}a^2\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/120*b^2*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*c
) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c) +
 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x - 3*c
) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*e^(-
11*d*x - 11*c) + e^(-13*d*x - 13*c)))) + 1/12*a*b*((27*e^(-d*x - c) - e^(-3
*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(
-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x
- 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/
d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3341 vs. $2(172) = 344$.
time = 0.36, size = 3341, normalized size = 18.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^16 + 80*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^15 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^16 - 20*(a
^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^14 + 20*(30*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^2 - a^2 - 11*a*b - 10*b^2)*sinh(d*x + c)^14 + 280*(10*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^3 - (a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c
)^13 - 20*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^12 + 20*(455*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)^4 - 91*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 - 11
*a^2 - 61*a*b - 137*b^2)*sinh(d*x + c)^12 + 80*(273*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^5 - 91*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^3 - 3*(11*a^2 + 61*
a*b + 137*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 - 20*(31*a^2 + 87*a*b + 390*
b^2)*cosh(d*x + c)^10 + 20*(2002*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 1001
```

*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4 - 66*(11*a^2 + 61*a*b + 137*b^2)*c
osh(d*x + c)^2 - 31*a^2 - 87*a*b - 390*b^2)*sinh(d*x + c)^10 + 40*(1430*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 1001*(a^2 + 11*a*b + 10*b^2)*cosh(d*x +
c)^5 - 110*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^3 - 5*(31*a^2 + 87*a*b
 + 390*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 2*(425*a^2 + 5649*b^2)*cosh(d*
x + c)^8 + 2*(32175*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 30030*(a^2 + 11*a
*b + 10*b^2)*cosh(d*x + c)^6 - 4950*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x +
c)^4 - 450*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^2 - 425*a^2 - 5649*b^2
)*sinh(d*x + c)^8 + 16*(3575*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 - 4290*(a^
2 + 11*a*b + 10*b^2)*cosh(d*x + c)^7 - 990*(11*a^2 + 61*a*b + 137*b^2)*cosh
(d*x + c)^5 - 150*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^3 - (425*a^2 +
5649*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(31*a^2 - 87*a*b + 390*b^2)*c
osh(d*x + c)^6 + 4*(10010*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 - 15015*(a^2
 + 11*a*b + 10*b^2)*cosh(d*x + c)^8 - 4620*(11*a^2 + 61*a*b + 137*b^2)*cosh
(d*x + c)^6 - 1050*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^4 - 14*(425*a^
2 + 5649*b^2)*cosh(d*x + c)^2 - 155*a^2 + 435*a*b - 1950*b^2)*sinh(d*x + c)
^6 + 8*(2730*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 - 5005*(a^2 + 11*a*b + 10
*b^2)*cosh(d*x + c)^9 - 1980*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^7 -
630*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^5 - 14*(425*a^2 + 5649*b^2)*c
osh(d*x + c)^3 - 15*(31*a^2 - 87*a*b + 390*b^2)*cosh(d*x + c))*sinh(d*x + c
)^5 - 20*(11*a^2 - 61*a*b + 137*b^2)*cosh(d*x + c)^4 + 20*(455*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^12 - 1001*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^10 -
495*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^8 - 210*(31*a^2 + 87*a*b + 39
0*b^2)*cosh(d*x + c)^6 - 7*(425*a^2 + 5649*b^2)*cosh(d*x + c)^4 - 15*(31*a^
2 - 87*a*b + 390*b^2)*cosh(d*x + c)^2 - 11*a^2 + 61*a*b - 137*b^2)*sinh(d*x
 + c)^4 + 16*(175*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^13 - 455*(a^2 + 11*a*b
+ 10*b^2)*cosh(d*x + c)^11 - 275*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^
9 - 150*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^7 - 7*(425*a^2 + 5649*b^2
)*cosh(d*x + c)^5 - 25*(31*a^2 - 87*a*b + 390*b^2)*cosh(d*x + c)^3 - 5*(11*
a^2 - 61*a*b + 137*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 20*(a^2 - 11*a*b +
 10*b^2)*cosh(d*x + c)^2 + 4*(150*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^14 - 45
5*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^12 - 330*(11*a^2 + 61*a*b + 137*b^2
)*cosh(d*x + c)^10 - 225*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^8 - 14*(
425*a^2 + 5649*b^2)*cosh(d*x + c)^6 - 75*(31*a^2 - 87*a*b + 390*b^2)*cosh(d
x + c)^4 - 30(11*a^2 - 61*a*b + 137*b^2)*cosh(d*x + c)^2 - 5*a^2 + 55*a*b
 - 50*b^2)*sinh(d*x + c)^2 + 5*a^2 - 10*a*b + 5*b^2 + 1200*(a*b*cosh(d*x +
c)^13 + 13*a*b*cosh(d*x + c)*sinh(d*x + c)^12 + a*b*sinh(d*x + c)^13 + 5*a*
b*cosh(d*x + c)^11 + (78*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(d*x + c)^11 + 10
*a*b*cosh(d*x + c)^9 + 11*(26*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*si
nh(d*x + c)^10 + 5*(143*a*b*cosh(d*x + c)^4 + 55*a*b*cosh(d*x + c)^2 + 2*a*
b)*sinh(d*x + c)^9 + 10*a*b*cosh(d*x + c)^7 + 3*(429*a*b*cosh(d*x + c)^5 +
275*a*b*cosh(d*x + c)^3 + 30*a*b*cosh(d*x + c))*sinh(d*x + c)^8 + 2*(858*a*
b*cosh(d*x + c)^6 + 825*a*b*cosh(d*x + c)^4 + 180*a*b*cosh(d*x + c)^2 + 5*a
*b)*sinh(d*x + c)^7 + 5*a*b*cosh(d*x + c)^5 + 2*(858*a*b*cosh(d*x + c)^7 +
1155*a*b*cosh(d*x + c)^5 + 420*a*b*cosh(d*x + c)^3 + 35*a*b*cosh(d*x + c))*

```
sinh(d*x + c)^6 + (1287*a*b*cosh(d*x + c)^8 + 2310*a*b*cosh(d*x + c)^6 + 12
60*a*b*cosh(d*x + c)^4 + 210*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(d*x + c)^5 +
 a*b*cosh(d*x + c)^3 + 5*(143*a*b*cosh(d*x + c)^9 + 330*a*b*cosh(d*x + c)^7
 + 252*a*b*cosh(d*x + c)^5 + 70*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*
sinh(d*x + c)^4 + (286*a*b*cosh(d*x + c)^10 + 825*a*b*cosh(d*x + c)^8 + 840
*a*b*cosh(d*x + c)^6 + 350*a*b*cosh(d*x + c)^4 + 50*a*b*cosh(d*x + c)^2 + a
*b)*sinh(d*x + c)^3 + (78*a*b*cosh(d*x + c)^11 + 275*a*b*cosh(d*x + c)^9 +
360*a*b*cosh(d*x + c)^7 + 210*a*b*cosh(d*x + c)^5 + 50*a*b*cosh(d*x + c)^3
+ 3*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + (13*a*b*cosh(d*x + c)^12 + 55*a*b*
cosh(d*x + c)^10 + 90*a*b*cosh(d*x + c)^8 + 70*a*b*cosh(d*x + c)^6 + 25*a*b
*cosh(d*x + c)^4 + 3*a*b*cosh(d*x + c)^2)*sinh(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^{3}\left(c+dx\right)\right)^{2}\sinh^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**3, x)
```

Giac [A]
time = 0.56, size = 289, normalized size = 1.59

$$\frac{1200\,ab\arctan\left(e^{(dx+c)}\right)+5\,a^{2}e^{(3\,dx+3\,c)}+10\,abe^{(3\,dx+3\,c)}+5\,b^{2}e^{(3\,dx+3\,c)}-45\,a^{2}e^{(dx+c)}-270\,abe^{(dx+c)}-225\,b^{2}e^{(dx+c)}-5\left(9\,a^{2}e^{(2\,dx+2\,c)}-54\,abe^{(2\,dx+2\,c)}+45\,b^{2}e^{(2\,dx+2\,c)}-a^{2}+2\,ab-b^{2}\right)e^{(-3\,dx-3\,c)}-\frac{16\,\left(15\,abe^{(9\,dx+9\,c)}+90\,b^{2}e^{(9\,dx+9\,c)}+30\,abe^{(7\,dx+7\,c)}+280\,b^{2}e^{(7\,dx+7\,c)}+428\,b^{2}e^{(5\,dx+5\,c)}-30\,abe^{(3\,dx+3\,c)}+280\,b^{2}e^{(3\,dx+3\,c)}-15\,abe^{(dx+c)}+90\,b^{2}e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/120*(1200*a*b*arctan(e^(d*x + c)) + 5*a^2*e^(3*d*x + 3*c) + 10*a*b*e^(3*d
*x + 3*c) + 5*b^2*e^(3*d*x + 3*c) - 45*a^2*e^(d*x + c) - 270*a*b*e^(d*x + c
) - 225*b^2*e^(d*x + c) - 5*(9*a^2*e^(2*d*x + 2*c) - 54*a*b*e^(2*d*x + 2*c)
 + 45*b^2*e^(2*d*x + 2*c) - a^2 + 2*a*b - b^2)*e^(-3*d*x - 3*c) - 16*(15*a*
b*e^(9*d*x + 9*c) + 90*b^2*e^(9*d*x + 9*c) + 30*a*b*e^(7*d*x + 7*c) + 280*b
^2*e^(7*d*x + 7*c) + 428*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(3*d*x + 3*c) + 280
*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c) + 90*b^2*e^(d*x + c))/(e^(2*d*x +
 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.44, size = 397, normalized size = 2.18

$$\frac{\mathrm{e}^{3\,c+3\,d\,x}\,{\left(a+b\right)}^2}{24\,d}-\frac{\mathrm{e}^{c+d\,x}\,\left(3\,a^2+18\,a\,b+15\,b^2\right)}{8\,d}-\frac{\mathrm{e}^{-c-d\,x}\,\left(3\,a^2-18\,a\,b+15\,b^2\right)}{8\,d}+\frac{\mathrm{e}^{-3\,c-3\,d\,x}\,{\left(a-b\right)}^2}{24\,d}+\frac{10\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}}-\frac{256\,b^2\,\mathrm{e}^{c+d\,x}}{15\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}+\frac{64\,b^2\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-\frac{32\,b^2\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}\,\left(6\,b^2+a\,b\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{4\,\mathrm{e}^{c+d\,x}\,\left(8\,b^2+3\,a\,b\right)}{3\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)^2,x)
```

```
[Out] (exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(c + d*x)*(18*a*b + 3*a^2 + 15*b^
2))/(8*d) - (exp(- c - d*x)*(3*a^2 - 18*a*b + 15*b^2))/(8*d) + (exp(- 3*c -
 3*d*x)*(a - b)^2)/(24*d) + (10*atan((a*b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(
a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2) - (256*b^2*exp(c + d*x))/(15*
d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (64*b
^2*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c
+ 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2*exp(c + d*x))/(5*d*(5*exp(2*c +
 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) +
exp(10*c + 10*d*x) + 1)) - (2*exp(c + d*x)*(a*b + 6*b^2))/(d*(exp(2*c + 2*d
*x) + 1)) + (4*exp(c + d*x)*(3*a*b + 8*b^2))/(3*d*(2*exp(2*c + 2*d*x) + exp
(4*c + 4*d*x) + 1))
```

3.59 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2 dx$

Optimal. Leaf size=129

$$-\frac{1}{2}\left(a^2+7b^2\right)x-\frac{4ab\log(\cosh(c+dx))}{d}+\frac{3b^2\tanh(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{2b^2\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5}{5}$$

```
[Out] -1/2*(a^2+7*b^2)*x-4*a*b*ln(cosh(d*x+c))/d+3*b^2*tanh(d*x+c)/d+a*b*tanh(d*x
+c)^2/d+2/3*b^2*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d+1/2*cosh(d*x+c)*sin
h(d*x+c)*(a^2+b^2+2*a*b*tanh(d*x+c))/d
```

Rubi [A]
time = 0.14, antiderivative size = 159, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 1818, 1816, 647, 31}

$$\frac{(a^2+7b^2)\tanh(c+dx)}{2d}+\frac{\sinh^2(c+dx)\left((a^2+b^2)\tanh(c+dx)+2ab\right)}{2d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{(a+b)(a+7b)\log(1-\tanh(c+dx))}{4d}-\frac{(a-7b)(a-b)\log(\tanh(c+dx)+1)}{4d}+\frac{b^2\tanh^5(c+dx)}{5d}+\frac{2b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] ((a + b)*(a + 7*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 7*b)*(a - b)*Log[1
 + Tanh[c + d*x]])/(4*d) + ((a^2 + 7*b^2)*Tanh[c + d*x])/(2*d) + (a*b*Tanh[
c + d*x]^2)/d + (2*b^2*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^5)/(5*d)
 + (Sinh[c + d*x]^2*(2*a*b + (a^2 + b^2)*Tanh[c + d*x]))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{x^2(a+bx^3)^2}{(1-x^2)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\sinh^2(c+dx)\left(2ab+(a^2+b^2)\tanh(c+dx)\right)}{2d}+\frac{\text{Subst}\left(\int\frac{x(-4ab}{}\right.}{}$$

$$=\frac{\sinh^2(c+dx)\left(2ab+(a^2+b^2)\tanh(c+dx)\right)}{2d}+\frac{\text{Subst}\left(\int\left(a^2+\right.\right.}{}$$

$$=\frac{(a^2+7b^2)\tanh(c+dx)}{2d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{2b^2\tanh^3(c+dx}{3d}$$

$$=\frac{(a^2+7b^2)\tanh(c+dx)}{2d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{2b^2\tanh^3(c+dx}{3d}$$

$$=\frac{(a+b)(a+7b)\log(1-\tanh(c+dx))}{4d}-\frac{(a-7b)(a-b)\log(1}{4d}$$

Mathematica [A]
time = 1.14, size = 137, normalized size = 1.06

$$\frac{-30a^2c-210b^2c-30a^2dx-210b^2dx+30ab\cosh(2(c+dx))-240ab\log(\cosh(c+dx))+15a^2\sinh(2(c+dx))+15b^2\sinh(2(c+dx))+232b^2\tanh(c+dx)+12b^2\operatorname{sech}^4(c+dx)\tanh(c+dx)-4b\operatorname{sech}^2(c+dx)(15a+16b\tanh(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

[Out] (-30*a^2*c - 210*b^2*c - 30*a^2*d*x - 210*b^2*d*x + 30*a*b*Cosh[2*(c + d*x)
] - 240*a*b*Log[Cosh[c + d*x]] + 15*a^2*Sinh[2*(c + d*x)] + 15*b^2*Sinh[2*(
c + d*x)] + 232*b^2*Tanh[c + d*x] + 12*b^2*Sech[c + d*x]^4*Tanh[c + d*x] -
4*b*Sech[c + d*x]^2*(15*a + 16*b*Tanh[c + d*x]))/(60*d)

Maple [A]
time = 1.63, size = 130, normalized size = 1.01

method	result
derivativedivides	$\frac{a^2\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh^4(dx+c)}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh^2(dx+c)\right)+b^2\left(\frac{\sinh^7(dx+c)}{2\cosh(dx+c)^5}-\frac{7dx}{2}-\frac{7c}{2}\right.}{d}$
default	$\frac{a^2\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh^4(dx+c)}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh^2(dx+c)\right)+b^2\left(\frac{\sinh^7(dx+c)}{2\cosh(dx+c)^5}-\frac{7dx}{2}-\frac{7c}{2}\right.}{d}$
risch	$-\frac{a^2x}{2}+4abx-\frac{7b^2x}{2}+\frac{e^{2dx+2c}a^2}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}+\frac{e^{-2dx-2c}ab}{4d}-\frac{e^{-2dx-2c}b}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(1/2*sinh(d*x+c)*cosh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(1/2*sinh(d*x+c)
^4/cosh(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2)+b^2*(1/2*sinh(d*x+c)^7/co
sh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/10*tanh(d*x+c
)^5))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(121) = 242$.
time = 0.49, size = 301, normalized size = 2.33

$$-\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{120}b^2\left(\frac{420(dx+c)}{d}+\frac{15e^{(-2dx-2c)}}{d}-\frac{1003e^{(-2dx-2c)}+3350e^{(-4dx-4c)}+5590e^{(-6dx-6c)}+3915e^{(-8dx-8c)}+1455e^{(-10dx-10c)}+15}{d\left(e^{(-2dx-2c)}+5e^{(-4dx-4c)}+10e^{(-6dx-6c)}+10e^{(-8dx-8c)}+5e^{(-10dx-10c)}+e^{(-12dx-12c)}\right)}\right)-\frac{1}{4}ab\left(\frac{16(dx+c)}{d}-\frac{e^{(-2dx-2c)}}{d}+\frac{16\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{2e^{(-2dx-2c)}-15e^{(-4dx-4c)}+1}{d\left(e^{(-2dx-2c)}+2e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")

[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/120*b^2*(420*(d
*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d*x
 - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*x -
 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))) - 1/
4*a*b*(16*(d*x + c)/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d
 - (2*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*
e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3649 vs. $2(121) = 242$.
time = 0.36, size = 3649, normalized size = 28.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")

[Out] 1/120*(15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^14 + 210*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^13 + 15*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^14 - 15*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^12 - 15*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 91*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - 5*a^2 - 10*a*b - 5*b^2)*sinh(d*x + c)^12 + 60*(91*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 3*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 - 15*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c)^10 + 15*(1001*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 20*(a^2 - 8*a*b + 7*b^2)*d*x - 66*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^2 + 9*a^2 - 10*a*b - 87*b^2)*sinh(d*x + c)^10 + 30*(1001*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 110*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^3 - 5*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 15*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^8 + 15*(3003*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 495*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^4 - 40*(a^2 - 8*a*b + 7*b^2)*d*x - 45*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c)^2 + 5*a^2 - 66*a*b - 251*b^2)*sinh(d*x + c)^8 + 120*(429*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 99*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^5 - 15*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c)^3 - (40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 5*(120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 1103*b^2)*cosh(d*x + c)^6 + 5*(9009*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 2772*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^6 - 630*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c)^4 - 120*(a^2 - 8*a*b + 7*b^2)*d*x - 84*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^2 - 15*a^2 - 198*a*b - 1103*b^2)*sinh(d*x + c)^6 + 30*(1001*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 - 396*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^7 - 126*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c)^5 - 28*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^3 - (120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 1103*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 5*(60*(a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b + 667*b^2)*cosh(d*x + c)^4 + 5*(3003*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 - 1485*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^8 - 630*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c)^6 - 210*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^4 - 60*(a^2 - 8*a*b + 7*b^2)*d*x - 15*(120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 1103*b^2)*cosh(d*x + c)^2 - 27*a^2 - 30*a*b - 667*b^2)*sinh(d*x + c)^4 + 20*(273*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 - 165*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*

```
x + c)^9 - 90*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh
(d*x + c)^7 - 42*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*
cosh(d*x + c)^5 - 5*(120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 110
3*b^2)*cosh(d*x + c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b +
667*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 7
5*a^2 - 150*a*b + 1003*b^2)*cosh(d*x + c)^2 + (1365*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^12 - 990*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*
cosh(d*x + c)^10 - 675*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*
b^2)*cosh(d*x + c)^8 - 420*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b +
 251*b^2)*cosh(d*x + c)^6 - 75*(120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 19
8*a*b + 1103*b^2)*cosh(d*x + c)^4 - 60*(a^2 - 8*a*b + 7*b^2)*d*x - 30*(60*(
a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b + 667*b^2)*cosh(d*x + c)^2 - 75*
a^2 + 150*a*b - 1003*b^2)*sinh(d*x + c)^2 - 15*a^2 + 30*a*b - 15*b^2 - 480*
(a*b*cosh(d*x + c)^12 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^11 + a*b*sinh(d*
x + c)^12 + 5*a*b*cosh(d*x + c)^10 + (66*a*b*cosh(d*x + c)^2 + 5*a*b)*sinh(
d*x + c)^10 + 10*a*b*cosh(d*x + c)^8 + 10*(22*a*b*cosh(d*x + c)^3 + 5*a*b*c
osh(d*x + c))*sinh(d*x + c)^9 + 5*(99*a*b*cosh(d*x + c)^4 + 45*a*b*cosh(d*x
 + c)^2 + 2*a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 8*(99*a*b*cosh(
d*x + c)^5 + 75*a*b*cosh(d*x + c)^3 + 10*a*b*cosh(d*x + c))*sinh(d*x + c)^7
 + 2*(462*a*b*cosh(d*x + c)^6 + 525*a*b*cosh(d*x + c)^4 + 140*a*b*cosh(d*x
+ c)^2 + 5*a*b)*sinh(d*x + c)^6 + 5*a*b*cosh(d*x + c)^4 + 4*(198*a*b*cosh(d
*x + c)^7 + 315*a*b*cosh(d*x + c)^5 + 140*a*b*cosh(d*x + c)^3 + 15*a*b*cosh
(d*x + c))*sinh(d*x + c)^5 + 5*(99*a*b*cosh(d*x + c)^8 + 210*a*b*cosh(d*x +
 c)^6 + 140*a*b*cosh(d*x + c)^4 + 30*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x +
c)^4 + a*b*cosh(d*x + c)^2 + 20*(11*a*b*cosh(d*x + c)^9 + 30*a*b*cosh(d*x +
 c)^7 + 28*a*b*cosh(d*x + c)^5 + 10*a*b*cosh(d*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh^{3}{\left(c + d x \right)}\right)^{2} \sinh^{2}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(121) = 242.
time = 0.55, size = 296, normalized size = 2.29

$$\frac{15\,a^{2}e^{\left(2\,dx+2\,c\right)}+30\,abe^{\left(2\,dx+2\,c\right)}+15\,b^{2}e^{\left(2\,dx+2\,c\right)}-480\,ab\log\left(e^{\left(2\,dx+2\,c\right)}+1\right)-60\,\left(a^{2}-8\,ab+7\,b^{2}\right)\left(dx+c\right)+15\,\left(2\,a^{2}e^{\left(2\,dx+2\,c\right)}-16\,abe^{\left(2\,dx+2\,c\right)}+14\,b^{2}e^{\left(2\,dx+2\,c\right)}-a^{2}+2\,ab-b^{2}\right)e^{\left(-2\,dx-2\,c\right)}+\frac{8\,\left(137\,abe^{\left(10\,dx+10\,c\right)}+625\,abe^{\left(8\,dx+8\,c\right)}-180\,b^{2}e^{\left(8\,dx+8\,c\right)}+1190\,abe^{\left(6\,dx+6\,c\right)}-480\,b^{2}e^{\left(6\,dx+6\,c\right)}+1190\,abe^{\left(4\,dx+4\,c\right)}-680\,b^{2}e^{\left(4\,dx+4\,c\right)}+625\,abe^{\left(2\,dx+2\,c\right)}-400\,b^{2}e^{\left(2\,dx+2\,c\right)}+137\,ab-116\,b^{2}\right)}{{\left(e^{\left(2\,dx+2\,c\right)}+1\right)}^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/120*(15*a^2*e^(2*d*x + 2*c) + 30*a*b*e^(2*d*x + 2*c) + 15*b^2*e^(2*d*x +
2*c) - 480*a*b*log(e^(2*d*x + 2*c) + 1) - 60*(a^2 - 8*a*b + 7*b^2)*(d*x + c
) + 15*(2*a^2*e^(2*d*x + 2*c) - 16*a*b*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x +
2*c) - a^2 + 2*a*b - b^2)*e^(-2*d*x - 2*c) + 8*(137*a*b*e^(10*d*x + 10*c) +
 625*a*b*e^(8*d*x + 8*c) - 180*b^2*e^(8*d*x + 8*c) + 1190*a*b*e^(6*d*x + 6*
c) - 480*b^2*e^(6*d*x + 6*c) + 1190*a*b*e^(4*d*x + 4*c) - 680*b^2*e^(4*d*x
+ 4*c) + 625*a*b*e^(2*d*x + 2*c) - 400*b^2*e^(2*d*x + 2*c) + 137*a*b - 116*
b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.36, size = 306, normalized size = 2.37

$$\frac{e^{2c+2dx}\,(a+b)^2}{8d}-\frac{4\,(3b^2+a\,b)}{d\,(e^{2c+2dx}+1)}-x\left(\frac{a^2}{2}-4\,a\,b+\frac{7\,b^2}{2}\right)+\frac{4\,(4b^2+a\,b)}{d\,(2\,e^{2c+2dx}+e^{4c+4dx}+1)}-\frac{64\,b^2}{3\,d\,(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1)}-\frac{e^{-2c-2dx}\,(a-b)^2}{8d}+\frac{16\,b^2}{d\,(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1)}-\frac{32\,b^2}{5\,d\,(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1)}-\frac{4\,a\,b\,\ln\left(e^{2c}\,e^{2dx}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)^2,x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - (4*(a*b + 3*b^2))/(d*(exp(2*c + 2*d*x)
 + 1)) - x*(a^2/2 - 4*a*b + (7*b^2)/2) + (4*(a*b + 4*b^2))/(d*(2*exp(2*c +
2*d*x) + exp(4*c + 4*d*x) + 1)) - (64*b^2)/(3*d*(3*exp(2*c + 2*d*x) + 3*exp
(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a - b)^2)/(8*
d) + (16*b^2)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d
*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4
*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x)
 + 1)) - (4*a*b*log(exp(2*c)*exp(2*d*x) + 1))/d
```

3.60 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=123

$$-\frac{3ab\text{ArcTan}(\sinh(c+dx))}{d}+\frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{3b^2\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^3(c+dx)}{d}+\frac{b^2\text{sech}$$

```
[Out] -3*a*b*arctan(sinh(d*x+c))/d+a^2*cosh(d*x+c)/d+b^2*cosh(d*x+c)/d+3*b^2*sech
(d*x+c)/d-b^2*sech(d*x+c)^3/d+1/5*b^2*sech(d*x+c)^5/d+3*a*b*sinh(d*x+c)/d-a
*b*sinh(d*x+c)*tanh(d*x+c)^2/d
```

Rubi [A]
time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3747, 2718, 2672, 294, 327, 209, 2670, 276}

$$\frac{a^2\cosh(c+dx)}{d}-\frac{3ab\text{ArcTan}(\sinh(c+dx))}{d}+\frac{3ab\sinh(c+dx)}{d}-\frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\text{sech}^5(c+dx)}{5d}-\frac{b^2\text{sech}^3(c+dx)}{d}+\frac{3b^2\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (-3*a*b*ArcTan[Sinh[c + d*x]])/d + (a^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*
x])/d + (3*b^2*Sech[c + d*x])/d - (b^2*Sech[c + d*x]^3)/d + (b^2*Sech[c + d
*x]^5)/(5*d) + (3*a*b*Sinh[c + d*x])/d - (a*b*Sinh[c + d*x]*Tanh[c + d*x]^2
)/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\left(i\int \left(ia^2\sinh(c+dx)+2iab\sinh(c+dx)\tanh^3(c+dx)+\right.\right.$$
$$= a^2\int \sinh(c+dx)\,dx + (2ab)\int \sinh(c+dx)\tanh^3(c+dx)\,dx$$
$$= \frac{a^2\cosh(c+dx)}{d} + \frac{(2ab)\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d} -$$
$$= \frac{a^2\cosh(c+dx)}{d} - \frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d} + \frac{(3ab)\text{Subs}}{}$$
$$= \frac{a^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d} + \frac{3b^2\text{sech}(c+dx)}{d} - \frac{b^2\text{sech}}{}$$
$$= -\frac{3ab\tan^{-1}(\sinh(c+dx))}{d} + \frac{a^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx}{d}$$

Mathematica [A]
time = 0.31, size = 90, normalized size = 0.73

$$\frac{5(a^2+b^2)\cosh(c+dx)+b\left(-5b\text{sech}^3(c+dx)+b\text{sech}^5(c+dx)+10a\left(-3\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\sinh(c+dx)\right)+5\text{sech}(c+dx)(3b+a\tanh(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (5*(a^2 + b^2)*Cosh[c + d*x] + b*(-5*b*Sech[c + d*x]^3 + b*Sech[c + d*x]^5
+ 10*a*(-3*ArcTan[Tanh[(c + d*x)/2]] + Sinh[c + d*x]) + 5*Sech[c + d*x]*(3*
b + a*Tanh[c + d*x])))/(5*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.82, size = 244, normalized size = 1.98

method	result
risch	$\frac{e^{dx+c}a^2}{2d}+\frac{ab\,e^{dx+c}}{d}+\frac{e^{dx+c}b^2}{2d}+\frac{e^{-dx-c}a^2}{2d}-\frac{e^{-dx-c}ab}{d}+\frac{e^{-dx-c}b^2}{2d}+\frac{2b\,e^{dx+c}\left(5a\,e^{8dx+8c}+15b\,e^{8dx+8c}+10a\,e^{6dx+6c}+\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*exp(d*x+c)*a^2+a*b/d*exp(d*x+c)+1/2/d*exp(d*x+c)*b^2+1/2/d*exp(-d*x-c
)*a^2-1/d*exp(-d*x-c)*a*b+1/2/d*exp(-d*x-c)*b^2+2/5*b*exp(d*x+c)*(5*a*exp(8
*d*x+8*c)+15*b*exp(8*d*x+8*c)+10*a*exp(6*d*x+6*c)+40*b*exp(6*d*x+6*c)+66*b*
exp(4*d*x+4*c)-10*a*exp(2*d*x+2*c)+40*b*exp(2*d*x+2*c)-5*a+15*b)/d/(1+exp(2
*d*x+2*c))^5+3*I*b*a/d*ln(exp(d*x+c)-I)-3*I*b*a/d*ln(exp(d*x+c)+I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(121) = 242$.
time = 0.48, size = 253, normalized size = 2.06

$$ab\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right)+\frac{1}{10}\,b^2\left(\frac{5\,e^{(-dx-c)}}{d}+\frac{85\,e^{(-2\,dx-2\,c)}+210\,e^{(-4\,dx-4\,c)}+314\,e^{(-6\,dx-6\,c)}+185\,e^{(-8\,dx-8\,c)}+65\,e^{(-10\,dx-10\,c)}+5}{d(e^{(-dx-c)}+5\,e^{(-3\,dx-3\,c)}+10\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+5\,e^{(-9\,dx-9\,c)}+e^{(-11\,dx-11\,c)})}\right)+\frac{a^2\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] a*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(-
4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c))
)) + 1/10*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x - 4*
c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c) +
5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^(-7*d
*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + a^2*cosh(d*x + c)/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2230 vs. $2(121) = 242$.
time = 0.37, size = 2230, normalized size = 18.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 60*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^11 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 30*(a^
2 + 2*a*b + 3*b^2)*cosh(d*x + c)^10 + 30*(11*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^2 + a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^10 + 100*(11*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 +
 5*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^8 + 5*(495*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 270*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a^2 + 18*a
*b + 47*b^2)*sinh(d*x + c)^8 + 40*(99*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 +
 90*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (15*a^2 + 18*a*b + 47*b^2)*cosh
(d*x + c))*sinh(d*x + c)^7 + 4*(25*a^2 + 91*b^2)*cosh(d*x + c)^6 + 4*(1155*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 1575*(a^2 + 2*a*b + 3*b^2)*cosh(d*x +
 c)^4 + 35*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^2 + 25*a^2 + 91*b^2)*si
nh(d*x + c)^6 + 8*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 945*(a^2 + 2*a
*b + 3*b^2)*cosh(d*x + c)^5 + 35*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^3
 + 3*(25*a^2 + 91*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*(15*a^2 - 18*a*b
+ 47*b^2)*cosh(d*x + c)^4 + 5*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 12
60*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^6 + 70*(15*a^2 + 18*a*b + 47*b^2)*co
sh(d*x + c)^4 + 12*(25*a^2 + 91*b^2)*cosh(d*x + c)^2 + 15*a^2 - 18*a*b + 47
*b^2)*sinh(d*x + c)^4 + 20*(55*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 180*(a
```

```
^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^7 + 14*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*
x + c)^5 + 4*(25*a^2 + 91*b^2)*cosh(d*x + c)^3 + (15*a^2 - 18*a*b + 47*b^2)
*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ 10*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 135*(a^2 + 2*a*b + 3*b^2)*c
osh(d*x + c)^8 + 14*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^6 + 6*(25*a^2
+ 91*b^2)*cosh(d*x + c)^4 + 3*(15*a^2 - 18*a*b + 47*b^2)*cosh(d*x + c)^2 +
3*a^2 - 6*a*b + 9*b^2)*sinh(d*x + c)^2 + 5*a^2 - 10*a*b + 5*b^2 - 60*(a*b*c
osh(d*x + c)^11 + 11*a*b*cosh(d*x + c)*sinh(d*x + c)^10 + a*b*sinh(d*x + c)
^11 + 5*a*b*cosh(d*x + c)^9 + 5*(11*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c
)^9 + 10*a*b*cosh(d*x + c)^7 + 15*(11*a*b*cosh(d*x + c)^3 + 3*a*b*cosh(d*x
+ c))*sinh(d*x + c)^8 + 10*(33*a*b*cosh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2
 + a*b)*sinh(d*x + c)^7 + 10*a*b*cosh(d*x + c)^5 + 14*(33*a*b*cosh(d*x + c)
^5 + 30*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(231
*a*b*cosh(d*x + c)^6 + 315*a*b*cosh(d*x + c)^4 + 105*a*b*cosh(d*x + c)^2 +
5*a*b)*sinh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + 10*(33*a*b*cosh(d*x + c)^7
 + 63*a*b*cosh(d*x + c)^5 + 35*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*s
inh(d*x + c)^4 + 5*(33*a*b*cosh(d*x + c)^8 + 84*a*b*cosh(d*x + c)^6 + 70*a*
b*cosh(d*x + c)^4 + 20*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^3 + a*b*cos
h(d*x + c) + 5*(11*a*b*cosh(d*x + c)^9 + 36*a*b*cosh(d*x + c)^7 + 42*a*b*co
sh(d*x + c)^5 + 20*a*b*cosh(d*x + c)^3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c)
^2 + (11*a*b*cosh(d*x + c)^10 + 45*a*b*cosh(d*x + c)^8 + 70*a*b*cosh(d*x +
c)^6 + 50*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)
)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 4*(15*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^11 + 75*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^9 + 10*(15*a^2 + 18*a*b
+ 47*b^2)*cosh(d*x + c)^7 + 6*(25*a^2 + 91*b^2)*cosh(d*x + c)^5 + 5*(15*a^2
 - 18*a*b + 47*b^2)*cosh(d*x + c)^3 + 15*(a^2 - 2*a*b + 3*b^2)*cosh(d*x + c
))*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10
 + d*sinh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 5*(11*d*cosh(d*x + c)^2 + d)*
sinh(d*x + c)^9 + 15*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
c)^8 + 10*d*cosh(d*x + c)^7 + 10*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^7 + 14*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 +
 5*d*cosh(d*x + c))*sinh(d*x + c)^6 + 10*d*cosh(d*x + c)^5 + 2*(231*d*cosh(
d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x
+ c)^5 + 10*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x +
c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(33*d*c
osh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^3 + 5*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c
)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^2 + d*cosh(d*x + c) + (11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)
^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^3(c+dx)\right)^2 \sinh(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x), x)
```

Giac [A]
time = 0.50, size = 203, normalized size = 1.65

$$-\frac{60\,ab\arctan\left(e^{(dx+c)}\right)-5\,a^2e^{(dx+c)}-10\,abe^{(dx+c)}-5\,b^2e^{(dx+c)}-5\,(a^2-2\,ab+b^2)e^{(-dx-c)}-\frac{4\,(5\,abe^{(9\,dx+9\,c)}+15\,b^2e^{(9\,dx+9\,c)}+10\,abe^{(7\,dx+7\,c)}+40\,b^2e^{(7\,dx+7\,c)}+66\,b^2e^{(5\,dx+5\,c)}-10\,abe^{(3\,dx+3\,c)}+40\,b^2e^{(3\,dx+3\,c)}-5\,abe^{(dx+c)}+15\,b^2e^{(dx+c)})}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/10*(60*a*b*arctan(e^(d*x + c)) - 5*a^2*e^(d*x + c) - 10*a*b*e^(d*x + c)
- 5*b^2*e^(d*x + c) - 5*(a^2 - 2*a*b + b^2)*e^(-d*x - c) - 4*(5*a*b*e^(9*d*
x + 9*c) + 15*b^2*e^(9*d*x + 9*c) + 10*a*b*e^(7*d*x + 7*c) + 40*b^2*e^(7*d*
x + 7*c) + 66*b^2*e^(5*d*x + 5*c) - 10*a*b*e^(3*d*x + 3*c) + 40*b^2*e^(3*d*
x + 3*c) - 5*a*b*e^(d*x + c) + 15*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)
/d
```

Mupad [B]
time = 1.29, size = 338, normalized size = 2.75

$$\frac{\mathrm{e}^{c+d\,x}\,{\left(a+b\right)}^2}{2\,d}+\frac{\mathrm{e}^{-c-d\,x}\,{\left(a-b\right)}^2}{2\,d}-\frac{6\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}}+\frac{72\,b^2\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}-\frac{64\,b^2\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{32\,b^2\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}+\frac{2\,\mathrm{e}^{c+d\,x}\,\left(3\,b^2+a\,b\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{4\,\mathrm{e}^{c+d\,x}\,\left(2\,b^2+a\,b\right)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)^2,x)
```

```
[Out] (exp(c + d*x)*(a + b)^2)/(2*d) + (exp(- c - d*x)*(a - b)^2)/(2*d) - (6*atan
((a*b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d
^2)^(1/2) + (72*b^2*exp(c + d*x))/(5*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) + 1)) - (64*b^2*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d
*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (3
2*b^2*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp
(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (2*exp(c +
d*x)*(a*b + 3*b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (4*exp(c + d*x)*(a*b + 2*b
^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.61 $\int \mathrm{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=98

$$\frac{ab\mathrm{ArcTan}(\sinh(c+dx))}{d}-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\mathrm{sech}(c+dx)}{d}+\frac{2b^2\mathrm{sech}^3(c+dx)}{3d}-\frac{b^2\mathrm{sech}^5(c+dx)}{5d}-$$

```
[Out] a*b*arctan(sinh(d*x+c))/d-a^2*arctanh(cosh(d*x+c))/d-b^2*sech(d*x+c)/d+2/3*
b^2*sech(d*x+c)^3/d-1/5*b^2*sech(d*x+c)^5/d-a*b*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3747, 3855, 2691, 2686, 200}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{ab\mathrm{ArcTan}(\sinh(c+dx))}{d}-\frac{ab\tanh(c+dx)\mathrm{sech}(c+dx)}{d}-\frac{b^2\mathrm{sech}^5(c+dx)}{5d}+\frac{2b^2\mathrm{sech}^3(c+dx)}{3d}-\frac{b^2\mathrm{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (a*b*ArcTan[Sinh[c + d*x]])/d - (a^2*ArcTanh[Cosh[c + d*x]])/d - (b^2*Sech[
c + d*x])/d + (2*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(5*d) -
 (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2 dx = i\int \left(-ia^2\operatorname{csch}(c+dx) - 2iab\operatorname{sech}(c+dx)\tanh^2(c+dx) - ib^2\operatorname{se}\right.$$

$$= a^2\int \operatorname{csch}(c+dx)\,dx + (2ab)\int \operatorname{sech}(c+dx)\tanh^2(c+dx)\,dx +$$

$$= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d} - \frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d} + (ab$$

$$= \frac{ab\tan^{-1}(\sinh(c+dx))}{d} - \frac{a^2\tanh^{-1}(\cosh(c+dx))}{d} - \frac{ab\operatorname{sech}(c}{}$$

$$= \frac{ab\tan^{-1}(\sinh(c+dx))}{d} - \frac{a^2\tanh^{-1}(\cosh(c+dx))}{d} - \frac{b^2\operatorname{sech}(c+}{d}$$

Mathematica [A]
time = 0.12, size = 106, normalized size = 1.08

$$\frac{2ab\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{b^2\operatorname{sech}(c+dx)}{d} + \frac{2b^2\operatorname{sech}^3(c+dx)}{3d} - \frac{b^2\operatorname{sech}^5(c+dx)}{5d} - \frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d + (a^2*Log[Tanh[(c + d*x)/2]])/d - (b^2
*Sech[c + d*x])/d + (2*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(
5*d) - (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Maple [C] Result contains complex when optimal does not.
time = 3.30, size = 187, normalized size = 1.91

method	result
risch	$-\frac{2b\,e^{dx+c}\left(15a\,e^{8dx+8c}+15b\,e^{8dx+8c}+30a\,e^{6dx+6c}+20b\,e^{6dx+6c}+58b\,e^{4dx+4c}-30a\,e^{2dx+2c}+20b\,e^{2dx+2c}-15a+15b\right)}{15d\left(1+e^{2dx+2c}\right)^5} + \frac{a^2\ln\left(e^{dx+c}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*b*exp(d*x+c)*(15*a*exp(8*d*x+8*c)+15*b*exp(8*d*x+8*c)+30*a*exp(6*d*x+
6*c)+20*b*exp(6*d*x+6*c)+58*b*exp(4*d*x+4*c)-30*a*exp(2*d*x+2*c)+20*b*exp(2
*d*x+2*c)-15*a+15*b)/d/(1+exp(2*d*x+2*c))^5+a^2/d*ln(exp(d*x+c)-1)-a^2/d*ln
(exp(d*x+c)+1)+I*b*a/d*ln(exp(d*x+c)+I)-I*b*a/d*ln(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(94) = 188$.
time = 0.47, size = 447, normalized size = 4.56

$$-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)-\frac{2}{15}\,b^2\left(\frac{15\,e^{(-dx-c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{20\,e^{(-3\,dx-3\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{58\,e^{(-5\,dx-5\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{20\,e^{(-7\,dx-7\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{15\,e^{(-9\,dx-9\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^
(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 2/15*b^2*(15*e^(-d*x - c)/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
- 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2
*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) + 1)) + 58*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-
4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c
) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*
e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^2*log(tanh(1/
2*d*x + 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(94) = 188$.
time = 0.38, size = 2498, normalized size = 25.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(30*(a*b + b^2)*cosh(d*x + c)^9 + 270*(a*b + b^2)*cosh(d*x + c)*sinh(
d*x + c)^8 + 30*(a*b + b^2)*sinh(d*x + c)^9 + 20*(3*a*b + 2*b^2)*cosh(d*x +
 c)^7 + 20*(54*(a*b + b^2)*cosh(d*x + c)^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^7
 + 116*b^2*cosh(d*x + c)^5 + 140*(18*(a*b + b^2)*cosh(d*x + c)^3 + (3*a*b +
```

2*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(945*(a*b + b^2)*cosh(d*x + c)^4 + 105*(3*a*b + 2*b^2)*cosh(d*x + c)^2 + 29*b^2)*sinh(d*x + c)^5 + 20*(189*(a*b + b^2)*cosh(d*x + c)^5 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^3 + 29*b^2*cosh(d*x + c))*sinh(d*x + c)^4 - 20*(3*a*b - 2*b^2)*cosh(d*x + c)^3 + 20*(126*(a*b + b^2)*cosh(d*x + c)^6 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^4 + 58*b^2*cosh(d*x + c)^2 - 3*a*b + 2*b^2)*sinh(d*x + c)^3 + 20*(54*(a*b + b^2)*cosh(d*x + c)^7 + 21*(3*a*b + 2*b^2)*cosh(d*x + c)^5 + 58*b^2*cosh(d*x + c)^3 - 3*(3*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 30*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x + c)^9 + a*b*sinh(d*x + c)^10 + 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 + 14*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^6 + 10*a*b*cosh(d*x + c)^4 + 4*(63*a*b*cosh(d*x + c)^5 + 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 + 35*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b*cosh(d*x + c)^7 + 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 + 28*a*b*cosh(d*x + c)^6 + 30*a*b*cosh(d*x + c)^4 + 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^2 + a*b + 10*(a*b*cosh(d*x + c)^9 + 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x + c)^5 + 4*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 30*(a*b - b^2)*cosh(d*x + c) + 15*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*sinh(d*x + c)^10 + 5*a^2*cosh(d*x + c)^8 + 5*(9*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 10*a^2*cosh(d*x + c)^6 + 40*(3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a^2*cosh(d*x + c)^4 + 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^6 + 10*a^2*cosh(d*x + c)^4 + 4*(63*a^2*cosh(d*x + c)^5 + 70*a^2*cosh(d*x + c)^3 + 15*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a^2*cosh(d*x + c)^6 + 35*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 5*a^2*cosh(d*x + c)^2 + 40*(3*a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a^2*cosh(d*x + c)^8 + 28*a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 + 12*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 10*(a^2*cosh(d*x + c)^9 + 4*a^2*cosh(d*x + c)^7 + 6*a^2*cosh(d*x + c)^5 + 4*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 15*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*sinh(d*x + c)^10 + 5*a^2*cosh(d*x + c)^8 + 5*(9*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 10*a^2*cosh(d*x + c)^6 + 40*(3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a^2*cosh(d*x + c)^4 + 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^6 + 10*a^2*cosh(d*x + c)^4 + 4*(63*a^2*cosh(d*x + c)^5 + 70*a^2*cosh(d*x + c)^3 + 15*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a^2*cosh(d*x + c)^6 + 35*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 5*a^2*cosh(d*x + c)^2 + 40*(3*a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a^2*cosh(d*x + c)^8 + 28*a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 + 12*a^2*cosh(d*x + c)^2

```
 + a^2)*sinh(d*x + c)^2 + a^2 + 10*(a^2*cosh(d*x + c)^9 + 4*a^2*cosh(d*x +
c)^7 + 6*a^2*cosh(d*x + c)^5 + 4*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*s
inh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(27*(a*b + b^2)*c
osh(d*x + c)^8 + 14*(3*a*b + 2*b^2)*cosh(d*x + c)^6 + 58*b^2*cosh(d*x + c)^
4 - 6*(3*a*b - 2*b^2)*cosh(d*x + c)^2 - 3*a*b + 3*b^2)*sinh(d*x + c))/(d*co
sh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 +
5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d
*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6
+ 10*(21*d*cosh(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*
(63*d*cosh(d*x + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x
 + c)^5 + 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x +
 c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7
 + 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6
+ 30*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*
cosh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^3\left(c+dx\right)\right)^2\operatorname{csch}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x), x)
```

Giac [A]
time = 0.50, size = 178, normalized size = 1.82

$$\frac{30\,ab\arctan\left(e^{(dx+c)}\right)-15\,a^2\log\left(e^{(dx+c)}+1\right)+15\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{2\,\left(15\,abe^{(9\,dx+9\,c)}+15\,b^2e^{(9\,dx+9\,c)}+30\,abe^{(7\,dx+7\,c)}+20\,b^2e^{(7\,dx+7\,c)}+58\,b^2e^{(5\,dx+5\,c)}-30\,abe^{(3\,dx+3\,c)}+20\,b^2e^{(3\,dx+3\,c)}-15\,abe^{(dx+c)}+15\,b^2e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/15*(30*a*b*arctan(e^(d*x + c)) - 15*a^2*log(e^(d*x + c) + 1) + 15*a^2*log
(abs(e^(d*x + c) - 1)) - 2*(15*a*b*e^(9*d*x + 9*c) + 15*b^2*e^(9*d*x + 9*c)
 + 30*a*b*e^(7*d*x + 7*c) + 20*b^2*e^(7*d*x + 7*c) + 58*b^2*e^(5*d*x + 5*c)
 - 30*a*b*e^(3*d*x + 3*c) + 20*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c) + 1
5*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 3.28, size = 522, normalized size = 5.33

$$\frac{a^2\,\ln\left(32\,a^6+32\,a^4\,b^2-32\,a^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-32\,a^4\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{d}-\frac{176\,b^2\,\mathrm{e}^{c+d\,x}}{15\,\left(d+3\,d\,\mathrm{e}^{2\,c+2\,d\,x}+3\,d\,\mathrm{e}^{4\,c+4\,d\,x}+d\,\mathrm{e}^{6\,c+6\,d\,x}\right)}-\frac{32\,b^2\,\mathrm{e}^{c+d\,x}}{5\,\left(d+5\,d\,\mathrm{e}^{2\,c+2\,d\,x}+10\,d\,\mathrm{e}^{4\,c+4\,d\,x}+10\,d\,\mathrm{e}^{6\,c+6\,d\,x}+5\,d\,\mathrm{e}^{8\,c+8\,d\,x}+d\,\mathrm{e}^{10\,c+10\,d\,x}\right)}-\frac{a^2\,\ln\left(-32\,a^6-32\,a^4\,b^2-32\,a^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-32\,a^4\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{d}-\frac{2\,b^2\,\mathrm{e}^{c+d\,x}}{d+d\,\mathrm{e}^{2\,c+2\,d\,x}}+\frac{16\,b^2\,\mathrm{e}^{c+d\,x}}{3\,\left(d+2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}\right)}+\frac{64\,b^2\,\mathrm{e}^{c+d\,x}}{5\,\left(d+4\,d\,\mathrm{e}^{2\,c+2\,d\,x}+6\,d\,\mathrm{e}^{4\,c+4\,d\,x}+4\,d\,\mathrm{e}^{6\,c+6\,d\,x}+d\,\mathrm{e}^{8\,c+8\,d\,x}\right)}-\frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d+d\,\mathrm{e}^{2\,c+2\,d\,x}}+\frac{4\,a\,b\,\mathrm{e}^{c+d\,x}}{d+2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}}-\frac{a\,b\,\left(\ln\left(32\,a^3\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\,a^5\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-a^5\,b\,32{}\mathrm{i}-a^3\,b^3\,32{}\mathrm{i}\right)\,1{}\mathrm{i}-\ln\left(32\,a^3\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\,a^5\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+a^5\,b\,32{}\mathrm{i}+a^3\,b^3\,32{}\mathrm{i}\right)\,1{}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x),x)
```

```
[Out] (a^2*log(32*a^6 + 32*a^4*b^2 - 32*a^6*exp(d*x)*exp(c) - 32*a^4*b^2*exp(d*x)
*exp(c)))/d - (176*b^2*exp(c + d*x))/(15*(d + 3*d*exp(2*c + 2*d*x) + 3*d*ex
p(4*c + 4*d*x) + d*exp(6*c + 6*d*x))) - (32*b^2*exp(c + d*x))/(5*(d + 5*d*e
xp(2*c + 2*d*x) + 10*d*exp(4*c + 4*d*x) + 10*d*exp(6*c + 6*d*x) + 5*d*exp(8
*c + 8*d*x) + d*exp(10*c + 10*d*x))) - (a^2*log(- 32*a^6 - 32*a^4*b^2 - 32*
a^6*exp(d*x)*exp(c) - 32*a^4*b^2*exp(d*x)*exp(c)))/d - (2*b^2*exp(c + d*x))
/(d + d*exp(2*c + 2*d*x)) + (16*b^2*exp(c + d*x))/(3*(d + 2*d*exp(2*c + 2*d
*x) + d*exp(4*c + 4*d*x))) + (64*b^2*exp(c + d*x))/(5*(d + 4*d*exp(2*c + 2*
d*x) + 6*d*exp(4*c + 4*d*x) + 4*d*exp(6*c + 6*d*x) + d*exp(8*c + 8*d*x))) -
 (2*a*b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x)) + (4*a*b*exp(c + d*x))/(d +
2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*b*(log(32*a^3*b^3*exp(d*x)*
exp(c) - a^3*b^3*32i - a^5*b*32i + 32*a^5*b*exp(d*x)*exp(c))*1i - log(a^5*b
*32i + a^3*b^3*32i + 32*a^3*b^3*exp(d*x)*exp(c) + 32*a^5*b*exp(d*x)*exp(c))
*1i))/d
```

3.62 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=47

$$-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] -a^2*coth(d*x+c)/d+a*b*tanh(d*x+c)^2/d+1/5*b^2*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3744, 276}

$$-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -((a^2*Coth[c + d*x])/d) + (a*b*Tanh[c + d*x]^2)/d + (b^2*Tanh[c + d*x]^5)/
(5*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^3\right)^2}{x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2}{x^2}+2abx+b^2x^4\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 94, normalized size = 2.00

$$-\frac{a^2\coth(c+dx)}{d}-\frac{ab\operatorname{sech}^2(c+dx)}{d}+\frac{b^2\tanh(c+dx)}{5d}-\frac{2b^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d}+\frac{b^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -((a^2*Coth[c + d*x])/d) - (a*b*Sech[c + d*x]^2)/d + (b^2*Tanh[c + d*x])/(5
*d) - (2*b^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) + (b^2*Sech[c + d*x]^4*Ta
nh[c + d*x])/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(45) = 90.
time = 3.20, size = 234, normalized size = 4.98

method	result
risch	$-\frac{2(5a^2e^{10dx+10c}+10ab\,e^{10dx+10c}+5b^2e^{10dx+10c}+25a^2e^{8dx+8c}+20ab\,e^{8dx+8c}-5b^2e^{8dx+8c}+50a^2e^{6dx+6c}+10b^2e^{6dx+6c}+50a^2e^{4dx+}}{5d\left(e^{2dx+2c}-1\right)\left(1+e^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(5*a^2*exp(10*d*x+10*c)+10*a*b*exp(10*d*x+10*c)+5*b^2*exp(10*d*x+10*c)
+25*a^2*exp(8*d*x+8*c)+20*a*b*exp(8*d*x+8*c)-5*b^2*exp(8*d*x+8*c)+50*a^2*ex
p(6*d*x+6*c)+10*b^2*exp(6*d*x+6*c)+50*a^2*exp(4*d*x+4*c)-20*a*b*exp(4*d*x+4
*c)-10*b^2*exp(4*d*x+4*c)+25*a^2*exp(2*d*x+2*c)-10*a*b*exp(2*d*x+2*c)+b^2*e
xp(2*d*x+2*c)+5*a^2-b^2)/d/(exp(2*d*x+2*c)-1)/(1+exp(2*d*x+2*c))^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(45) = 90.
time = 0.27, size = 256, normalized size = 5.45

$$\frac{2}{5}b^2\left(\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{5\,e^{(-8\,dx-8\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{1}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)+\frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}-1)}-\frac{4\,ab}{d(e^{(dx+c)}+e^{(-dx-c)})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 2/5*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^
(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1)) - 4*a*b/(d*(e^(
d*x + c) + e^(-d*x - c))^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(45) = 90$.
time = 0.35, size = 518, normalized size = 11.02

$$-\frac{4\,\left(\left(5\,a^{2}+5\,ab+2\,b^{2}\right)\cosh\left(dx+c\right)^{5}+5\,\left(5\,a^{2}+5\,ab+2\,b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+\left(5\,ab+3\,b^{2}\right)\sinh\left(dx+c\right)^{5}+\left(25\,a^{2}+5\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)^{3}+\left(10\,\left(5\,ab+3\,b^{2}\right)\cosh\left(dx+c\right)^{2}+15\,ab-3\,b^{2}\right)\sinh\left(dx+c\right)^{3}+\left(10\,\left(5\,a^{2}+5\,ab+2\,b^{2}\right)\cosh\left(dx+c\right)^{3}+3\,\left(25\,a^{2}+5\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+10\,\left(5\,a^{2}-ab\right)\cosh\left(dx+c\right)+\left(5\,\left(5\,ab+3\,b^{2}\right)\cosh\left(dx+c\right)^{4}+9\,\left(5\,ab-b^{2}\right)\cosh\left(dx+c\right)^{2}+10\,ab+10\,b^{2}\right)\sinh\left(dx+c\right)\right)}{5\,\left(d\cosh\left(dx+c\right)^{7}+7\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{7}+3\,d\cosh\left(dx+c\right)^{5}+\left(21\,d\cosh\left(dx+c\right)^{2}+5\,d\right)\sinh\left(dx+c\right)^{5}+5\,\left(7\,d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+d\cosh\left(dx+c\right)^{3}+\left(35\,d\cosh\left(dx+c\right)^{4}+50\,d\cosh\left(dx+c\right)^{2}+9\,d\right)\sinh\left(dx+c\right)^{3}+3\,\left(7\,d\cosh\left(dx+c\right)^{5}+10\,d\cosh\left(dx+c\right)^{3}+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-5\,d\cosh\left(dx+c\right)+\left(7\,d\cosh\left(dx+c\right)^{6}+25\,d\cosh\left(dx+c\right)^{4}+27\,d\cosh\left(dx+c\right)^{2}+5\,d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -4/5*((5*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^5 + 5*(5*a^2 + 5*a*b + 2*b^2)*c
osh(d*x + c)*sinh(d*x + c)^4 + (5*a*b + 3*b^2)*sinh(d*x + c)^5 + (25*a^2 +
5*a*b - 2*b^2)*cosh(d*x + c)^3 + (10*(5*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a
*b - 3*b^2)*sinh(d*x + c)^3 + (10*(5*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 +
 3*(25*a^2 + 5*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 - a*
b)*cosh(d*x + c) + (5*(5*a*b + 3*b^2)*cosh(d*x + c)^4 + 9*(5*a*b - b^2)*cos
h(d*x + c)^2 + 10*a*b + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cos
h(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + (21*
d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 3*d*cos
h(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 5
0*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + 3*(7*d*cosh(d*x + c)^5 + 10*d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 - 5*d*cosh(d*x + c) + (7
*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5*d)*sin
h(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^{3}\left(c + dx\right)\right)^{2}\operatorname{csch}^{2}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(45) = 90$.
time = 0.52, size = 122, normalized size = 2.60

$$-\frac{2\left(\frac{5\,a^{2}}{e^{(2\,dx+2\,c)}-1}+\frac{10\,abe^{(8\,dx+8\,c)}+5\,b^{2}e^{(8\,dx+8\,c)}+30\,abe^{(6\,dx+6\,c)}+30\,abe^{(4\,dx+4\,c)}+10\,b^{2}e^{(4\,dx+4\,c)}+10\,abe^{(2\,dx+2\,c)}+b^{2}}{\left(e^{(2\,dx+2\,c)}+1\right)^{5}}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^2/(e^(2*d*x + 2*c) - 1) + (10*a*b*e^(8*d*x + 8*c) + 5*b^2*e^(8*d*
x + 8*c) + 30*a*b*e^(6*d*x + 6*c) + 30*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*
x + 4*c) + 10*a*b*e^(2*d*x + 2*c) + b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.26, size = 483, normalized size = 10.28

$$-\frac{\frac{2\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(b^2+2\,a\,b\right)}{5\,d}-\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+a\,b\right)}{5\,d}-\frac{2\,\left(2\,a\,b-b^2\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a\,b-b^2\right)}{5\,d}+\frac{12\,b^2\,{\mathrm{e}}^{4\,c+4\,d\,x}}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{\frac{2\,b^2}{5\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(b^2+2\,a\,b\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b-b^2\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{\frac{2\,\left(a\,b-b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+2\,a\,b\right)}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(b^2+2\,a\,b\right)}{5\,d}-\frac{2\,\left(b^2+a\,b\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a\,b-b^2\right)}{5\,d}+\frac{6\,b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{2\,a^2}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,\left(b^2+2\,a\,b\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^2,x)
```

```
[Out] - ((2*exp(8*c + 8*d*x)*(2*a*b + b^2))/(5*d) - (8*exp(2*c + 2*d*x)*(a*b + b^
2))/(5*d) - (2*(2*a*b - b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(a*b - b^2))/(5*d
) + (12*b^2*exp(4*c + 4*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d
*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) -
((2*b^2)/(5*d) + (2*exp(4*c + 4*d*x)*(2*a*b + b^2))/(5*d) + (4*exp(2*c + 2*
d*x)*(a*b - b^2))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) + 1) - ((2*(a*b - b^2))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2)
)/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*exp(6*c + 6*d*x)
*(2*a*b + b^2))/(5*d) - (2*(a*b + b^2))/(5*d) + (6*exp(4*c + 4*d*x)*(a*b -
b^2))/(5*d) + (6*b^2*exp(2*c + 2*d*x))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4
*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*a^2)/(d*(exp(
2*c + 2*d*x) - 1)) - (2*(2*a*b + b^2))/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.63 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=107

$$\frac{ab\text{ArcTan}(\sinh(c+dx))}{d}+\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}-\frac{b^2\text{sech}^3(c+dx)}{3d}+\frac{b^2\text{sech}}{}$$

```
[Out] a*b*arctan(sinh(d*x+c))/d+1/2*a^2*arctanh(cosh(d*x+c))/d-1/2*a^2*coth(d*x+c
)*csch(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d+1/5*b^2*sech(d*x+c)^5/d+a*b*sech(d*
x+c)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3747, 3853, 3855, 2686, 14}

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{ab\text{ArcTan}(\sinh(c+dx))}{d}+\frac{ab\tanh(c+dx)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^5(c+dx)}{5d}-\frac{b^2\text{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (a*b*ArcTan[Sinh[c + d*x]])/d + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (a^2*C
oth[c + d*x]*Csch[c + d*x])/(2*d) - (b^2*Sech[c + d*x]^3)/(3*d) + (b^2*Sech
[c + d*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= -\left(i\int \left(ia^2\operatorname{csch}^3(c+dx)+2iab\operatorname{sech}^3(c+dx)+ib^2\operatorname{sech}^3(c+d\right.\right.\\ &= a^2\int \operatorname{csch}^3(c+dx)\,dx+(2ab)\int \operatorname{sech}^3(c+dx)\,dx+b^2\int \operatorname{sech}^3\\ &= -\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}\\ &= \frac{ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c\\ &= \frac{ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c\end{aligned}$$

Mathematica [A]
time = 0.13, size = 138, normalized size = 1.29

$$\frac{2ab\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}+\frac{b^2\operatorname{sech}^5(c+dx)}{5d}+\frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d - (a^2*Csch[(c + d*x)/2]^2)/(8*d) - (a^
2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d) - (b^2*Se
ch[c + d*x]^3)/(3*d) + (b^2*Sech[c + d*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tan
h[c + d*x])/d
```

Maple [C] Result contains complex when optimal does not.
time = 3.72, size = 312, normalized size = 2.92

method	result

risch	$-\frac{e^{dx+c}\left(15a^2e^{12dx+12c}-30ab\,e^{12dx+12c}+90a^2e^{10dx+10c}+40b^2e^{10dx+10c}+225a^2e^{8dx+8c}+90ab\,e^{8dx+8c}-96b^2e^{8dx+8c}+300a^2e^{6dx+}\right.}{15d\left(1+e^{2dx+2c}\right)^5\left(e^{2dx+2c}-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*exp(d*x+c)*(15*a^2*exp(12*d*x+12*c)-30*a*b*exp(12*d*x+12*c)+90*a^2*ex
p(10*d*x+10*c)+40*b^2*exp(10*d*x+10*c)+225*a^2*exp(8*d*x+8*c)+90*a*b*exp(8*
d*x+8*c)-96*b^2*exp(8*d*x+8*c)+300*a^2*exp(6*d*x+6*c)+112*b^2*exp(6*d*x+6*c
)+225*a^2*exp(4*d*x+4*c)-90*a*b*exp(4*d*x+4*c)-96*b^2*exp(4*d*x+4*c)+90*a^2
*exp(2*d*x+2*c)+40*b^2*exp(2*d*x+2*c)+15*a^2+30*a*b)/d/(1+exp(2*d*x+2*c))^5
/(exp(2*d*x+2*c)-1)^2+I*b*a/d*ln(exp(d*x+c)+I)-I*b*a/d*ln(exp(d*x+c)-I)+1/2
*a^2/d*ln(exp(d*x+c)+1)-1/2*a^2/d*ln(exp(d*x+c)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(99) = 198$.
time = 0.49, size = 378, normalized size = 3.53

$$-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)-\frac{8}{15}\,b^2\left(\frac{5\,e^{(-3\,dx-3\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}-\frac{2\,e^{(-5\,dx-5\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}+\frac{5\,e^{(-7\,dx-7\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^
(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d
 - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 8/15*b^2*(5*e^(-3*d*x - 3*c)/(d*(5
*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
 - 8*c) + e^(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2
*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4
*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
 + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4642 vs. $2(99) = 198$.
time = 0.39, size = 4642, normalized size = 43.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/30*(30*(a^2 - 2*a*b)*cosh(d*x + c)^13 + 390*(a^2 - 2*a*b)*cosh(d*x + c)*
sinh(d*x + c)^12 + 30*(a^2 - 2*a*b)*sinh(d*x + c)^13 + 20*(9*a^2 + 4*b^2)*c
```

osh(d*x + c)^11 + 20*(117*(a^2 - 2*a*b)*cosh(d*x + c)^2 + 9*a^2 + 4*b^2)*si
nh(d*x + c)^11 + 220*(39*(a^2 - 2*a*b)*cosh(d*x + c)^3 + (9*a^2 + 4*b^2)*co
sh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^
9 + 2*(10725*(a^2 - 2*a*b)*cosh(d*x + c)^4 + 550*(9*a^2 + 4*b^2)*cosh(d*x +
 c)^2 + 225*a^2 + 90*a*b - 96*b^2)*sinh(d*x + c)^9 + 6*(6435*(a^2 - 2*a*b)*
cosh(d*x + c)^5 + 550*(9*a^2 + 4*b^2)*cosh(d*x + c)^3 + 9*(75*a^2 + 30*a*b
- 32*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 8*(75*a^2 + 28*b^2)*cosh(d*x + c
)^7 + 8*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^6 + 825*(9*a^2 + 4*b^2)*cosh(d*x
+ c)^4 + 27*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^2 + 75*a^2 + 28*b^2)*s
inh(d*x + c)^7 + 8*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^7 + 1155*(9*a^2 + 4*b^
2)*cosh(d*x + c)^5 + 63*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^3 + 7*(75*
a^2 + 28*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(75*a^2 - 30*a*b - 32*b^2)
*cosh(d*x + c)^5 + 6*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^8 + 1540*(9*a^2 + 4*
b^2)*cosh(d*x + c)^6 + 126*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^4 + 28*
(75*a^2 + 28*b^2)*cosh(d*x + c)^2 + 75*a^2 - 30*a*b - 32*b^2)*sinh(d*x + c)
^5 + 2*(10725*(a^2 - 2*a*b)*cosh(d*x + c)^9 + 3300*(9*a^2 + 4*b^2)*cosh(d*x
 + c)^7 + 378*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^5 + 140*(75*a^2 + 28
*b^2)*cosh(d*x + c)^3 + 15*(75*a^2 - 30*a*b - 32*b^2)*cosh(d*x + c))*sinh(d
x + c)^4 + 20(9*a^2 + 4*b^2)*cosh(d*x + c)^3 + 4*(2145*(a^2 - 2*a*b)*cosh
(d*x + c)^10 + 825*(9*a^2 + 4*b^2)*cosh(d*x + c)^8 + 126*(75*a^2 + 30*a*b -
 32*b^2)*cosh(d*x + c)^6 + 70*(75*a^2 + 28*b^2)*cosh(d*x + c)^4 + 15*(75*a^
2 - 30*a*b - 32*b^2)*cosh(d*x + c)^2 + 45*a^2 + 20*b^2)*sinh(d*x + c)^3 + 4
(585(a^2 - 2*a*b)*cosh(d*x + c)^11 + 275*(9*a^2 + 4*b^2)*cosh(d*x + c)^9
+ 54*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^7 + 42*(75*a^2 + 28*b^2)*cosh
(d*x + c)^5 + 15*(75*a^2 - 30*a*b - 32*b^2)*cosh(d*x + c)^3 + 15*(9*a^2 + 4
*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 60*(a*b*cosh(d*x + c)^14 + 14*a*b*co
sh(d*x + c)*sinh(d*x + c)^13 + a*b*sinh(d*x + c)^14 + 3*a*b*cosh(d*x + c)^1
2 + (91*a*b*cosh(d*x + c)^2 + 3*a*b)*sinh(d*x + c)^12 + a*b*cosh(d*x + c)^1
0 + 4*(91*a*b*cosh(d*x + c)^3 + 9*a*b*cosh(d*x + c))*sinh(d*x + c)^11 + (10
01*a*b*cosh(d*x + c)^4 + 198*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^10 -
5*a*b*cosh(d*x + c)^8 + 2*(1001*a*b*cosh(d*x + c)^5 + 330*a*b*cosh(d*x + c)
^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c)^9 + (3003*a*b*cosh(d*x + c)^6 + 148
5*a*b*cosh(d*x + c)^4 + 45*a*b*cosh(d*x + c)^2 - 5*a*b)*sinh(d*x + c)^8 - 5
*a*b*cosh(d*x + c)^6 + 8*(429*a*b*cosh(d*x + c)^7 + 297*a*b*cosh(d*x + c)^5
 + 15*a*b*cosh(d*x + c)^3 - 5*a*b*cosh(d*x + c))*sinh(d*x + c)^7 + (3003*a*
b*cosh(d*x + c)^8 + 2772*a*b*cosh(d*x + c)^6 + 210*a*b*cosh(d*x + c)^4 - 14
0*a*b*cosh(d*x + c)^2 - 5*a*b)*sinh(d*x + c)^6 + a*b*cosh(d*x + c)^4 + 2*(1
001*a*b*cosh(d*x + c)^9 + 1188*a*b*cosh(d*x + c)^7 + 126*a*b*cosh(d*x + c)^
5 - 140*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + (1001
*a*b*cosh(d*x + c)^10 + 1485*a*b*cosh(d*x + c)^8 + 210*a*b*cosh(d*x + c)^6
- 350*a*b*cosh(d*x + c)^4 - 75*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^4 +
 3*a*b*cosh(d*x + c)^2 + 4*(91*a*b*cosh(d*x + c)^11 + 165*a*b*cosh(d*x + c)
^9 + 30*a*b*cosh(d*x + c)^7 - 70*a*b*cosh(d*x + c)^5 - 25*a*b*cosh(d*x + c)
^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (91*a*b*cosh(d*x + c)^12 + 198*a*
b*cosh(d*x + c)^10 + 45*a*b*cosh(d*x + c)^8 - 140*a*b*cosh(d*x + c)^6 - 75*

```
a*b*cosh(d*x + c)^4 + 6*a*b*cosh(d*x + c)^2 + 3*a*b)*sinh(d*x + c)^2 + a*b
+ 2*(7*a*b*cosh(d*x + c)^13 + 18*a*b*cosh(d*x + c)^11 + 5*a*b*cosh(d*x + c)
^9 - 20*a*b*cosh(d*x + c)^7 - 15*a*b*cosh(d*x + c)^5 + 2*a*b*cosh(d*x + c)^
3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c
)) + 30*(a^2 + 2*a*b)*cosh(d*x + c) - 15*(a^2*cosh(d*x + c)^14 + 14*a^2*cos
h(d*x + c)*sinh(d*x + c)^13 + a^2*sinh(d*x + c)^14 + 3*a^2*cosh(d*x + c)^12
 + (91*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^12 + a^2*cosh(d*x + c)^10
 + 4*(91*a^2*cosh(d*x + c)^3 + 9*a^2*cosh(d*x + c))*sinh(d*x + c)^11 + (100
1*a^2*cosh(d*x + c)^4 + 198*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^10 - 5
*a^2*cosh(d*x + c)^8 + 2*(1001*a^2*cosh(d*x + c)^5 + 330*a^2*cosh(d*x + c)^
3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^9 + (3003*a^2*cosh(d*x + c)^6 + 1485
*a^2*cosh(d*x + c)^4 + 45*a^2*cosh(d*x + c)^2 - 5*a^2)*sinh(d*x + c)^8 - 5*
a^2*cosh(d*x + c)^6 + 8*(429*a^2*cosh(d*x + c)^7 + 297*a^2*cosh(d*x + c)^5
+ 15*a^2*cosh(d*x + c)^3 - 5*a^2*cosh(d*x + c))*sinh(d*x + c)^7 + (3003*a^2
*cosh(d*x + c)^8 + 2772*a^2*cosh(d*x + c)^6 + 210*a^2*cosh(d*x + c)^4 - 140
*a^2*cosh(d*x + c)^2 - 5*a^2)*sinh(d*x + c)^6 + a^2*cosh(d*x + c)^4 + 2*(10
01*a^2*cosh(d*x + c)^9 + 1188*a^2*cosh(d*x + c)^7 + 126*a^2*cosh(d*x + c)^5
 - 140*a^2*cosh(d*x + c)^3 - 15*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + (1001*
a^2*cosh(d*x + c)^10 + 1485*a^2*cosh(d*x + c)^8 + 210*a^2*cosh(d*x + c)^6 -
 350*a^2*cosh(d*x + c)^4 - 75*a^2*cosh(d*x + c)...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^3\left(c + dx\right)\right)^2 \operatorname{csch}^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**3, x)
```

Giac [**A**]
time = 0.52, size = 192, normalized size = 1.79

$$\frac{60\,ab\arctan\left(e^{(dx+c)}\right) + 15\,a^2\log\left(e^{(dx+c)}+1\right) - 15\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{30\,\left(a^2e^{(3\,dx+3\,c)}+a^2e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2} + \frac{4\,\left(15\,abe^{(9\,dx+9\,c)}+30\,abe^{(7\,dx+7\,c)}-20\,b^2e^{(7\,dx+7\,c)}+8\,b^2e^{(5\,dx+5\,c)}-30\,abe^{(3\,dx+3\,c)}-20\,b^2e^{(3\,dx+3\,c)}-15\,abe^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*arctan(e^(d*x + c)) + 15*a^2*log(e^(d*x + c) + 1) - 15*a^2*log
(abs(e^(d*x + c) - 1)) - 30*(a^2*e^(3*d*x + 3*c) + a^2*e^(d*x + c))/(e^(2*d
*x + 2*c) - 1)^2 + 4*(15*a*b*e^(9*d*x + 9*c) + 30*a*b*e^(7*d*x + 7*c) - 20*
b^2*e^(7*d*x + 7*c) + 8*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(3*d*x + 3*c) - 20*b
^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 2.90, size = 561, normalized size = 5.24

$$\frac{a^2\,e^{c+d\,x}}{d-d\,e^{2\,c+2\,d\,x}}+\frac{136\,b^2\,e^{c+d\,x}}{15\,\left(d+3\,d\,e^{2\,c+2\,d\,x}+3\,d\,e^{4\,c+4\,d\,x}+d\,e^{6\,c+6\,d\,x}\right)}+\frac{32\,b^2\,e^{c+d\,x}}{5\,\left(d+5\,d\,e^{2\,c+2\,d\,x}+10\,d\,e^{4\,c+4\,d\,x}+10\,d\,e^{6\,c+6\,d\,x}+5\,d\,e^{8\,c+8\,d\,x}+d\,e^{10\,c+10\,d\,x}\right)}-\frac{a^2\,\ln\left(4\,a^6\,e^{d\,x}\,e^c-16\,a^4\,b^2-4\,a^6+16\,a^4\,b^2\,e^{d\,x}\,e^c\right)}{2\,d}+\frac{a^2\,\ln\left(4\,a^6+16\,a^4\,b^2+4\,a^6\,e^{d\,x}\,e^c+16\,a^4\,b^2\,e^{d\,x}\,e^c\right)}{2\,d}-\frac{2\,a^2\,e^{c+d\,x}}{d-2\,d\,e^{2\,c+2\,d\,x}+d\,e^{4\,c+4\,d\,x}}-\frac{8\,b^2\,e^{c+d\,x}}{3\,\left(d+2\,d\,e^{2\,c+2\,d\,x}+d\,e^{4\,c+4\,d\,x}\right)}-\frac{64\,b^2\,e^{c+d\,x}}{5\,\left(d+4\,d\,e^{2\,c+2\,d\,x}+6\,d\,e^{4\,c+4\,d\,x}+4\,d\,e^{6\,c+6\,d\,x}+d\,e^{8\,c+8\,d\,x}\right)}+\frac{2\,a\,b\,e^{c+d\,x}}{d+d\,e^{2\,c+2\,d\,x}}-\frac{4\,a\,b\,e^{c+d\,x}}{d+2\,d\,e^{2\,c+2\,d\,x}+d\,e^{4\,c+4\,d\,x}}-\frac{a\,b\,\left(\ln\left(32\,a^3\,b^3\,e^{d\,x}\,e^c+8\,a^5\,b\,e^{d\,x}\,e^c-a^5\,b\,8{}\mathrm{i}-a^3\,b^3\,32{}\mathrm{i}\right)\,1{}\mathrm{i}-\ln\left(32\,a^3\,b^3\,e^{d\,x}\,e^c+8\,a^5\,b\,e^{d\,x}\,e^c+a^5\,b\,8{}\mathrm{i}+a^3\,b^3\,32{}\mathrm{i}\right)\,1{}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^3,x)
```

```
[Out] (a^2*exp(c + d*x))/(d - d*exp(2*c + 2*d*x)) + (136*b^2*exp(c + d*x))/(15*(d
 + 3*d*exp(2*c + 2*d*x) + 3*d*exp(4*c + 4*d*x) + d*exp(6*c + 6*d*x))) + (32
*b^2*exp(c + d*x))/(5*(d + 5*d*exp(2*c + 2*d*x) + 10*d*exp(4*c + 4*d*x) + 1
0*d*exp(6*c + 6*d*x) + 5*d*exp(8*c + 8*d*x) + d*exp(10*c + 10*d*x))) - (a^2
*log(4*a^6*exp(d*x)*exp(c) - 16*a^4*b^2 - 4*a^6 + 16*a^4*b^2*exp(d*x)*exp(c
)))/(2*d) + (a^2*log(4*a^6 + 16*a^4*b^2 + 4*a^6*exp(d*x)*exp(c) + 16*a^4*b^
2*exp(d*x)*exp(c)))/(2*d) - (2*a^2*exp(c + d*x))/(d - 2*d*exp(2*c + 2*d*x)
+ d*exp(4*c + 4*d*x)) - (8*b^2*exp(c + d*x))/(3*(d + 2*d*exp(2*c + 2*d*x) +
 d*exp(4*c + 4*d*x))) - (64*b^2*exp(c + d*x))/(5*(d + 4*d*exp(2*c + 2*d*x)
+ 6*d*exp(4*c + 4*d*x) + 4*d*exp(6*c + 6*d*x) + d*exp(8*c + 8*d*x))) + (2*a
*b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x)) - (4*a*b*exp(c + d*x))/(d + 2*d*e
xp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*b*(log(32*a^3*b^3*exp(d*x)*exp(c
) - a^3*b^3*32i - a^5*b*8i + 8*a^5*b*exp(d*x)*exp(c))*1i - log(a^5*b*8i + a
^3*b^3*32i + 32*a^3*b^3*exp(d*x)*exp(c) + 8*a^5*b*exp(d*x)*exp(c))*1i))/d
```

3.64 $\int \operatorname{csch}^4(c+dx) \left(a+b\tanh^3(c+dx)\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}+\frac{2ab\log(\tanh(c+dx))}{d}-\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] a^2*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d+2*a*b*ln(tanh(d*x+c))/d-a*b*tanh(
d*x+c)^2/d+1/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3744, 1816}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}+\frac{2ab\log(\tanh(c+dx))}{d}-\frac{b^2\tanh^5(c+dx)}{5d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) + (2*a*b*Log[Tanh[c + d
*x]])/d - (a*b*Tanh[c + d*x]^2)/d + (b^2*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh
[c + d*x]^5)/(5*d)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{(1-x^2)(a+bx^3)^2}{x^4}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int\left(\frac{a^2}{x^4}-\frac{a^2}{x^2}+\frac{2ab}{x}-2abx+b^2x^2-b^2x^4\right)dx,x,\tanh(c+\right.}{d}$$
$$=\frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}+\frac{2ab\log(\tanh(c+dx))}{d}-$$

Mathematica [A]
time = 0.13, size = 147, normalized size = 1.52

$$\frac{2a^2\coth(c+dx)}{3d}-\frac{a^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{2ab\log(\cosh(c+dx))}{d}+\frac{2ab\log(\sinh(c+dx))}{d}+\frac{ab\operatorname{sech}^2(c+dx)}{d}+\frac{2b^2\tanh(c+dx)}{15d}+\frac{b^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{15d}-\frac{b^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (2*a^2*Coth[c + d*x])/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (
2*a*b*Log[Cosh[c + d*x]])/d + (2*a*b*Log[Sinh[c + d*x]])/d + (a*b*Sech[c +
d*x]^2)/d + (2*b^2*Tanh[c + d*x])/(15*d) + (b^2*Sech[c + d*x]^2*Tanh[c + d*
x])/(15*d) - (b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(91) = 182.
time = 3.40, size = 302, normalized size = 3.11

method	result
risch	$-\frac{4\left(-15ab\,\mathrm{e}^{14dx+14c}+15a^2\mathrm{e}^{12dx+12c}+15b^2\mathrm{e}^{12dx+12c}+70a^2\mathrm{e}^{10dx+10c}+45ab\,\mathrm{e}^{10dx+10c}-50b^2\mathrm{e}^{10dx+10c}+125a^2\mathrm{e}^{8dx+8c}+65b^2\mathrm{e}^{8dx+8c}\right.}{15d\left(1+\mathrm{e}^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/15*(-15*a*b*exp(14*d*x+14*c)+15*a^2*exp(12*d*x+12*c)+15*b^2*exp(12*d*x+1
2*c)+70*a^2*exp(10*d*x+10*c)+45*a*b*exp(10*d*x+10*c)-50*b^2*exp(10*d*x+10*c
)+125*a^2*exp(8*d*x+8*c)+65*b^2*exp(8*d*x+8*c)+100*a^2*exp(6*d*x+6*c)-45*a*
b*exp(6*d*x+6*c)-44*b^2*exp(6*d*x+6*c)+25*a^2*exp(4*d*x+4*c)+17*b^2*exp(4*d
*x+4*c)-10*a^2*exp(2*d*x+2*c)+15*a*b*exp(2*d*x+2*c)-2*b^2*exp(2*d*x+2*c)-5*
a^2-b^2)/d/(1+exp(2*d*x+2*c))^5/(exp(2*d*x+2*c)-1)^3-2*a*b/d*ln(1+exp(2*d*x
+2*c))+2*a*b/d*ln(exp(2*d*x+2*c)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(91) = 182.
time = 0.50, size = 468, normalized size = 4.82

$$2ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)+\frac{4}{15}b^2\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}-\frac{5e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{15e^{(-6dx-6c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{1}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+\frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x -
2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)
+ 1))) + 4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*
d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*
c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*
c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-1
0*d*x - 10*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) -
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4125 vs. $2(91) = 182$.
time = 0.37, size = 4125, normalized size = 42.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 2/15*(30*a*b*cosh(d*x + c)^14 + 420*a*b*cosh(d*x + c)*sinh(d*x + c)^13 + 30
*a*b*sinh(d*x + c)^14 - 30*(a^2 + b^2)*cosh(d*x + c)^12 + 30*(91*a*b*cosh(d
*x + c)^2 - a^2 - b^2)*sinh(d*x + c)^12 + 120*(91*a*b*cosh(d*x + c)^3 - 3*(
a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^11 - 10*(14*a^2 + 9*a*b - 10*b^2)*c
osh(d*x + c)^10 + 10*(3003*a*b*cosh(d*x + c)^4 - 198*(a^2 + b^2)*cosh(d*x +
 c)^2 - 14*a^2 - 9*a*b + 10*b^2)*sinh(d*x + c)^10 + 20*(3003*a*b*cosh(d*x +
 c)^5 - 330*(a^2 + b^2)*cosh(d*x + c)^3 - 5*(14*a^2 + 9*a*b - 10*b^2)*cosh(
d*x + c))*sinh(d*x + c)^9 - 10*(25*a^2 + 13*b^2)*cosh(d*x + c)^8 + 10*(9009
*a*b*cosh(d*x + c)^6 - 1485*(a^2 + b^2)*cosh(d*x + c)^4 - 45*(14*a^2 + 9*a*
b - 10*b^2)*cosh(d*x + c)^2 - 25*a^2 - 13*b^2)*sinh(d*x + c)^8 + 80*(1287*a
*b*cosh(d*x + c)^7 - 297*(a^2 + b^2)*cosh(d*x + c)^5 - 15*(14*a^2 + 9*a*b -
 10*b^2)*cosh(d*x + c)^3 - (25*a^2 + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^7
 - 2*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^6 + 2*(45045*a*b*cosh(d*x +
c)^8 - 13860*(a^2 + b^2)*cosh(d*x + c)^6 - 1050*(14*a^2 + 9*a*b - 10*b^2)*c
osh(d*x + c)^4 - 140*(25*a^2 + 13*b^2)*cosh(d*x + c)^2 - 100*a^2 + 45*a*b +
 44*b^2)*sinh(d*x + c)^6 + 4*(15015*a*b*cosh(d*x + c)^9 - 5940*(a^2 + b^2)*
cosh(d*x + c)^7 - 630*(14*a^2 + 9*a*b - 10*b^2)*cosh(d*x + c)^5 - 140*(25*a
^2 + 13*b^2)*cosh(d*x + c)^3 - 3*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c))
*sinh(d*x + c)^5 - 2*(25*a^2 + 17*b^2)*cosh(d*x + c)^4 + 2*(15015*a*b*cosh(
d*x + c)^10 - 7425*(a^2 + b^2)*cosh(d*x + c)^8 - 1050*(14*a^2 + 9*a*b - 10*
```

b^2)*cosh(d*x + c)^6 - 350*(25*a^2 + 13*b^2)*cosh(d*x + c)^4 - 15*(100*a^2
- 45*a*b - 44*b^2)*cosh(d*x + c)^2 - 25*a^2 - 17*b^2)*sinh(d*x + c)^4 + 8*(
1365*a*b*cosh(d*x + c)^11 - 825*(a^2 + b^2)*cosh(d*x + c)^9 - 150*(14*a^2 +
 9*a*b - 10*b^2)*cosh(d*x + c)^7 - 70*(25*a^2 + 13*b^2)*cosh(d*x + c)^5 - 5
*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^3 - (25*a^2 + 17*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 2*(10*a^2 - 15*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(136
5*a*b*cosh(d*x + c)^12 - 990*(a^2 + b^2)*cosh(d*x + c)^10 - 225*(14*a^2 + 9
*a*b - 10*b^2)*cosh(d*x + c)^8 - 140*(25*a^2 + 13*b^2)*cosh(d*x + c)^6 - 15
*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^4 - 6*(25*a^2 + 17*b^2)*cosh(d*x
 + c)^2 + 10*a^2 - 15*a*b + 2*b^2)*sinh(d*x + c)^2 + 10*a^2 + 2*b^2 - 15*(a
*b*cosh(d*x + c)^16 + 16*a*b*cosh(d*x + c)*sinh(d*x + c)^15 + a*b*sinh(d*x
+ c)^16 + 2*a*b*cosh(d*x + c)^14 + 2*(60*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*
x + c)^14 - 2*a*b*cosh(d*x + c)^12 + 28*(20*a*b*cosh(d*x + c)^3 + a*b*cosh(
d*x + c))*sinh(d*x + c)^13 + 2*(910*a*b*cosh(d*x + c)^4 + 91*a*b*cosh(d*x +
 c)^2 - a*b)*sinh(d*x + c)^12 - 6*a*b*cosh(d*x + c)^10 + 8*(546*a*b*cosh(d*
x + c)^5 + 91*a*b*cosh(d*x + c)^3 - 3*a*b*cosh(d*x + c))*sinh(d*x + c)^11 +
 2*(4004*a*b*cosh(d*x + c)^6 + 1001*a*b*cosh(d*x + c)^4 - 66*a*b*cosh(d*x +
 c)^2 - 3*a*b)*sinh(d*x + c)^10 + 4*(2860*a*b*cosh(d*x + c)^7 + 1001*a*b*co
sh(d*x + c)^5 - 110*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x + c))*sinh(d*x +
c)^9 + 6*(2145*a*b*cosh(d*x + c)^8 + 1001*a*b*cosh(d*x + c)^6 - 165*a*b*cos
h(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 6*a*b*cosh(d*x + c
)^6 + 16*(715*a*b*cosh(d*x + c)^9 + 429*a*b*cosh(d*x + c)^7 - 99*a*b*cosh(d
*x + c)^5 - 45*a*b*cosh(d*x + c)^3)*sinh(d*x + c)^7 + 2*(4004*a*b*cosh(d*x
+ c)^10 + 3003*a*b*cosh(d*x + c)^8 - 924*a*b*cosh(d*x + c)^6 - 630*a*b*cosh
(d*x + c)^4 + 3*a*b)*sinh(d*x + c)^6 + 2*a*b*cosh(d*x + c)^4 + 4*(1092*a*b*
cosh(d*x + c)^11 + 1001*a*b*cosh(d*x + c)^9 - 396*a*b*cosh(d*x + c)^7 - 378
*a*b*cosh(d*x + c)^5 + 9*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(910*a*b*co
sh(d*x + c)^12 + 1001*a*b*cosh(d*x + c)^10 - 495*a*b*cosh(d*x + c)^8 - 630*
a*b*cosh(d*x + c)^6 + 45*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^4 - 2*a*b
*cosh(d*x + c)^2 + 8*(70*a*b*cosh(d*x + c)^13 + 91*a*b*cosh(d*x + c)^11 - 5
5*a*b*cosh(d*x + c)^9 - 90*a*b*cosh(d*x + c)^7 + 15*a*b*cosh(d*x + c)^3 + a
*b*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(60*a*b*cosh(d*x + c)^14 + 91*a*b*cos
h(d*x + c)^12 - 66*a*b*cosh(d*x + c)^10 - 135*a*b*cosh(d*x + c)^8 + 45*a*b*
cosh(d*x + c)^4 + 6*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^2 - a*b + 4*(4
*a*b*cosh(d*x + c)^15 + 7*a*b*cosh(d*x + c)^13 - 6*a*b*cosh(d*x + c)^11 - 1
5*a*b*cosh(d*x + c)^9 + 9*a*b*cosh(d*x + c)^5 + 2*a*b*cosh(d*x + c)^3 - a*b
*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*
x + c))) + 15*(a*b*cosh(d*x + c)^16 + 16*a*b*cosh(d*x + c)*sinh(d*x + c)^15
 + a*b*sinh(d*x + c)^16 + 2*a*b*cosh(d*x + c)^14 + 2*(60*a*b*cosh(d*x + c)^
2 + a*b)*sinh(d*x + c)^14 - 2*a*b*cosh(d*x + c)^12 + 28*(20*a*b*cosh(d*x +
c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^13 + 2*(910*a*b*cosh(d*x + c)^4 + 9
1*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^12 - 6*a*b*cosh(d*x + c)^10 + 8*
(546*a*b*cosh(d*x + c)^5 + 91*a*b*cosh(d*x + c)^3 - 3*a*b*cosh(d*x + c))*si
nh(d*x + c)^11 + 2*(4004*a*b*cosh(d*x + c)^6 + 1001*a*b*cosh(d*x + c)^4 - 6
6*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^10 + 4*(2860*a*b*cosh(d*x + c)

```
^7 + 1001*a*b*cosh(d*x + c)^5 - 110*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x +
 c))*sinh(d*x + c)^9 + 6*(2145*a*b*cosh(d*x + c...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^3\left(c + dx\right)\right)^2 \operatorname{csch}^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(91) = 182.
time = 0.52, size = 249, normalized size = 2.57

$$-\frac{60\,ab\log\left(e^{(2\,dx+2\,c)}+1\right) - 60\,ab\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{10\left(11\,abe^{(6\,dx+6\,c)} - 33\,abe^{(4\,dx+4\,c)} + 12\,a^2e^{(2\,dx+2\,c)} + 33\,abe^{(2\,dx+2\,c)} - 4\,a^2 - 11\,ab\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3} - \frac{137\,abe^{(10\,dx+10\,c)} + 805\,abe^{(8\,dx+8\,c)} + 1730\,abe^{(6\,dx+6\,c)} - 120\,b^2e^{(6\,dx+6\,c)} + 1730\,abe^{(4\,dx+4\,c)} + 40\,b^2e^{(4\,dx+4\,c)} + 805\,abe^{(2\,dx+2\,c)} - 40\,b^2e^{(2\,dx+2\,c)} + 137\,ab - 8\,b^2}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*a*b*log(e^(2*d*x + 2*c) + 1) - 60*a*b*log(abs(e^(2*d*x + 2*c) - 1
)) + 10*(11*a*b*e^(6*d*x + 6*c) - 33*a*b*e^(4*d*x + 4*c) + 12*a^2*e^(2*d*x
+ 2*c) + 33*a*b*e^(2*d*x + 2*c) - 4*a^2 - 11*a*b)/(e^(2*d*x + 2*c) - 1)^3 -
 (137*a*b*e^(10*d*x + 10*c) + 805*a*b*e^(8*d*x + 8*c) + 1730*a*b*e^(6*d*x +
 6*c) - 120*b^2*e^(6*d*x + 6*c) + 1730*a*b*e^(4*d*x + 4*c) + 40*b^2*e^(4*d*
x + 4*c) + 805*a*b*e^(2*d*x + 2*c) - 40*b^2*e^(2*d*x + 2*c) + 137*a*b - 8*b
^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 0.26, size = 344, normalized size = 3.55

$$\frac{40\,b^2}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}-\frac{4\,a^2}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,a^2}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{4\,\left(b^2+a\,b\right)}{d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{16\,b^2}{d\,\left(4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}+\frac{32\,b^2}{5\,d\,\left(5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1\right)}-\frac{4\,\mathrm{atan}\left(\frac{a\,b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}+\frac{4\,a\,b}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^4,x)
```

```
[Out] (40*b^2)/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
 1)) - (4*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^2)/(3
*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(
a*b + b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (16*b^2)/(d*(
4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*
d*x) + 1)) + (32*b^2)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (4*atan((
a*b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))
/(-d^2)^(1/2) + (4*a*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.65 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=275

$$\frac{3}{8}a\left(a^2+63b^2\right)x+\frac{3b(3a^2+5b^2)\log(\cosh(c+dx))}{d}-\frac{18ab^2\tanh(c+dx)}{d}-\frac{b(3a^2+10b^2)\tanh^2(c+dx)}{2d}-\frac{3ab^2\text{t}}{}$$

```
[Out] 3/8*a*(a^2+63*b^2)*x+3*b*(3*a^2+5*b^2)*ln(cosh(d*x+c))/d-18*a*b^2*tanh(d*x+
c)/d-1/2*b*(3*a^2+10*b^2)*tanh(d*x+c)^2/d-3*a*b^2*tanh(d*x+c)^3/d-3/2*b^3*t
anh(d*x+c)^4/d-3/5*a*b^2*tanh(d*x+c)^5/d-1/2*b^3*tanh(d*x+c)^6/d-1/8*b^3*ta
nh(d*x+c)^8/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)*(a*(a^2+3*b^2)+b*(3*a^2+b^2)*ta
nh(d*x+c))/d-1/8*cosh(d*x+c)*sinh(d*x+c)*(a*(5*a^2+51*b^2)+2*b*(15*a^2+11*b
^2)*tanh(d*x+c))/d
```

Rubi [A]
time = 0.34, antiderivative size = 306, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$,
Rules used = {3744, 1818, 1816, 647, 31}

$$-\frac{3b(3a^2+5b^2)\tanh^2(c+dx)}{2d}-\frac{3a(a^2+63b^2)\tanh(c+dx)}{8d}-\frac{3(a+b)\left(a^2+23ab+40b^2\right)\log(1-\tanh(c+dx))}{16d}+\frac{3(a-b)\left(a^2-23ab+40b^2\right)\log(\tanh(c+dx)+1)}{16d}+\frac{\sinh^4(c+dx)\left(a(a^2+3b^2)\tanh(c+dx)+b(3a^2+b^2)\right)}{4d}-\frac{\sinh^2(c+dx)\tanh(c+dx)\left(4b(6a^2+5b^2)\tanh(c+dx)+a(a^2+39b^2)\right)}{8d}-\frac{3ab^2\tanh^5(c+dx)}{5d}-\frac{3ab^2\tanh^3(c+dx)}{d}-\frac{b^3\tanh^8(c+dx)}{8d}-\frac{b^3\tanh^6(c+dx)}{2d}-\frac{3b^3\tanh^4(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (-3*(a + b)*(a^2 + 23*a*b + 40*b^2)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a
- b)*(a^2 - 23*a*b + 40*b^2)*Log[1 + Tanh[c + d*x]])/(16*d) - (3*a*(a^2 + 6
3*b^2)*Tanh[c + d*x])/(8*d) - (3*b*(3*a^2 + 5*b^2)*Tanh[c + d*x]^2)/(2*d) -
 (3*a*b^2*Tanh[c + d*x]^3)/d - (3*b^3*Tanh[c + d*x]^4)/(2*d) - (3*a*b^2*Tan
h[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*x]^6)/(2*d) - (b^3*Tanh[c + d*x]^8)/(
8*d) + (Sinh[c + d*x]^4*(b*(3*a^2 + b^2) + a*(a^2 + 3*b^2)*Tanh[c + d*x]))/
(4*d) - (Sinh[c + d*x]^2*Tanh[c + d*x]*(a*(a^2 + 39*b^2) + 4*b*(6*a^2 + 5*b
^2)*Tanh[c + d*x]))/(8*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^3)^3}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^4(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{4d} + \frac{\text{Sub}}{}$$

$$= \frac{\sinh^4(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{4d} - \frac{\text{sinl}}{}$$

$$= \frac{\sinh^4(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{4d} - \frac{\text{sinl}}{}$$

$$= -\frac{3a(a^2+63b^2)\tanh(c+dx)}{8d} - \frac{3b(3a^2+5b^2)\tanh^2(c+dx)}{2d}$$

$$= -\frac{3a(a^2+63b^2)\tanh(c+dx)}{8d} - \frac{3b(3a^2+5b^2)\tanh^2(c+dx)}{2d}$$

$$= -\frac{3(a+b)\left(a^2+23ab+40b^2\right)\log(1-\tanh(c+dx))}{16d} + \frac{3(a-}{}$$

Mathematica [A]
time = 6.20, size = 294, normalized size = 1.07

$$\frac{3a(a^2+63b^2)(c+dx)}{8d}-\frac{b(15a^2+11b^2)\cosh(2(c+dx))}{8d}+\frac{b(3a^2+b^2)\cosh(4(c+dx))}{32d}+\frac{3(3a^2b+5b^3)\log(\cosh(c+dx))}{d}+\frac{b(3a^2+20b^2)\,\mathrm{sech}^2(c+dx)}{2d}-\frac{15b^3\mathrm{sech}^4(c+dx)}{4d}+\frac{b^3\mathrm{sech}^6(c+dx)}{d}-\frac{b^3\mathrm{sech}^8(c+dx)}{8d}-\frac{a(a^2+12b^2)\sinh(2(c+dx))}{4d}+\frac{a(a^2+3b^2)\sinh(4(c+dx))}{32d}-\frac{108ab^2\tanh(c+dx)}{5d}+\frac{21ab^2\mathrm{sech}^2(c+dx)\tanh(c+dx)}{5d}-\frac{3ab^2\mathrm{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (3*a*(a^2 + 63*b^2)*(c + d*x))/(8*d) - (b*(15*a^2 + 11*b^2)*Cosh[2*(c + d*x
)])/(8*d) + (b*(3*a^2 + b^2)*Cosh[4*(c + d*x)])/(32*d) + (3*(3*a^2*b + 5*b^
3)*Log[Cosh[c + d*x]])/d + (b*(3*a^2 + 20*b^2)*Sech[c + d*x]^2)/(2*d) - (15
*b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]^6)/d - (b^3*Sech[c + d*x]^
8)/(8*d) - (a*(a^2 + 12*b^2)*Sinh[2*(c + d*x)])/(4*d) + (a*(a^2 + 3*b^2)*Si
nh[4*(c + d*x)])/(32*d) - (108*a*b^2*Tanh[c + d*x])/(5*d) + (21*a*b^2*Sech[
c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(
5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(259) = 518$.
time = 3.14, size = 679, normalized size = 2.47

method	result
risch	$-9a^2bx-\frac{15b\,\mathrm{e}^{2dx+2c}a^2}{16d}-\frac{11b^3\mathrm{e}^{-2dx-2c}}{16d}+\frac{b^3\mathrm{e}^{-4dx-4c}}{64d}-15b^3x-\frac{\mathrm{e}^{-4dx-4c}a^3}{64d}-\frac{30b^3c}{d}+\frac{15b^3\ln(1+\mathrm{e}^{2dx+2c})}{d}+3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -9*a^2*b*x-15/16*b/d*exp(2*d*x+2*c)*a^2-11/16*b^3/d*exp(-2*d*x-2*c)+1/64*b^
3/d*exp(-4*d*x-4*c)-15*b^3*x-1/64/d*exp(-4*d*x-4*c)*a^3-30*b^3/d*c+15*b^3/d
*ln(1+exp(2*d*x+2*c))+3/64/d*exp(-4*d*x-4*c)*a^2*b+9*b/d*ln(1+exp(2*d*x+2*c
))*a^2+3/2/d*exp(-2*d*x-2*c)*a*b^2-3/64*b^2/d*exp(-4*d*x-4*c)*a+3/8*a^3*x+1
89/8*a*b^2*x-15/16*b/d*exp(-2*d*x-2*c)*a^2-18/d*a^2*b*c+1/64*b^3/d*exp(4*d*
x+4*c)-11/16*b^3/d*exp(2*d*x+2*c)+1/64/d*exp(4*d*x+4*c)*a^3-1/8/d*exp(2*d*x
+2*c)*a^3+1/8/d*exp(-2*d*x-2*c)*a^3+3/64*b^2/d*exp(4*d*x+4*c)*a-3/2/d*exp(2
*d*x+2*c)*a*b^2+3/64/d*exp(4*d*x+4*c)*a^2*b+2/5*b*(15*a^2*exp(14*d*x+14*c)+
150*a*b*exp(14*d*x+14*c)+100*b^2*exp(14*d*x+14*c)+90*a^2*exp(12*d*x+12*c)+9
00*a*b*exp(12*d*x+12*c)+450*b^2*exp(12*d*x+12*c)+225*a^2*exp(10*d*x+10*c)+2
430*a*b*exp(10*d*x+10*c)+1060*b^2*exp(10*d*x+10*c)+300*a^2*exp(8*d*x+8*c)+3
780*a*b*exp(8*d*x+8*c)+1340*b^2*exp(8*d*x+8*c)+225*a^2*exp(6*d*x+6*c)+3618*
a*b*exp(6*d*x+6*c)+1060*b^2*exp(6*d*x+6*c)+90*a^2*exp(4*d*x+4*c)+2124*a*b*e
xp(4*d*x+4*c)+450*b^2*exp(4*d*x+4*c)+15*a^2*exp(2*d*x+2*c)+714*a*b*exp(2*d*
x+2*c)+100*b^2*exp(2*d*x+2*c)+108*a*b)/d/(1+exp(2*d*x+2*c))^8
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(259) = 518$.

time = 0.50, size = 647, normalized size = 2.35

$$\frac{1}{64}\,a^{3}\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{3}{320}\,ab^{2}\left(\frac{2520\,(dx+c)}{d}+\frac{5\,\left(32\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}\right)}{d}-\frac{135\,e^{(-2\,dx-2\,c)}+5358\,e^{(-4\,dx-4\,c)}+18190\,e^{(-6\,dx-6\,c)}+28455\,e^{(-8\,dx-8\,c)}+19995\,e^{(-10\,dx-10\,c)}+6560\,e^{(-12\,dx-12\,c)}-5}{d(e^{(-4\,dx-4\,c)}+5\,e^{(-6\,dx-6\,c)}+10\,e^{(-8\,dx-8\,c)}+10\,e^{(-10\,dx-10\,c)}+5\,e^{(-12\,dx-12\,c)}+e^{(-14\,dx-14\,c)})}\right)+\frac{1}{64}\,b^{3}\left(\frac{960\,(dx+c)}{d}-\frac{44\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}+\frac{960\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{36\,e^{(-2\,dx-2\,c)}+324\,e^{(-4\,dx-4\,c)}-1384\,e^{(-6\,dx-6\,c)}-9126\,e^{(-8\,dx-8\,c)}-24112\,e^{(-10\,dx-10\,c)}-31868\,e^{(-12\,dx-12\,c)}-25912\,e^{(-14\,dx-14\,c)}-11169\,e^{(-16\,dx-16\,c)}-2516\,e^{(-18\,dx-18\,c)}-1}{d(e^{(-4\,dx-4\,c)}+8\,e^{(-6\,dx-6\,c)}+28\,e^{(-8\,dx-8\,c)}+56\,e^{(-10\,dx-10\,c)}+70\,e^{(-12\,dx-12\,c)}+56\,e^{(-14\,dx-14\,c)}+28\,e^{(-16\,dx-16\,c)}+8\,e^{(-18\,dx-18\,c)}+e^{(-20\,dx-20\,c)})}\right)+\frac{3}{64}\,a^{2}b\left(\frac{192\,(dx+c)}{d}-\frac{20\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}+\frac{192\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{18\,e^{(-2\,dx-2\,c)}+39\,e^{(-4\,dx-4\,c)}-108\,e^{(-6\,dx-6\,c)}-1}{d(e^{(-4\,dx-4\,c)}+2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/320*a*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*d*
x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4
*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x -
10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6
*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) +
e^(-14*d*x - 14*c)))) + 1/64*b^3*(960*(d*x + c)/d - (44*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c))/d + 960*log(e^(-2*d*x - 2*c) + 1)/d - (36*e^(-2*d*x - 2*c
) + 324*e^(-4*d*x - 4*c) - 1384*e^(-6*d*x - 6*c) - 9126*e^(-8*d*x - 8*c) -
24112*e^(-10*d*x - 10*c) - 31868*e^(-12*d*x - 12*c) - 25912*e^(-14*d*x - 14
*c) - 11169*e^(-16*d*x - 16*c) - 2516*e^(-18*d*x - 18*c) - 1)/(d*(e^(-4*d*x
 - 4*c) + 8*e^(-6*d*x - 6*c) + 28*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c)
+ 70*e^(-12*d*x - 12*c) + 56*e^(-14*d*x - 14*c) + 28*e^(-16*d*x - 16*c) + 8
*e^(-18*d*x - 18*c) + e^(-20*d*x - 20*c)))) + 3/64*a^2*b*(192*(d*x + c)/d -
 (20*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)
/d - (18*e^(-2*d*x - 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)
/(d*(e^(-4*d*x - 4*c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 12323 vs. $2(259) = 518$.
time = 0.48, size = 12323, normalized size = 44.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^24 + 120*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^23 + 5*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sinh(d*x + c)^24 - 180*(a^2*b + 2*a*b^2 + b^3)*cosh(d*x + c)^2
2 - 60*(3*a^2*b + 6*a*b^2 + 3*b^3 - 23*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^22 + 440*(23*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^3 - 9*(a^2*b + 2*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^21 -
60*(3*a^3 + 33*a^2*b + 57*a*b^2 + 27*b^3 - 2*(a^3 - 24*a^2*b + 63*a*b^2 - 4
0*b^3)*d*x)*cosh(d*x + c)^20 + 30*(1771*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^4 - 6*a^3 - 66*a^2*b - 114*a*b^2 - 54*b^3 + 4*(a^3 - 24*a^2*b +
63*a*b^2 - 40*b^3)*d*x - 1386*(a^2*b + 2*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^20 + 120*(1771*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 -
```

2310*(a^2*b + 2*a*b^2 + b^3)*cosh(d*x + c)^3 - 10*(3*a^3 + 33*a^2*b + 57*a*
b^2 + 27*b^3 - 2*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c))*s
inh(d*x + c)^19 - 20*(40*a^3 + 297*a^2*b - 354*a*b^2 - 335*b^3 - 48*(a^3 -
24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^18 + 20*(33649*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 65835*(a^2*b + 2*a*b^2 + b^3)*cosh(d
*x + c)^4 - 40*a^3 - 297*a^2*b + 354*a*b^2 + 335*b^3 + 48*(a^3 - 24*a^2*b +
63*a*b^2 - 40*b^3)*d*x - 570*(3*a^3 + 33*a^2*b + 57*a*b^2 + 27*b^3 - 2*(a^
3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^18 +
360*(4807*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 13167*(a^2*b +
2*a*b^2 + b^3)*cosh(d*x + c)^5 - 190*(3*a^3 + 33*a^2*b + 57*a*b^2 + 27*b^3
- 2*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^3 - (40*a^3 + 2
97*a^2*b - 354*a*b^2 - 335*b^3 - 48*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*
x)*cosh(d*x + c))*sinh(d*x + c)^17 - 15*(105*a^3 + 441*a^2*b - 6213*a*b^2 -
2925*b^3 - 224*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^16
+ 15*(245157*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 - 895356*(a^2*
b + 2*a*b^2 + b^3)*cosh(d*x + c)^6 - 19380*(3*a^3 + 33*a^2*b + 57*a*b^2 + 2
7*b^3 - 2*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^4 - 105*a
^3 - 441*a^2*b + 6213*a*b^2 + 2925*b^3 + 224*(a^3 - 24*a^2*b + 63*a*b^2 - 4
0*b^3)*d*x - 204*(40*a^3 + 297*a^2*b - 354*a*b^2 - 335*b^3 - 48*(a^3 - 24*a
^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 80*(8171
9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 - 383724*(a^2*b + 2*a*b^2
+ b^3)*cosh(d*x + c)^7 - 11628*(3*a^3 + 33*a^2*b + 57*a*b^2 + 27*b^3 - 2*(
a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^5 - 204*(40*a^3 + 29
7*a^2*b - 354*a*b^2 - 335*b^3 - 48*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x
)*cosh(d*x + c)^3 - 3*(105*a^3 + 441*a^2*b - 6213*a*b^2 - 2925*b^3 - 224*(a
^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^15 - 4
0*(36*a^3 - 9*a^2*b - 7290*a*b^2 - 2861*b^3 - 168*(a^3 - 24*a^2*b + 63*a*b^
2 - 40*b^3)*d*x)*cosh(d*x + c)^14 + 40*(245157*(a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*cosh(d*x + c)^10 - 1438965*(a^2*b + 2*a*b^2 + b^3)*cosh(d*x + c)^8 - 58
140*(3*a^3 + 33*a^2*b + 57*a*b^2 + 27*b^3 - 2*(a^3 - 24*a^2*b + 63*a*b^2 -
40*b^3)*d*x)*cosh(d*x + c)^6 - 1530*(40*a^3 + 297*a^2*b - 354*a*b^2 - 335*b
^3 - 48*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^4 - 36*a^3
+ 9*a^2*b + 7290*a*b^2 + 2861*b^3 + 168*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3
)*d*x - 45*(105*a^3 + 441*a^2*b - 6213*a*b^2 - 2925*b^3 - 224*(a^3 - 24*a^2
*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 80*(156009
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^11 - 1119195*(a^2*b + 2*a*b^
2 + b^3)*cosh(d*x + c)^9 - 58140*(3*a^3 + 33*a^2*b + 57*a*b^2 + 27*b^3 - 2*
(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^7 - 2142*(40*a^3 +
297*a^2*b - 354*a*b^2 - 335*b^3 - 48*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d
*x)*cosh(d*x + c)^5 - 105*(105*a^3 + 441*a^2*b - 6213*a*b^2 - 2925*b^3 - 22
4*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^3 - 7*(36*a^3 - 9
*a^2*b - 7290*a*b^2 - 2861*b^3 - 168*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d
*x)*cosh(d*x + c))*sinh(d*x + c)^13 + 40*(141*a^2*b + 12096*a*b^2 + 3679*b^
3 + 210*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^12 + 20*(67
6039*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 - 5819814*(a^2*b + 2*

```
a*b^2 + b^3)*cosh(d*x + c)^10 - 377910*(3*a^3 + 33*a^2*b + 57*a*b^2 + 27*b^
3 - 2*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^8 - 18564*(40
*a^3 + 297*a^2*b - 354*a*b^2 - 335*b^3 - 48*(a^3 - 24*a^2*b + 63*a*b^2 - 40
*b^3)*d*x)*cosh(d*x + c)^6 - 1365*(105*a^3 + 441*a^2*b - 6213*a*b^2 - 2925*
b^3 - 224*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^4 + 282*a
^2*b + 24192*a*b^2 + 7358*b^3 + 420*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*
x - 182*(36*a^3 - 9*a^2*b - 7290*a*b^2 - 2861*b^3 - 168*(a^3 - 24*a^2*b + 6
3*a*b^2 - 40*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 80*(156009*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^13 - 1587222*(a^2*b + 2*a*b^2 + b^
3)*cosh(d*x + c)^11 - 125970*(3*a^3 + 33*a^2*b + 57*a*b^2 + 27*b^3 - 2*(a^3
 - 24*a^2*b + 63*a*b^2 - 40*b^3)*d*x)*cosh(d*x ...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 694 vs. 2(259) = 518.
time = 0.93, size = 694, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
[Out] 1/2240*(35*a^3*e^(4*d*x + 4*c) + 105*a^2*b*e^(4*d*x + 4*c) + 105*a*b^2*e^(4
*d*x + 4*c) + 35*b^3*e^(4*d*x + 4*c) - 280*a^3*e^(2*d*x + 2*c) - 2100*a^2*b
*e^(2*d*x + 2*c) - 3360*a*b^2*e^(2*d*x + 2*c) - 1540*b^3*e^(2*d*x + 2*c) +
840*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*(d*x + c) - 35*(18*a^3*e^(4*d*x +
4*c) - 432*a^2*b*e^(4*d*x + 4*c) + 1134*a*b^2*e^(4*d*x + 4*c) - 720*b^3*e^(
4*d*x + 4*c) - 8*a^3*e^(2*d*x + 2*c) + 60*a^2*b*e^(2*d*x + 2*c) - 96*a*b^2*
e^(2*d*x + 2*c) + 44*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*b + 3*a*b^2 - b^3)*e
^(-4*d*x - 4*c) + 6720*(3*a^2*b + 5*b^3)*log(e^(2*d*x + 2*c) + 1) - 8*(6849
*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(16*d*x + 16*c) + 53112*a^2*b*e^(14*
d*x + 14*c) - 16800*a*b^2*e^(14*d*x + 14*c) + 80120*b^3*e^(14*d*x + 14*c) +
 181692*a^2*b*e^(12*d*x + 12*c) - 100800*a*b^2*e^(12*d*x + 12*c) + 269220*b
^3*e^(12*d*x + 12*c) + 358344*a^2*b*e^(10*d*x + 10*c) - 272160*a*b^2*e^(10*
d*x + 10*c) + 520520*b^3*e^(10*d*x + 10*c) + 445830*a^2*b*e^(8*d*x + 8*c) -
 423360*a*b^2*e^(8*d*x + 8*c) + 648970*b^3*e^(8*d*x + 8*c) + 358344*a^2*b*e
^(6*d*x + 6*c) - 405216*a*b^2*e^(6*d*x + 6*c) + 520520*b^3*e^(6*d*x + 6*c)
```

```
+ 181692*a^2*b*e^(4*d*x + 4*c) - 237888*a*b^2*e^(4*d*x + 4*c) + 269220*b^3*
e^(4*d*x + 4*c) + 53112*a^2*b*e^(2*d*x + 2*c) - 79968*a*b^2*e^(2*d*x + 2*c)
 + 80120*b^3*e^(2*d*x + 2*c) + 6849*a^2*b - 12096*a*b^2 + 11415*b^3)/(e^(2*
d*x + 2*c) + 1)^8)/d
```

Mupad [B]
time = 1.70, size = 682, normalized size = 2.48

$$x\left(\frac{3a^3}{8}-9a^2b+\frac{189ab^2}{8}-15b^3\right)-\frac{4\left(71b^3+12ab^2\right)}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}-\frac{256b^3}{d\left(6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1\right)}+\frac{\ln\left(e^{2c}e^{2dx}+1\right)\left(9a^2b+15b^3\right)}{d}+\frac{2\left(3a^2b+30ab^2+20b^3\right)}{d\left(e^{2c+2dx}+1\right)}+\frac{32\left(50b^3+3ab^2\right)}{5d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)}+\frac{128b^3}{d\left(7e^{2c+2dx}+21e^{4c+4dx}+35e^{6c+6dx}+35e^{8c+8dx}+21e^{10c+10dx}+7e^{12c+12dx}+e^{14c+14dx}+1\right)}-\frac{2\left(3a^2b+30ab^2+50b^3\right)}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}-\frac{32b^3}{d\left(8e^{2c+2dx}+28e^{4c+4dx}+56e^{6c+6dx}+70e^{8c+8dx}+56e^{10c+10dx}+28e^{12c+12dx}+8e^{14c+14dx}+e^{16c+16dx}+1\right)}+\frac{e^{4c+4dx}(a+b)^3}{64d}-\frac{e^{-4c-4dx}(a-b)^3}{64d}+\frac{8\left(23b^3+9ab^2\right)}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}-\frac{e^{2c+2dx}(a+b)^2(2a+11b)}{16d}+\frac{e^{-2c-2dx}(a-b)^2(2a-11b)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)^3,x)
```

```
[Out] x*((189*a*b^2)/8 - 9*a^2*b + (3*a^3)/8 - 15*b^3) - (4*(12*a*b^2 + 71*b^3))/
(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
+ 8*d*x) + 1)) - (256*b^3)/(d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 2
0*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c
+ 12*d*x) + 1)) + (log(exp(2*c)*exp(2*d*x) + 1)*(9*a^2*b + 15*b^3))/d + (2*
(30*a*b^2 + 3*a^2*b + 20*b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (32*(3*a*b^2 +
50*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (128*b^3)/(d*(7*exp(2*
c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x
) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1))
 - (2*(30*a*b^2 + 3*a^2*b + 50*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d
*x) + 1)) - (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(
6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c +
12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) + (exp(4*c + 4*d*
x)*(a + b)^3)/(64*d) - (exp(- 4*c - 4*d*x)*(a - b)^3)/(64*d) + (8*(9*a*b^2
+ 23*b^3))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
 1)) - (exp(2*c + 2*d*x)*(a + b)^2*(2*a + 11*b))/(16*d) + (exp(- 2*c - 2*d*
x)*(a - b)^2*(2*a - 11*b))/(16*d)
```

3.66 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=351

$$\frac{15a^2b\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{1155b^3\text{ArcTan}(\sinh(c+dx))}{128d}-\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c+dx)}{d}+\frac{a^3\cosh}{3}$$

```
[Out] 15/2*a^2*b*arctan(sinh(d*x+c))/d+1155/128*b^3*arctan(sinh(d*x+c))/d-a^3*cos
h(d*x+c)/d-12*a*b^2*cosh(d*x+c)/d+1/3*a^3*cosh(d*x+c)^3/d+a*b^2*cosh(d*x+c)
^3/d-18*a*b^2*sech(d*x+c)/d+4*a*b^2*sech(d*x+c)^3/d-3/5*a*b^2*sech(d*x+c)^5
/d-15/2*a^2*b*sinh(d*x+c)/d-1155/128*b^3*sinh(d*x+c)/d+5/2*a^2*b*sinh(d*x+c
)^3/d+385/128*b^3*sinh(d*x+c)^3/d-3/2*a^2*b*sinh(d*x+c)^3*tanh(d*x+c)^2/d-2
31/128*b^3*sinh(d*x+c)^3*tanh(d*x+c)^2/d-33/64*b^3*sinh(d*x+c)^3*tanh(d*x+c
)^4/d-11/48*b^3*sinh(d*x+c)^3*tanh(d*x+c)^6/d-1/8*b^3*sinh(d*x+c)^3*tanh(d*
x+c)^8/d
```

Rubi [A]
time = 0.25, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3747, 2713, 2672, 294, 308, 209, 2670, 276}

$$\frac{a^3\cosh^3(c+dx)}{3d}-\frac{a^3\cosh(c+dx)}{d}+\frac{15a^2b\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{5a^2b\sinh^3(c+dx)}{2d}-\frac{15a^2b\sinh(c+dx)}{2d}-\frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}+\frac{ab^2\cosh^3(c+dx)}{d}-\frac{12ab^2\cosh(c+dx)}{d}-\frac{3ab^2\text{sech}^5(c+dx)}{5d}+\frac{4ab^2\text{sech}^3(c+dx)}{d}-\frac{18ab^2\text{sech}(c+dx)}{d}+\frac{1155b^3\text{ArcTan}(\sinh(c+dx))}{128d}+\frac{385b^3\sinh^3(c+dx)}{128d}-\frac{1155b^3\sinh(c+dx)}{128d}-\frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d}-\frac{11b^3\sinh^3(c+dx)\tanh^6(c+dx)}{48d}-\frac{33b^3\sinh^3(c+dx)\tanh^4(c+dx)}{64d}-\frac{231b^3\sinh^3(c+dx)\tanh^2(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (15*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (1155*b^3*ArcTan[Sinh[c + d*x]])/(
128*d) - (a^3*Cosh[c + d*x])/d - (12*a*b^2*Cosh[c + d*x])/d + (a^3*Cosh[c +
 d*x]^3)/(3*d) + (a*b^2*Cosh[c + d*x]^3)/d - (18*a*b^2*Sech[c + d*x])/d + (
4*a*b^2*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c + d*x]^5)/(5*d) - (15*a^2*b*Si
nh[c + d*x])/(2*d) - (1155*b^3*Sinh[c + d*x])/(128*d) + (5*a^2*b*Sinh[c + d
*x]^3)/(2*d) + (385*b^3*Sinh[c + d*x]^3)/(128*d) - (3*a^2*b*Sinh[c + d*x]^3
*Tanh[c + d*x]^2)/(2*d) - (231*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/(128*d)
 - (33*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^4)/(64*d) - (11*b^3*Sinh[c + d*x]^
3*Tanh[c + d*x]^6)/(48*d) - (b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^8)/(8*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = i\int\left(-ia^3\sinh^3(c+dx)-3ia^2b\sinh^3(c+dx)\tanh^3(c+dx)\right.$$

$$= a^3\int\sinh^3(c+dx)\,dx+\left(3a^2b\right)\int\sinh^3(c+dx)\tanh^3(c+dx$$

$$= -\frac{a^3\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cosh(c+dx)\right)}{d}+\frac{\left(3a^2b\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a^3\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}-\frac{3a^2b\sinh^3(c+dx)\,\text{ta}}{2d}$$

$$= -\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+$$

$$= -\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+$$

$$= \frac{15a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c}{d}$$

$$= \frac{15a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c}{d}$$

$$= \frac{15a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\cosh(c+dx)}{d}-\frac{12ab^2\cosh(c}{d}$$

$$= \frac{15a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{1155b^3\tan^{-1}(\sinh(c+dx))}{128d}-\frac{a}{}$$

Mathematica [A]
time = 6.45, size = 291, normalized size = 0.83

$$\frac{15b(64a^2+77b^2)\,\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}-\frac{3a(a^2+15b^2)\cosh(c+dx)}{4d}+\frac{a(a^2+3b^2)\cosh(3(c+dx))}{12d}-\frac{18ab^2\text{sech}(c+dx)}{d}+\frac{4ab^2\text{sech}^3(c+dx)}{d}-\frac{3ab^2\text{sech}^5(c+dx)}{5d}-\frac{3b(9a^2+7b^2)\sinh(c+dx)}{4d}-\frac{3\text{sech}^2(c+dx)\left(64a^2b\sinh(c+dx)+255b^3\sinh(c+dx)\right)}{128d}+\frac{b(3a^2+b^2)\sinh(3(c+dx))}{12d}+\frac{515b^3\text{sech}^3(c+dx)\tanh(c+dx)}{192d}-\frac{41b^3\text{sech}^5(c+dx)\tanh(c+dx)}{48d}+\frac{b^3\text{sech}^7(c+dx)\tanh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (15*b*(64*a^2 + 77*b^2)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (3*a*(a^2 + 15*
b^2)*Cosh[c + d*x])/(4*d) + (a*(a^2 + 3*b^2)*Cosh[3*(c + d*x)])/(12*d) - (1
8*a*b^2*Sech[c + d*x])/d + (4*a*b^2*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c +
d*x]^5)/(5*d) - (3*b*(9*a^2 + 7*b^2)*Sinh[c + d*x])/(4*d) - (3*Sech[c + d*x
]^2*(64*a^2*b*Sinh[c + d*x] + 255*b^3*Sinh[c + d*x]))/(128*d) + (b*(3*a^2 +
 b^2)*Sinh[3*(c + d*x)])/(12*d) + (515*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(
192*d) - (41*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d) + (b^3*Sech[c + d*x]
^7*Tanh[c + d*x])/(8*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.84, size = 675, normalized size = 1.92

method	result
risch	$\frac{e^{3dx+3c}a^3}{24d} + \frac{e^{3dx+3c}a^2b}{8d} + \frac{a\,b^2 e^{3dx+3c}}{8d} + \frac{e^{3dx+3c}b^3}{24d} - \frac{3\,e^{dx+c}a^3}{8d} - \frac{27\,e^{dx+c}a^2b}{8d} - \frac{45a\,e^{dx+c}b^2}{8d} - \frac{21b^3 e^{dx+c}}{8d} - \frac{3\,e^{-dx-}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*a^3+1/8/d*exp(3*d*x+3*c)*a^2*b+1/8*a*b^2/d*exp(3*d*x+
3*c)+1/24/d*exp(3*d*x+3*c)*b^3-3/8/d*exp(d*x+c)*a^3-27/8/d*exp(d*x+c)*a^2*b
-45/8*a/d*exp(d*x+c)*b^2-21/8*b^3/d*exp(d*x+c)-3/8/d*exp(-d*x-c)*a^3+27/8/d
*exp(-d*x-c)*a^2*b-45/8*a/d*exp(-d*x-c)*b^2+21/8/d*exp(-d*x-c)*b^3+1/24/d*e
xp(-3*d*x-3*c)*a^3-1/8/d*exp(-3*d*x-3*c)*a^2*b+1/8*a*b^2/d*exp(-3*d*x-3*c)-
1/24/d*exp(-3*d*x-3*c)*b^3-1/960*b*exp(d*x+c)*(2880*a^2*exp(14*d*x+14*c)+34
560*a*b*exp(14*d*x+14*c)+11475*b^2*exp(14*d*x+14*c)+14400*a^2*exp(12*d*x+12
*c)+211200*a*b*exp(12*d*x+12*c)+36775*b^2*exp(12*d*x+12*c)+25920*a^2*exp(10
*d*x+10*c)+590592*a*b*exp(10*d*x+10*c)+67715*b^2*exp(10*d*x+10*c)+14400*a^2
*exp(8*d*x+8*c)+957696*a*b*exp(8*d*x+8*c)+27055*b^2*exp(8*d*x+8*c)-14400*a^
2*exp(6*d*x+6*c)+957696*a*b*exp(6*d*x+6*c)-27055*b^2*exp(6*d*x+6*c)-25920*a
^2*exp(4*d*x+4*c)+590592*a*b*exp(4*d*x+4*c)-67715*b^2*exp(4*d*x+4*c)-14400*
a^2*exp(2*d*x+2*c)+211200*a*b*exp(2*d*x+2*c)-36775*b^2*exp(2*d*x+2*c)-2880*
a^2+34560*a*b-11475*b^2)/d/(1+exp(2*d*x+2*c))^8-1155/128*I*b^3/d*ln(exp(d*x
+c)-I)+1155/128*I*b^3/d*ln(exp(d*x+c)+I)-15/2*I*b/d*ln(exp(d*x+c)-I)*a^2+15
/2*I*b/d*ln(exp(d*x+c)+I)*a^2
```

Maxima [A]
time = 0.49, size = 604, normalized size = 1.72

$$\frac{1}{192}b^3\left(\frac{8\left(63\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}-\frac{3465\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{440\,e^{(-2\,dx-2\,c)}+6103\,e^{(-4\,dx-4\,c)}+21019\,e^{(-6\,dx-6\,c)}+41207\,e^{(-8\,dx-8\,c)}+40243\,e^{(-10\,dx-10\,c)}+22589\,e^{(-12\,dx-12\,c)}+505\,e^{(-14\,dx-14\,c)}-3331\,e^{(-16\,dx-16\,c)}-1791\,e^{(-18\,dx-18\,c)}-8}{d\left(e^{(-3\,dx-3\,c)}+8\,e^{(-5\,dx-5\,c)}+28\,e^{(-7\,dx-7\,c)}+56\,e^{(-9\,dx-9\,c)}+70\,e^{(-11\,dx-11\,c)}+56\,e^{(-13\,dx-13\,c)}+28\,e^{(-15\,dx-15\,c)}+8\,e^{(-17\,dx-17\,c)}+e^{(-19\,dx-19\,c)}\right)}\right)-\frac{1}{40}ab^2\left(\frac{5\left(45\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}+\frac{200\,e^{(-2\,dx-2\,c)}+2515\,e^{(-4\,dx-4\,c)}+6680\,e^{(-6\,dx-6\,c)}+9073\,e^{(-8\,dx-8\,c)}+5600\,e^{(-10\,dx-10\,c)}+1665\,e^{(-12\,dx-12\,c)}-5}{d\left(e^{(-3\,dx-3\,c)}+5\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+10\,e^{(-9\,dx-9\,c)}+5\,e^{(-11\,dx-11\,c)}+e^{(-13\,dx-13\,c)}\right)}\right)+\frac{1}{8}a^2b\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)}\right)}\right)+\frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/192*b^3*(8*(63*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 3465*arctan(e^(-d*x -
 c))/d - (440*e^(-2*d*x - 2*c) + 6103*e^(-4*d*x - 4*c) + 21019*e^(-6*d*x -
6*c) + 41207*e^(-8*d*x - 8*c) + 40243*e^(-10*d*x - 10*c) + 22589*e^(-12*d*x
 - 12*c) + 505*e^(-14*d*x - 14*c) - 3331*e^(-16*d*x - 16*c) - 1791*e^(-18*d
*x - 18*c) - 8)/(d*(e^(-3*d*x - 3*c) + 8*e^(-5*d*x - 5*c) + 28*e^(-7*d*x -
7*c) + 56*e^(-9*d*x - 9*c) + 70*e^(-11*d*x - 11*c) + 56*e^(-13*d*x - 13*c)
+ 28*e^(-15*d*x - 15*c) + 8*e^(-17*d*x - 17*c) + e^(-19*d*x - 19*c)))) - 1/
40*a*b^2*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*c)
+ 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c) + 5
600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x - 3*c)
```

```
+ 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*e^(-11
*d*x - 11*c) + e^(-13*d*x - 13*c)))) + 1/8*a^2*b*((27*e^(-d*x - c) - e^(-3*
d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(-
4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x -
 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d
 - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8462 vs. $2(325) = 650$.
time = 0.44, size = 8462, normalized size = 24.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/960*(40*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^22 + 880*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^21 + 40*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*sinh(d*x + c)^22 - 40*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*
cosh(d*x + c)^20 - 40*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3 - 231*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^20 + 800*(77*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 - (a^3 + 57*a^2*b + 111*a*b^2 + 55*b
^3)*cosh(d*x + c))*sinh(d*x + c)^19 - 5*(424*a^3 + 4440*a^2*b + 15960*a*b^2
 + 5599*b^3)*cosh(d*x + c)^18 + 5*(58520*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
sh(d*x + c)^4 - 424*a^3 - 4440*a^2*b - 15960*a*b^2 - 5599*b^3 - 1520*(a^3 +
 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^18 + 30*(351
12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 - 1520*(a^3 + 57*a^2*b +
 111*a*b^2 + 55*b^3)*cosh(d*x + c)^3 - 3*(424*a^3 + 4440*a^2*b + 15960*a*b^
2 + 5599*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 15*(712*a^3 + 4840*a^2*b +
26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^16 + 15*(198968*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*cosh(d*x + c)^6 - 12920*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*co
sh(d*x + c)^4 - 712*a^3 - 4840*a^2*b - 26584*a*b^2 - 5665*b^3 - 51*(424*a^3
 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 +
 240*(28424*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 2584*(a^3 + 5
7*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^5 - 17*(424*a^3 + 4440*a^2*b +
15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^3 - (712*a^3 + 4840*a^2*b + 26584*a*b
^2 + 5665*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 - 3*(9040*a^3 + 36400*a^2*b
+ 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^14 + 3*(4263600*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*cosh(d*x + c)^8 - 516800*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^
3)*cosh(d*x + c)^6 - 5100*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*c
osh(d*x + c)^4 - 9040*a^3 - 36400*a^2*b - 344944*a*b^2 - 45265*b^3 - 600*(7
12*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^14 + 2*(9948400*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 - 1550400
*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^7 - 21420*(424*a^3 + 4
440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^5 - 4200*(712*a^3 + 4840*
```

```
a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^3 - 21*(9040*a^3 + 36400*a^2*
b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 3*(14000*a^
3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*cosh(d*x + c)^12 + (25865840*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 - 5038800*(a^3 + 57*a^2*b +
111*a*b^2 + 55*b^3)*cosh(d*x + c)^8 - 92820*(424*a^3 + 4440*a^2*b + 15960*a
*b^2 + 5599*b^3)*cosh(d*x + c)^6 - 27300*(712*a^3 + 4840*a^2*b + 26584*a*b^
2 + 5665*b^3)*cosh(d*x + c)^4 - 42000*a^3 - 56400*a^2*b - 1628016*a*b^2 - 6
1215*b^3 - 273*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^12 + 4*(7054320*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c)^11 - 1679600*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^
9 - 39780*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^7 -
 16380*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^5 - 27
3*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^3 - 9*(
14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^11 - 3*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 20405*b^3)*cosh(d*x
+ c)^10 + (25865840*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 - 7390
240*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(d*x + c)^10 - 218790*(424*a^
3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*x + c)^8 - 120120*(712*a^3
+ 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x + c)^6 - 3003*(9040*a^3 + 3
6400*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x + c)^4 - 42000*a^3 + 56400*
a^2*b - 1628016*a*b^2 + 61215*b^3 - 198*(14000*a^3 + 18800*a^2*b + 542672*a
*b^2 + 20405*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(9948400*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^13 - 3359200*(a^3 + 57*a^2*b + 111*a*b^
2 + 55*b^3)*cosh(d*x + c)^11 - 121550*(424*a^3 + 4440*a^2*b + 15960*a*b^2 +
 5599*b^3)*cosh(d*x + c)^9 - 85800*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 56
65*b^3)*cosh(d*x + c)^7 - 3003*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 452
65*b^3)*cosh(d*x + c)^5 - 330*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 204
05*b^3)*cosh(d*x + c)^3 - 15*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 2040
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - 3*(9040*a^3 - 36400*a^2*b + 344944*
a*b^2 - 45265*b^3)*cosh(d*x + c)^8 + 3*(4263600*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^14 - 1679600*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*cosh(
d*x + c)^12 - 72930*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*cosh(d*
x + c)^10 - 64350*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*cosh(d*x
+ c)^8 - 3003*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*cosh(d*x
+ c)^6 - 495*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*cosh(d*x
+ c)^4 - 9040*a^3 + 36400*a^2*b - 344944*a*b^2 + 45265*b^3 - 45*(14000*a^3
- 18800*a^2*b + 542672*a*b^2 - 20405*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8
+ 24*(284240*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.82, size = 580, normalized size = 1.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/960*(40*a^3*e^(3*d*x + 3*c) + 120*a^2*b*e^(3*d*x + 3*c) + 120*a*b^2*e^(3*
d*x + 3*c) + 40*b^3*e^(3*d*x + 3*c) - 360*a^3*e^(d*x + c) - 3240*a^2*b*e^(d
*x + c) - 5400*a*b^2*e^(d*x + c) - 2520*b^3*e^(d*x + c) + 225*(64*a^2*b + 7
7*b^3)*arctan(e^(d*x + c)) - 40*(9*a^3*e^(2*d*x + 2*c) - 81*a^2*b*e^(2*d*x
+ 2*c) + 135*a*b^2*e^(2*d*x + 2*c) - 63*b^3*e^(2*d*x + 2*c) - a^3 + 3*a^2*b
 - 3*a*b^2 + b^3)*e^(-3*d*x - 3*c) - (2880*a^2*b*e^(15*d*x + 15*c) + 34560*
a*b^2*e^(15*d*x + 15*c) + 11475*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*e^(13*d
*x + 13*c) + 211200*a*b^2*e^(13*d*x + 13*c) + 36775*b^3*e^(13*d*x + 13*c) +
 25920*a^2*b*e^(11*d*x + 11*c) + 590592*a*b^2*e^(11*d*x + 11*c) + 67715*b^3
*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) + 957696*a*b^2*e^(9*d*x +
9*c) + 27055*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) + 957696*a*b
^2*e^(7*d*x + 7*c) - 27055*b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^(5*d*x + 5*c
) + 590592*a*b^2*e^(5*d*x + 5*c) - 67715*b^3*e^(5*d*x + 5*c) - 14400*a^2*b*
e^(3*d*x + 3*c) + 211200*a*b^2*e^(3*d*x + 3*c) - 36775*b^3*e^(3*d*x + 3*c)
- 2880*a^2*b*e^(d*x + c) + 34560*a*b^2*e^(d*x + c) - 11475*b^3*e^(d*x + c))
/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B]
time = 1.60, size = 757, normalized size = 2.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)^3,x)
```

```
[Out] (exp(3*c + 3*d*x)*(a + b)^3)/(24*d) + (exp(- 3*c - 3*d*x)*(a - b)^3)/(24*d)
 + (15*atan((exp(d*x)*exp(c)*(77*b^3*(d^2)^(1/2) + 64*a^2*b*(d^2)^(1/2)))/(
d*(5929*b^6 + 9856*a^2*b^4 + 4096*a^4*b^2)^(1/2)))*(5929*b^6 + 9856*a^2*b^4
 + 4096*a^4*b^2)^(1/2))/(64*(d^2)^(1/2)) - (3*exp(- c - d*x)*(a - b)^2*(a -
 7*b))/(8*d) - (exp(c + d*x)*(6144*a*b^2 + 11005*b^3))/(120*d*(3*exp(2*c +
2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(768*a
*b^2 + 3365*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*
c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (596*b^3*exp(c + d*x))/(3*d*(6*exp(2*
c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x
```

```
) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (3*exp(c + d*x)*(a +
b)^2*(a + 7*b))/(8*d) - (3*exp(c + d*x)*(768*a*b^2 + 64*a^2*b + 255*b^3))/(
64*d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(144*a*b^2 + 1625*b^3))/(15*
d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8
*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (112*b^3*exp(c + d*x))/(d*(7*exp(2
*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*
x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)
) + (exp(c + d*x)*(3072*a*b^2 + 576*a^2*b + 4355*b^3))/(96*d*(2*exp(2*c + 2
*d*x) + exp(4*c + 4*d*x) + 1)) + (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*
x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*e
xp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c
 + 16*d*x) + 1))
```

3.67 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=220

$$-\frac{1}{2}a\left(a^2+21b^2\right)x-\frac{b(6a^2+5b^2)\log(\cosh(c+dx))}{d}+\frac{9ab^2\tanh(c+dx)}{d}+\frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d}+\frac{2ab^2\mathrm{t}}{}$$

```
[Out] -1/2*a*(a^2+21*b^2)*x-b*(6*a^2+5*b^2)*ln(cosh(d*x+c))/d+9*a*b^2*tanh(d*x+c)
/d+1/2*b*(3*a^2+4*b^2)*tanh(d*x+c)^2/d+2*a*b^2*tanh(d*x+c)^3/d+3/4*b^3*tanh
(d*x+c)^4/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/3*b^3*tanh(d*x+c)^6/d+1/8*b^3*tanh(
d*x+c)^8/d+1/2*cosh(d*x+c)*sinh(d*x+c)*(a*(a^2+3*b^2)+b*(3*a^2+b^2)*tanh(d*
x+c))/d
```

Rubi [A]
time = 0.22, antiderivative size = 241, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 1818, 1816, 647, 31}

$$\frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d}+\frac{a(a^2+21b^2)\tanh(c+dx)}{2d}+\frac{\sinh^2(c+dx)\left(a(a^2+3b^2)\tanh(c+dx)+b(3a^2+b^2)\right)}{2d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{2ab^2\tanh^3(c+dx)}{d}+\frac{(a+b)^2(a+10b)\log(1-\tanh(c+dx))}{4d}-\frac{(a-10b)(a-b)^2\log(\tanh(c+dx)+1)}{4d}+\frac{b^3\tanh^8(c+dx)}{8d}+\frac{b^3\tanh^6(c+dx)}{3d}+\frac{3b^3\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] ((a + b)^2*(a + 10*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 10*b)*(a - b)^2
*Log[1 + Tanh[c + d*x]])/(4*d) + (a*(a^2 + 21*b^2)*Tanh[c + d*x])/(2*d) + (
b*(3*a^2 + 4*b^2)*Tanh[c + d*x]^2)/(2*d) + (2*a*b^2*Tanh[c + d*x]^3)/d + (3
*b^3*Tanh[c + d*x]^4)/(4*d) + (3*a*b^2*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c
 + d*x]^6)/(3*d) + (b^3*Tanh[c + d*x]^8)/(8*d) + (Sinh[c + d*x]^2*(b*(3*a^2
 + b^2) + a*(a^2 + 3*b^2)*Tanh[c + d*x]))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^3)^3}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\sinh^2(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{2d} + \frac{\text{Subs}}{}$$

$$= \frac{\sinh^2(c+dx)\left(b(3a^2+b^2)+a(a^2+3b^2)\tanh(c+dx)\right)}{2d} + \frac{\text{Subs}}{}$$

$$= \frac{a(a^2+21b^2)\tanh(c+dx)}{2d} + \frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d} + \frac{2ab}{}$$

$$= \frac{a(a^2+21b^2)\tanh(c+dx)}{2d} + \frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d} + \frac{2ab}{}$$

$$= \frac{(a+b)^2(a+10b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-10b)(a-b)^2\,\text{lo}}{4}$$

Mathematica [A]
time = 6.19, size = 244, normalized size = 1.11

$$-\frac{a(a^2+21b^2)(c+dx)}{2d} + \frac{b(3a^2+b^2)\cosh(2(c+dx))}{4d} + \frac{(-6a^2b-5b^3)\log(\cosh(c+dx))}{d} - \frac{b(3a^2+10b^2)\operatorname{sech}^2(c+dx)}{2d} + \frac{5b^3\operatorname{sech}^4(c+dx)}{2d} - \frac{5b^3\operatorname{sech}^6(c+dx)}{6d} + \frac{b^3\operatorname{sech}^8(c+dx)}{8d} + \frac{a(a^2+3b^2)\sinh(2(c+dx))}{4d} + \frac{58ab^2\tanh(c+dx)}{5d} - \frac{16ab^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d} + \frac{3ab^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] -1/2*(a*(a^2 + 21*b^2)*(c + d*x))/d + (b*(3*a^2 + b^2)*Cosh[2*(c + d*x)])/(
4*d) + ((-6*a^2*b - 5*b^3)*Log[Cosh[c + d*x]])/d - (b*(3*a^2 + 10*b^2)*Sech
[c + d*x]^2)/(2*d) + (5*b^3*Sech[c + d*x]^4)/(2*d) - (5*b^3*Sech[c + d*x]^6
)/(6*d) + (b^3*Sech[c + d*x]^8)/(8*d) + (a*(a^2 + 3*b^2)*Sinh[2*(c + d*x)])
/(4*d) + (58*a*b^2*Tanh[c + d*x])/(5*d) - (16*a*b^2*Sech[c + d*x]^2*Tanh[c
+ d*x])/(5*d) + (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

Maple [A]
time = 1.95, size = 206, normalized size = 0.94

method	result
derivativedivides	$\frac{a^{3}\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^{2}b\left(\frac{\sinh^{4}(dx+c)}{2\cosh(dx+c)^{2}}-2\ln(\cosh(dx+c))+\tanh^{2}(dx+c)\right)+3a\,b^{2}\left(\frac{\sinh^{7}(dx+c)}{2\cosh(dx+c)^{5}}-\frac{7dx}{2}\right.}{}$
default	$\frac{a^{3}\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^{2}b\left(\frac{\sinh^{4}(dx+c)}{2\cosh(dx+c)^{2}}-2\ln(\cosh(dx+c))+\tanh^{2}(dx+c)\right)+3a\,b^{2}\left(\frac{\sinh^{7}(dx+c)}{2\cosh(dx+c)^{5}}-\frac{7dx}{2}\right.}{}$
risch	$-\frac{a^{3}x}{2}+6a^{2}bx-\frac{21a\,b^{2}x}{2}+5b^{3}x+\frac{\mathrm{e}^{2dx+2c}a^{3}}{8d}+\frac{3b\,\mathrm{e}^{2dx+2c}a^{2}}{8d}+\frac{3\,\mathrm{e}^{2dx+2c}a\,b^{2}}{8d}+\frac{b^{3}\mathrm{e}^{2dx+2c}}{8d}-\frac{\mathrm{e}^{-2dx-2}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sinh(d*x+c)*cosh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(1/2*sinh(d*x+
c)^4/cosh(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2)+3*a*b^2*(1/2*sinh(d*x+c
)^7/cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/10*tanh
(d*x+c)^5)+b^3*(1/2*sinh(d*x+c)^10/cosh(d*x+c)^8-5*ln(cosh(d*x+c))+5/2*tanh
(d*x+c)^2+5/4*tanh(d*x+c)^4+5/6*tanh(d*x+c)^6+5/8*tanh(d*x+c)^8))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(206) = 412.
time = 0.51, size = 544, normalized size = 2.47

$$-\frac{1}{8}a^{3}\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{40}ab^{2}\left(\frac{420(dx+c)}{d}+\frac{15e^{(-2dx-2c)}}{d}-\frac{1003e^{(-2dx-2c)}+3350e^{(-4dx-4c)}+5590e^{(-6dx-6c)}+3915e^{(-8dx-8c)}+1455e^{(-10dx-10c)}+15}{d(e^{(-2dx-2c)}+5e^{(-4dx-4c)}+10e^{(-6dx-6c)}+10e^{(-8dx-8c)}+5e^{(-10dx-10c)}+e^{(-12dx-12c)})}\right)-\frac{1}{24}b^{3}\left(\frac{120(dx+c)}{d}-\frac{3e^{(-2dx-2c)}}{d}+\frac{120\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{24e^{(-2dx-2c)}-396e^{(-4dx-4c)}-1752e^{(-6dx-6c)}-4430e^{(-8dx-8c)}-5464e^{(-10dx-10c)}-4556e^{(-12dx-12c)}-1896e^{(-14dx-14c)}-477e^{(-16dx-16c)}+3}{d(e^{(-2dx-2c)}+8e^{(-4dx-4c)}+28e^{(-6dx-6c)}+56e^{(-8dx-8c)}+70e^{(-10dx-10c)}+56e^{(-12dx-12c)}+28e^{(-14dx-14c)}+8e^{(-16dx-16c)}+e^{(-18dx-18c)})}\right)-\frac{3}{8}a^{2}b\left(\frac{16(dx+c)}{d}-\frac{e^{(-2dx-2c)}}{d}+\frac{16\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{2e^{(-2dx-2c)}-15e^{(-4dx-4c)}+1}{d(e^{(-2dx-2c)}+2e^{(-4dx-4c)}+e^{(-6dx-6c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/40*a*b^2*(420*(
d*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d*
x - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*x
- 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*
c) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))) - 1
```

```
/24*b^3*(120*(d*x + c)/d - 3*e^(-2*d*x - 2*c)/d + 120*log(e^(-2*d*x - 2*c)
+ 1)/d - (24*e^(-2*d*x - 2*c) - 396*e^(-4*d*x - 4*c) - 1752*e^(-6*d*x - 6*c
) - 4430*e^(-8*d*x - 8*c) - 5464*e^(-10*d*x - 10*c) - 4556*e^(-12*d*x - 12*
c) - 1896*e^(-14*d*x - 14*c) - 477*e^(-16*d*x - 16*c) + 3)/(d*(e^(-2*d*x -
2*c) + 8*e^(-4*d*x - 4*c) + 28*e^(-6*d*x - 6*c) + 56*e^(-8*d*x - 8*c) + 70*
e^(-10*d*x - 10*c) + 56*e^(-12*d*x - 12*c) + 28*e^(-14*d*x - 14*c) + 8*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c)))) - 3/8*a^2*b*(16*(d*x + c)/d - e^(-2*
d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d - (2*e^(-2*d*x - 2*c) - 15*e^
(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*e^(-4*d*x - 4*c) + e^(-6*d*x -
 6*c))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9862 vs. $2(206) = 412$.
time = 0.45, size = 9862, normalized size = 44.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^20 + 300*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^19 + 15*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*sinh(d*x + c)^20 + 60*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (
a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^18 + 30*(4*a^3 + 12*
a^2*b + 12*a*b^2 + 4*b^3 - 2*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x + 95*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^18 + 180*(95
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 6*(2*a^3 + 6*a^2*b + 6*a
*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c))*sin
h(d*x + c)^17 + 15*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*
a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^16 + 15*(4845*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3
- 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x + 612*(2*a^3 + 6*a^2*b + 6*a*
b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^2)*si
nh(d*x + c)^16 + 240*(969*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 +
 204*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b
^3)*d*x)*cosh(d*x + c)^3 + (27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a
^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^15 + 2
40*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10
*b^3)*d*x)*cosh(d*x + c)^14 + 120*(4845*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^6 + 1530*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b +
21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^4 + 6*a^3 - 12*a^2*b - 186*a*b^2 - 72
*b^3 - 14*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x + 15*(27*a^3 + 39*a^2*b
- 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d
*x + c)^2)*sinh(d*x + c)^14 + 240*(4845*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^7 + 2142*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b +
```

```
21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^5 + 35*(27*a^3 + 39*a^2*b - 207*a*b^2
 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^3 +
 14*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 1
0*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^13 + 10*(63*a^3 - 639*a^2*b - 6195
*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x
+ c)^12 + 10*(188955*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 1113
84*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3
)*d*x)*cosh(d*x + c)^6 + 2730*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32
*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^4 + 63*a^3 - 639*a
^2*b - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x
 + 2184*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2
 - 10*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 120*(20995*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 15912*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*
b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^7 + 546*(27*a
^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^
3)*d*x)*cosh(d*x + c)^5 + 728*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3
 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^3 + (63*a^3 - 639*a^2*b
 - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^11 - 40*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*
(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^10 + 20*(138567*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 131274*(2*a^3 + 6*a^2*b + 6
*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^8
+ 6006*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a
*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^6 + 12012*(3*a^3 - 6*a^2*b - 93*a*b^2 - 3
6*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^4 - 468*a
^2*b - 4872*a*b^2 - 1324*b^3 - 210*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x
 + 33*(63*a^3 - 639*a^2*b - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 2
1*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 40*(62985*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^11 + 72930*(2*a^3 + 6*a^2*b + 6*a*b
^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^9 + 42
90*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2
 - 10*b^3)*d*x)*cosh(d*x + c)^7 + 12012*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^
3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^5 + 55*(63*a^
3 - 639*a^2*b - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10
*b^3)*d*x)*cosh(d*x + c)^3 - 10*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^
3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 2*(
315*a^3 + 3195*a^2*b + 46977*a*b^2 + 10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*
a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^8 + 2*(944775*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*cosh(d*x + c)^12 + 1312740*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3
- 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^10 + 96525*(27*a^3 + 39*
a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*
cosh(d*x + c)^8 + 360360*(3*a^3 - 6*a^2*b - 93*...
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(206) = 412.
time = 0.78, size = 577, normalized size = 2.62

$$\frac{105 a^3 e^{(2 dx+2 c)} + 315 a^2 b e^{(2 dx+2 c)} + 315 a b^2 e^{(2 dx+2 c)} + 105 b^3 e^{(2 dx+2 c)} - 420 (a^3 - 12 a^2 b + 21 a b^2 - 10 b^3)(dx+c) + 105 (2 a^3 e^{(2 dx+2 c)} - 24 a^2 b e^{(2 dx+2 c)} + 42 a b^2 e^{(2 dx+2 c)} - 20 b^3 e^{(2 dx+2 c)} - a^3 + 3 a^2 b - 3 a b^2 + b^3) e^{(-2 dx-2 c)} - 840 (6 a^2 b + 5 b^3) \log(e^{(2 dx+2 c)} + 1) + \frac{13698 a^2 b e^{(16 dx+16 c)} + 11415 b^3 e^{(16 dx+16 c)} + 104544 a^2 b e^{(14 dx+14 c)} - 30240 a b^2 e^{(14 dx+14 c)} + 74520 b^3 e^{(14 dx+14 c)} + 353304 a^2 b e^{(12 dx+12 c)} - 171360 a b^2 e^{(12 dx+12 c)} + 252420 b^3 e^{(12 dx+12 c)} + 691488 a^2 b e^{(10 dx+10 c)} - 446880 a b^2 e^{(10 dx+10 c)} + 476840 b^3 e^{(10 dx+10 c)} + 858060 a^2 b e^{(8 dx+8 c)} - 682080 a b^2 e^{(8 dx+8 c)} + 601930 b^3 e^{(8 dx+8 c)} + 691488 a^2 b e^{(6 dx+6 c)} - 644448 a b^2 e^{(6 dx+6 c)} + 476840 b^3 e^{(6 dx+6 c)} + 353304 a^2 b e^{(4 dx+4 c)} - 374304 a b^2 e^{(4 dx+4 c)} + 252420 b^3 e^{(4 dx+4 c)} + 104544 a^2 b e^{(2 dx+2 c)} - 125664 a b^2 e^{(2 dx+2 c)} + 74520 b^3 e^{(2 dx+2 c)} + 13698 a^2 b - 19488 a b^2 + 11415 b^3}{(e^{(2 dx+2 c)} + 1)^8}}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/840*(105*a^3*e^(2*d*x + 2*c) + 315*a^2*b*e^(2*d*x + 2*c) + 315*a*b^2*e^(2
*d*x + 2*c) + 105*b^3*e^(2*d*x + 2*c) - 420*(a^3 - 12*a^2*b + 21*a*b^2 - 10
*b^3)*(d*x + c) + 105*(2*a^3*e^(2*d*x + 2*c) - 24*a^2*b*e^(2*d*x + 2*c) + 4
2*a*b^2*e^(2*d*x + 2*c) - 20*b^3*e^(2*d*x + 2*c) - a^3 + 3*a^2*b - 3*a*b^2
+ b^3)*e^(-2*d*x - 2*c) - 840*(6*a^2*b + 5*b^3)*log(e^(2*d*x + 2*c) + 1) +
(13698*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(16*d*x + 16*c) + 104544*a^2*b
*e^(14*d*x + 14*c) - 30240*a*b^2*e^(14*d*x + 14*c) + 74520*b^3*e^(14*d*x +
14*c) + 353304*a^2*b*e^(12*d*x + 12*c) - 171360*a*b^2*e^(12*d*x + 12*c) + 2
52420*b^3*e^(12*d*x + 12*c) + 691488*a^2*b*e^(10*d*x + 10*c) - 446880*a*b^2
*e^(10*d*x + 10*c) + 476840*b^3*e^(10*d*x + 10*c) + 858060*a^2*b*e^(8*d*x +
 8*c) - 682080*a*b^2*e^(8*d*x + 8*c) + 601930*b^3*e^(8*d*x + 8*c) + 691488*
a^2*b*e^(6*d*x + 6*c) - 644448*a*b^2*e^(6*d*x + 6*c) + 476840*b^3*e^(6*d*x
+ 6*c) + 353304*a^2*b*e^(4*d*x + 4*c) - 374304*a*b^2*e^(4*d*x + 4*c) + 2524
20*b^3*e^(4*d*x + 4*c) + 104544*a^2*b*e^(2*d*x + 2*c) - 125664*a*b^2*e^(2*d
*x + 2*c) + 74520*b^3*e^(2*d*x + 2*c) + 13698*a^2*b - 19488*a*b^2 + 11415*b
^3)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B]
time = 0.56, size = 617, normalized size = 2.80

$$\frac{8 (29 b^3 + 6 a b^2)}{d (4 e^{2c+2dx} + 6 e^{4c+4dx} + 4 e^{6c+6dx} + e^{8c+8dx} + 1)} + \frac{736 b^3}{3 d (6 e^{2c+2dx} + 15 e^{4c+4dx} + 20 e^{6c+6dx} + 15 e^{8c+8dx} + 6 e^{10c+10dx} + e^{12c+12dx} + 1)} - \frac{\ln(e^{2c} e^{2dx} + 1)(6 a^2 b + 5 b^3)}{d} - \frac{2 (3 a^2 b + 18 a b^2 + 10 b^3)}{d (e^{2c+2dx} + 1)} - \frac{96 (15 b^3 + a b^2)}{5 d (5 e^{2c+2dx} + 10 e^{4c+4dx} + 10 e^{6c+6dx} + 5 e^{8c+8dx} + e^{10c+10dx} + 1)} - \frac{128 b^3}{d (7 e^{2c+2dx} + 21 e^{4c+4dx} + 35 e^{6c+6dx} + 35 e^{8c+8dx} + 21 e^{10c+10dx} + 7 e^{12c+12dx} + e^{14c+14dx} + 1)} - \frac{x (a-b)^2 (a - 10 b)}{2} + \frac{6 (a^2 b + 8 a b^2 + 10 b^3)}{d (2 e^{2c+2dx} + e^{4c+4dx} + 1)} + \frac{32 b^3}{d (8 e^{2c+2dx} + 28 e^{4c+4dx} + 56 e^{6c+6dx} + 70 e^{8c+8dx} + 56 e^{10c+10dx} + 28 e^{12c+12dx} + 8 e^{14c+14dx} + e^{16c+16dx} + 1)} + \frac{e^{2c+2dx} (a+b)^3}{8 d} - \frac{e^{-2c-2dx} (a-b)^3}{8 d} - \frac{16 (25 b^3 + 12 a b^2)}{3 d (3 e^{2c+2dx} + 3 e^{4c+4dx} + e^{6c+6dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)^3,x)
```

```
[Out] (8*(6*a*b^2 + 29*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(
6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (736*b^3)/(3*d*(6*exp(2*c + 2*d*x)
```

```
+ 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(1
0*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (log(exp(2*c)*exp(2*d*x) + 1)*(6
*a^2*b + 5*b^3))/d - (2*(18*a*b^2 + 3*a^2*b + 10*b^3))/(d*(exp(2*c + 2*d*x)
 + 1)) - (96*(a*b^2 + 15*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*
x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) -
(128*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x
) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + ex
p(14*c + 14*d*x) + 1)) - (x*(a - b)^2*(a - 10*b))/2 + (6*(8*a*b^2 + a^2*b +
 10*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (32*b^3)/(d*(8*
exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c +
 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d
*x) + exp(16*c + 16*d*x) + 1)) + (exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - (exp(
- 2*c - 2*d*x)*(a - b)^3)/(8*d) - (16*(12*a*b^2 + 25*b^3))/(3*d*(3*exp(2*c
+ 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

3.68 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=269

$$-\frac{9a^2b\text{ArcTan}(\sinh(c+dx))}{2d}-\frac{315b^3\text{ArcTan}(\sinh(c+dx))}{128d}+\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{9ab^2\text{sech}(c}{d}$$

```
[Out] -9/2*a^2*b*arctan(sinh(d*x+c))/d-315/128*b^3*arctan(sinh(d*x+c))/d+a^3*cosh
(d*x+c)/d+3*a*b^2*cosh(d*x+c)/d+9*a*b^2*sech(d*x+c)/d-3*a*b^2*sech(d*x+c)^3
/d+3/5*a*b^2*sech(d*x+c)^5/d+9/2*a^2*b*sinh(d*x+c)/d+315/128*b^3*sinh(d*x+c
)/d-3/2*a^2*b*sinh(d*x+c)*tanh(d*x+c)^2/d-105/128*b^3*sinh(d*x+c)*tanh(d*x+
c)^2/d-21/64*b^3*sinh(d*x+c)*tanh(d*x+c)^4/d-3/16*b^3*sinh(d*x+c)*tanh(d*x+
c)^6/d-1/8*b^3*sinh(d*x+c)*tanh(d*x+c)^8/d
```

Rubi [A]
time = 0.18, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3747, 2718, 2672, 294, 327, 209, 2670, 276}

$$\frac{a^3\cosh(c+dx)}{d}-\frac{9a^2b\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{9a^2b\sinh(c+dx)}{2d}-\frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\text{sech}^5(c+dx)}{5d}-\frac{3ab^2\text{sech}^3(c+dx)}{d}+\frac{9ab^2\text{sech}(c+dx)}{d}-\frac{315b^3\text{ArcTan}(\sinh(c+dx))}{128d}+\frac{315b^3\sinh(c+dx)}{128d}-\frac{b^3\sinh(c+dx)\tanh^8(c+dx)}{8d}-\frac{3b^3\sinh(c+dx)\tanh^6(c+dx)}{16d}-\frac{21b^3\sinh(c+dx)\tanh^4(c+dx)}{64d}-\frac{105b^3\sinh(c+dx)\tanh^2(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (-9*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) - (315*b^3*ArcTan[Sinh[c + d*x]])/(1
28*d) + (a^3*Cosh[c + d*x])/d + (3*a*b^2*Cosh[c + d*x])/d + (9*a*b^2*Sech[c
 + d*x])/d - (3*a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d)
+ (9*a^2*b*Sinh[c + d*x])/(2*d) + (315*b^3*Sinh[c + d*x])/(128*d) - (3*a^2*
b*Sinh[c + d*x]*Tanh[c + d*x]^2)/(2*d) - (105*b^3*Sinh[c + d*x]*Tanh[c + d*
x]^2)/(128*d) - (21*b^3*Sinh[c + d*x]*Tanh[c + d*x]^4)/(64*d) - (3*b^3*Sinh
[c + d*x]*Tanh[c + d*x]^6)/(16*d) - (b^3*Sinh[c + d*x]*Tanh[c + d*x]^8)/(8*
d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = -\left(i\int \left(ia^3\sinh(c+dx)+3ia^2b\sinh(c+dx)\tanh^3(c+dx)+3\right.\right.$$

$$= a^3\int \sinh(c+dx)\,dx + \left(3a^2b\right)\int \sinh(c+dx)\tanh^3(c+dx)\,dx$$

$$= \frac{a^3\cosh(c+dx)}{d} + \frac{\left(3a^2b\right)\operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d} -$$

$$= \frac{a^3\cosh(c+dx)}{d} - \frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d} - \frac{b^3\sinh(c+}{}$$

$$= \frac{a^3\cosh(c+dx)}{d} + \frac{3ab^2\cosh(c+dx)}{d} + \frac{9ab^2\operatorname{sech}(c+dx)}{d} - \frac{3ab}{}$$

$$= -\frac{9a^2b\tan^{-1}(\sinh(c+dx))}{2d} + \frac{a^3\cosh(c+dx)}{d} + \frac{3ab^2\cosh(c+d}{d}$$

$$= -\frac{9a^2b\tan^{-1}(\sinh(c+dx))}{2d} + \frac{a^3\cosh(c+dx)}{d} + \frac{3ab^2\cosh(c+d}{d}$$

$$= -\frac{9a^2b\tan^{-1}(\sinh(c+dx))}{2d} + \frac{a^3\cosh(c+dx)}{d} + \frac{3ab^2\cosh(c+d}{d}$$

$$= -\frac{9a^2b\tan^{-1}(\sinh(c+dx))}{2d} - \frac{315b^3\tan^{-1}(\sinh(c+dx))}{128d} + \frac{a^3\text{co}}{}$$

Mathematica [A]
time = 5.79, size = 168, normalized size = 0.62

$$\frac{640a(a^2+3b^2)\cosh(c+dx)+b\left(-90(64a^2+35b^2)\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+640(3a^2+b^2)\sinh(c+dx)-80b^2\operatorname{sech}^7(c+dx)\tanh(c+dx)-30b\operatorname{sech}^3(c+dx)(64a+35b\tanh(c+dx))+8b\operatorname{sech}^5(c+dx)(48a+55b\tanh(c+dx))+5\operatorname{sech}(c+dx)\left(1152ab+(192a^2+325b^2)\tanh(c+dx)\right)\right)}{640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (640*a*(a^2 + 3*b^2)*Cosh[c + d*x] + b*(-90*(64*a^2 + 35*b^2)*ArcTan[Tanh[(
c + d*x)/2]] + 640*(3*a^2 + b^2)*Sinh[c + d*x] - 80*b^2*Sech[c + d*x]^7*Tan
h[c + d*x] - 30*b*Sech[c + d*x]^3*(64*a + 35*b*Tanh[c + d*x]) + 8*b*Sech[c
+ d*x]^5*(48*a + 55*b*Tanh[c + d*x]) + 5*Sech[c + d*x]*(1152*a*b + (192*a^2
 + 325*b^2)*Tanh[c + d*x])))/(640*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.82, size = 535, normalized size = 1.99

method	result

risch	$\frac{e^{dx+c}a^3}{2d} + \frac{3\,e^{dx+c}a^2b}{2d} + \frac{3a\,e^{dx+c}b^2}{2d} + \frac{b^3e^{dx+c}}{2d} + \frac{e^{-dx-c}a^3}{2d} - \frac{3\,e^{-dx-c}a^2b}{2d} + \frac{3a\,e^{-dx-c}b^2}{2d} - \frac{e^{-dx-c}b^3}{2d} + \frac{b\,e^{dx+c}(960}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*exp(d*x+c)*a^3+3/2/d*exp(d*x+c)*a^2*b+3/2*a/d*exp(d*x+c)*b^2+1/2*b^3/
d*exp(d*x+c)+1/2/d*exp(-d*x-c)*a^3-3/2/d*exp(-d*x-c)*a^2*b+3/2*a/d*exp(-d*x
-c)*b^2-1/2/d*exp(-d*x-c)*b^3+1/320*b*exp(d*x+c)*(960*a^2*exp(14*d*x+14*c)+
5760*a*b*exp(14*d*x+14*c)+1625*b^2*exp(14*d*x+14*c)+4800*a^2*exp(12*d*x+12*
c)+32640*a*b*exp(12*d*x+12*c)+3925*b^2*exp(12*d*x+12*c)+8640*a^2*exp(10*d*x
+10*c)+88704*a*b*exp(10*d*x+10*c)+9065*b^2*exp(10*d*x+10*c)+4800*a^2*exp(8*
d*x+8*c)+143232*a*b*exp(8*d*x+8*c)+1645*b^2*exp(8*d*x+8*c)-4800*a^2*exp(6*d
*x+6*c)+143232*a*b*exp(6*d*x+6*c)-1645*b^2*exp(6*d*x+6*c)-8640*a^2*exp(4*d*
x+4*c)+88704*a*b*exp(4*d*x+4*c)-9065*b^2*exp(4*d*x+4*c)-4800*a^2*exp(2*d*x+
2*c)+32640*a*b*exp(2*d*x+2*c)-3925*b^2*exp(2*d*x+2*c)-960*a^2+5760*a*b-1625
*b^2)/d/(1+exp(2*d*x+2*c))^8+9/2*I*b/d*ln(exp(d*x+c)-I)*a^2+315/128*I*b^3/d
*ln(exp(d*x+c)-I)-9/2*I*b/d*ln(exp(d*x+c)+I)*a^2-315/128*I*b^3/d*ln(exp(d*x
+c)+I)
```

Maxima [A]
time = 0.52, size = 484, normalized size = 1.80

$$\frac{1}{64}\,b^3\left(\frac{315\,\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{32\,e^{(-dx-c)}}{d} + \frac{581\,e^{(-2\,dx-2\,c)} + 1681\,e^{(-4\,dx-4\,c)} + 3605\,e^{(-6\,dx-6\,c)} + 2569\,e^{(-8\,dx-8\,c)} + 1463\,e^{(-10\,dx-10\,c)} - 917\,e^{(-12\,dx-12\,c)} - 529\,e^{(-14\,dx-14\,c)} - 293\,e^{(-16\,dx-16\,c)} + 32}{d(e^{(-dx-c)} + 8\,e^{(-3\,dx-3\,c)} + 28\,e^{(-5\,dx-5\,c)} + 56\,e^{(-7\,dx-7\,c)} + 70\,e^{(-9\,dx-9\,c)} + 56\,e^{(-11\,dx-11\,c)} + 28\,e^{(-13\,dx-13\,c)} + 8\,e^{(-15\,dx-15\,c)} + e^{(-17\,dx-17\,c)})}\right) + \frac{3}{2}\,a^2b\left(\frac{6\,\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + 1}{d(e^{(-dx-c)} + 2\,e^{(-3\,dx-3\,c)} + e^{(-5\,dx-5\,c)})}\right) + \frac{3}{10}\,ab^2\left(\frac{5\,e^{(-dx-c)}}{d} + \frac{85\,e^{(-2\,dx-2\,c)} + 210\,e^{(-4\,dx-4\,c)} + 314\,e^{(-6\,dx-6\,c)} + 185\,e^{(-8\,dx-8\,c)} + 65\,e^{(-10\,dx-10\,c)} + 5}{d(e^{(-dx-c)} + 5\,e^{(-3\,dx-3\,c)} + 10\,e^{(-5\,dx-5\,c)} + 10\,e^{(-7\,dx-7\,c)} + 5\,e^{(-9\,dx-9\,c)} + e^{(-11\,dx-11\,c)})}\right) + \frac{a^3\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(315*arctan(e^(-d*x - c))/d - 32*e^(-d*x - c)/d + (581*e^(-2*d*x -
 2*c) + 1681*e^(-4*d*x - 4*c) + 3605*e^(-6*d*x - 6*c) + 2569*e^(-8*d*x - 8*
c) + 1463*e^(-10*d*x - 10*c) - 917*e^(-12*d*x - 12*c) - 529*e^(-14*d*x - 14
*c) - 293*e^(-16*d*x - 16*c) + 32)/(d*(e^(-d*x - c) + 8*e^(-3*d*x - 3*c) +
28*e^(-5*d*x - 5*c) + 56*e^(-7*d*x - 7*c) + 70*e^(-9*d*x - 9*c) + 56*e^(-11
*d*x - 11*c) + 28*e^(-13*d*x - 13*c) + 8*e^(-15*d*x - 15*c) + e^(-17*d*x -
17*c)))) + 3/2*a^2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*
d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) +
e^(-5*d*x - 5*c)))) + 3/10*a*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) +
 210*e^(-4*d*x - 4*c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^
(-10*d*x - 10*c) + 5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x
- 5*c) + 10*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) +
 a^3*cosh(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6410 vs. $2(249) = 498$.

time = 0.43, size = 6410, normalized size = 23.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(160*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^18 + 2880*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^17 + 160*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sinh(d*x + c)^18 + 45*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61
*b^3)*cosh(d*x + c)^16 + 45*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3 + 544*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 240*(544
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(32*a^3 + 96*a^2*b + 2
24*a*b^2 + 61*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 15*(384*a^3 + 960*a^2*
b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^14 + 15*(32640*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^4 + 384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3 +
360*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^14 + 210*(6528*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 120*(32*a
^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^3 + (384*a^3 + 960*a^2*b
+ 3328*a*b^2 + 475*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 + 3*(4480*a^3 + 736
0*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^12 + 3*(990080*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 27300*(32*a^3 + 96*a^2*b + 224*a*b^2
+ 61*b^3)*cosh(d*x + c)^4 + 4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3
+ 455*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^12 + 12*(424320*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 16
380*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^5 + 455*(384*a^3
 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^3 + 3*(4480*a^3 + 7360*a
^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + 3*(6720*a^
3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^10 + 3*(2333760*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 120120*(32*a^3 + 96*a^2*b + 2
24*a*b^2 + 61*b^3)*cosh(d*x + c)^6 + 5005*(384*a^3 + 960*a^2*b + 3328*a*b^2
 + 475*b^3)*cosh(d*x + c)^4 + 6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^
3 + 66*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^10 + 10*(777920*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9
 + 51480*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^7 + 3003*(3
84*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^5 + 66*(4480*a^3 +
 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^3 + 3*(6720*a^3 + 3840*
a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(6720*a^
3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*cosh(d*x + c)^8 + 3*(2333760*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 193050*(32*a^3 + 96*a^2*b + 2
24*a*b^2 + 61*b^3)*cosh(d*x + c)^8 + 15015*(384*a^3 + 960*a^2*b + 3328*a*b^
2 + 475*b^3)*cosh(d*x + c)^6 + 495*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4
515*b^3)*cosh(d*x + c)^4 + 6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3 +
 45*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^8 + 24*(212160*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^11 +
```

```
21450*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^9 + 2145*(384*
a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^7 + 99*(4480*a^3 + 73
60*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^5 + 15*(6720*a^3 + 3840*a^
2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^3 + (6720*a^3 - 3840*a^2*b + 67
904*a*b^2 - 1295*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 3*(4480*a^3 - 7360*a
^2*b + 43008*a*b^2 - 4515*b^3)*cosh(d*x + c)^6 + 3*(990080*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 120120*(32*a^3 + 96*a^2*b + 224*a*b^2 +
61*b^3)*cosh(d*x + c)^10 + 15015*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^
3)*cosh(d*x + c)^8 + 924*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*c
osh(d*x + c)^6 + 210*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(
d*x + c)^4 + 4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 4515*b^3 + 28*(6720*a^3
- 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 6
*(228480*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^13 + 32760*(32*a^3 +
 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c)^11 + 5005*(384*a^3 + 960*a^2*
b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^9 + 396*(4480*a^3 + 7360*a^2*b + 43
008*a*b^2 + 4515*b^3)*cosh(d*x + c)^7 + 126*(6720*a^3 + 3840*a^2*b + 67904*
a*b^2 + 1295*b^3)*cosh(d*x + c)^5 + 28*(6720*a^3 - 3840*a^2*b + 67904*a*b^2
 - 1295*b^3)*cosh(d*x + c)^3 + 3*(4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 451
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(384*a^3 - 960*a^2*b + 3328*a*b^
2 - 475*b^3)*cosh(d*x + c)^4 + 15*(32640*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
sh(d*x + c)^14 + 5460*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*cosh(d*x + c
)^12 + 1001*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*cosh(d*x + c)^10 +
 99*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*cosh(d*x + c)^8 + 42*(
6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*cosh(d*x + c)^6 + 14*(6720*
a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*cosh(d*x + c)^4 + 384*a^3 - 960*
a^2*b + 3328*a*b^2 - 475*b^3 + 3*(4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 451
5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 12*(10880*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)^15 + 2100*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*c
osh(d*x + c)^13 + 455*(384*a^3 + 960*a^2*b + 33...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^{3}\left(c + dx\right)\right)^{3} \sinh\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*sinh(c + d*x), x)
```

Giac [A]
time = 0.68, size = 463, normalized size = 1.72

$$\frac{160\,a^{3}e^{(dx+c)} + 480\,a^{2}be^{(dx+c)} + 480\,ab^{2}e^{(dx+c)} + 160\,b^{3}e^{(dx+c)} - 45\,(64\,a^{2}b + 35\,b^{3})\arctan\left(e^{(dx+c)}\right) + 160\,(a^{3} - 3\,a^{2}b + 3\,ab^{2} - b^{3})e^{(-dx-c)} + \frac{960\,a^{2}be^{(15\,dx+15\,c)} + 5760\,ab^{2}e^{(15\,dx+15\,c)} + 1625\,b^{3}e^{(15\,dx+15\,c)} + 4800\,a^{2}be^{(13\,dx+13\,c)} + 32640\,ab^{2}e^{(13\,dx+13\,c)} + 3925\,b^{3}e^{(13\,dx+13\,c)} + 8640\,a^{2}be^{(11\,dx+11\,c)} + 88704\,ab^{2}e^{(11\,dx+11\,c)} + 9065\,b^{3}e^{(11\,dx+11\,c)} + 4800\,a^{2}be^{(9\,dx+9\,c)} + 143232\,ab^{2}e^{(9\,dx+9\,c)} + 1645\,b^{3}e^{(9\,dx+9\,c)} - 4800\,a^{2}be^{(7\,dx+7\,c)} + 143232\,ab^{2}e^{(7\,dx+7\,c)} - 1645\,b^{3}e^{(7\,dx+7\,c)} - 8640\,a^{2}be^{(5\,dx+5\,c)} + 88704\,ab^{2}e^{(5\,dx+5\,c)} - 9065\,b^{3}e^{(5\,dx+5\,c)} - 4800\,a^{2}be^{(3\,dx+3\,c)} + 32640\,ab^{2}e^{(3\,dx+3\,c)} - 3925\,b^{3}e^{(3\,dx+3\,c)} - 960\,a^{2}be^{(dx+c)} + 5760\,ab^{2}e^{(dx+c)} - 1625\,b^{3}e^{(dx+c)}}{\left(e^{(2\,dx+2\,c)} + 1\right)^{8}}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/320*(160*a^3*e^(d*x + c) + 480*a^2*b*e^(d*x + c) + 480*a*b^2*e^(d*x + c)
+ 160*b^3*e^(d*x + c) - 45*(64*a^2*b + 35*b^3)*arctan(e^(d*x + c)) + 160*(a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*e^(-d*x - c) + (960*a^2*b*e^(15*d*x + 15*c) +
 5760*a*b^2*e^(15*d*x + 15*c) + 1625*b^3*e^(15*d*x + 15*c) + 4800*a^2*b*e^(
13*d*x + 13*c) + 32640*a*b^2*e^(13*d*x + 13*c) + 3925*b^3*e^(13*d*x + 13*c)
 + 8640*a^2*b*e^(11*d*x + 11*c) + 88704*a*b^2*e^(11*d*x + 11*c) + 9065*b^3*
e^(11*d*x + 11*c) + 4800*a^2*b*e^(9*d*x + 9*c) + 143232*a*b^2*e^(9*d*x + 9*
c) + 1645*b^3*e^(9*d*x + 9*c) - 4800*a^2*b*e^(7*d*x + 7*c) + 143232*a*b^2*e
^(7*d*x + 7*c) - 1645*b^3*e^(7*d*x + 7*c) - 8640*a^2*b*e^(5*d*x + 5*c) + 88
704*a*b^2*e^(5*d*x + 5*c) - 9065*b^3*e^(5*d*x + 5*c) - 4800*a^2*b*e^(3*d*x
+ 3*c) + 32640*a*b^2*e^(3*d*x + 3*c) - 3925*b^3*e^(3*d*x + 3*c) - 960*a^2*b
*e^(d*x + c) + 5760*a*b^2*e^(d*x + c) - 1625*b^3*e^(d*x + c))/(e^(2*d*x + 2
*c) + 1)^8)/d
```

Mupad [B]
time = 1.49, size = 707, normalized size = 2.63

$$\frac{e^{c+dx}\,(a+b)^3}{2\,d}+\frac{e^{-c-dx}\,(a-b)^3}{2\,d}-\frac{9\,\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\,\left(35\,b^3\,\sqrt{d^2}+64\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{4096\,a^4\,b^2+4480\,a^2\,b^4+1225\,b^6}}\right)\,\sqrt{4096\,a^4\,b^2+4480\,a^2\,b^4+1225\,b^6}}{64\,\sqrt{d^2}}+\frac{e^{c+dx}\,\left(2455\,b^3+1728\,a\,b^2\right)}{40\,d\,\left(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1\right)}-\frac{e^{c+dx}\,\left(2605\,b^3+768\,a\,b^2\right)}{20\,d\,\left(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}-\frac{188\,b^3\,e^{c+dx}}{d\,\left(6\,e^{2c+2dx}+15\,e^{4c+4dx}+20\,e^{6c+6dx}+15\,e^{8c+8dx}+6\,e^{10c+10dx}+e^{12c+12dx}+1\right)}+\frac{e^{c+dx}\,\left(192\,a^2\,b+1152\,a\,b^2+325\,b^3\right)}{64\,d\,\left(e^{2c+2dx}+1\right)}+\frac{2\,e^{c+dx}\,\left(475\,b^3+48\,a\,b^2\right)}{5\,d\,\left(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1\right)}+\frac{112\,b^3\,e^{c+dx}}{d\,\left(7\,e^{2c+2dx}+21\,e^{4c+4dx}+35\,e^{6c+6dx}+35\,e^{8c+8dx}+21\,e^{10c+10dx}+7\,e^{12c+12dx}+e^{14c+14dx}+1\right)}-\frac{e^{c+dx}\,\left(192\,a^2\,b+768\,a\,b^2+745\,b^3\right)}{32\,d\,\left(2\,e^{2c+2dx}+e^{4c+4dx}+1\right)}-\frac{32\,b^3\,e^{c+dx}}{d\,\left(8\,e^{2c+2dx}+28\,e^{4c+4dx}+56\,e^{6c+6dx}+70\,e^{8c+8dx}+56\,e^{10c+10dx}+28\,e^{12c+12dx}+8\,e^{14c+14dx}+e^{16c+16dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)^3,x)
```

```
[Out] (exp(c + d*x)*(a + b)^3)/(2*d) + (exp(- c - d*x)*(a - b)^3)/(2*d) - (9*atan
((exp(d*x)*exp(c)*(35*b^3*(d^2)^(1/2) + 64*a^2*b*(d^2)^(1/2)))/(d*(1225*b^6
 + 4480*a^2*b^4 + 4096*a^4*b^2)^(1/2)))*(1225*b^6 + 4480*a^2*b^4 + 4096*a^4
*b^2)^(1/2))/(64*(d^2)^(1/2)) + (exp(c + d*x)*(1728*a*b^2 + 2455*b^3))/(40*
d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(
c + d*x)*(768*a*b^2 + 2605*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*
d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (188*b^3*exp(c + d*x))
/(d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*ex
p(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (exp(c +
 d*x)*(1152*a*b^2 + 192*a^2*b + 325*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) + (
2*exp(c + d*x)*(48*a*b^2 + 475*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c
+ 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) +
1)) + (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) +
 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(
12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (exp(c + d*x)*(768*a*b^2 + 192*
a^2*b + 745*b^3))/(32*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (32*
b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c
 + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*
d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1))
```

3.69 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=219

$$\frac{3a^2b\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{35b^3\text{ArcTan}(\sinh(c+dx))}{128d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3ab^2\text{sech}(c+dx)}{d}+\frac{2ab^2}{}$$

```
[Out] 3/2*a^2*b*arctan(sinh(d*x+c))/d+35/128*b^3*arctan(sinh(d*x+c))/d-a^3*arctan
h(cosh(d*x+c))/d-3*a*b^2*sech(d*x+c)/d+2*a*b^2*sech(d*x+c)^3/d-3/5*a*b^2*se
ch(d*x+c)^5/d-3/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d-35/128*b^3*sech(d*x+c)*ta
nh(d*x+c)/d-35/192*b^3*sech(d*x+c)*tanh(d*x+c)^3/d-7/48*b^3*sech(d*x+c)*tan
h(d*x+c)^5/d-1/8*b^3*sech(d*x+c)*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.20, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3747, 3855, 2691, 2686, 200}

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\text{ArcTan}(\sinh(c+dx))}{2d}-\frac{3a^2b\tanh(c+dx)\text{sech}(c+dx)}{2d}-\frac{3ab^2\text{sech}^5(c+dx)}{5d}+\frac{2ab^2\text{sech}^3(c+dx)}{d}-\frac{3ab^2\text{sech}(c+dx)}{d}+\frac{35b^3\text{ArcTan}(\sinh(c+dx))}{128d}-\frac{b^3\tanh^7(c+dx)\text{sech}(c+dx)}{8d}-\frac{7b^3\tanh^5(c+dx)\text{sech}(c+dx)}{48d}-\frac{35b^3\tanh^3(c+dx)\text{sech}(c+dx)}{192d}-\frac{35b^3\tanh(c+dx)\text{sech}(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (3*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (35*b^3*ArcTan[Sinh[c + d*x]])/(128
*d) - (a^3*ArcTanh[Cosh[c + d*x]])/d - (3*a*b^2*Sech[c + d*x])/d + (2*a*b^2
*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c + d*x]^5)/(5*d) - (3*a^2*b*Sech[c + d
*x]*Tanh[c + d*x])/(2*d) - (35*b^3*Sech[c + d*x]*Tanh[c + d*x])/(128*d) - (
35*b^3*Sech[c + d*x]*Tanh[c + d*x]^3)/(192*d) - (7*b^3*Sech[c + d*x]*Tanh[c
 + d*x]^5)/(48*d) - (b^3*Sech[c + d*x]*Tanh[c + d*x]^7)/(8*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx &= i\int\left(-ia^3\operatorname{csch}(c+dx)-3ia^2b\operatorname{sech}(c+dx)\tanh^2(c+dx)-3ia\right. \\
&= a^3\int\operatorname{csch}(c+dx)\,dx+\left(3a^2b\right)\int\operatorname{sech}(c+dx)\tanh^2(c+dx)\,dx \\
&= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}- \\
&= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3a^2b\operatorname{sec}}{} \\
&= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3ab^2\operatorname{sec}}{} \\
&= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{3ab^2\operatorname{sec}}{} \\
&= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{35b^3\tan^{-1}(\sinh(c+dx))}{128d}-\frac{a^3\tanh}{}
\end{aligned}
$$

Mathematica [A]
time = 2.11, size = 154, normalized size = 0.70

$$\frac{30\left(b\left(192a^2+35b^2\right)\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+64a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+240b^3\operatorname{sech}^7(c+dx)\tanh(c+dx)-8b^2\operatorname{sech}^5(c+dx)(144a+125b\tanh(c+dx))+10b^2\operatorname{sech}^3(c+dx)(384a+163b\tanh(c+dx))-45b\operatorname{sech}(c+dx)\left(128ab+\left(64a^2+31b^2\right)\tanh(c+dx)\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

[Out] (30*(b*(192*a^2 + 35*b^2)*ArcTan[Tanh[(c + d*x)/2]] + 64*a^3*Log[Tanh[(c + d*x)/2]]) + 240*b^3*Sech[c + d*x]^7*Tanh[c + d*x] - 8*b^2*Sech[c + d*x]^5*(144*a + 125*b*Tanh[c + d*x]) + 10*b^2*Sech[c + d*x]^3*(384*a + 163*b*Tanh[c + d*x]) - 45*b*Sech[c + d*x]*(128*a*b + (64*a^2 + 31*b^2)*Tanh[c + d*x]))/ (1920*d)

Maple [C] Result contains complex when optimal does not.
time = 3.67, size = 440, normalized size = 2.01

method	result
risch	$-\frac{b\,e^{dx+c}\left(2880a^2e^{14dx+14c}+5760ab\,e^{14dx+14c}+1395b^2e^{14dx+14c}+14400a^2e^{12dx+12c}+24960ab\,e^{12dx+12c}+455b^2e^{12dx+12c}+25920\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)

[Out] -1/960*b*exp(d*x+c)*(2880*a^2*exp(14*d*x+14*c)+5760*a*b*exp(14*d*x+14*c)+1395*b^2*exp(14*d*x+14*c)+14400*a^2*exp(12*d*x+12*c)+24960*a*b*exp(12*d*x+12*c)+455*b^2*exp(12*d*x+12*c)+25920*a^2*exp(10*d*x+10*c)+62592*a*b*exp(10*d*x+10*c)+8995*b^2*exp(10*d*x+10*c)+14400*a^2*exp(8*d*x+8*c)+103296*a*b*exp(8*d*x+8*c)-5425*b^2*exp(8*d*x+8*c)-14400*a^2*exp(6*d*x+6*c)+103296*a*b*exp(6*d*x+6*c)+5425*b^2*exp(6*d*x+6*c)-25920*a^2*exp(4*d*x+4*c)+62592*a*b*exp(4*d*x+4*c)-8995*b^2*exp(4*d*x+4*c)-14400*a^2*exp(2*d*x+2*c)+24960*a*b*exp(2*d*x+2*c)-455*b^2*exp(2*d*x+2*c)-2880*a^2+5760*a*b-1395*b^2)/d/(1+exp(2*d*x+2*c))^8+3/2*I*b/d*ln(exp(d*x+c)+I)*a^2+35/128*I*b^3/d*ln(exp(d*x+c)+I)-3/2*I*b/d*ln(exp(d*x+c)-I)*a^2-35/128*I*b^3/d*ln(exp(d*x+c)-I)+a^3/d*ln(exp(d*x+c)-1)-a^3/d*ln(exp(d*x+c)+1)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(203) = 406.
time = 0.50, size = 654, normalized size = 2.99

$$-\frac{1}{192}b^3\left(\frac{105\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{279e^{(-dx-c)}+91e^{(-3dx-3c)}+1799e^{(-5dx-5c)}-1085e^{(-7dx-7c)}+1085e^{(-9dx-9c)}-1799e^{(-11dx-11c)}-91e^{(-13dx-13c)}-279e^{(-15dx-15c)}}{d\left(8e^{(-2dx-2c)}+28e^{(-4dx-4c)}+56e^{(-6dx-6c)}+70e^{(-8dx-8c)}+56e^{(-10dx-10c)}+28e^{(-12dx-12c)}+8e^{(-14dx-14c)}+e^{(-16dx-16c)}+1\right)}\right)-3a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}\right)-\frac{2}{5}ab^2\left(\frac{15e^{(-dx-c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{20e^{(-3dx-3c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{58e^{(-5dx-5c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{20e^{(-7dx-7c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{15e^{(-9dx-9c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)+\frac{a^3\log\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] -1/192*b^3*(105*arctan(e^(-d*x - c))/d + (279*e^(-d*x - c) + 91*e^(-3*d*x - 3*c) + 1799*e^(-5*d*x - 5*c) - 1085*e^(-7*d*x - 7*c) + 1085*e^(-9*d*x - 9*c) - 1799*e^(-11*d*x - 11*c) - 91*e^(-13*d*x - 13*c) - 279*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 2/5*a*b^2*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d

```
*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 58*e^(
-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1))) + a^3*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7127 vs. $2(203) = 406$.
time = 0.47, size = 7127, normalized size = 32.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/960*(45*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^15 + 675*(64*a^2*b
 + 128*a*b^2 + 31*b^3)*cosh(d*x + c)*sinh(d*x + c)^14 + 45*(64*a^2*b + 128*
a*b^2 + 31*b^3)*sinh(d*x + c)^15 + 5*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cos
h(d*x + c)^13 + 5*(2880*a^2*b + 4992*a*b^2 + 91*b^3 + 945*(64*a^2*b + 128*a
*b^2 + 31*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 65*(315*(64*a^2*b + 128*
a*b^2 + 31*b^3)*cosh(d*x + c)^3 + (2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d
*x + c))*sinh(d*x + c)^12 + (25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x
 + c)^11 + (61425*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^4 + 25920*a
^2*b + 62592*a*b^2 + 8995*b^3 + 390*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^11 + 11*(12285*(64*a^2*b + 128*a*b^2 + 31*b^3)*c
osh(d*x + c)^5 + 130*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*x + c)^3 + (
25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + (14
400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^9 + (225225*(64*a^2*b +
128*a*b^2 + 31*b^3)*cosh(d*x + c)^6 + 3575*(2880*a^2*b + 4992*a*b^2 + 91*b^
3)*cosh(d*x + c)^4 + 14400*a^2*b + 103296*a*b^2 - 5425*b^3 + 55*(25920*a^2*
b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 3*(96525*(64
*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^7 + 2145*(2880*a^2*b + 4992*a*b^
2 + 91*b^3)*cosh(d*x + c)^5 + 55*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cos
h(d*x + c)^3 + 3*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c))*sin
h(d*x + c)^8 - (14400*a^2*b - 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^7 + (2
89575*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^8 + 8580*(2880*a^2*b +
4992*a*b^2 + 91*b^3)*cosh(d*x + c)^6 + 330*(25920*a^2*b + 62592*a*b^2 + 899
5*b^3)*cosh(d*x + c)^4 - 14400*a^2*b + 103296*a*b^2 + 5425*b^3 + 36*(14400*
a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + (225225
*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^9 + 8580*(2880*a^2*b + 4992*
a*b^2 + 91*b^3)*cosh(d*x + c)^7 + 462*(25920*a^2*b + 62592*a*b^2 + 8995*b^3
```

```
)*cosh(d*x + c)^5 + 84*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c
)^3 - 7*(14400*a^2*b - 103296*a*b^2 - 5425*b^3)*cosh(d*x + c))*sinh(d*x + c
)^6 - (25920*a^2*b - 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^5 + (135135*(64*
a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^10 + 6435*(2880*a^2*b + 4992*a*b^
2 + 91*b^3)*cosh(d*x + c)^8 + 462*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*co
sh(d*x + c)^6 + 126*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^4
 - 25920*a^2*b + 62592*a*b^2 - 8995*b^3 - 21*(14400*a^2*b - 103296*a*b^2 -
5425*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (61425*(64*a^2*b + 128*a*b^2 +
 31*b^3)*cosh(d*x + c)^11 + 3575*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*
x + c)^9 + 330*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^7 + 126
*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^5 - 35*(14400*a^2*b
- 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^3 - 5*(25920*a^2*b - 62592*a*b^2 +
 8995*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 5*(2880*a^2*b - 4992*a*b^2 + 91
*b^3)*cosh(d*x + c)^3 + (20475*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c
)^12 + 1430*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*x + c)^10 + 165*(2592
0*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^8 + 84*(14400*a^2*b + 10329
6*a*b^2 - 5425*b^3)*cosh(d*x + c)^6 - 35*(14400*a^2*b - 103296*a*b^2 - 5425
*b^3)*cosh(d*x + c)^4 - 14400*a^2*b + 24960*a*b^2 - 455*b^3 - 10*(25920*a^2
*b - 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (4725*(64*a
^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^13 + 390*(2880*a^2*b + 4992*a*b^2
+ 91*b^3)*cosh(d*x + c)^11 + 55*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh
(d*x + c)^9 + 36*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^7 -
21*(14400*a^2*b - 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^5 - 10*(25920*a^2*
b - 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^3 - 15*(2880*a^2*b - 4992*a*b^2 +
 91*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 15*((192*a^2*b + 35*b^3)*cosh(d*x
 + c)^16 + 16*(192*a^2*b + 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^15 + (192*a^
2*b + 35*b^3)*sinh(d*x + c)^16 + 8*(192*a^2*b + 35*b^3)*cosh(d*x + c)^14 +
8*(192*a^2*b + 35*b^3 + 15*(192*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^14 + 112*(5*(192*a^2*b + 35*b^3)*cosh(d*x + c)^3 + (192*a^2*b + 35*b^3)
*cosh(d*x + c))*sinh(d*x + c)^13 + 28*(192*a^2*b + 35*b^3)*cosh(d*x + c)^12
 + 28*(65*(192*a^2*b + 35*b^3)*cosh(d*x + c)^4 + 192*a^2*b + 35*b^3 + 26*(1
92*a^2*b + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 112*(39*(192*a^2*b +
 35*b^3)*cosh(d*x + c)^5 + 26*(192*a^2*b + 35*b^3)*cosh(d*x + c)^3 + 3*(192
*a^2*b + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + 56*(192*a^2*b + 35*b^3)*
cosh(d*x + c)^10 + 56*(143*(192*a^2*b + 35*b^3)*cosh(d*x + c)^6 + 143*(192*
a^2*b + 35*b^3)*cosh(d*x + c)^4 + 192*a^2*b + 35*b^3 + 33*(192*a^2*b + 35*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 16*(715*(192*a^2*b + 35*b^3)*cosh(d
*x + c)^7 + 1001*(192*a^2*b + 35*b^3)*cosh(d*x + c)^5 + 385*(192*a^2*b + 35
*b^3)*cosh(d*x + c)^3 + 35*(192*a^2*b + 35*b^3)*cosh(d*x + c))*sinh(d*x + c
)^9 + 70*(192*a^2*b + 35*b^3)*cosh(d*x + c)^8 + 2*(6435*(192*a^2*b + 35*b^3
)*cosh(d*x + c)^8 + 12012*(192*a^2*b + 35*b^3)*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^3\left(c+dx\right)\right)^3\operatorname{csch}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(203) = 406.
time = 0.66, size = 414, normalized size = 1.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/960*(960*a^3*log(e^(d*x + c) + 1) - 960*a^3*log(abs(e^(d*x + c) - 1)) -
15*(192*a^2*b + 35*b^3)*arctan(e^(d*x + c)) + (2880*a^2*b*e^(15*d*x + 15*c)
 + 5760*a*b^2*e^(15*d*x + 15*c) + 1395*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*
e^(13*d*x + 13*c) + 24960*a*b^2*e^(13*d*x + 13*c) + 455*b^3*e^(13*d*x + 13*
c) + 25920*a^2*b*e^(11*d*x + 11*c) + 62592*a*b^2*e^(11*d*x + 11*c) + 8995*b
^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) + 103296*a*b^2*e^(9*d*x
+ 9*c) - 5425*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) + 103296*a*
b^2*e^(7*d*x + 7*c) + 5425*b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^(5*d*x + 5*c
) + 62592*a*b^2*e^(5*d*x + 5*c) - 8995*b^3*e^(5*d*x + 5*c) - 14400*a^2*b*e^
(3*d*x + 3*c) + 24960*a*b^2*e^(3*d*x + 3*c) - 455*b^3*e^(3*d*x + 3*c) - 288
0*a^2*b*e^(d*x + c) + 5760*a*b^2*e^(d*x + c) - 1395*b^3*e^(d*x + c))/(e^(2*
d*x + 2*c) + 1)^8)/d
```

Mupad [B]
time = 6.40, size = 671, normalized size = 3.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x),x)
```

```
[Out] (a^3*log(exp(c + d*x) - 1))/d - (a^3*log(exp(c + d*x) + 1))/d - (exp(c + d*
x)*(4224*a*b^2 + 4445*b^3))/(120*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
 + exp(6*c + 6*d*x) + 1)) - (b*log(exp(c + d*x) - 1i)*(192*a^2 + 35*b^2)*1i
)/(128*d) + (b*log(exp(c + d*x) + 1i)*(192*a^2 + 35*b^2)*1i)/(128*d) + (exp
(c + d*x)*(768*a*b^2 + 1925*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4
*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (532*b^3*exp(c + d*x)
```

)/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15
*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (3*ex
p(c + d*x)*(128*a*b^2 + 64*a^2*b + 31*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) -
 (2*exp(c + d*x)*(144*a*b^2 + 1225*b^3))/(15*d*(5*exp(2*c + 2*d*x) + 10*exp
(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*
x) + 1)) - (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d
*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7
*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) + (exp(c + d*x)*(1536*a*b^2
+ 576*a^2*b + 931*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
+ (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*e
xp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c
 + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1))

3.70 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=71

$$-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^8(c+dx)}{8d}$$

```
[Out] -a^3*coth(d*x+c)/d+3/2*a^2*b*tanh(d*x+c)^2/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/8*
b^3*tanh(d*x+c)^8/d
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3744, 276}

$$-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] -((a^3*Coth[c + d*x])/d) + (3*a^2*b*Tanh[c + d*x]^2)/(2*d) + (3*a*b^2*Tanh[
c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^8)/(8*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^3)^3}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^2}+3a^2bx+3ab^2x^4+b^3x^7\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= -\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3ab^2\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.53, size = 113, normalized size = 1.59

$$\frac{-40a^3\coth(c+dx)+b\left(-20b^2\operatorname{sech}^6(c+dx)+5b^2\operatorname{sech}^8(c+dx)+24ab\tanh(c+dx)+6b\operatorname{sech}^4(c+dx)(5b+4a\tanh(c+dx))-4\operatorname{sech}^2(c+dx)\left(15a^2+5b^2+12ab\tanh(c+dx)\right)\right)}{40d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (-40*a^3*Coth[c + d*x] + b*(-20*b^2*Sech[c + d*x]^6 + 5*b^2*Sech[c + d*x]^8
 + 24*a*b*Tanh[c + d*x] + 6*b*Sech[c + d*x]^4*(5*b + 4*a*Tanh[c + d*x]) - 4
*Sech[c + d*x]^2*(15*a^2 + 5*b^2 + 12*a*b*Tanh[c + d*x])))/(40*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(65) = 130$.
time = 3.45, size = 508, normalized size = 7.15

method	result
risch	$-\frac{2\left(-3a\,b^2+30a\,b^2 e^{14dx+14c}+135a^2b\,e^{12dx+12c}+30a\,b^2e^{12dx+12c}+75a^2b\,e^{10dx+10c}+30a\,b^2e^{10dx+10c}+15a^2b\,e^{16dx+16c}+15a\,b^2e^{16}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(-3*a*b^2+30*a*b^2*exp(14*d*x+14*c)+135*a^2*b*exp(12*d*x+12*c)+30*a*b^
2*exp(12*d*x+12*c)+75*a^2*b*exp(10*d*x+10*c)+30*a*b^2*exp(10*d*x+10*c)+15*a
^2*b*exp(16*d*x+16*c)+15*a*b^2*exp(16*d*x+16*c)+75*a^2*b*exp(14*d*x+14*c)-7
5*a^2*b*exp(4*d*x+4*c)-15*a^2*b*exp(2*d*x+2*c)+5*a^3-12*a*b^2*exp(8*d*x+8*c
)-54*a*b^2*exp(6*d*x+6*c)-30*a*b^2*exp(4*d*x+4*c)-75*a^2*b*exp(8*d*x+8*c)-6
*a*b^2*exp(2*d*x+2*c)-135*a^2*b*exp(6*d*x+6*c)+5*b^3*exp(16*d*x+16*c)-5*b^3
*exp(14*d*x+14*c)+40*a^3*exp(2*d*x+2*c)+140*a^3*exp(12*d*x+12*c)+35*b^3*exp
(12*d*x+12*c)+280*a^3*exp(10*d*x+10*c)+40*a^3*exp(14*d*x+14*c)-5*b^3*exp(2*
d*x+2*c)+5*a^3*exp(16*d*x+16*c)+35*b^3*exp(8*d*x+8*c)+140*a^3*exp(4*d*x+4*c
)+5*b^3*exp(4*d*x+4*c)-35*b^3*exp(10*d*x+10*c)+350*a^3*exp(8*d*x+8*c)+280*a
^3*exp(6*d*x+6*c)-35*b^3*exp(6*d*x+6*c))/d/(1+exp(2*d*x+2*c))^8/(exp(2*d*x+
2*c)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(65) = 130$.
time = 0.28, size = 679, normalized size = 9.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -2*b^3*(e^(-2*d*x - 2*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*
e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*
d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-6*d*x
 - 6*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c)
+ 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e
^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-10*d*x - 10*c)/(d*(8*e
^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x
- 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*
c) + e^(-16*d*x - 16*c) + 1)) + e^(-14*d*x - 14*c)/(d*(8*e^(-2*d*x - 2*c) +
 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-1
0*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x -
 16*c) + 1))) + 6/5*a*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*
c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) +
1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^
(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1
)) - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(65) = 130$.
time = 0.33, size = 1192, normalized size = 16.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -2/5*((10*a^3 + 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 8*(15*a^2*b
+ 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (10*a^3 + 15*a^2*b + 12
*a*b^2 + 5*b^3)*sinh(d*x + c)^8 + 2*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*
cosh(d*x + c)^6 + 2*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3 + 14*(10*a^3 + 15
*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(15*a^2
*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 27*(5*a^2*b + 2*a*b^2)*cosh(d*x +
c))*sinh(d*x + c)^5 + 20*(14*a^3 + 3*a^2*b + 2*b^3)*cosh(d*x + c)^4 + 10*(7
*(10*a^3 + 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 28*a^3 + 6*a^2*b
+ 4*b^3 + 3*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(7*(15*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 45*(5*a^2*b
 + 2*a*b^2)*cosh(d*x + c)^3 + 15*(7*a^2*b + 2*a*b^2 + b^3)*cosh(d*x + c))*s
inh(d*x + c)^3 + 350*a^3 - 75*a^2*b - 12*a*b^2 + 35*b^3 + 2*(280*a^3 - 30*a
^2*b - 12*a*b^2 - 35*b^3)*cosh(d*x + c)^2 + 2*(14*(10*a^3 + 15*a^2*b + 12*a
*b^2 + 5*b^3)*cosh(d*x + c)^6 + 15*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*c
osh(d*x + c)^4 + 280*a^3 - 30*a^2*b - 12*a*b^2 - 35*b^3 + 60*(14*a^3 + 3*a^
2*b + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(15*a^2*b + 18*a*b^2 +
 5*b^3)*cosh(d*x + c)^7 + 27*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^5 + 30*(7*a^
```

```
2*b + 2*a*b^2 + b^3)*cosh(d*x + c)^3 + 21*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)
)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 +
 d*sinh(d*x + c)^10 + 6*d*cosh(d*x + c)^8 + 3*(15*d*cosh(d*x + c)^2 + 2*d)*
sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 + 8*d*cosh(d*x + c))*sinh(d*x + c
)^7 + 13*d*cosh(d*x + c)^6 + (210*d*cosh(d*x + c)^4 + 168*d*cosh(d*x + c)^2
 + 13*d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d*x + c)^5 + 224*d*cosh(d*x + c)^3
 + 81*d*cosh(d*x + c))*sinh(d*x + c)^5 + 8*d*cosh(d*x + c)^4 + (210*d*cosh(
d*x + c)^6 + 420*d*cosh(d*x + c)^4 + 195*d*cosh(d*x + c)^2 + 8*d)*sinh(d*x
+ c)^4 + 4*(30*d*cosh(d*x + c)^7 + 112*d*cosh(d*x + c)^5 + 135*d*cosh(d*x +
 c)^3 + 48*d*cosh(d*x + c))*sinh(d*x + c)^3 - 14*d*cosh(d*x + c)^2 + (45*d*
cosh(d*x + c)^8 + 168*d*cosh(d*x + c)^6 + 195*d*cosh(d*x + c)^4 + 48*d*cosh
(d*x + c)^2 - 14*d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 + 32*d*cosh(d*
x + c)^7 + 81*d*cosh(d*x + c)^5 + 96*d*cosh(d*x + c)^3 + 42*d*cosh(d*x + c)
)*sinh(d*x + c) - 14*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^3(c + dx)\right)^3 \operatorname{csch}^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(65) = 130.
time = 0.69, size = 311, normalized size = 4.38

$$-\frac{2\left(\frac{5\,a^3}{e^{(2\,dx+2\,c)}-1} + \frac{15\,a^2be^{(14\,dx+14\,c)}+15\,ab^2e^{(14\,dx+14\,c)}+5\,b^3e^{(14\,dx+14\,c)}+90\,a^2be^{(12\,dx+12\,c)}+45\,ab^2e^{(12\,dx+12\,c)}+225\,a^2be^{(10\,dx+10\,c)}+75\,ab^2e^{(10\,dx+10\,c)}+35\,b^3e^{(10\,dx+10\,c)}+300\,a^2be^{(8\,dx+8\,c)}+105\,ab^2e^{(8\,dx+8\,c)}+225\,a^2be^{(6\,dx+6\,c)}+93\,ab^2e^{(6\,dx+6\,c)}+35\,b^3e^{(6\,dx+6\,c)}+90\,a^2be^{(4\,dx+4\,c)}+39\,ab^2e^{(4\,dx+4\,c)}+15\,a^2be^{(2\,dx+2\,c)}+9\,ab^2e^{(2\,dx+2\,c)}+5\,b^3e^{(2\,dx+2\,c)}+3\,ab^2}{\left(e^{(2\,dx+2\,c)}+1\right)^8}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^3/(e^(2*d*x + 2*c) - 1) + (15*a^2*b*e^(14*d*x + 14*c) + 15*a*b^2*
e^(14*d*x + 14*c) + 5*b^3*e^(14*d*x + 14*c) + 90*a^2*b*e^(12*d*x + 12*c) +
45*a*b^2*e^(12*d*x + 12*c) + 225*a^2*b*e^(10*d*x + 10*c) + 75*a*b^2*e^(10*d
*x + 10*c) + 35*b^3*e^(10*d*x + 10*c) + 300*a^2*b*e^(8*d*x + 8*c) + 105*a*b
^2*e^(8*d*x + 8*c) + 225*a^2*b*e^(6*d*x + 6*c) + 93*a*b^2*e^(6*d*x + 6*c) +
 35*b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 39*a*b^2*e^(4*d*x + 4*
c) + 15*a^2*b*e^(2*d*x + 2*c) + 9*a*b^2*e^(2*d*x + 2*c) + 5*b^3*e^(2*d*x +
2*c) + 3*a*b^2)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [**B**]
time = 1.38, size = 1515, normalized size = 21.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^2,x)
```

```
[Out] ((3*a*b^2 - 15*a^2*b + 7*b^3)/(28*d) - (exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b
 + b^3))/(4*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((9*a*b^2 + 1
5*a^2*b - 35*b^3)/(140*d) + (exp(6*c + 6*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4
*d) + (3*exp(2*c + 2*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(28*d) - (3*exp(4*c
+ 4*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(28*d))/(4*exp(2*c + 2*d*x) + 6*exp(
4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((exp(10*c + 10
*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*d) - (3*a*b^2 + 9*a^2*b + 7*b^3)/(28*d)
 + (5*exp(6*c + 6*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(14*d) - (5*exp(8*c + 8
*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(28*d) + (exp(2*c + 2*d*x)*(9*a*b^2 - 1
5*a^2*b + 35*b^3))/(28*d) + (exp(4*c + 4*d*x)*(9*a*b^2 + 15*a^2*b - 35*b^3)
)/(14*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) +
 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - ((9
*a^2*b - 3*a*b^2 + 7*b^3)/(28*d) + (exp(4*c + 4*d*x)*(3*a*b^2 + 3*a^2*b + b
^3))/(4*d) - (exp(2*c + 2*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(14*d))/(3*exp
(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((9*a*b^2 - 15
*a^2*b + 35*b^3)/(140*d) + (exp(8*c + 8*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*
d) + (3*exp(4*c + 4*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(14*d) - (exp(6*c + 6
*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(7*d) + (exp(2*c + 2*d*x)*(9*a*b^2 + 15
*a^2*b - 35*b^3))/(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*ex
p(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((exp(12*c
+ 12*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*d) - (3*a*b^2 + 15*a^2*b - 7*b^3)/(
28*d) - (3*exp(2*c + 2*d*x)*(3*a*b^2 + 9*a^2*b + 7*b^3))/(14*d) + (15*exp(8
*c + 8*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(28*d) - (3*exp(10*c + 10*d*x)*(3*
a*b^2 - 15*a^2*b + 7*b^3))/(14*d) + (3*exp(4*c + 4*d*x)*(9*a*b^2 - 15*a^2*b
 + 35*b^3))/(28*d) + (exp(6*c + 6*d*x)*(9*a*b^2 + 15*a^2*b - 35*b^3))/(7*d)
)/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(
8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14
*d*x) + 1) + ((3*a^2*b - 3*a*b^2 + b^3)/(4*d) - (exp(14*c + 14*d*x)*(3*a*b^
2 + 3*a^2*b + b^3))/(4*d) + (3*exp(4*c + 4*d*x)*(3*a*b^2 + 9*a^2*b + 7*b^3)
)/(4*d) + (exp(2*c + 2*d*x)*(3*a*b^2 + 15*a^2*b - 7*b^3))/(4*d) - (3*exp(10
*c + 10*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(4*d) + (exp(12*c + 12*d*x)*(3*a*
b^2 - 15*a^2*b + 7*b^3))/(4*d) - (exp(6*c + 6*d*x)*(9*a*b^2 - 15*a^2*b + 35
*b^3))/(4*d) - (exp(8*c + 8*d*x)*(9*a*b^2 + 15*a^2*b - 35*b^3))/(4*d))/(8*e
xp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c +
8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*
x) + exp(16*c + 16*d*x) + 1) - (2*a^3)/(d*(exp(2*c + 2*d*x) - 1)) - (3*a*b^
2 + 3*a^2*b + b^3)/(4*d*(exp(2*c + 2*d*x) + 1))
```

3.71 $\int \mathrm{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=232

$$\frac{3a^2b\mathrm{ArcTan}(\sinh(c+dx))}{2d}+\frac{5b^3\mathrm{ArcTan}(\sinh(c+dx))}{128d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\mathrm{csch}(c+$$

```
[Out] 3/2*a^2*b*arctan(sinh(d*x+c))/d+5/128*b^3*arctan(sinh(d*x+c))/d+1/2*a^3*arc
tanh(cosh(d*x+c))/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d-a*b^2*sech(d*x+c)^3/d
+3/5*a*b^2*sech(d*x+c)^5/d+3/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d+5/128*b^3*se
ch(d*x+c)*tanh(d*x+c)/d-5/64*b^3*sech(d*x+c)^3*tanh(d*x+c)/d-5/48*b^3*sech(
d*x+c)^3*tanh(d*x+c)^3/d-1/8*b^3*sech(d*x+c)^3*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.22, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3747, 3853, 3855, 2686, 14, 2691}

$$\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\mathrm{csch}(c+dx)}{2d}+\frac{3a^2b\mathrm{ArcTan}(\sinh(c+dx))}{2d}+\frac{3a^2b\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}+\frac{3ab^2\mathrm{sech}^5(c+dx)}{5d}-\frac{ab^2\mathrm{sech}^3(c+dx)}{d}+\frac{5b^3\mathrm{ArcTan}(\sinh(c+dx))}{128d}-\frac{b^3\tanh^5(c+dx)\mathrm{sech}^3(c+dx)}{8d}-\frac{5b^3\tanh^3(c+dx)\mathrm{sech}^3(c+dx)}{48d}-\frac{5b^3\tanh(c+dx)\mathrm{sech}^3(c+dx)}{64d}+\frac{5b^3\tanh(c+dx)\mathrm{sech}(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (3*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) + (5*b^3*ArcTan[Sinh[c + d*x]])/(128*
d) + (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) - (a^3*Coth[c + d*x]*Csch[c + d*x])
/(2*d) - (a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (3*a
^2*b*Sech[c + d*x]*Tanh[c + d*x])/(2*d) + (5*b^3*Sech[c + d*x]*Tanh[c + d*x
])/(128*d) - (5*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(64*d) - (5*b^3*Sech[c +
 d*x]^3*Tanh[c + d*x]^3)/(48*d) - (b^3*Sech[c + d*x]^3*Tanh[c + d*x]^5)/(8*
d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3747

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = -\left(i\int \left(ia^3\operatorname{csch}^3(c+dx)+3ia^2b\operatorname{sech}^3(c+dx)+3iab^2\operatorname{sech}^3(c\right.\right.$$

$$= a^3\int \operatorname{csch}^3(c+dx)\,dx+\left(3a^2b\right)\int \operatorname{sech}^3(c+dx)\,dx+\left(3ab^2\right)\int$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx}{2d}$$

$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth}{}$$

$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth}{}$$

$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth}{}$$

$$= \frac{3a^2b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{5b^3\tan^{-1}(\sinh(c+dx))}{128d}+\frac{a^3\tanh}{}$$

Mathematica [A]
time = 6.27, size = 243, normalized size = 1.05

$$\frac{b(192a^2+5b^2)\,\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}-\frac{a^3\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^3\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{ab^2\text{sech}^3(c+dx)}{d}+\frac{3ab^2\text{sech}^5(c+dx)}{5d}+\frac{\text{sech}^2(c+dx)\,(192a^2b\sinh(c+dx)+5b^3\sinh(c+dx))}{128d}-\frac{59b^3\text{sech}^3(c+dx)\tanh(c+dx)}{192d}+\frac{17b^3\text{sech}^5(c+dx)\tanh(c+dx)}{48d}-\frac{b^3\text{sech}^7(c+dx)\tanh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (b*(192*a^2 + 5*b^2)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (a^3*Csch[(c + d*x
)/2]^2)/(8*d) - (a^3*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^3*Sech[(c + d*x)/2]
^2)/(8*d) - (a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (
Sech[c + d*x]^2*(192*a^2*b*Sinh[c + d*x] + 5*b^3*Sinh[c + d*x]))/(128*d) -
(59*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d) + (17*b^3*Sech[c + d*x]^5*Ta
nh[c + d*x])/(48*d) - (b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(8*d)
```

Maple [C] Result contains complex when optimal does not.
time = 5.02, size = 645, normalized size = 2.78

method	result
risch	$-\frac{e^{dx+c}\left(4608a\,b^2e^{14dx+14c}+23040a^2b\,e^{12dx+12c}-10752a\,b^2e^{12dx+12c}+17280a^2b\,e^{10dx+10c}-1536a\,b^2e^{10dx+10c}-8640a^2b\,e^{16dx+16}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/960*exp(d*x+c)*(4608*a*b^2*exp(14*d*x+14*c)+23040*a^2*b*exp(12*d*x+12*c)
-10752*a*b^2*exp(12*d*x+12*c)+17280*a^2*b*exp(10*d*x+10*c)-1536*a*b^2*exp(1
0*d*x+10*c)-8640*a^2*b*exp(16*d*x+16*c)+7680*a*b^2*exp(16*d*x+16*c)+8640*a^
2*b*exp(2*d*x+2*c)+2880*a^2*b+960*a^3+75*b^3-1536*a*b^2*exp(8*d*x+8*c)-1075
2*a*b^2*exp(6*d*x+6*c)+4608*a*b^2*exp(4*d*x+4*c)-17280*a^2*b*exp(8*d*x+8*c)
+7680*a*b^2*exp(2*d*x+2*c)-23040*a^2*b*exp(6*d*x+6*c)-2880*a^2*b*exp(18*d*x
+18*c)-75*b^3*exp(18*d*x+18*c)+2135*b^3*exp(16*d*x+16*c)-8520*b^3*exp(14*d*
x+14*c)+8640*a^3*exp(2*d*x+2*c)+80640*a^3*exp(12*d*x+12*c)+19760*b^3*exp(12
*d*x+12*c)+120960*a^3*exp(10*d*x+10*c)+34560*a^3*exp(14*d*x+14*c)-2135*b^3*
exp(2*d*x+2*c)+8640*a^3*exp(16*d*x+16*c)+30950*b^3*exp(8*d*x+8*c)+34560*a^3
*exp(4*d*x+4*c)+8520*b^3*exp(4*d*x+4*c)+960*a^3*exp(18*d*x+18*c)-30950*b^3*
exp(10*d*x+10*c)+120960*a^3*exp(8*d*x+8*c)+80640*a^3*exp(6*d*x+6*c)-19760*b
^3*exp(6*d*x+6*c))/d/(1+exp(2*d*x+2*c))^8/(exp(2*d*x+2*c)-1)^2+1/2*a^3/d*ln
(exp(d*x+c)+1)-1/2*a^3/d*ln(exp(d*x+c)-1)+3/2*I*b/d*ln(exp(d*x+c)+I)*a^2+5/
128*I*b^3/d*ln(exp(d*x+c)+I)-3/2*I*b/d*ln(exp(d*x+c)-I)*a^2-5/128*I*b^3/d*l
n(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(212) = 424$.
time = 0.49, size = 586, normalized size = 2.53

$$-\frac{1}{192}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15e^{(-dx-c)}-397e^{(-3dx-3c)}+895e^{(-5dx-5c)}-1765e^{(-7dx-7c)}+1765e^{(-9dx-9c)}-895e^{(-11dx-11c)}+397e^{(-13dx-13c)}-15e^{(-15dx-15c)}}{d(8e^{(-2dx-2c)}+28e^{(-4dx-4c)}+56e^{(-6dx-6c)}+70e^{(-8dx-8c)}+56e^{(-10dx-10c)}+28e^{(-12dx-12c)}+8e^{(-14dx-14c)}+e^{(-16dx-16c)}+1)}\right)-3a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)+\frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3dx-3c)}\right)}{d(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1)}\right)-\frac{8}{5}ab^2\left(\frac{5e^{(-3dx-3c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}-\frac{2e^{(-5dx-5c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{5e^{(-7dx-7c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/192*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) - 397*e^(-3*d*x -
3*c) + 895*e^(-5*d*x - 5*c) - 1765*e^(-7*d*x - 7*c) + 1765*e^(-9*d*x - 9*c)
 - 895*e^(-11*d*x - 11*c) + 397*e^(-13*d*x - 13*c) - 15*e^(-15*d*x - 15*c))
/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^
(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d
*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-d*x - c))/d -
(e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)
 + 1))) + 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e
^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) -
 1))) - 8/5*a*b^2*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-7*d
*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10985 vs. $2(212) = 424$.
time = 0.44, size = 10985, normalized size = 47.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/960*(15*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^19 + 285*(64*a^3 - 19
2*a^2*b - 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^18 + 15*(64*a^3 - 192*a^2*b -
5*b^3)*sinh(d*x + c)^19 + 5*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*
cosh(d*x + c)^17 + 5*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3 + 513*(6
4*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^17 + 85*(171*(64*
a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^3 + (1728*a^3 - 1728*a^2*b + 1536*a*
b^2 + 427*b^3)*cosh(d*x + c))*sinh(d*x + c)^16 + 24*(1440*a^3 + 192*a*b^2 -
 355*b^3)*cosh(d*x + c)^15 + 4*(14535*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x
 + c)^4 + 8640*a^3 + 1152*a*b^2 - 2130*b^3 + 170*(1728*a^3 - 1728*a^2*b + 1
536*a*b^2 + 427*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^15 + 20*(8721*(64*a^3 -
 192*a^2*b - 5*b^3)*cosh(d*x + c)^5 + 170*(1728*a^3 - 1728*a^2*b + 1536*a*b
^2 + 427*b^3)*cosh(d*x + c)^3 + 18*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*
x + c))*sinh(d*x + c)^14 + 16*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3
)*cosh(d*x + c)^13 + 4*(101745*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^6
 + 2975*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^4 + 20
160*a^3 + 5760*a^2*b - 2688*a*b^2 + 4940*b^3 + 630*(1440*a^3 + 192*a*b^2 -
355*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 52*(14535*(64*a^3 - 192*a^2*b
```

- 5*b^3)*cosh(d*x + c)^7 + 595*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^
3)*cosh(d*x + c)^5 + 210*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^3 +
4*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c))*sinh(d*x +
c)^12 + 2*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^1
1 + 2*(566865*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^8 + 30940*(1728*a^
3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^6 + 16380*(1440*a^3 +
192*a*b^2 - 355*b^3)*cosh(d*x + c)^4 + 60480*a^3 + 8640*a^2*b - 768*a*b^2 -
15475*b^3 + 624*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^11 + 22*(62985*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x +
c)^9 + 4420*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^7
+ 3276*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^5 + 208*(5040*a^3 + 1
440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^3 + (60480*a^3 + 8640*a^2*b
- 768*a*b^2 - 15475*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + 2*(60480*a^3 -
8640*a^2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c)^9 + 2*(692835*(64*a^3 - 1
92*a^2*b - 5*b^3)*cosh(d*x + c)^10 + 60775*(1728*a^3 - 1728*a^2*b + 1536*a*
b^2 + 427*b^3)*cosh(d*x + c)^8 + 60060*(1440*a^3 + 192*a*b^2 - 355*b^3)*cos
h(d*x + c)^6 + 5720*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x
+ c)^4 + 60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3 + 55*(60480*a^3 +
8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 2*(5
66865*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^11 + 60775*(1728*a^3 - 172
8*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)^9 + 77220*(1440*a^3 + 192*a*b
^2 - 355*b^3)*cosh(d*x + c)^7 + 10296*(5040*a^3 + 1440*a^2*b - 672*a*b^2 +
1235*b^3)*cosh(d*x + c)^5 + 165*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475
*b^3)*cosh(d*x + c)^3 + 9*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*
cosh(d*x + c))*sinh(d*x + c)^8 + 16*(5040*a^3 - 1440*a^2*b - 672*a*b^2 - 12
35*b^3)*cosh(d*x + c)^7 + 4*(188955*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x +
c)^12 + 24310*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*cosh(d*x + c)
^10 + 38610*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^8 + 6864*(5040*a
^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^6 + 165*(60480*a^3 +
8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^4 + 20160*a^3 - 5760*a^2*
b - 2688*a*b^2 - 4940*b^3 + 18*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 4*(101745*(64*a^3 - 192*a^2*b - 5*b
^3)*cosh(d*x + c)^13 + 15470*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)
*cosh(d*x + c)^11 + 30030*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^9
+ 6864*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^7 + 231
*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^5 + 42*(604
80*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c)^3 + 28*(5040*a^3
- 1440*a^2*b - 672*a*b^2 - 1235*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 24*(
1440*a^3 + 192*a*b^2 + 355*b^3)*cosh(d*x + c)^5 + 4*(43605*(64*a^3 - 192*a^
2*b - 5*b^3)*cosh(d*x + c)^14 + 7735*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 +
427*b^3)*cosh(d*x + c)^12 + 18018*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x
+ c)^10 + 5148*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c
)^8 + 231*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*cosh(d*x + c)^6
+ 63*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*cosh(d*x + c)^4 + 864
0*a^3 + 1152*a*b^2 + 2130*b^3 + 84*(5040*a^3 - 1440*a^2*b - 672*a*b^2 - 123

```
5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 4*(14535*(64*a^3 - 192*a^2*b - 5*
b^3)*cosh(d*x + c)^15 + 2975*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)
*cosh(d*x + c)^13 + 8190*(1440*a^3 + 192*a*b^2 - 355*b^3)*cosh(d*x + c)^11
+ 2860*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*cosh(d*x + c)^9 + 165
*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^3\left(c+dx\right)\right)^3\operatorname{csch}^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(212) = 424.
time = 0.67, size = 426, normalized size = 1.84

$$\frac{480\,a^3\log\left(e^{(dx+c)}+1\right)-480\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right)+15\,(192\,a^2b+5\,b^3)\arctan\left(e^{(dx+c)}\right)-\frac{960\,\left(a^3e^{(3\,dx+3\,c)}+a^3e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}+\frac{2880\,a^2be^{(15\,dx+15\,c)}+75\,b^3e^{(15\,dx+15\,c)}+14400\,a^2be^{(13\,dx+13\,c)}-7680\,ab^2e^{(13\,dx+13\,c)}-1985\,b^3e^{(13\,dx+13\,c)}+25920\,a^2be^{(11\,dx+11\,c)}-19968\,ab^2e^{(11\,dx+11\,c)}+4475\,b^3e^{(11\,dx+11\,c)}+14400\,a^2be^{(9\,dx+9\,c)}-21504\,ab^2e^{(9\,dx+9\,c)}-8825\,b^3e^{(9\,dx+9\,c)}-14400\,a^2be^{(7\,dx+7\,c)}-21504\,ab^2e^{(7\,dx+7\,c)}+8825\,b^3e^{(7\,dx+7\,c)}-25920\,a^2be^{(5\,dx+5\,c)}-19968\,ab^2e^{(5\,dx+5\,c)}-4475\,b^3e^{(5\,dx+5\,c)}-14400\,a^2be^{(3\,dx+3\,c)}-7680\,ab^2e^{(3\,dx+3\,c)}+1985\,b^3e^{(3\,dx+3\,c)}-2880\,a^2be^{(dx+c)}-75\,b^3e^{(dx+c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^8}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/960*(480*a^3*log(e^(d*x + c) + 1) - 480*a^3*log(abs(e^(d*x + c) - 1)) + 1
5*(192*a^2*b + 5*b^3)*arctan(e^(d*x + c)) - 960*(a^3*e^(3*d*x + 3*c) + a^3*
e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2 + (2880*a^2*b*e^(15*d*x + 15*c) + 75*b
^3*e^(15*d*x + 15*c) + 14400*a^2*b*e^(13*d*x + 13*c) - 7680*a*b^2*e^(13*d*x
 + 13*c) - 1985*b^3*e^(13*d*x + 13*c) + 25920*a^2*b*e^(11*d*x + 11*c) - 199
68*a*b^2*e^(11*d*x + 11*c) + 4475*b^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*
d*x + 9*c) - 21504*a*b^2*e^(9*d*x + 9*c) - 8825*b^3*e^(9*d*x + 9*c) - 14400
*a^2*b*e^(7*d*x + 7*c) - 21504*a*b^2*e^(7*d*x + 7*c) + 8825*b^3*e^(7*d*x +
7*c) - 25920*a^2*b*e^(5*d*x + 5*c) - 19968*a*b^2*e^(5*d*x + 5*c) - 4475*b^3
*e^(5*d*x + 5*c) - 14400*a^2*b*e^(3*d*x + 3*c) - 7680*a*b^2*e^(3*d*x + 3*c)
 + 1985*b^3*e^(3*d*x + 3*c) - 2880*a^2*b*e^(d*x + c) - 75*b^3*e^(d*x + c))/
(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B]
time = 8.05, size = 731, normalized size = 3.15

$$\frac{a^3\ln\left(\mathrm{e}^{c+dx}+1\right)}{2\,d}-\frac{a^3\ln\left(\mathrm{e}^{c+dx}-1\right)}{2\,d}+\frac{\mathrm{e}^{c+dx}\,\left(192\,a^2\,b+5\,b^3\right)}{64\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{\mathrm{e}^{c+dx}\,\left(2245\,b^3+3264\,a\,b^2\right)}{120\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}-\frac{\mathrm{e}^{c+dx}\,\left(1325\,b^3+768\,a\,b^2\right)}{20\,d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-\frac{500\,b^3\,\mathrm{e}^{c+dx}}{3\,d\,\left(6\,\mathrm{e}^{2\,c+2\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}+20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}+6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}+\frac{2\,\mathrm{e}^{c+dx}\,\left(1025\,b^3+144\,a\,b^2\right)}{15\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}+\frac{112\,b^3\,\mathrm{e}^{c+dx}}{d\,\left(7\,\mathrm{e}^{2\,c+2\,d\,x}+21\,\mathrm{e}^{4\,c+4\,d\,x}+35\,\mathrm{e}^{6\,c+6\,d\,x}+35\,\mathrm{e}^{8\,c+8\,d\,x}+21\,\mathrm{e}^{10\,c+10\,d\,x}+7\,\mathrm{e}^{12\,c+12\,d\,x}+\mathrm{e}^{14\,c+14\,d\,x}+1\right)}-\frac{\mathrm{e}^{c+dx}\,\left(576\,a^2\,b+768\,a\,b^2+251\,b^3\right)}{96\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{32\,b^3\,\mathrm{e}^{c+dx}}{d\,\left(8\,\mathrm{e}^{2\,c+2\,d\,x}+28\,\mathrm{e}^{4\,c+4\,d\,x}+56\,\mathrm{e}^{6\,c+6\,d\,x}+70\,\mathrm{e}^{8\,c+8\,d\,x}+56\,\mathrm{e}^{10\,c+10\,d\,x}+28\,\mathrm{e}^{12\,c+12\,d\,x}+8\,\mathrm{e}^{14\,c+14\,d\,x}+\mathrm{e}^{16\,c+16\,d\,x}+1\right)}-\frac{a^3\,\mathrm{e}^{c+dx}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^3\,\mathrm{e}^{c+dx}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{b\,\ln\left(\mathrm{e}^{c+dx}-\mathrm{i}\right)\,\left(192\,a^2+5\,b^2\right)\,1{}\mathrm{i}}{128\,d}+\frac{b\,\ln\left(\mathrm{e}^{c+dx}+1{}\mathrm{i}\right)\,\left(192\,a^2+5\,b^2\right)\,1{}\mathrm{i}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^3,x)
```

```
[Out] (a^3*log(exp(c + d*x) + 1))/(2*d) - (a^3*log(exp(c + d*x) - 1))/(2*d) + (ex
p(c + d*x)*(192*a^2*b + 5*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*
x)*(3264*a*b^2 + 2245*b^3))/(120*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
 + exp(6*c + 6*d*x) + 1)) - (b*log(exp(c + d*x) - 1i)*(192*a^2 + 5*b^2)*1i)
/(128*d) + (b*log(exp(c + d*x) + 1i)*(192*a^2 + 5*b^2)*1i)/(128*d) - (exp(c
 + d*x)*(768*a*b^2 + 1325*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d
*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (500*b^3*exp(c + d*x))/
(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*e
xp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (2*exp(
c + d*x)*(144*a*b^2 + 1025*b^3))/(15*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4
*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1))
 + (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35
*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*
c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (exp(c + d*x)*(768*a*b^2 + 576*a^2
*b + 251*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (32*b^3
*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c +
6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x
) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (a^3*exp(c + d*x))/(d
*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*ex
p(2*c + 2*d*x) + 1))
```

3.72 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=138

$$\frac{a^3\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{3a^2b\log(\tanh(c+dx))}{d}-\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{ab^2\tanh^3(c+dx)}{d}-3ab^2\,\mathrm{ta}$$

```
[Out] a^3*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d+3*a^2*b*ln(tanh(d*x+c))/d-3/2*a^2
*b*tanh(d*x+c)^2/d+a*b^2*tanh(d*x+c)^3/d-3/5*a*b^2*tanh(d*x+c)^5/d+1/6*b^3*
tanh(d*x+c)^6/d-1/8*b^3*tanh(d*x+c)^8/d
```

Rubi [A]

time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3744, 1816}

$$-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}-\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3a^2b\log(\tanh(c+dx))}{d}-\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{ab^2\tanh^3(c+dx)}{d}-\frac{b^3\tanh^8(c+dx)}{8d}+\frac{b^3\tanh^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (a^3*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a^2*b*Log[Tanh[c +
 d*x]])/d - (3*a^2*b*Tanh[c + d*x]^2)/(2*d) + (a*b^2*Tanh[c + d*x]^3)/d - (
3*a*b^2*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^6)/(6*d) - (b^3*Tanh[c
+ d*x]^8)/(8*d)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int\frac{(1-x^2)\,(a+bx^3)^3}{x^4}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\operatorname{Subst}\left(\int\left(\frac{a^3}{x^4}-\frac{a^3}{x^2}+\frac{3a^2b}{x}-3a^2bx+3ab^2x^2-3ab^2x^4+b^3x^5\right.\right.}{d}$$

$$=\frac{a^3\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{3a^2b\log(\tanh(c+dx))}{d}$$

Mathematica [A]
time = 0.14, size = 213, normalized size = 1.54

$$\frac{2a^3\coth(c+dx)}{3d}-\frac{a^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{3a^2b\log(\cosh(c+dx))}{d}+\frac{3a^2b\log(\sinh(c+dx))}{d}+\frac{3a^2b\operatorname{sech}^2(c+dx)}{2d}-\frac{b^3\operatorname{sech}^4(c+dx)}{4d}+\frac{b^3\operatorname{sech}^6(c+dx)}{3d}-\frac{b^3\operatorname{sech}^8(c+dx)}{8d}+\frac{2ab^2\tanh(c+dx)}{5d}+\frac{ab^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d}-\frac{3ab^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (2*a^3*Coth[c + d*x])/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (
3*a^2*b*Log[Cosh[c + d*x]])/d + (3*a^2*b*Log[Sinh[c + d*x]])/d + (3*a^2*b*S
ech[c + d*x]^2)/(2*d) - (b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]^6)
/(3*d) - (b^3*Sech[c + d*x]^8)/(8*d) + (2*a*b^2*Tanh[c + d*x])/(5*d) + (a*b
^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tanh[c +
 d*x])/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(128) = 256$.
time = 3.69, size = 565, normalized size = 4.09

method	result
risch	$-\frac{2\left(-6a\,b^2-240a\,b^2\mathrm{e}^{14dx+14c}+270a^2b\,\mathrm{e}^{12dx+12c}+96a\,b^2\mathrm{e}^{12dx+12c}-270a^2b\,\mathrm{e}^{10dx+10c}+180a\,b^2\mathrm{e}^{10dx+10c}+90a\,b^2\mathrm{e}^{18dx+18c}-30a\,b^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(-6*a*b^2-240*a*b^2*exp(14*d*x+14*c)+270*a^2*b*exp(12*d*x+12*c)+96*a*
b^2*exp(12*d*x+12*c)-270*a^2*b*exp(10*d*x+10*c)+180*a*b^2*exp(10*d*x+10*c)+
90*a*b^2*exp(18*d*x+18*c)-30*a*b^2*exp(16*d*x+16*c)+360*a^2*b*exp(14*d*x+14
*c)+135*a^2*b*exp(4*d*x+4*c)+45*a^2*b*exp(2*d*x+2*c)-10*a^3-108*a*b^2*exp(8
*d*x+8*c)+48*a*b^2*exp(4*d*x+4*c)-360*a^2*b*exp(8*d*x+8*c)-30*a*b^2*exp(2*d
*x+2*c)-135*a^2*b*exp(18*d*x+18*c)+30*b^3*exp(18*d*x+18*c)-130*b^3*exp(16*d
*x+16*c)+310*b^3*exp(14*d*x+14*c)-50*a^3*exp(2*d*x+2*c)-45*a^2*b*exp(20*d*x
```

```
+20*c)+1400*a^3*exp(12*d*x+12*c)-490*b^3*exp(12*d*x+12*c)+1540*a^3*exp(10*d
*x+10*c)+760*a^3*exp(14*d*x+14*c)+230*a^3*exp(16*d*x+16*c)-310*b^3*exp(8*d*
x+8*c)-40*a^3*exp(4*d*x+4*c)-30*b^3*exp(4*d*x+4*c)+30*a^3*exp(18*d*x+18*c)+
490*b^3*exp(10*d*x+10*c)+980*a^3*exp(8*d*x+8*c)+280*a^3*exp(6*d*x+6*c)+130*
b^3*exp(6*d*x+6*c))/d/(1+exp(2*d*x+2*c))^8/(exp(2*d*x+2*c)-1)^3+3*a^2*b/d*l
n(exp(2*d*x+2*c)-1)-3*b/d*ln(1+exp(2*d*x+2*c))*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(128) = 256$.
time = 0.50, size = 997, normalized size = 7.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3*a^2*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x
 - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c
) + 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*
d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
+ 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(
-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c
) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c)
- 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) - 4/3*b^3*(3*e^(-4*d*x - 4*c
)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e
^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*
d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) - 4*e^(-6*d*x - 6*c)/(d*(8*e^(-2*d*x
 - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) +
 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(
-16*d*x - 16*c) + 1)) + 10*e^(-8*d*x - 8*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(
-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x -
 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c)
+ 1)) - 4*e^(-10*d*x - 10*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) +
 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(
-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 3*e^(-1
2*d*x - 12*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x -
 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c)
 + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9459 vs. $2(128) = 256$.

time = 0.48, size = 9459, normalized size = 68.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(90*a^2*b*cosh(d*x + c)^20 + 1800*a^2*b*cosh(d*x + c)*sinh(d*x + c)^19
 + 90*a^2*b*sinh(d*x + c)^20 - 30*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(
d*x + c)^18 + 30*(570*a^2*b*cosh(d*x + c)^2 - 2*a^3 + 9*a^2*b - 6*a*b^2 - 2
*b^3)*sinh(d*x + c)^18 + 540*(190*a^2*b*cosh(d*x + c)^3 - (2*a^3 - 9*a^2*b
+ 6*a*b^2 + 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 20*(23*a^3 - 3*a*b^2 -
 13*b^3)*cosh(d*x + c)^16 + 10*(43605*a^2*b*cosh(d*x + c)^4 - 46*a^3 + 6*a*
b^2 + 26*b^3 - 459*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^16 + 160*(8721*a^2*b*cosh(d*x + c)^5 - 153*(2*a^3 - 9*a^2*b + 6*
a*b^2 + 2*b^3)*cosh(d*x + c)^3 - 2*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c
))*sinh(d*x + c)^15 - 20*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x +
 c)^14 + 20*(174420*a^2*b*cosh(d*x + c)^6 - 4590*(2*a^3 - 9*a^2*b + 6*a*b^2
 + 2*b^3)*cosh(d*x + c)^4 - 76*a^3 - 36*a^2*b + 24*a*b^2 - 31*b^3 - 120*(23
*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 40*(174420*a^2
*b*cosh(d*x + c)^7 - 6426*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)
^5 - 280*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^3 - 7*(76*a^3 + 36*a^2*b
 - 24*a*b^2 + 31*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 4*(700*a^3 + 135*a^
2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^12 + 4*(2834325*a^2*b*cosh(d*x + c)
^8 - 139230*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^6 - 9100*(23*
a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^4 - 700*a^3 - 135*a^2*b - 48*a*b^2 +
245*b^3 - 455*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^12 + 16*(944775*a^2*b*cosh(d*x + c)^9 - 59670*(2*a^3 - 9*a^2*b +
6*a*b^2 + 2*b^3)*cosh(d*x + c)^7 - 5460*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*
x + c)^5 - 455*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^3 - 3*
(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c))*sinh(d*x + c)^11
- 20*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^10 + 4*(4157010
*a^2*b*cosh(d*x + c)^10 - 328185*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d
*x + c)^8 - 40040*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^6 - 5005*(76*a^
3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^4 - 770*a^3 + 135*a^2*b - 9
0*a*b^2 - 245*b^3 - 66*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^10 + 40*(377910*a^2*b*cosh(d*x + c)^11 - 36465*(2*a^3
 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^9 - 5720*(23*a^3 - 3*a*b^2 - 13
*b^3)*cosh(d*x + c)^7 - 1001*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d
*x + c)^5 - 22*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^3 -
 5*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c))*sinh(d*x + c)^9
- 4*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*cosh(d*x + c)^8 + 4*(2834325
*a^2*b*cosh(d*x + c)^12 - 328185*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d
*x + c)^10 - 64350*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^8 - 15015*(76*
a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^6 - 495*(700*a^3 + 135*a^
```

```
2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^4 - 490*a^3 + 180*a^2*b + 54*a*b^2
+ 155*b^3 - 225*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^8 + 32*(218025*a^2*b*cosh(d*x + c)^13 - 29835*(2*a^3 - 9*a^2*b
 + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^11 - 7150*(23*a^3 - 3*a*b^2 - 13*b^3)*cos
h(d*x + c)^9 - 2145*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^7
 - 99*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^5 - 75*(154*
a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^3 - (490*a^3 - 180*a^2*b
- 54*a*b^2 - 155*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(28*a^3 + 13*b^3)
*cosh(d*x + c)^6 + 4*(872100*a^2*b*cosh(d*x + c)^14 - 139230*(2*a^3 - 9*a^2
*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^12 - 40040*(23*a^3 - 3*a*b^2 - 13*b^3)*
cosh(d*x + c)^10 - 15015*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x +
 c)^8 - 924*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^6 - 10
50*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^4 - 140*a^3 - 65*
b^3 - 28*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 8*(174420*a^2*b*cosh(d*x + c)^15 - 32130*(2*a^3 - 9*a^2*b + 6*a
*b^2 + 2*b^3)*cosh(d*x + c)^13 - 10920*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x
 + c)^11 - 5005*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^9 - 3
96*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^7 - 630*(154*a^
3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^5 - 28*(490*a^3 - 180*a^2*b
 - 54*a*b^2 - 155*b^3)*cosh(d*x + c)^3 - 15*(28*a^3 + 13*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(40*a^3 - 135*a^2*b - 48*a*b^2 + 30*b^3)*cosh(d*x + c
)^4 + 2*(218025*a^2*b*cosh(d*x + c)^16 - 45900*(2*a^3 - 9*a^2*b + 6*a*b^2 +
 2*b^3)*cosh(d*x + c)^14 - 18200*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^
12 - 10010*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^10 - 990*(
700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^8 - 2100*(154*a^3 -
 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^6 - 140*(490*a^3 - 180*a^2*b -
 54*a*b^2 - 155*b^3)*cosh(d*x + c)^4 + 40*a^3 - 135*a^2*b - 48*a*b^2 + 30*b
^3 - 150*(28*a^3 + 13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(12825*a^2*
b*cosh(d*x + c)^17 - 3060*(2*a^3 - 9*a^2*b + 6*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^{3}\left(c + dx\right)\right)^{3} \operatorname{csch}^{4}\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(128) = 256.
time = 0.67, size = 437, normalized size = 3.17

$$-\frac{2520\,a^{2}b\log\left(e^{\left(2\,dx+2\,c\right)}+1\right)-2520\,a^{2}b\log\left(\left|e^{\left(2\,dx+2\,c\right)}-1\right|\right)+\frac{140\,\left(33\,a^{2}be^{\left(6\,dx+6\,c\right)}-99\,a^{2}be^{\left(4\,dx+4\,c\right)}+24\,a^{3}e^{\left(2\,dx+2\,c\right)}+99\,a^{2}be^{\left(2\,dx+2\,c\right)}-8\,a^{3}-33\,a^{2}b\right)}{\left(e^{\left(2\,dx+2\,c\right)}-1\right)^{3}}-\frac{6849\,a^{2}be^{\left(16\,dx+16\,c\right)}+59832\,a^{2}be^{\left(14\,dx+14\,c\right)}+222012\,a^{2}be^{\left(12\,dx+12\,c\right)}-10080\,ab^{2}e^{\left(12\,dx+12\,c\right)}-3360\,b^{3}e^{\left(12\,dx+12\,c\right)}+459144\,a^{2}be^{\left(10\,dx+10\,c\right)}-26880\,ab^{2}e^{\left(10\,dx+10\,c\right)}+4480\,b^{3}e^{\left(10\,dx+10\,c\right)}+580230\,a^{2}be^{\left(8\,dx+8\,c\right)}-23520\,ab^{2}e^{\left(8\,dx+8\,c\right)}-11200\,b^{3}e^{\left(8\,dx+8\,c\right)}+459144\,a^{2}be^{\left(6\,dx+6\,c\right)}-10752\,ab^{2}e^{\left(6\,dx+6\,c\right)}+4480\,b^{3}e^{\left(6\,dx+6\,c\right)}+222012\,a^{2}be^{\left(4\,dx+4\,c\right)}-8736\,ab^{2}e^{\left(4\,dx+4\,c\right)}-3360\,b^{3}e^{\left(4\,dx+4\,c\right)}+59832\,a^{2}be^{\left(2\,dx+2\,c\right)}-5376\,ab^{2}e^{\left(2\,dx+2\,c\right)}+6849\,a^{2}b-672\,ab^{2}}{\left(e^{\left(2\,dx+2\,c\right)}+1\right)^{8}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/840*(2520*a^2*b*log(e^(2*d*x + 2*c) + 1) - 2520*a^2*b*log(abs(e^(2*d*x +
 2*c) - 1)) + 140*(33*a^2*b*e^(6*d*x + 6*c) - 99*a^2*b*e^(4*d*x + 4*c) + 24
*a^3*e^(2*d*x + 2*c) + 99*a^2*b*e^(2*d*x + 2*c) - 8*a^3 - 33*a^2*b)/(e^(2*d
*x + 2*c) - 1)^3 - (6849*a^2*b*e^(16*d*x + 16*c) + 59832*a^2*b*e^(14*d*x +
14*c) + 222012*a^2*b*e^(12*d*x + 12*c) - 10080*a*b^2*e^(12*d*x + 12*c) - 33
60*b^3*e^(12*d*x + 12*c) + 459144*a^2*b*e^(10*d*x + 10*c) - 26880*a*b^2*e^(
10*d*x + 10*c) + 4480*b^3*e^(10*d*x + 10*c) + 580230*a^2*b*e^(8*d*x + 8*c)
- 23520*a*b^2*e^(8*d*x + 8*c) - 11200*b^3*e^(8*d*x + 8*c) + 459144*a^2*b*e^
(6*d*x + 6*c) - 10752*a*b^2*e^(6*d*x + 6*c) + 4480*b^3*e^(6*d*x + 6*c) + 22
2012*a^2*b*e^(4*d*x + 4*c) - 8736*a*b^2*e^(4*d*x + 4*c) - 3360*b^3*e^(4*d*x
 + 4*c) + 59832*a^2*b*e^(2*d*x + 2*c) - 5376*a*b^2*e^(2*d*x + 2*c) + 6849*a
^2*b - 672*a*b^2)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B]
time = 0.54, size = 646, normalized size = 4.68

$$\frac{96\,(10\,b^3+a\,b^2)}{5\,d\,(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1)}-\frac{640\,b^3}{3\,d\,(6\,e^{2c+2dx}+15\,e^{4c+4dx}+20\,e^{6c+6dx}+15\,e^{8c+8dx}+6\,e^{10c+10dx}+e^{12c+12dx}+1)}-\frac{4\,(25\,b^3+12\,a\,b^2)}{d\,(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1)}+\frac{128\,b^3}{d\,(7\,e^{2c+2dx}+21\,e^{4c+4dx}+35\,e^{6c+6dx}+35\,e^{8c+8dx}+21\,e^{10c+10dx}+7\,e^{12c+12dx}+e^{14c+14dx}+1)}-\frac{2\,(3\,a^2\,b+6\,a\,b^2+2\,b^3)}{d\,(2\,e^{2c+2dx}+e^{4c+4dx}+1)}-\frac{6\,\mathrm{atan}\left(\frac{a^2\,b\,e^{2c}\,e^{2dx}\,\sqrt{-d^2}}{d\,\sqrt{a^4\,b^2}}\right)\,\sqrt{a^4\,b^2}}{\sqrt{-d^2}}-\frac{32\,b^3}{d\,(8\,e^{2c+2dx}+28\,e^{4c+4dx}+56\,e^{6c+6dx}+70\,e^{8c+8dx}+56\,e^{10c+10dx}+28\,e^{12c+12dx}+8\,e^{14c+14dx}+e^{16c+16dx}+1)}-\frac{4\,a^3}{d\,(e^{4c+4dx}-2\,e^{2c+2dx}+1)}-\frac{8\,a^3}{3\,d\,(3\,e^{2c+2dx}-3\,e^{4c+4dx}+e^{6c+6dx}-1)}+\frac{8\,(11\,b^3+15\,a\,b^2)}{3\,d\,(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1)}+\frac{6\,a^2\,b}{d\,(e^{2c+2dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^4,x)
```

```
[Out] (96*(a*b^2 + 10*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (640*b^3)
/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*
exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (4*(12
*a*b^2 + 25*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c +
 6*d*x) + exp(8*c + 8*d*x) + 1)) + (128*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*ex
p(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c +
10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (2*(6*a*b^2 + 3
*a^2*b + 2*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (6*atan(
(a^2*b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(a^4*b^2)^(1/2)))*(a^4*b^2)^(1/
2))/(-d^2)^(1/2) - (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) +
56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(
12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (4*a^3)/
(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^3)/(3*d*(3*exp(2*c +
 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) + (8*(15*a*b^2 + 11*b
^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
 + (6*a^2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.73 $\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=491

$$-\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{4/3}b^{2/3}-b^2\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)^3 d}-\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3 d}+\frac{3a(a+5b)}{}$$

```
[Out] -3/16*a*(a-5*b)*ln(1-tanh(d*x+c))/(a+b)^3/d+3/16*a*(a+5*b)*ln(1+tanh(d*x+c)
)/(a-b)^3/d-1/3*a^(2/3)*b^(1/3)*(a^4+7*a^2*b^2+b^4+3*a^(2/3)*b^(4/3)*(2*a^2
+b^2))*ln(a^(1/3)+b^(1/3)*tanh(d*x+c))/(a^2-b^2)^3/d+1/6*a^(2/3)*b^(1/3)*(a
^4+7*a^2*b^2+b^4+3*a^(2/3)*b^(4/3)*(2*a^2+b^2))*ln(a^(2/3)-a^(1/3)*b^(1/3)*
tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/(a^2-b^2)^3/d-a^2*b*(a^2+2*b^2)*ln(a+b*t
anh(d*x+c)^3)/(a^2-b^2)^3/d-1/3*a^(2/3)*b^(1/3)*(a^2+3*a^(4/3)*b^(2/3)-b^2)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a^(1/3)*3^(1/2))/(a^(4/3)+a^(2/
3)*b^(2/3)+b^(4/3))^3/d*3^(1/2)+1/16/(a+b)/d/(1-tanh(d*x+c))^2+1/16*(-5*a+b
)/(a+b)^2/d/(1-tanh(d*x+c))-1/16/(a-b)/d/(1+tanh(d*x+c))^2+1/16*(5*a+b)/(a-
b)^2/d/(1+tanh(d*x+c))
```

Rubi [A]

time = 0.65, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3744, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{a^2b(a^2+2b^2)\log\left(a+b\tanh^3(c+dx)\right)}{d\left(a^2-b^2\right)^3}-\frac{a^{2/3}\sqrt[3]{b}\left(3a^{4/3}b^{2/3}+a^2-b^2\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}d\left(a^{2/3}b^{2/3}+a^{4/3}+b^{4/3}\right)^3}+\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+3a^{2/3}b^{4/3}(2a^2+b^2)+b^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6d\left(a^2-b^2\right)^3}-\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+3a^{2/3}b^{4/3}(2a^2+b^2)+b^4\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3d\left(a^2-b^2\right)^3}-\frac{5a-b}{16d(a+b)^2(1-\tanh(c+dx))}+\frac{5a+b}{16d(a-b)^2(\tanh(c+dx)+1)}+\frac{1}{16d(a+b)(1-\tanh(c+dx))^2}-\frac{1}{16d(a-b)(\tanh(c+dx)+1)^2}-\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16d(a+b)^3}+\frac{3a(a+5b)\log(\tanh(c+dx)+1)}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -((a^(2/3)*b^(1/3)*(a^2 + 3*a^(4/3)*b^(2/3) - b^2)*ArcTan[(a^(1/3) - 2*b^(1
/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^(4/3) + a^(2/3)*b^(2/3)
+ b^(4/3))^3*d)) - (3*a*(a - 5*b)*Log[1 - Tanh[c + d*x]])/(16*(a + b)^3*d)
+ (3*a*(a + 5*b)*Log[1 + Tanh[c + d*x]])/(16*(a - b)^3*d) - (a^(2/3)*b^(1/3
)*(a^4 + 7*a^2*b^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(1/3) + b
^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^3*d) + (a^(2/3)*b^(1/3)*(a^4 + 7*a^2*
b^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*
Tanh[c + d*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^3*d) - (a^2*b*(a^2
 + 2*b^2)*Log[a + b*Tanh[c + d*x]^3])/((a^2 - b^2)^3*d) + 1/(16*(a + b)*d*(
1 - Tanh[c + d*x])^2) - (5*a - b)/(16*(a + b)^2*d*(1 - Tanh[c + d*x])) - 1/
(16*(a - b)*d*(1 + Tanh[c + d*x])^2) + (5*a + b)/(16*(a - b)^2*d*(1 + Tanh[
c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8(a+b)(-1+x)^3} + \frac{-5a+b}{16(a+b)^2(-1+x)^2} - \frac{3a(a-5b)}{16(a+b)^3(-1+x)} + \frac{1}{8(a-b)(1+x)^3} + \frac{-}{16(a-b}\right.\right.}{}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} + \frac{}{16(a}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} + \frac{}{16(a}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^2b(c}{}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^{2/3}\sqrt[3]{}}{}$$

$$= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^{2/3}\sqrt[3]{}}{}$$

$$= -\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{4/3}b^{2/3}-b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)^3d} - \frac{3a(a-5b)\log(1-}{16(a+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.21, size = 645, normalized size = 1.31

$$\frac{-32ab\,\text{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2-3b\#1^2+a\#1^3+b\#1^3\&,\frac{-6a^3c-12ab^2c-6a^3dx-12ab^2dx+3a^3\log\left(e^{2(c+dx)}-\#1\right)+6ab^2\log\left(e^{2(c+dx)}-\#1\right)-8a^3c\#1+4a^2bc\#1+8ab^2c\#1-4b^3c\#1-8a^3dx\#1+4a^2bdx\#1+8ab^2dx\#1-4b^3dx\#1+4a^3\log\left(e^{2(c+dx)}-\#1\right)\#1-2a^2b\log\left(e^{2(c+dx)}-\#1\right)\#1-4ab^2\log\left(e^{2(c+dx)}-\#1\right)\#1+2b^3\log\left(e^{2(c+dx)}-\#1\right)\#1-10a^3c\#1^2+20a^2bc\#1^2-20ab^2c\#1^2+4b^3c\#1^2-10a^3dx\#1^2+20a^2bdx\#1^2-20ab^2dx\#1^2+4b^3dx\#1^2+5a^3\log\left(e^{2(c+dx)}-\#1\right)\#1^2-10a^2b\log\left(e^{2(c+dx)}-\#1\right)\#1^2+10ab^2\log\left(e^{2(c+dx)}-\#1\right)\#1^2-2b^3\log\left(e^{2(c+dx)}-\#1\right)\#1^2}{a-b+2a\#1+2b\#1+a\#1^2-b\#1^2}\&\right]+3(4b(5a^3+5a^2b+ab^2+b^3)\cosh(2(c+dx))-(a-b)b(a+b)^2\cosh(4(c+dx))-8a(a^3+a^2b+2ab^2+2b^3)\sinh(2(c+dx))+a(a-b)\left(12(a^2-6ab+5b^2)(c+dx)+(a+b)^2\sinh(4(c+dx))\right))}{96(a-b)^2(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-32*a*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b
*#1^3 & , (-6*a^3*c - 12*a*b^2*c - 6*a^3*d*x - 12*a*b^2*d*x + 3*a^3*Log[E^(
2*(c + d*x)) - #1] + 6*a*b^2*Log[E^(2*(c + d*x)) - #1] - 8*a^3*c*#1 + 4*a^2
*b*c*#1 + 8*a*b^2*c*#1 - 4*b^3*c*#1 - 8*a^3*d*x*#1 + 4*a^2*b*d*x*#1 + 8*a*b
^2*d*x*#1 - 4*b^3*d*x*#1 + 4*a^3*Log[E^(2*(c + d*x)) - #1]*#1 - 2*a^2*b*Log
[E^(2*(c + d*x)) - #1]*#1 - 4*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b^3*Lo
g[E^(2*(c + d*x)) - #1]*#1 - 10*a^3*c*#1^2 + 20*a^2*b*c*#1^2 - 20*a*b^2*c*#
1^2 + 4*b^3*c*#1^2 - 10*a^3*d*x*#1^2 + 20*a^2*b*d*x*#1^2 - 20*a*b^2*d*x*#1^
2 + 4*b^3*d*x*#1^2 + 5*a^3*Log[E^(2*(c + d*x)) - #1]*#1^2 - 10*a^2*b*Log[E^
(2*(c + d*x)) - #1]*#1^2 + 10*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1^2 - 2*b^3*
Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2)
 & ] + 3*(4*b*(5*a^3 + 5*a^2*b + a*b^2 + b^3)*Cosh[2*(c + d*x)] - (a - b)*b
*(a + b)^2*Cosh[4*(c + d*x)] - 8*a*(a^3 + a^2*b + 2*a*b^2 + 2*b^3)*Sinh[2*(
c + d*x)] + a*(a - b)*(12*(a^2 - 6*a*b + 5*b^2)*(c + d*x) + (a + b)^2*Sinh[
4*(c + d*x)])))/(96*(a - b)^2*(a + b)^3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.42, size = 452, normalized size = 0.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8/(32*a-32*b)/(tanh(1/2*d*x+1/2*c)+1)^4+32/(64*a-64*b)/(tanh(1/2*d*x+
1/2*c)+1)^3-1/8*(-a-5*b)/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/8*(3*a+3*b)/(a
-b)^2/(tanh(1/2*d*x+1/2*c)+1)+3/8*a*(a+5*b)/(a-b)^3*ln(tanh(1/2*d*x+1/2*c)+
1)-1/3*a*b/(a-b)^3/(a+b)^3*sum((3*a^2*(a^2+2*b^2)*_R^5+3*a*b*(-2*a^2-b^2)*_
R^4+2*(4*a^4+13*a^2*b^2+b^4)*_R^3+12*a*b*(a^2+2*b^2)*_R^2+(a^4-8*a^2*b^2-2*
b^4)*_R+6*a^3*b+3*a*b^3)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+8/(32*a+32*b)/(tan
h(1/2*d*x+1/2*c)-1)^4+32/(64*a+64*b)/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(a-5*b)/
(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(3*a-3*b)/(a+b)^2/(tanh(1/2*d*x+1/2*c
)-1)-3/8*a*(a-5*b)/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] -6*a^4*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (d*x + c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)) - 12*a^2*b^3*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (d*x + c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)) + 10*a^4*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) - 20*a^3*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 20*a^2*b^3*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) - 4*a*b^4*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 8*a^4*b*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) - 4*a^3*b^2*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) - 8*a^2*b^3*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 4*a*b^4*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) - 1/64*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 - 24*(a^4*d*e^(4*c) - 7*a^3*b*d*e^(4*c) + 11*a^2*b^2*d*e^(4*c) - 5*a*b^3*d*e^(4*c))*x*e^(4*d*x) - (a^4*e^(8*c) - 2*a^2*b^2*e^(8*c) + b^4*e^(8*c))*e^(8*d*x) + 4*(2*a^4*e^(6*c) - 3*a^3*b*e^(6*c) - a^2*b^2*e^(6*c) + 3*a*b^3*e^(6*c) - b^4*e^(6*c))*e^(6*d*x) - 4*(2*a^4*e^(2*c) + 7*a^3*b*e^(2*c) + 9*a^2*b^2*e^(2*c) + 5*a*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x))*e^(-4*d*x)/(a^5*d*e^(4*c) + a^4*b*d*e^(4*c) - 2*a^3*b^2*d*e^(4*c) - 2*a^2*b^3*d*e^(4*c) + a*b^4*d*e^(4*c) + b^5*d*e^(4*c))

Fricas [C] Result contains complex when optimal does not.
time = 1.49, size = 17123, normalized size = 34.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] 1/192*(3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(d*x + c)^
8 + 24*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(d*x + c)*si
nh(d*x + c)^7 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(
d*x + c)^8 + 72*(a^5 + 8*a^4*b + 18*a^3*b^2 + 16*a^2*b^3 + 5*a*b^4)*d*x*cos
h(d*x + c)^4 - 12*(2*a^5 - 5*a^4*b + 2*a^3*b^2 + 4*a^2*b^3 - 4*a*b^4 + b^5)
*cosh(d*x + c)^6 - 12*(2*a^5 - 5*a^4*b + 2*a^3*b^2 + 4*a^2*b^3 - 4*a*b^4 +
b^5 - 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(d*x + c)^2
)*sinh(d*x + c)^6 + 24*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^
5)*cosh(d*x + c)^3 - 3*(2*a^5 - 5*a^4*b + 2*a^3*b^2 + 4*a^2*b^3 - 4*a*b^4 +
 b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 3*a^5 - 3*a^4*b + 6*a^3*b^2 + 6*a^2*
b^3 - 3*a*b^4 - 3*b^5 + 6*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4
- b^5)*cosh(d*x + c)^4 + 12*(a^5 + 8*a^4*b + 18*a^3*b^2 + 16*a^2*b^3 + 5*a*
b^4)*d*x - 30*(2*a^5 - 5*a^4*b + 2*a^3*b^2 + 4*a^2*b^3 - 4*a*b^4 + b^5)*cos
h(d*x + c)^2)*sinh(d*x + c)^4 + 24*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3
+ a*b^4 - b^5)*cosh(d*x + c)^5 + 12*(a^5 + 8*a^4*b + 18*a^3*b^2 + 16*a^2*b^
3 + 5*a*b^4)*d*x*cosh(d*x + c) - 10*(2*a^5 - 5*a^4*b + 2*a^3*b^2 + 4*a^2*b^
3 - 4*a*b^4 + b^5)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 12*(2*a^5 + 5*a^4*b +
 2*a^3*b^2 - 4*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^2 + 32*((a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*d*cosh(d*x + c)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*s
inh(d*x + c)^4)*(6*(1/2)^(2/3)*(a^2*b^2/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^
4*d^2 - b^6*d^2) - 3*(a^4*b + 2*a^2*b^3)^2/(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4
*d - b^6*d)^2)*(-I*sqrt(3) + 1)/(27*(a^4*b + 2*a^2*b^3)*a^2*b^2/((a^6*d^2 -
 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)*(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*
d - b^6*d)) - a^2*b/(a^6*d^3 - 3*a^4*b^2*d^3 + 3*a^2*b^4*d^3 - b^6*d^3) - 5
4*(a^4*b + 2*a^2*b^3)^3/(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)^3 - (a^
6 + 24*a^4*b^2 + 3*a^2*b^4 - b^6)*a^2*b/((a^2 - b^2)^6*d^3))^(1/3) - (1/2)^
(1/3)*(27*(a^4*b + 2*a^2*b^3)*a^2*b^2/((a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4
*d^2 - b^6*d^2)*(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)) - a^2*b/(a^6*d
^3 - 3*a^4*b^2*d^3 + 3*a^2*b^4*d^3 - b^6*d^3) - 54*(a^4*b + 2*a^2*b^3)^3/(a
^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)^3 - (a^6 + 24*a^4*b^2 + 3*a^2*b^4
 - b^6)*a^2*b/((a^2 - b^2)^6*d^3))^(1/3)*(I*sqrt(3) + 1) - 6*(a^4*b + 2*a^2
*b^3)/(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d))*log(-a^7 - 4*a^6*b - 14*
a^5*b^2 - 40*a^4*b^3 - 11*a^3*b^4 - 10*a^2*b^5 - a*b^6 - 1/2*(a^9 + 5*a^8*b
 - a^7*b^2 - 14*a^6*b^3 - 3*a^5*b^4 + 12*a^4*b^5 + 5*a^3*b^6 - 2*a^2*b^7 -
2*a*b^8 - b^9)*(6*(1/2)^(2/3)*(a^2*b^2/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4
*d^2 - b^6*d^2) - 3*(a^4*b + 2*a^2*b^3)^2/(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*
d - b^6*d)^2)*(-I*sqrt(3) + 1)/(27*(a^4*b + 2*a^2*b^3)*a^2*b^2/((a^6*d^2 -
3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)*(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d
 - b^6*d)) - a^2*b/(a^6*d^3 - 3*a^4*b^2*d^3 + 3*a^2*b^4*d^3 - b^6*d^3) - 54
*(a^4*b + 2*a^2*b^3)^3/(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)^3 - (a^6
```

```
 + 24*a^4*b^2 + 3*a^2*b^4 - b^6)*a^2*b/((a^2 - b^2)^6*d^3))^(1/3) - (1/2)^(
1/3)*(27*(a^4*b + 2*a^2*b^3)*a^2*b^2/((a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*
d^2 - b^6*d^2)*(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)) - a^2*b/(a^6*d^
3 - 3*a^4*b^2*d^3 + 3*a^2*b^4*d^3 - b^6*d^3) - 54*(a^4*b + 2*a^2*b^3)^3/(a^
6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)^3 - (a^6 + 24*a^4*b^2 + 3*a^2*b^4
- b^6)*a^2*b/((a^2 - b^2)^6*d^3))^(1/3)*(I*sqrt(3) + 1) - 6*(a^4*b + 2*a^2*
b^3)/(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d))^2*d^2 - (a^8 + 5*a^7*b +
45*a^6*b^2 + 45*a^5*b^3 + 96*a^4*b^4 + 30*a^3*b^5 + 20*a^2*b^6 + a*b^7)*(6*
(1/2)^(2/3)*(a^2*b^2/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2) -
3*(a^4*b + 2*a^2*b^3)^2/(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)^2)*(-I*
sqrt(3) + 1)/(27*(a^4*b + 2*a^2*b^3)*a^2*b^2/((a^6*d^2 - 3*a^4*b^2*d^2 + 3*
a^2*b^4*d^2 - b^6*d^2)*(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)) - a^2*b
/(a^6*d^3 - 3*a^4*b^2*d^3 + 3*a^2*b^4*d^3 - b^6*d^3) - 54*(a^4*b + 2*a^2*b^
3)^3/(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)^3 - (a^6 + 24*a^4*b^2 + 3*
a^2*b^4 - b^6)*a^2*b/((a^2 - b^2)^6*d^3))^(1/3) - (1/2)^(1/3)*(27*(a^4*b +
2*a^2*b^3)*a^2*b^2/((a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)*(a^
6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)) - a^2*b/(a^6*d^3 - 3*a^4*b^2*d^3
+ 3*a^2*b^4*d^3 - b^6*d^3) - 54*(a^4*b + 2*a^2*b^3)^3/(a^6*d - 3*a^4*b^2*d
+ 3*a^2*b^4*d - b^6*d)^3 - (a^6 + 24*a^4*b^2 + 3*a^2*b^4 - b^6)*a^2*b/((a^2
 - b^2)^6*d^3))^(1/3)*(I*sqrt(3) + 1) - 6*(a^4*b + 2*a^2*b^3)/(a^6*d - 3*a^
4*b^2*d + 3*a^2*b^4*d - b^6*d))*d - (a^7 + 24*a^5*b^2 + 3*a^3*b^4 - a*b^6)*
cosh(d*x + c)^2 - 2*(a^7 + 24*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(d*x + c)*si
nh(d*x + c) - (a^7 + 24*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sinh(d*x + c)^2) + 12*
(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.67, size = 338, normalized size = 0.69

$$\frac{\frac{24\,(a^2+5\,ab)(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(18\,a^2e^{(4\,dx+4\,c)}+90\,abe^{(4\,dx+4\,c)}-8\,a^2e^{(2\,dx+2\,c)}+4\,abe^{(2\,dx+2\,c)}+4\,b^2e^{(2\,dx+2\,c)}+a^2-2\,ab+b^2\right)e^{(-4\,dx-4\,c)}}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{64\,(a^4b+2\,a^2b^3)\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}}{a^2+2\,ab+b^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 + 5*a*b)*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (18*a^2*
e^(4*d*x + 4*c) + 90*a*b*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 4*a*b*e^
(2*d*x + 2*c) + 4*b^2*e^(2*d*x + 2*c) + a^2 - 2*a*b + b^2)*e^(-4*d*x - 4*c)
```

```
/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 64*(a^4*b + 2*a^2*b^3)*log(abs(a*e^(6*d*
x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) +
3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6) + (a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a*e^(2*d*x + 2*c)
 + 4*b*e^(2*d*x + 2*c))/(a^2 + 2*a*b + b^2))/d
```

Mupad [B]
time = 4.36, size = 2500, normalized size = 5.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^3),x)
```

```
[Out] symsum(log(- root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3
- 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z
- a^2*b, z, k)*((96*(a^2*b^10*d + 20*a^3*b^9*d - 89*a^4*b^8*d + 270*a^5*b^7
*d - 417*a^6*b^6*d + 408*a^7*b^5*d - 190*a^8*b^4*d + 58*a^9*b^3*d - 7*a^10*
b^2*d - a^2*b^10*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*
b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*
a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 52*a^3*b^9*d*exp(2*root(81*a^4*b^2
*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b
^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) +
 59*a^4*b^8*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d
^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b
^2*d*z - a^2*b, z, k))*exp(2*d*x) - 218*a^5*b^7*d*exp(2*root(81*a^4*b^2*d^3
*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d
^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 241
*a^6*b^6*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*
z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*
d*z - a^2*b, z, k))*exp(2*d*x) + 220*a^7*b^5*d*exp(2*root(81*a^4*b^2*d^3*z^
3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*
z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 298*a^
8*b^4*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3
 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z
 - a^2*b, z, k))*exp(2*d*x) + 50*a^9*b^3*d*exp(2*root(81*a^4*b^2*d^3*z^3 -
81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2
- 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - a^10*b^2*d
*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a
^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*
b, z, k))*exp(2*d*x)))/((a + b)*(a^2 - b^2)*(a*b^2 - a^2*b - a^3 + b^3)*(3*
a*b^2 + 3*a^2*b + a^3 + b^3)*(a*b^4 + a^4*b + a^5 + b^5 - 2*a^2*b^3 - 2*a^3
*b^2)) - (288*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3
 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z
 - a^2*b, z, k)*(a^7*b*d^2 - 8*a^2*b^6*d^2 + 16*a^3*b^5*d^2 - 41*a^4*b^4*d^
```

2 + 37*a^5*b^3*d^2 - 5*a^6*b^2*d^2 + 18*a^2*b^6*d^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 14*a^3*b^5*d^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 79*a^4*b^4*d^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 81*a^5*b^3*d^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - a^6*b^2*d^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + a^7*b*d^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x)))/((a + b)^2*(a - b)*(a*b^2 - a^2*b - a^3 + b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))) - (32*(22*a^4*b^7 - 4*a^3*b^8 - 68*a^5*b^6 + 85*a^6*b^5 - 56*a^7*b^4 + 10*a^8*b^3 + 2*a^9*b^2 + 6*a^3*b^8*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 10*a^4*b^7*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 54*a^5*b^6*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 101*a^6*b^5*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 56*a^7*b^4*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 12*a^8*b^3*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 4*a^9*b^2*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x)))/((a + b)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a*b^4 + a^4*b + a^5 + ...

3.74 $\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=33

$$i\text{Int}\left(-\frac{i\sinh^3(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] I*Unintegrable(-I*sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] I*Defer[Int][((-I)*Sinh[c + d*x]^3)/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = i\int -\frac{i\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 826 vs. 2(33) = 66.
time = 0.39, size = 826, normalized size = 25.03

$$\frac{-9a(a^2+3b^2)\cosh(c+dx)+a^3\cosh(3(c+dx))-ab^2\cosh(3(c+dx))-2ab\,\text{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3b\#1^4+a\#1^6+b\#1^6\&,\ \ldots\right]\ \ldots}{\ldots}$$

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-9*a*(a^2 + 3*b^2)*Cosh[c + d*x] + a^3*Cosh[3*(c + d*x)] - a*b^2*Cosh[3*(c
 + d*x)] - 2*a*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4
+ a*#1^6 + b*#1^6 & , (3*a^2*c + 3*a*b*c + 3*b^2*c + 3*a^2*d*x + 3*a*b*d*x
+ 3*b^2*d*x + 6*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c
 + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*b^2*Log[-Cosh
[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2
```

```
]*#1] + 2*a^2*c*#1^2 - 2*b^2*c*#1^2 + 2*a^2*d*x*#1^2 - 2*b^2*d*x*#1^2 + 4*a
^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh
[(c + d*x)/2]*#1]*#1^2 - 4*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
 Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a^2*c*#1^4 - 3*a*b*c
*#1^4 + 3*b^2*c*#1^4 + 3*a^2*d*x*#1^4 - 3*a*b*d*x*#1^4 + 3*b^2*d*x*#1^4 + 6
*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Si
nh[(c + d*x)/2]*#1]*#1^4 - 6*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
 + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 6*b^2*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]
*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ] + 27*a^2*b
*Sinh[c + d*x] + 9*b^3*Sinh[c + d*x] - a^2*b*Sinh[3*(c + d*x)] + b^3*Sinh[3
*(c + d*x)])/(12*(a - b)^2*(a + b)^2*d)
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(30) = 60$.
time = 7.43, size = 289, normalized size = 8.76

method	result
derivativedivides	$-\frac{ab\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\left(2a^2+b^2\right)\_R^4-6ab\_R^3+2\left(4a^2+5b^2\right)\_R^2-6ab\_R+2a^2+b^2\right)1}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right.}{3(a+b)^2(a-b)^2}$
default	$-\frac{ab\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\left(2a^2+b^2\right)\_R^4-6ab\_R^3+2\left(4a^2+5b^2\right)\_R^2-6ab\_R+2a^2+b^2\right)1}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right.}{3(a+b)^2(a-b)^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a*b/(a+b)^2/(a-b)^2*sum(((2*a^2+b^2)*_R^4-6*a*b*_R^3+2*(4*a^2+5*b
^2)*_R^2-6*a*b*_R+2*a^2+b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*
x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-8/(16*a-16*b)/
(tanh(1/2*d*x+1/2*c)+1)^2+16/3/(tanh(1/2*d*x+1/2*c)+1)^3/(16*a-16*b)-1/2*(2
*b+a)/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)-16/3/(tanh(1/2*d*x+1/2*c)-1)^3/(16*a+
16*b)-8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^2*(2*b-a)/(tanh(1/2
*d*x+1/2*c)-1))
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/24*(a^3 + a^2*b - a*b^2 - b^3 + (a^3*e^(6*c) - a^2*b*e^(6*c) - a*b^2*e^(6
*c) + b^3*e^(6*c))*e^(6*d*x) - 9*(a^3*e^(4*c) - 3*a^2*b*e^(4*c) + 3*a*b^2*e
^(4*c) - b^3*e^(4*c))*e^(4*d*x) - 9*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^
2*e^(2*c) + b^3*e^(2*c))*e^(2*d*x))*e^(-3*d*x)/(a^4*d*e^(3*c) - 2*a^2*b^2*d
*e^(3*c) + b^4*d*e^(3*c)) - 1/8*integrate(16*(3*(a^3*b*e^(5*c) - a^2*b^2*e^
(5*c) + a*b^3*e^(5*c))*e^(5*d*x) + 2*(a^3*b*e^(3*c) - a*b^3*e^(3*c))*e^(3*d
*x) + 3*(a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*e^(d*x))/(a^5 - a^4*b - 2*a^3
*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5*e^(6*c) + a^4*b*e^(6*c) - 2*a^3*b^2*e
^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c) + b^5*e^(6*c))*e^(6*d*x) + 3*(a^
5*e^(4*c) - a^4*b*e^(4*c) - 2*a^3*b^2*e^(4*c) + 2*a^2*b^3*e^(4*c) + a*b^4*e
^(4*c) - b^5*e^(4*c))*e^(4*d*x) + 3*(a^5*e^(2*c) + a^4*b*e^(2*c) - 2*a^3*b^
2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c) + b^5*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**A**] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 4.53, size = 62017, normalized size = 1879.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/24*((a^3 - a^2*b - a*b^2 + b^3)*cosh(d*x + c)^6 + 6*(a^3 - a^2*b - a*b^2
+ b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(d*x
 + c)^6 - 9*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^4 - 3*(3*a^3 - 9*
a^2*b + 9*a*b^2 - 3*b^3 - 5*(a^3 - a^2*b - a*b^2 + b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^4 + 4*(5*(a^3 - a^2*b - a*b^2 + b^3)*cosh(d*x + c)^3 - 9*(a^3 -
 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*sqrt(2/3)*sqrt
(1/6)*((a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^3 + 3*(a^4 - 2*a^2*b^2 + b^4
)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x +
c)*sinh(d*x + c)^2 + (a^4 - 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^3)*sqrt(-(810*
a^6*b^2 + 2754*a^4*b^4 + 810*a^2*b^6 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*
a^4*b^6 + 5*a^2*b^8 - b^10)*((5*a^2*b^2/(a^8*d^4 - 4*a^6*b^2*d^4 + 6*a^4*b^
4*d^4 - 4*a^2*b^6*d^4 + b^8*d^4) + 9*(5*a^6*b^2 + 17*a^4*b^4 + 5*a^2*b^6)^2
/(a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^2 + 5*a^2*b^8*d^
2 - b^10*d^2)^2)*(-I*sqrt(3) + 1)/(-1/1458*a^2*b^2/(a^10*d^6 - 5*a^8*b^2*d^
6 + 10*a^6*b^4*d^6 - 10*a^4*b^6*d^6 + 5*a^2*b^8*d^6 - b^10*d^6) - 5/162*(5*
a^6*b^2 + 17*a^4*b^4 + 5*a^2*b^6)*a^2*b^2/((a^10*d^2 - 5*a^8*b^2*d^2 + 10*a
^6*b^4*d^2 - 10*a^4*b^6*d^2 + 5*a^2*b^8*d^2 - b^10*d^2)*(a^8*d^4 - 4*a^6*b^
2*d^4 + 6*a^4*b^4*d^4 - 4*a^2*b^6*d^4 + b^8*d^4)) - 1/27*(5*a^6*b^2 + 17*a^
4*b^4 + 5*a^2*b^6)^3/(a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^
6*d^2 + 5*a^2*b^8*d^2 - b^10*d^2)^3 + 1/1458*(a^10 - 30*a^8*b^2 - 700*a^6*b
^4 - 700*a^4*b^6 - 30*a^2*b^8 + b^10)*a^2*b^2/((a^2 - b^2)^10*d^6))^(1/3) +
 81*(-1/1458*a^2*b^2/(a^10*d^6 - 5*a^8*b^2*d^6 + 10*a^6*b^4*d^6 - 10*a^4*b^
6*d^6 + 5*a^2*b^8*d^6 - b^10*d^6) - 5/162*(5*a^6*b^2 + 17*a^4*b^4 + 5*a^2*b
```

```
^6)*a^2*b^2/((a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^2 +
5*a^2*b^8*d^2 - b^10*d^2)*(a^8*d^4 - 4*a^6*b^2*d^4 + 6*a^4*b^4*d^4 - 4*a^2*
b^6*d^4 + b^8*d^4)) - 1/27*(5*a^6*b^2 + 17*a^4*b^4 + 5*a^2*b^6)^3/(a^10*d^2
 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^2 + 5*a^2*b^8*d^2 - b^10*d
^2)^3 + 1/1458*(a^10 - 30*a^8*b^2 - 700*a^6*b^4 - 700*a^4*b^6 - 30*a^2*b^8
+ b^10)*a^2*b^2/((a^2 - b^2)^10*d^6))^(1/3)*(I*sqrt(3) + 1) + 54*(5*a^6*b^2
 + 17*a^4*b^4 + 5*a^2*b^6)/(a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*
a^4*b^6*d^2 + 5*a^2*b^8*d^2 - b^10*d^2))*d^2 + 3*sqrt(1/3)*(a^10 - 5*a^8*b^
2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*d^2*sqrt((6480*a^14*b^2 + 1
79820*a^12*b^4 + 1584360*a^10*b^6 + 2835972*a^8*b^8 + 1584360*a^6*b^10 + 17
9820*a^4*b^12 + 6480*a^2*b^14 - (a^20 - 10*a^18*b^2 + 45*a^16*b^4 - 120*a^1
4*b^6 + 210*a^12*b^8 - 252*a^10*b^10 + 210*a^8*b^12 - 120*a^6*b^14 + 45*a^4
*b^16 - 10*a^2*b^18 + b^20)*((5*a^2*b^2/(a^8*d^4 - 4*a^6*b^2*d^4 + 6*a^4*b^
4*d^4 - 4*a^2*b^6*d^4 + b^8*d^4) + 9*(5*a^6*b^2 + 17*a^4*b^4 + 5*a^2*b^6)^2
/(a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^2 + 5*a^2*b^8*d^
2 - b^10*d^2)^2)*(-I*sqrt(3) + 1)/(-1/1458*a^2*b^2/(a^10*d^6 - 5*a^8*b^2*d^
6 + 10*a^6*b^4*d^6 - 10*a^4*b^6*d^6 + 5*a^2*b^8*d^6 - b^10*d^6) - 5/162*(5*
a^6*b^2 + 17*a^4*b^4 + 5*a^2*b^6)*a^2*b^2/((a^10*d^2 - 5*a^8*b^2*d^2 + 10*a
^6*b^4*d^2 - 10*a^4*b^6*d^2 + 5*a^2*b^8*d^2 - b^10*d^2)*(a^8*d^4 - 4*a^6*b^
2*d^4 + 6*a^4*b^4*d^4 - 4*a^2*b^6*d^4 + b^8*d^4)) - 1/27*(5*a^6*b^2 + 17*a^
4*b^4 + 5*a^2*b^6)^3/(a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^
6*d^2 + 5*a^2*b^8*d^2 - b^10*d^2)^3 + 1/1458*(a^10 - 30*a^8*b^2 - 700*a^6*b
^4 - 700*a^4*b^6 - 30*a^2*b^8 + b^10)*a^2*b^2/((a^2 - b^2)^10*d^6))^(1/3) +
 81*(-1/1458*a^2*b^2/(a^10*d^6 - 5*a^8*b^2*d^6 + 10*a^6*b^4*d^6 - 10*a^4*b^
6*d^6 + 5*a^2*b^8*d^6 - b^10*d^6) - 5/162*(5*a^6*b^2 + 17*a^4*b^4 + 5*a^2*b
^6)*a^2*b^2/((a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^2 +
5*a^2*b^8*d^2 - b^10*d^2)*(a^8*d^4 - 4*a^6*b^2*d^4 + 6*a^4*b^4*d^4 - 4*a^2*
b^6*d^4 + b^8*d^4)) - 1/27*(5*a^6*b^2 + 17*a^4*b^4 + 5*a^2*b^6)^3/(a^10*d^2
 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^2 + 5*a^2*b^8*d^2 - b^10*d
^2)^3 + 1/1458*(a^10 - 30*a^8*b^2 - 700*a^6*b^4 - 700*a^4*b^6 - 30*a^2*b^8
+ b^10)*a^2*b^2/((a^2 - b^2)^10*d^6))^(1/3)*(I*sqrt(3) + 1) + 54*(5*a^6*b^2
 + 17*a^4*b^4 + 5*a^2*b^6)/(a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*
a^4*b^6*d^2 + 5*a^2*b^8*d^2 - b^10*d^2))^2*d^4 + 108*(5*a^16*b^2 - 8*a^14*b
^4 - 30*a^12*b^6 + 95*a^10*b^8 - 95*a^8*b^10 + 30*a^6*b^12 + 8*a^4*b^14 - 5
*a^2*b^16)*((5*a^2*b^2/(a^8*d^4 - 4*a^6*b^2*d^4 + 6*a^4*b^4*d^4 - 4*a^2*b^6
*d^4 + b^8*d^4) + 9*(5*a^6*b^2 + 17*a^4*b^4 + 5*a^2*b^6)^2/(a^10*d^2 - 5*a^
8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^2 + 5*a^2*b^8*d^2 - b^10*d^2)^2)*
(-I*sqrt(3) + 1)/(-1/1458*a^2*b^2/(a^10*d^6 - 5*a^8*b^2*d^6 + 10*a^6*b^4*d^
6 - 10*a^4*b^6*d^6 + 5*a^2*b^8*d^6 - b^10*d^6) - 5/162*(5*a^6*b^2 + 17*a^4*
b^4 + 5*a^2*b^6)*a^2*b^2/((a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a
^4*b^6*d^2 + 5*a^2*b^8*d^2 - b^10*d^2)*(a^8*d^4 - 4*a^6*b^2*d^4 + 6*a^4*b^4
*d^4 - 4*a^2*b^6*d^4 + b^8*d^4)) - 1/27*(5*a^6*b^2 + 17*a^4*b^4 + 5*a^2*b^6
)^3/(a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^2 + 5*a^2*b^8
*d^2 - b^10*d^2)^3 + 1/1458*(a^10 - 30*a^8*b^2 ...
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**3), x)
```

Giac [A] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 1.53, size = 303, normalized size = 9.18

$$-\frac{\frac{\left(9\,ae^{(2\,dx+2\,c)}+9\,be^{(2\,dx+2\,c)}-a+b\right)e^{(-3\,dx-3\,c)}}{a^2-2\,ab+b^2}-\frac{a^2e^{(3\,dx+3\,c)}+2\,abe^{(3\,dx+3\,c)}+b^2e^{(3\,dx+3\,c)}-9\,a^2e^{(dx+c)}+9\,b^2e^{(dx+c)}}{a^3+3\,a^2b+3\,ab^2+b^3}}{24\,d}-\frac{\frac{6\,(a^3b+a^2b^2+ab^3)(dx+c)}{a-b}-\frac{(a^3b+a^2b^2+ab^3)\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a-b}}{(a^4-2\,a^2b^2+b^4)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/24*((9*a*e^(2*d*x + 2*c) + 9*b*e^(2*d*x + 2*c) - a + b)*e^(-3*d*x - 3*c)
/(a^2 - 2*a*b + b^2) - (a^2*e^(3*d*x + 3*c) + 2*a*b*e^(3*d*x + 3*c) + b^2*e
^(3*d*x + 3*c) - 9*a^2*e^(d*x + c) + 9*b^2*e^(d*x + c))/(a^3 + 3*a^2*b + 3*
a*b^2 + b^3))/d - (6*(a^3*b + a^2*b^2 + a*b^3)*(d*x + c)/(a - b) - (a^3*b +
 a^2*b^2 + a*b^3)*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*
d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c
) + a - b))/(a - b))/((a^4 - 2*a^2*b^2 + b^4)*d^2)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^3),x)
```

```
[Out] \text{Hanged}
```

3.75 $\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=384

$$\frac{a^{2/3}\sqrt[3]{b}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\left(a^2-b^2\right)^2d}+\frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d}-\frac{(a+2b)\log(}{4(}$$

```
[Out] 1/4*(a-2*b)*ln(1-tanh(d*x+c))/(a+b)^2/d-1/4*(a+2*b)*ln(1+tanh(d*x+c))/(a-b)
^2/d+1/3*a^(2/3)*b^(1/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(a^(1/3)+b^(1/3)*t
anh(d*x+c))/(a^2-b^2)^2/d-1/6*a^(2/3)*b^(1/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/(a^2-b^2)^2/
d+1/3*b*(2*a^2+b^2)*ln(a+b*tanh(d*x+c)^3)/(a^2-b^2)^2/d+1/3*a^(2/3)*b^(1/3)
*(a^2-3*a^(2/3)*b^(4/3)+2*b^2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a
^(1/3)*3^(1/2))/(a^2-b^2)^2/d*3^(1/2)+1/4/(a+b)/d/(1-tanh(d*x+c))-1/4/(a-b)
/d/(1+tanh(d*x+c))
```

Rubi [A]
time = 0.44, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3744, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{b(2a^2+b^2)\log\left(a+b\tanh^3(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{a^{2/3}\sqrt[3]{b}\left(-3a^{2/3}b^{4/3}+a^2+2b^2\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}d\left(a^2-b^2\right)^2}-\frac{a^{2/3}\sqrt[3]{b}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6d\left(a^2-b^2\right)^2}+\frac{a^{2/3}\sqrt[3]{b}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\tanh(c+dx))}-\frac{1}{4d(a-b)(\tanh(c+dx)+1)}+\frac{(a-2b)\log(1-\tanh(c+dx))}{4d(a+b)^2}-\frac{(a+2b)\log(\tanh(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (a^(2/3)*b^(1/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^(1
/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^2 - b^2)^2*d) + ((a - 2*
b)*Log[1 - Tanh[c + d*x]])/(4*(a + b)^2*d) - ((a + 2*b)*Log[1 + Tanh[c + d*
x]])/(4*(a - b)^2*d) + (a^(2/3)*b^(1/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*L
og[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^2*d) - (a^(2/3)*b^(1/3)
*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d
*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^2*d) + (b*(2*a^2 + b^2)*Log[
a + b*Tanh[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Tanh[c +
d*x])) - 1/(4*(a - b)*d*(1 + Tanh[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2} + \frac{a-2b}{4(a+b)^2(-1+x)} + \frac{1}{4(a-b)(1+x)^2} + \frac{-a-2b}{4(a-b)^2(1+x)} + \frac{b(3a^2b-a(a^2+}{(a^2-}\right.\right.}{d}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{}{4(a+b)d(1}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{}{4(a+b)d(1}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{b(2a^2+b^2)}{}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{a^{2/3}\sqrt[3]{b}\,(a^2}{}$$

$$= \frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{a^{2/3}\sqrt[3]{b}\,(a^2}{}$$

$$= \frac{a^{2/3}\sqrt[3]{b}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\left(a^2-b^2\right)^2 d} + \frac{(a-2b)\log(1-\tan}{4(a+b)^2 d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 2.51, size = 423, normalized size = 1.10

$$-\frac{6(a^2-3ab+2b^2)(c+dx)+3b(a+b)\cosh(2(c+dx))+4b\text{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2-3b\#1^2+a\#1^3+b\#1^3\&,\frac{4a^2c+2b^2c+4a^2dx+2b^2dx-2a^2\log\left(e^{2(c+dx)}-\#1\right)-b^2\log\left(e^{2(c+dx)}-\#1\right)+4a^2c\#1-4b^2c\#1+4a^2dx\#1-4b^2dx\#1-2a^2\log\left(e^{2(c+dx)}-\#1\right)\#1+2b^2\log\left(e^{2(c+dx)}-\#1\right)\#1+8a^2c\#1^2-8abc\#1^2+2b^2c\#1^2+8a^2dx\#1^2-8abdx\#1^2+2b^2dx\#1^2-4a^2\log\left(e^{2(c+dx)}-\#1\right)\#1^2+4ab\log\left(e^{2(c+dx)}-\#1\right)\#1^2-b^2\log\left(e^{2(c+dx)}-\#1\right)\#1^2}{a-b+2a\#1+2b\#1+a\#1^2-b\#1^2}\&\right]-3a(a+b)\sinh(2(c+dx))}{12(a-b)(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -1/12*(6*(a^2 - 3*a*b + 2*b^2)*(c + d*x) + 3*b*(a + b)*Cosh[2*(c + d*x)] +
4*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b*#1^3
 & , (4*a^2*c + 2*b^2*c + 4*a^2*d*x + 2*b^2*d*x - 2*a^2*Log[E^(2*(c + d*x))
 - #1] - b^2*Log[E^(2*(c + d*x)) - #1] + 4*a^2*c*#1 - 4*b^2*c*#1 + 4*a^2*d*
x*#1 - 4*b^2*d*x*#1 - 2*a^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b^2*Log[E^(2*(
c + d*x)) - #1]*#1 + 8*a^2*c*#1^2 - 8*a*b*c*#1^2 + 2*b^2*c*#1^2 + 8*a^2*d*x
*#1^2 - 8*a*b*d*x*#1^2 + 2*b^2*d*x*#1^2 - 4*a^2*Log[E^(2*(c + d*x)) - #1]*#
1^2 + 4*a*b*Log[E^(2*(c + d*x)) - #1]*#1^2 - b^2*Log[E^(2*(c + d*x)) - #1]*
#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2) & ] - 3*a*(a + b)*Sinh[2*
(c + d*x)])/((a - b)*(a + b)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.00, size = 301, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-2b+a)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}+\frac{b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+\right.}\right.}{}}{}$
default	$\frac{\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-2b+a)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}+\frac{b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+\right.}\right.}{}}{}$
risch	$\frac{xb}{(a+b)^2}-\frac{ax}{2(a+b)^2}+\frac{e^{2dx+2c}}{8(a+b)d}-\frac{e^{-2dx-2c}}{8(a-b)d}-\frac{4a^2bd^3x}{a^4d^3-2a^2b^2d^3+b^4d^3}-\frac{2b^3d^3x}{a^4d^3-2a^2b^2d^3+b^4d^3}-\frac{4a^2bcd^2}{a^4d^3-2a^2b^2d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/(8*a+8*b)/(tanh(1/2*d*x+1/2*c)-1)^2+8/(16*a+16*b)/(tanh(1/2*d*x+1/2*
c)-1)+1/2*(-2*b+a)/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/3*b/(a-b)^2/(a+b)^2*
sum((a*(2*a^2+b^2)*_R^5-3*a^2*b*_R^4+6*a*(a^2+b^2)*_R^3+4*b*(2*a^2+b^2)*_R^
2-3*a*_R*b^2+3*a^2*b)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c
)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-4/(8*a-8*b)/(tanh(1/2
*d*x+1/2*c)+1)^2+8/(16*a-16*b)/(tanh(1/2*d*x+1/2*c)+1)+1/2/(a-b)^2*(-2*b-a)
*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 4*a^2*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3
*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4
*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^4 - 2*a^2*b^2 + b^4) - (d*x
 + c)/((a^4 - 2*a^2*b^2 + b^4)*d)) + 2*b^3*(integrate(((a + b)*e^(4*d*x + 4
*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*
d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b)
, x)/(a^4 - 2*a^2*b^2 + b^4) - (d*x + c)/((a^4 - 2*a^2*b^2 + b^4)*d)) - 8*a
^2*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*
x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*b^2 - b^
3) + 8*a*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)
*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*
b^2 - b^3) - 2*b^3*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(
a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*
b - a*b^2 - b^3) - 4*a^2*b*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*
c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^
3 + a^2*b - a*b^2 - b^3) + 4*b^3*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*
x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b),
x)/(a^3 + a^2*b - a*b^2 - b^3) - 1/8*(4*(a^2*d*e^(2*c) - 3*a*b*d*e^(2*c) +
2*b^2*d*e^(2*c))*x*e^(2*d*x) + a^2 + 2*a*b + b^2 - (a^2*e^(4*c) - b^2*e^(4*
c))*e^(4*d*x))*e^(-2*d*x)/(a^3*d*e^(2*c) + a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c
) - b^3*d*e^(2*c))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.35, size = 10695, normalized size = 27.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/72*(36*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x*cosh(d*x + c)^2 - 9*(a^3 -
a^2*b - a*b^2 + b^3)*cosh(d*x + c)^4 - 36*(a^3 - a^2*b - a*b^2 + b^3)*cosh(
d*x + c)*sinh(d*x + c)^3 - 9*(a^3 - a^2*b - a*b^2 + b^3)*sinh(d*x + c)^4 +
9*a^3 + 9*a^2*b - 9*a*b^2 - 9*b^3 - 4*((a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x +
 c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 - 2*
a^2*b^2 + b^4)*d*sinh(d*x + c)^2)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)
 - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/
```

18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*
b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 +
1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2
 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*
d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4
*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) +
1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*
a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*log(1/18*(a^5 + 2*a^4*b - 2*a^3
*b^2 - 4*a^2*b^3 + a*b^4 + 2*b^5)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)
 - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/
18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*
b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 +
1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2
 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*
d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4
*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) +
1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*
a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + a^3 + 2*a^2*b + 2*a*b^2
 + 4*b^3 - 1/3*(a^4 + 3*a^3*b + 13*a^2*b^2 + 6*a*b^3 + 4*b^4)*((b^2/(a^4*d^
2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4
d)^2)(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^
2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d
 - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/
54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3
)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) +
 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 -
 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(
1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d +
 (a^3 + 8*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + 8*a*b^2)*cosh(d*x + c)*sinh(d*x
 + c) + (a^3 + 8*a*b^2)*sinh(d*x + c)^2) + 18*(2*(a^3 + 4*a^2*b + 5*a*b^2 +
 2*b^3)*d*x - 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - 2*(18*(2*a^2*b + b^3)*cosh(d*x + c)^2 + 36*(2*a^2*b + b^3)*cosh(d*x + c
)*sinh(d*x + c) + 18*(2*a^2*b + b^3)*sinh(d*x + c)^2 - ((a^4 - 2*a^2*b^2 +
b^4)*d*cosh(d*x + c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x
 + c) + (a^4 - 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^2)*((b^2/(a^4*d^2 - 2*a^2*b
^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*
sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2
)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^
2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8
*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3) - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4
*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^
2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*
d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqr
t(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d)) - 3*sqrt(1/3)*
((a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*co

```
sh(d*x + c)*sinh(d*x + c) + (a^4 - 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^2)*sqrt
((288*a^4*b^2 + 720*a^2*b^4 - 36*b^6 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(
a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2
/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27
*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^
2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^(1/3)
- 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d
- 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4
*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2
*b/((a^2 - b^2)^4*d^3))^(1/3)*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d -
2*a^2*b^2*d + b^4*d))^2*d^2 + 12*(2*a^6*b - 3*a^4*b^3 + b^7)*((b^2/(a^4*d^2
 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*
d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2
 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**3),x)
[Out] Integral(sinh(c + d*x)**2/(a + b*tanh(c + d*x)**3), x)
```

Giac [A]
time = 0.55, size = 206, normalized size = 0.54

$$-\frac{\frac{12\,(dx+c)(a+2\,b)}{a^2-2\,ab+b^2}-\frac{3\,\left(2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}-a+b\right)e^{(-2\,dx-2\,c)}}{a^2-2\,ab+b^2}-\frac{8\,\left(2\,a^2b+b^3\right)\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4}-\frac{3\,e^{(2\,dx+2\,c)}}{a+b}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
[Out] -1/24*(12*(d*x + c)*(a + 2*b)/(a^2 - 2*a*b + b^2) - 3*(2*a*e^(2*d*x + 2*c)
+ 4*b*e^(2*d*x + 2*c) - a + b)*e^(-2*d*x - 2*c)/(a^2 - 2*a*b + b^2) - 8*(2*
a^2*b + b^3)*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x +
 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a
 - b))/(a^4 - 2*a^2*b^2 + b^4) - 3*e^(2*d*x + 2*c)/(a + b))/d
```

Mupad [B]
time = 2.81, size = 2100, normalized size = 5.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^3),x)

[Out] symsum(log(root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a
^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k)*((2304*root(54*a^2*b^2
*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*
z^2 - 9*b^2*d*z + b, z, k)*(146*a^5*b^5*d^2 - 133*a^4*b^6*d^2 - 24*a^3*b^7*
d^2 - 12*a^6*b^4*d^2 + 22*a^7*b^3*d^2 + a^8*b^2*d^2 + 32*a^3*b^7*d^2*exp(2*
root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^
2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 577*a^4*b^6*d^2*exp
(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2
*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 548*a^5*b^5*d^2*
exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*
d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 70*a^6*b^4*d^
2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*
b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 68*a^7*b^3*
d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^
2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + a^8*b^2*d
^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2
*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x)))/((a + b)^8
*(a - b)^2*(a^2 - 2*a*b + b^2)) + (1536*(24*a^3*b^8*d + 105*a^4*b^7*d - 156
*a^5*b^6*d + 51*a^6*b^5*d - 30*a^7*b^4*d + 6*a^8*b^3*d - 32*a^3*b^8*d*exp(2
*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z
^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) - 509*a^4*b^7*d*exp(
2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*
z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) - 350*a^5*b^6*d*exp
(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2
*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 64*a^6*b^5*d*exp
(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2
*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) - 50*a^7*b^4*d*exp
(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2
*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 13*a^8*b^3*d*exp
(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2
*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x)))/((a + b)^3*(a^2
- b^2)*(a - b)*(a^2 - 2*a*b + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^2)) + (2
56*(72*a^5*b^5 - 45*a^4*b^6 - 24*a^3*b^7 - 9*a^6*b^4 + 6*a^7*b^3 + 32*a^3*b
^7*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2
*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 393*a^4*b^
6*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*
b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 86*a^5*b^5*
exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*
d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 57*a^6*b^4*ex
p(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^
2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 8*a^7*b^3*exp(2
*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z
^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x)))/((a + b)^2*(a^2 -
```

```
b^2)*(a - b)*(a^2 - 2*a*b + b^2)*(2*a*b + a^2 + b^2)^2*(3*a*b^2 + 3*a^2*b +
 a^3 + b^3)))*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 5
4*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k), k, 1, 3) - (x*(a +
 2*b))/(2*(a - b)^2) + exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/
(8*d*(a - b))
```

3.76 $\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=31

$$-i\text{Int}\left(\frac{i\sinh(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] -I*Unintegrable(I*sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-I)*Defer[Int][(I*Sinh[c + d*x])/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\left(i\int \frac{i\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx\right)$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 409 vs. 2(31) = 62.
time = 0.16, size = 409, normalized size = 13.19

$$\frac{6a\cosh(c+dx)+b\text{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3b\#1^4+a\#1^6+b\#1^6\&,\frac{2ac+bc+2adx+bdx+4a\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)+2b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)+2ac\#1^4-bc\#1^4+2adx\#1^4-bdx\#1^4+4a\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4-2b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4}{a\#1+b\#1+2a\#1^3-2b\#1^3+a\#1^5+b\#1^5}\&\right]-6b\sinh(c+dx)}{6(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (6*a*Cosh[c + d*x] + b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b
*#1^4 + a*#1^6 + b*#1^6 & , (2*a*c + b*c + 2*a*d*x + b*d*x + 4*a*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1] + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#
1 - Sinh[(c + d*x)/2]*#1] + 2*a*c*#1^4 - b*c*#1^4 + 2*a*d*x*#1^4 - b*d*x*#1
^4 + 4*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
- Sinh[(c + d*x)/2]*#1]*#1^4 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/
```

```
2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*
#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ] - 6*b*Sinh[c + d*x])/(6*(a - b)*(a +
 b)*d)
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(28) = 56$.
time = 5.37, size = 159, normalized size = 5.13

method	result
derivativedivides	$\frac{\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4a-2\_R^3b+6\_R^2a-2\_Rb+a\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3(a-b)(a+b)}-\frac{}{(4}}{d}$
default	$\frac{\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4a-2\_R^3b+6\_R^2a-2\_Rb+a\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3(a-b)(a+b)}-\frac{}{(4}}{d}$
risch	$\frac{\mathrm{e}^{dx+c}}{2(a+b)d}+\frac{\mathrm{e}^{-dx-c}}{2(a-b)d}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^8d^6-2187a^6b^2d^6+2187a^4b^4d^6-729a^2b^6d^6\right)\_Z^6+\left(1458a^4b^2d^4+729a^2b^4\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b/(a-b)/(a+b)*sum((_R^4*a-2*_R^3*b+6*_R^2*a-2*_R*b+a)/(_R^5*a+2*_R
^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*
_Z^3*b+3*_Z^2*a+a))-4/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)-1)+4/(4*a-4*b)/(tanh(1
/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*((a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-d*x)/(a^2*d*e^c - b^2*d
*e^c) + 1/2*integrate(4*((2*a*b*e^(5*c) - b^2*e^(5*c))*e^(5*d*x) + (2*a*b*e
^c + b^2*e^c)*e^(d*x))/(a^3 - a^2*b - a*b^2 + b^3 + (a^3*e^(6*c) + a^2*b*e^
(6*c) - a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) + 3*(a^3*e^(4*c) - a^2*b*e^(
4*c) - a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 3*(a^3*e^(2*c) + a^2*b*e^(2
*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [A] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 1.99, size = 40923, normalized size = 1320.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(2/3)*sqrt(1/2)*((a^2 - b^2)*d*cosh(d*x + c) + (a^2 - b^2)*d*sinh
(d*x + c))*sqrt(-(108*a^2*b^2 + 54*b^4 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*((b^2/(a^6*d^4 - 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^4) + 3*(2*a^2*b^2
+ b^4)^2/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^2)*(-I*sqrt(3)
 + 1)/(-1/1458*b^2/(a^8*d^6 - 3*a^6*b^2*d^6 + 3*a^4*b^4*d^6 - a^2*b^6*d^6)
- 1/54*(2*a^2*b^2 + b^4)*b^2/((a^6*d^4 - 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^
6*d^4)*(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)) - 1/27*(2*a^2*b
^2 + b^4)^3/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^3 - 1/1458*
(a^6 - 3*a^4*b^2 - 24*a^2*b^4 - b^6)*b^2/((a^2 - b^2)^6*a^2*d^6))^(1/3) + 2
7*(-1/1458*b^2/(a^8*d^6 - 3*a^6*b^2*d^6 + 3*a^4*b^4*d^6 - a^2*b^6*d^6) - 1/
54*(2*a^2*b^2 + b^4)*b^2/((a^6*d^4 - 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^
4)*(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)) - 1/27*(2*a^2*b^2 +
 b^4)^3/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^3 - 1/1458*(a^6
 - 3*a^4*b^2 - 24*a^2*b^4 - b^6)*b^2/((a^2 - b^2)^6*a^2*d^6))^(1/3)*(I*sqrt
(3) + 1) + 18*(2*a^2*b^2 + b^4)/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 -
b^6*d^2))*d^2 + 3*sqrt(1/3)*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d^2*sqrt((4
32*a^6*b^2 + 2592*a^4*b^4 + 5184*a^2*b^6 + 540*b^8 - (a^12 - 6*a^10*b^2 + 1
5*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*b^10 + b^12)*((b^2/(a^6*d^4 - 3
*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^4) + 3*(2*a^2*b^2 + b^4)^2/(a^6*d^2 -
3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^2)*(-I*sqrt(3) + 1)/(-1/1458*b^2/(
a^8*d^6 - 3*a^6*b^2*d^6 + 3*a^4*b^4*d^6 - a^2*b^6*d^6) - 1/54*(2*a^2*b^2 +
b^4)*b^2/((a^6*d^4 - 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^4)*(a^6*d^2 - 3*
a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^6*d^2
 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^3 - 1/1458*(a^6 - 3*a^4*b^2 - 2
4*a^2*b^4 - b^6)*b^2/((a^2 - b^2)^6*a^2*d^6))^(1/3) + 27*(-1/1458*b^2/(a^8*
d^6 - 3*a^6*b^2*d^6 + 3*a^4*b^4*d^6 - a^2*b^6*d^6) - 1/54*(2*a^2*b^2 + b^4)
*b^2/((a^6*d^4 - 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^4)*(a^6*d^2 - 3*a^4*
b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^6*d^2 - 3
*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^3 - 1/1458*(a^6 - 3*a^4*b^2 - 24*a^
2*b^4 - b^6)*b^2/((a^2 - b^2)^6*a^2*d^6))^(1/3)*(I*sqrt(3) + 1) + 18*(2*a^2
*b^2 + b^4)/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2))^2*d^4 + 36
*(2*a^8*b^2 - 5*a^6*b^4 + 3*a^4*b^6 + a^2*b^8 - b^10)*((b^2/(a^6*d^4 - 3*a^
4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^4) + 3*(2*a^2*b^2 + b^4)^2/(a^6*d^2 - 3*a
^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^2)*(-I*sqrt(3) + 1)/(-1/1458*b^2/(a^8
*d^6 - 3*a^6*b^2*d^6 + 3*a^4*b^4*d^6 - a^2*b^6*d^6) - 1/54*(2*a^2*b^2 + b^4
)*b^2/((a^6*d^4 - 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^4)*(a^6*d^2 - 3*a^4
*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^6*d^2 -
```

```
3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^3 - 1/1458*(a^6 - 3*a^4*b^2 - 24*a
^2*b^4 - b^6)*b^2/((a^2 - b^2)^6*a^2*d^6))^(1/3) + 27*(-1/1458*b^2/(a^8*d^6
 - 3*a^6*b^2*d^6 + 3*a^4*b^4*d^6 - a^2*b^6*d^6) - 1/54*(2*a^2*b^2 + b^4)*b^
2/((a^6*d^4 - 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^4)*(a^6*d^2 - 3*a^4*b^2
*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^6*d^2 - 3*a^
4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^3 - 1/1458*(a^6 - 3*a^4*b^2 - 24*a^2*b
^4 - b^6)*b^2/((a^2 - b^2)^6*a^2*d^6))^(1/3)*(I*sqrt(3) + 1) + 18*(2*a^2*b^
2 + b^4)/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2))*d^2)/((a^12 -
 6*a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*b^10 + b^12)*d^4
)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d^2))*log(1/36*sqrt(2/3)*sqrt(1/2)
*((4*a^12 + 3*a^11*b + a^10*b^2 - 3*a^9*b^3 - 26*a^8*b^4 - 9*a^7*b^5 + 32*a
^6*b^6 + 15*a^5*b^7 - 10*a^4*b^8 - 6*a^3*b^9 - a^2*b^10)*((b^2/(a^6*d^4 - 3
*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^4) + 3*(2*a^2*b^2 + b^4)^2/(a^6*d^2 -
3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^2)*(-I*sqrt(3) + 1)/(-1/1458*b^2/(
a^8*d^6 - 3*a^6*b^2*d^6 + 3*a^4*b^4*d^6 - a^2*b^6*d^6) - 1/54*(2*a^2*b^2 +
b^4)*b^2/((a^6*d^4 - 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^4)*(a^6*d^2 - 3*
a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^6*d^2
 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^3 - 1/1458*(a^6 - 3*a^4*b^2 - 2
4*a^2*b^4 - b^6)*b^2/((a^2 - b^2)^6*a^2*d^6))^(1/3) + 27*(-1/1458*b^2/(a^8*
d^6 - 3*a^6*b^2*d^6 + 3*a^4*b^4*d^6 - a^2*b^6*d^6) - 1/54*(2*a^2*b^2 + b^4)
*b^2/((a^6*d^4 - 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 - b^6*d^4)*(a^6*d^2 - 3*a^4*
b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^6*d^2 - 3
*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2)^3 - 1/1458*(a^6 - 3*a^4*b^2 - 24*a^
2*b^4 - b^6)*b^2/((a^2 - b^2)^6*a^2*d^6))^(1/3)*(I*sqrt(3) + 1) + 18*(2*a^2
*b^2 + b^4)/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 - b^6*d^2))^2*d^5 - 6*
(a^10 + a^9*b + 71*a^8*b^2 + 50*a^7*b^3 + 267*a^6*b^4 + 141*a^5*b^5 + 140*a
^4*b^6 + 50*a^3*b^7 + 7*a^2*b^8 + a*b^9)*((b^2/(a^6*d^4 - 3*a^4*b^2*d^4 + 3
*a^2*b^4*d^4 - b^6*d^4) + 3*(2*a^2*b^2 + b^4)^2/(a^6*d^2 - 3*a^4*b^2*d^2 +
3*a^2*b^4*d^2 - b^6*d^2)^2)*(-I*sqrt(3) + 1)/(-1/1458*b^2/(a^8*d^6 - 3*a^6*
b^2*d^6 + 3*a^4*b^4*d^6 - a^2*b^6*d^6) - 1/54*(...
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**3), x)
```

Giac [A] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 1.06, size = 169, normalized size = 5.45

$$\frac{\frac{e^{(dx+c)}}{a+b}+\frac{e^{(-dx-c)}}{a-b}}{2\,d}+\frac{\frac{6\,(2\,ab+b^2)(dx+c)}{a-b}-\frac{(2\,ab+b^2)\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a-b}}{3\,(a^2-b^2)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out] 1/2*(e^(d*x + c)/(a + b) + e^(-d*x - c)/(a - b))/d + 1/3*(6*(2*a*b + b^2)*(d*x + c)/(a - b) - (2*a*b + b^2)*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b))/(a - b))/((a^2 - b^2)*d^2)

Mupad [A]
time = 87.99, size = 2500, normalized size = 80.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^3),x)

[Out] exp(- c - d*x)/(2*(a*d - b*d)) + symsum(log((81920*a^2*b^5*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 221184*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^8*d^3 - 3538944*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^3*b^7*d^3 + 1990656*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^4*b^6*d^3 + 3538944*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^5*b^5*d^3 - 2211840*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^6*b^4*d^3 + 7962624*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^3*b^9*d^5 + 15925248*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^4*b^8*d^5 - 7962624*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^5*b^7*d^5 - 31850496*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^6*b^6*d^5 - 7962624*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^7*b^5*d^5 + 15925248*ro ot(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^8*b^4*d^5 + 7962624*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^9*b^3*d^5 + 98304*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)*a^2*b^6*d - 98304*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)*a^3*b^5*d + 24576*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)*a^4*b^4*d + 8192*a*b^6*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 368640*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^2*b^7*d^2*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) - 2285568*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^3*b^6*d^2*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) - 5013504*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^4*b^5*d^2*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) - 368640*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^5*b^4*d^2*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 8626176*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^3*b^8*d^4*exp(d*x)*ex...

3.77 $\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=31

$$i\text{Int}\left(-\frac{i\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] I*Unintegrable(-I*csch(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] I*Defer[Int][((-I)*Csch[c + d*x])/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = i\int -\frac{i\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 319 vs. 2(31) = 62.
time = 0.12, size = 319, normalized size = 10.29

$$\frac{6\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-b\text{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3b\#1^4+a\#1^6+b\#1^6\&,\frac{c+dx+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)-2c\#1^2-2dx\#1^2-4\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2+c\#1^4+dx\#1^4+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4}{a\#1+b\#1+2a\#1^3-2b\#1^3+a\#1^5+b\#1^5}\&\right]}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (6*Log[Tanh[(c + d*x)/2]] - b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^
4 - 3*b*#1^4 + a*#1^6 + b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 -
 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c +
d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*
#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ])/(6*a*d)
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(28) = 56$.
time = 4.54, size = 96, normalized size = 3.10

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}}{d}$
risch	$-\frac{\ln(e^{dx+c}+1)}{da}+2\left(\sum_{\_R=\mathrm{RootOf}\left((46656a^8d^6-46656a^6b^2d^6)\_Z^6+3888a^4b^2d^4\_Z^4-108a^2b^2d^2\_Z^2+b^2\right)}\_R\ln\left(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tanh(1/2*d*x+1/2*c))-4/3/a*b*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b
+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2
*a+a)))
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(5*d*x + 5*c) - 2*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^
2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(4*c) - a*b*e^(4
*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [A] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 2.03, size = 20085, normalized size = 647.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(2/3)*sqrt(1/6)*a*d*sqrt(((a^4 - a^2*b^2)*((b^4/(a^4*d^2 - a^2*b^
2*d^2)^2 + b^2/(a^6*d^4 - a^4*b^2*d^4))*(-I*sqrt(3) + 1)/(-1/729*b^6/(a^4*d
```

```
^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2
*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4
*d^6))^(1/3) + 81*(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d
^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*
d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3)*(I*sqrt(3) + 1) + 18*b^2/(
a^4*d^2 - a^2*b^2*d^2))*d^2 + 3*sqrt(1/3)*(a^4 - a^2*b^2)*d^2*sqrt(-((a^8 -
 2*a^6*b^2 + a^4*b^4)*((b^4/(a^4*d^2 - a^2*b^2*d^2)^2 + b^2/(a^6*d^4 - a^4*
b^2*d^4))*(-I*sqrt(3) + 1)/(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^
4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 -
 a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3) + 81*(-1/729*b^6/
(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a
^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)
^2*a^4*d^6))^(1/3)*(I*sqrt(3) + 1) + 18*b^2/(a^4*d^2 - a^2*b^2*d^2))^2*d^4
- 1296*a^2*b^2 + 324*b^4 - 36*(a^4*b^2 - a^2*b^4)*((b^4/(a^4*d^2 - a^2*b^2*
d^2)^2 + b^2/(a^6*d^4 - a^4*b^2*d^4))*(-I*sqrt(3) + 1)/(-1/729*b^6/(a^4*d^2
 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d
^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d
^6))^(1/3) + 81*(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4
 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^
6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3)*(I*sqrt(3) + 1) + 18*b^2/(a^
4*d^2 - a^2*b^2*d^2))*d^2)/((a^8 - 2*a^6*b^2 + a^4*b^4)*d^4)) - 54*b^2)/((a
^4 - a^2*b^2)*d^2))*log(1/324*sqrt(2/3)*sqrt(1/6)*((a^6 - a^4*b^2)*((b^4/(a
^4*d^2 - a^2*b^2*d^2)^2 + b^2/(a^6*d^4 - a^4*b^2*d^4))*(-I*sqrt(3) + 1)/(-1
/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^
4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a
^2 - b^2)^2*a^4*d^6))^(1/3) + 81*(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/
486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8
*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3)*(I*sqrt(3)
+ 1) + 18*b^2/(a^4*d^2 - a^2*b^2*d^2))^2*d^5 - 18*(a^4 + 2*a^2*b^2)*((b^4/(
a^4*d^2 - a^2*b^2*d^2)^2 + b^2/(a^6*d^4 - a^4*b^2*d^4))*(-I*sqrt(3) + 1)/(-
1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a
^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((
a^2 - b^2)^2*a^4*d^6))^(1/3) + 81*(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1
/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^
8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3)*(I*sqrt(3)
 + 1) + 18*b^2/(a^4*d^2 - a^2*b^2*d^2))*d^3 - 324*(2*a*b + b^2)*d - 3*sqrt(
1/3)*((a^6 - a^4*b^2)*((b^4/(a^4*d^2 - a^2*b^2*d^2)^2 + b^2/(a^6*d^4 - a^4*
b^2*d^4))*(-I*sqrt(3) + 1)/(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^
4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 -
 a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3) + 81*(-1/729*b^6/
(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a
^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)
^2*a^4*d^6))^(1/3)*(I*sqrt(3) + 1) + 18*b^2/(a^4*d^2 - a^2*b^2*d^2))*d^5 +
18*(a^4 - a^2*b^2)*d^3)*sqrt(-((a^8 - 2*a^6*b^2 + a^4*b^4)*((b^4/(a^4*d^2 -
 a^2*b^2*d^2)^2 + b^2/(a^6*d^4 - a^4*b^2*d^4))*(-I*sqrt(3) + 1)/(-1/729*b^6
```

```
/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 -
a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2
)^2*a^4*d^6))^(1/3) + 81*(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/
((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a
^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3)*(I*sqrt(3) + 1) + 1
8*b^2/(a^4*d^2 - a^2*b^2*d^2))^2*d^4 - 1296*a^2*b^2 + 324*b^4 - 36*(a^4*b^2
 - a^2*b^4)*((b^4/(a^4*d^2 - a^2*b^2*d^2)^2 + b^2/(a^6*d^4 - a^4*b^2*d^4))*
(-I*sqrt(3) + 1)/(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^
4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d
^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3) + 81*(-1/729*b^6/(a^4*d^2 -
 a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2
)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6
))^(1/3)*(I*sqrt(3) + 1) + 18*b^2/(a^4*d^2 - a^2*b^2*d^2))*d^2)/((a^8 - 2*a
^6*b^2 + a^4*b^4)*d^4)))*sqrt(((a^4 - a^2*b^2)*((b^4/(a^4*d^2 - a^2*b^2*d^2
)^2 + b^2/(a^6*d^4 - a^4*b^2*d^4))*(-I*sqrt(3) + 1)/(-1/729*b^6/(a^4*d^2 -
a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)
) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6)
)^(1/3) + 81*(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 -
a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6)
+ 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3)*(I*...
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**3), x)
```

Giac [A] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.85, size = 146, normalized size = 4.71

$$-\frac{\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}}{d}-\frac{\frac{6\,(dx+c)b}{a-b}-\frac{b\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a-b}}{3\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -(log(e^(d*x + c) + 1)/a - log(abs(e^(d*x + c) - 1))/a)/d - 1/3*(6*(d*x + c
)*b/(a - b) - b*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*
x + 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c)
+ a - b))/(a - b))/(a*d^2)
```

Mupad [A]
time = 16.50, size = 2500, normalized size = 80.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)),x)
```

```
[Out] symsum(log(-(1409286144*b^6*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8
*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 1342177
28*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^
2*b^2*d^2*z^2 - b^2, z, k)*b^7*d + 1879048192*root(729*a^6*b^2*d^6*z^6 - 72
9*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*a*b^6
*d - 2818572288*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^
4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^7*d^3 - 40869298176*root(72
9*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*
z^2 - b^2, z, k)^3*a^3*b^6*d^3 + 28185722880*root(729*a^6*b^2*d^6*z^6 - 729
*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^4*
b^5*d^3 + 15502147584*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*
b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^5*b^4*d^3 + 18119393280*r
oot(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^
2*d^2*z^2 - b^2, z, k)^5*a^4*b^7*d^5 + 235552112640*root(729*a^6*b^2*d^6*z^
6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)
^5*a^5*b^6*d^5 + 14495514624*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 2
43*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^6*b^5*d^5 - 219244
658688*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 2
7*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^7*b^4*d^5 - 48922361856*root(729*a^6*b^2
*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2
, z, k)^5*a^8*b^3*d^5 - 32614907904*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*
z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^6*b^7*d^7 -
 179381993472*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*
z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^7*b^6*d^7 - 16307453952*root(729*
a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^
2 - b^2, z, k)^7*a^8*b^5*d^7 + 179381993472*root(729*a^6*b^2*d^6*z^6 - 729*
a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^9*b
^4*d^7 + 48922361856*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b
^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^10*b^3*d^7 - 1912602624*ro
ot(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2
*d^2*z^2 - b^2, z, k)*a^2*b^5*d - 100663296*root(729*a^6*b^2*d^6*z^6 - 729*
a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*a^3*b^4
*d + 738197504*a*b^5*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^
6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 268435456*root
(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d
^2*z^2 - b^2, z, k)^2*a*b^7*d^2*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729
*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 291
```

58801408*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^2*b^6*d^2*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 29125246976*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^3*b^5*d^2*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 2113929216*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^4*b^4*d^2*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 4831838208*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^3*b^7*d^4*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 165490458624*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^4*b^6*d^4*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 283870494720*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^5*b^5*d^4*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 132573560832*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^6*b^4*d^4*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 2717908992*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^7*b^3*d^4*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 21743271936*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^5*b^7*d^6*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) - 154920812544*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4...

3.78 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}-\frac{\coth(c+dx)}{ad}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d}-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{}\right.}{}$$

```
[Out] -coth(d*x+c)/a/d+1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*tanh(d*x+c))/a^(4/3)/d-1/6*
b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/a^(4/
3)/d+1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a^(1/3)*3^(1/2)
)/a^(4/3)/d*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3744, 331, 298, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(4/3)*d) - Coth[c + d*x]/(a*d) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*Tan
h[c + d*x]])/(3*a^(4/3)*d) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c
+ d*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*a^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{\coth(c+dx)}{ad} - \frac{b\operatorname{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, \tanh(c+dx)\right)}{ad}\\
&= -\frac{\coth(c+dx)}{ad} + \frac{b^{2/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \tanh(c+dx)\right)}{3a^{4/3}d} - \frac{b^{2/3}\operatorname{Subst}\left(\int\right.}{}\\
&= -\frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\operatorname{Subst}\left(\int \frac{-\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{a}}\right.}{}\\
&= -\frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\right.}{}\\
&= \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}d} - \frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c\right.}{3a^{4/3}d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 190, normalized size = 1.21

$$
-\frac{3\coth(c+dx)+2b\operatorname{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2-3b\#1^2+a\#1^3+b\#1^3\&, \frac{-c-dx-\log(-\cosh(c+dx)-\sinh(c+dx)+\cosh(c+dx)\#1-\sinh(c+dx)\#1)+c\#1+dx\#1+\log(-\cosh(c+dx)-\sinh(c+dx)+\cosh(c+dx)\#1-\sinh(c+dx)\#1)\#1}{a+b+2a\#1-2b\#1+a\#1^2+b\#1^2}\&\right]}{3ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -1/3*(3*Coth[c + d*x] + 2*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*
b*#1^2 + a*#1^3 + b*#1^3 & , (-c - d*x - Log[-Cosh[c + d*x] - Sinh[c + d*x]
 + Cosh[c + d*x]*#1 - Sinh[c + d*x]*#1] + c*#1 + d*x*#1 + Log[-Cosh[c + d*x
] - Sinh[c + d*x] + Cosh[c + d*x]*#1 - Sinh[c + d*x]*#1]*#1)/(a + b + 2*a*#
1 - 2*b*#1 + a*#1^2 + b*#1^2) & ])/(a*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.23, size = 116, normalized size = 0.74

method	result

derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{1}{2a\tanh\left(\right.}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{1}{2a\tanh\left(\right.}}{d}$
risch	$-\frac{2}{da\left(e^{2dx+2c}-1\right)}+4\left(\sum_{\_R=\mathrm{RootOf}\left(1728a^4d^3\_Z^3-b\right)}\_R\ln\left(e^{2dx+2c}+\frac{288a^3d^2\_R^2}{(a+b)\left(\frac{b}{a+b}+\frac{a}{a+b}\right)}-\frac{24a^2d\_}{(a+b)\left(\frac{b}{a+b}+\right.}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)+2/3/a*b*sum((_R^3-_R)/(_R^5*a+2*_R^3*a+4*_R
^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*
_Z^2*a+a))-1/2/a/tanh(1/2*d*x+1/2*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d*
x + 2*c))/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(4*
c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(123) = 246.
time = 0.40, size = 640, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) +
sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(b/a)^(1/3)*arctan(-1/3*(sqrt(3)*b*cosh(
d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x +
 c)^2 - (sqrt(3)*a*cosh(d*x + c)^2 + 2*sqrt(3)*a*cosh(d*x + c)*sinh(d*x + c
```

```
) + sqrt(3)*a*sinh(d*x + c)^2 + sqrt(3)*a)*(b/a)^(2/3) - (sqrt(3)*b*cosh(d*
x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c
)^2 - sqrt(3)*b)*(b/a)^(1/3))/b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2 - 1)*(b/a)^(1/3)*log((a + b)*cosh(d*x + c)^4 + 4
*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b
)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) - 2*(a*c
osh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a)*(
b/a)^(2/3) + 2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sin
h(d*x + c)^2 + a)*(b/a)^(1/3) + a + b) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b/a)^(1/3)*log((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + 2*
a*(b/a)^(2/3) - 2*a*(b/a)^(1/3) + a - b) + 12)/(a*d*cosh(d*x + c)^2 + 2*a*d
*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**3),x)
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**3), x)
```

Giac [A]
time = 0.47, size = 21, normalized size = 0.13

$$-\frac{2}{ad(e^{(2\,dx+2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
[Out] -2/(a*d*(e^(2*d*x + 2*c) - 1))
```

Mupad [B]
time = 8.71, size = 669, normalized size = 4.26

$$\frac{b^{1/3}\ln\left(a^{1/3}-b^{1/3}+a^{1/3}e^{2c+2dx}+b^{1/3}e^{2c+2dx}\right)}{3a^{4/3}d}-\frac{2}{a\,d\,\left(e^{2c+2dx}-1\right)}+\frac{b^{1/3}\ln\left(\frac{256b^3\left(19a^2b-24ab^2+6a^3-b^3+8a^3e^{2c+2dx}+b^3e^{2c+2dx}+70ab^2e^{2c+2dx}+113a^2be^{2c+2dx}\right)}{a^4(a+b)^6}+\frac{b^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1536b^3d\left(8a^2-8b^2+15a^2e^{2c+2dx}+15b^2e^{2c+2dx}+66abe^{2c+2dx}\right)}{a^2(a+b)^6}+\frac{768b^{7/3}d\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(24a^2b-19ab^2+a^3-6b^3+a^3e^{2c+2dx}+8b^3e^{2c+2dx}+113ab^2e^{2c+2dx}+70a^2be^{2c+2dx}\right)}{a^{7/3}(a+b)^6}\right)}{3a^{4/3}d}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3a^{4/3}d}-\frac{b^{1/3}\ln\left(\frac{256b^3\left(19a^2b-24ab^2+6a^3-b^3+8a^3e^{2c+2dx}+b^3e^{2c+2dx}+70ab^2e^{2c+2dx}+113a^2be^{2c+2dx}\right)}{a^4(a+b)^6}-\frac{b^{1/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1536b^3d\left(8a^2-8b^2+15a^2e^{2c+2dx}+15b^2e^{2c+2dx}+66abe^{2c+2dx}\right)}{a^2(a+b)^6}-\frac{768b^{7/3}d\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(24a^2b-19ab^2+a^3-6b^3+a^3e^{2c+2dx}+8b^3e^{2c+2dx}+113ab^2e^{2c+2dx}+70a^2be^{2c+2dx}\right)}{a^{7/3}(a+b)^6}\right)}{3a^{4/3}d}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3a^{4/3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)),x)
[Out] (b^(1/3)*log(a^(1/3) - b^(1/3) + a^(1/3)*exp(2*c + 2*d*x) + b^(1/3)*exp(2*c
 + 2*d*x)))/(3*a^(4/3)*d) - 2/(a*d*(exp(2*c + 2*d*x) - 1)) + (b^(1/3)*log((
```

```
256*b^3*(19*a^2*b - 24*a*b^2 + 6*a^3 - b^3 + 8*a^3*exp(2*c + 2*d*x) + b^3*e
xp(2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c + 2*d*x)))/
(a^4*(a + b)^6) + (b^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((1536*b^3*d*(8*a^2 - 8*b
^2 + 15*a^2*exp(2*c + 2*d*x) + 15*b^2*exp(2*c + 2*d*x) + 66*a*b*exp(2*c + 2
*d*x)))/(a^2*(a + b)^6) + (768*b^(7/3)*d*((3^(1/2)*1i)/2 - 1/2)*(24*a^2*b -
 19*a*b^2 + a^3 - 6*b^3 + a^3*exp(2*c + 2*d*x) + 8*b^3*exp(2*c + 2*d*x) + 1
13*a*b^2*exp(2*c + 2*d*x) + 70*a^2*b*exp(2*c + 2*d*x)))/(a^(7/3)*(a + b)^6)
))/(3*a^(4/3)*d))*((3^(1/2)*1i)/2 - 1/2))/(3*a^(4/3)*d) - (b^(1/3)*log((256
*b^3*(19*a^2*b - 24*a*b^2 + 6*a^3 - b^3 + 8*a^3*exp(2*c + 2*d*x) + b^3*exp(
2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c + 2*d*x)))/(a^
4*(a + b)^6) - (b^(1/3)*((3^(1/2)*1i)/2 + 1/2)*((1536*b^3*d*(8*a^2 - 8*b^2
+ 15*a^2*exp(2*c + 2*d*x) + 15*b^2*exp(2*c + 2*d*x) + 66*a*b*exp(2*c + 2*d*
x)))/(a^2*(a + b)^6) - (768*b^(7/3)*d*((3^(1/2)*1i)/2 + 1/2)*(24*a^2*b - 19
*a*b^2 + a^3 - 6*b^3 + a^3*exp(2*c + 2*d*x) + 8*b^3*exp(2*c + 2*d*x) + 113*
a*b^2*exp(2*c + 2*d*x) + 70*a^2*b*exp(2*c + 2*d*x)))/(a^(7/3)*(a + b)^6)))/
(3*a^(4/3)*d))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/3)*d)
```

3.79 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=33

$$-i\text{Int}\left(\frac{i\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

```
[Out] -I*Unintegrable(I*csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-I)*Defer[Int][(I*Csch[c + d*x]^3)/(a + b*Tanh[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\left(i\int \frac{i\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx\right)$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(33) = 66.
time = 0.28, size = 201, normalized size = 6.09

$$-\frac{16b\text{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3b\#1^4+a\#1^6+b\#1^6\&,\frac{c\#1+dx\#1+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1}{a+b+2a\#1^2-2b\#1^2+a\#1^4+b\#1^4}\&\right]+3\left(\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+4\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\right)}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -1/24*(16*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*#
1^6 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(a + b + 2*a*#1^2 -
2*b*#1^2 + a*#1^4 + b*#1^4) & ] + 3*(Csch[(c + d*x)/2]^2 + 4*Log[Tanh[(c +
d*x)/2]] + Sech[(c + d*x)/2]^2))/(a*d)
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(30) = 60$.
time = 4.85, size = 136, normalized size = 4.12

method	result
derivativedivides	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2\right.}{\_R^5a+2\_}\right.}{3a}}{d}$
default	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2\right.}{\_R^5a+2\_}\right.}{3a}}{d}$
risch	$-\frac{e^{dx+c}\left(1+e^{2dx+2c}\right)}{da\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{dx+c}+1\right)}{2da}-\frac{\ln\left(e^{dx+c}-1\right)}{2da}+8\left(\sum_{\_R=\mathrm{RootOf}\left(191102976d^6\_Z^6a^{10}+1728a^4d^2\_Z^2b^2+c\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-1/8/a/tanh(1/2*d*x+1/2*c)^2-1/2/a*ln(tanh(
1/2*d*x+1/2*c))-1/3/a*b*sum((_R^4-2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)
*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -8*b*integrate(e^(3*d*x + 3*c)/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(
6*d*x) + 3*(a^2*e^(4*c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(
2*c))*e^(2*d*x)), x) - (e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c)
 - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) -
 1/2*log((e^(d*x + c) - 1)*e^(-c))/(a*d)
```

Fricas [A] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 5.91, size = 24389, normalized size = 739.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] 1/32*(24*sqrt(1/2)*sqrt(1/3)*(1/12)^(3/4)*(1/27)^(1/4)*(a^11*d^7*e^(4*d*x + 4*c) - 2*a^11*d^7*e^(2*d*x + 2*c) + a^11*d^7)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))*sqrt((4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))^2*a^8*b^2*d^4 + 16*a^4*b^2 + 16*a^2*b^4 + 16*b^6 - 8*(a^6*b^2 - a^4*b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))*d^2 - (8*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))*a^8*b^2*d^4 + (a^12 - a^10*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))^2*d^6 - 4*(a^6*b^2 - a^4*b^4)*d^2)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))^2*a^6*d^4 + 12*b^2)/(a^6*d^4)))/(a^4*b^2 + 2*a^2*b^4 + b^6))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))^2*a^6*d^4 + 16*b^2)/(a^6*d^4))*((((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))^2*a^6*d^4 + 12*b^2)/(a^6*d^4))^(3/4)*arctan(1/128*(27*sqrt(1/2)*sqrt(1/3)*(1/12)^(3/4)*(1/27)^(3/4)*(sqrt(1/3)*((a^19 - a^18*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))^2*d^11 + 2*(a^16*b + a^15*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))*d^9)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))^2*a^6*d^4 + 16*b^2)/(a^6*d^4))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))^2*a^6*d^4 + 12*b^2)/(a^6*d^4)) + 4*sqrt(1/3)*((a^16*b + a^15*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3

```
)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^
4)/(a^10*d^6))^(1/3)))^2*d^9 - 2*(a^11*b^2 - a^10*b^3 - a^9*b^4 + a^8*b^5)*
d^5)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*
b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*
((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))^2*a^6*d^4
 + 16*b^2)/(a^6*d^4)))*sqrt((4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^
2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sq
rt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6
))^(1/3)))^2*a^8*b^2*d^4 + 16*a^4*b^2 + 16*a^2*b^4 + 16*b^6 - 8*(a^6*b^2 -
a^4*b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^
2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((
a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))*d^2 - (8*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a
^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*((a^2 + b^2)*
b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))*a^8*b^2*d^4 + (a^12 -
a^10*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b
^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/(a^6*d^4*(
(a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3)))^2*d^6 - 4*
(a^6*b^2 - a^4*b^4)*d^2)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^
2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6))^(1/3) - 2*(1/2)^(2/3)*b^2*(-I*sq
rt(3) + 1)/(a^6*d^4*((a^2 + b^2)*b^2/(a^10*d^6) - (a^2*b^2 - b^4)/(a^10*d^6
))^(1/3)))^2*a^6*d^4 + 12*b^2)/(a^6*d^4)))/(a^4*b^2 + 2*a^2*b^4 + b^6))*sqr
t((3*sqrt(1/2)*sqrt(1/3)*(1/12)^(1/4)*(1/27)^(1/4)*(4*(a^13*b - a^9*b^5)*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((a^2 + b^2)*b^2/(a^...
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**3), x)
```

Giac [A] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.65, size = 68, normalized size = 2.06

$$\frac{\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}-\frac{2\left(e^{(3\,dx+3\,c)}+e^{(dx+c)}\right)}{a\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*(log(e^(d*x + c) + 1)/a - log(abs(e^(d*x + c) - 1))/a - 2*(e^(3*d*x + 3
*c) + e^(d*x + c))/(a*(e^(2*d*x + 2*c) - 1)^2))/d
```

Mupad [A]
time = 26.92, size = 2500, normalized size = 75.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)),x)
```

```
[Out] exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - (2*exp(c + d*x))/(a*d - 2*a*d*e
xp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) + symsum(log((570425344*a^4*b^6*exp
(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)
) - 33554432*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z,
 k)*a*b^10*d - 553648128*a^2*b^8*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^
4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 167772160*a^3*b^7*exp(d*x)*exp(root
(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 16777216*b
^10*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4
, z, k)) + 192937984*a^5*b^5*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^
2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 2617245696*root(729*a^10*d^6*z^6 + 27*a
^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^5*b^8*d^3 - 150994944*root(729*a^
10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^6*b^7*d^3 - 1384
120320*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*
a^7*b^6*d^3 + 2415919104*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b
^2 - b^4, z, k)^3*a^8*b^5*d^3 - 3498049536*root(729*a^10*d^6*z^6 + 27*a^4*b
^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^9*b^4*d^3 + 5435817984*root(729*a^10*
d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^8*b^7*d^5 + 6794772
48*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^9*
b^6*d^5 - 70665633792*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2
- b^4, z, k)^5*a^10*b^5*d^5 + 52319748096*root(729*a^10*d^6*z^6 + 27*a^4*b^
2*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^11*b^4*d^5 + 12230590464*root(729*a^10
*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^12*b^3*d^5 + 32614
907904*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^7*
a^11*b^6*d^7 + 146767085568*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^
2*b^2 - b^4, z, k)^7*a^12*b^5*d^7 - 130459631616*root(729*a^10*d^6*z^6 + 27
*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^7*a^13*b^4*d^7 - 48922361856*root(7
29*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^7*a^14*b^3*d^7
+ 67108864*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k
)*a^2*b^9*d - 427819008*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^
2 - b^4, z, k)*a^3*b^8*d - 822083584*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2
*z^2 + a^2*b^2 - b^4, z, k)*a^4*b^7*d + 436207616*root(729*a^10*d^6*z^6 + 2
7*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)*a^5*b^6*d + 754974720*root(729*a^1
0*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)*a^6*b^5*d + 25165824*
root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)*a^7*b^4*d
```

- 25165824*a*b^9*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 234881024*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^3*b^9*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 2592079872*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^4*b^8*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 2860515328*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^5*b^7*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 2919235584*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^6*b^6*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 2357198848*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^7*b^5*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 528482304*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^2*a^8*b^4*d^2*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 301989888*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^6*b^8*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 9965666304*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^7*b^7*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 33671872512*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^8*b^6*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 6568280064*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^9*b^5*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 29293019136*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^10*b^4*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 679477248*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^4*a^11*b^3*d^4*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 72024588288*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^6*a^10*b^6*d^6*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 27179089920*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^6*a^11*b^5*d^6*exp...

3.80 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal. Leaf size=215

$$-\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\right.}{3}$$

```
[Out] coth(d*x+c)/a/d-1/3*coth(d*x+c)^3/a/d-b*ln(tanh(d*x+c))/a^2/d-1/3*b^(1/3)*l
n(a^(1/3)+b^(1/3)*tanh(d*x+c))/a^(4/3)/d+1/6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/a^(4/3)/d+1/3*b*ln(a+b*tanh(d*x+c)^
3)/a^2/d-1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a^(1/3)*3^(
1/2))/a^(4/3)/d*3^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3744, 1848, 1885, 12, 298, 31, 648, 631, 210, 642, 266}

$$-\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d}+\frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d}-\frac{b\log(\tanh(c+dx))}{a^2d}-\frac{\coth^3(c+dx)}{3ad}+\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(4/3)*d)) + Coth[c + d*x]/(a*d) - Coth[c + d*x]^3/(3*a*d) - (b*Lo
g[Tanh[c + d*x]])/(a^2*d) - (b^(1/3)*Log[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/
(3*a^(4/3)*d) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d*x] + b^(2
/3)*Tanh[c + d*x]^2])/(6*a^(4/3)*d) + (b*Log[a + b*Tanh[c + d*x]^3])/(3*a^2
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^3)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{bx(a+bx)}{a^2(a+bx^3)}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}+\frac{b\operatorname{Subst}\left(\int \frac{x(a+bx)}{a+bx^3}\,dx, x\right.}{a^2d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}+\frac{b\operatorname{Subst}\left(\int \frac{ax}{a+bx^3}\,dx, x, \mathrm{t}\right.}{a^2d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}+\frac{b\log\left(a+b\tanh^3(c+\right.}{3a^2d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}+\frac{b\log\left(a+b\tanh^3(c+\right.}{3a^2d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\mathrm{ta}\right.}{3a^{4/3}d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\mathrm{ta}\right.}{3a^{4/3}d}$$

$$= -\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh}{a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.44, size = 322, normalized size = 1.50

$$\frac{-a\coth(c+dx)\left(-2+\operatorname{csch}^2(c+dx)\right)+3b(c+dx-\log(\sinh(c+dx)))+b\operatorname{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2-3b\#1^2+a\#1^3+b\#1^3\&,\frac{-2ac+2bc-2adx+2bdx+a\log\left(e^{2(c+dx)}-\#1\right)-b\log\left(e^{2(c+dx)}-\#1\right)-8ac\#1-4bc\#1-8adx\#1-4bdx\#1+4a\log\left(e^{2(c+dx)}-\#1\right)\#1+2b\log\left(e^{2(c+dx)}-\#1\right)\#1+2ac\#1^2+2bc\#1^2+2adx\#1^2+2bdx\#1^2-a\log\left(e^{2(c+dx)}-\#1\right)\#1^2-b\log\left(e^{2(c+dx)}-\#1\right)\#1^2}{a-b+2a\#1+2b\#1+a\#1^2-b\#1^2}\&\right]}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-(a*Coth[c + d*x]*(-2 + Csch[c + d*x]^2)) + 3*b*(c + d*x - Log[Sinh[c + d*
x]]) + b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b
*#1^3 & , (-2*a*c + 2*b*c - 2*a*d*x + 2*b*d*x + a*Log[E^(2*(c + d*x)) - #1]
 - b*Log[E^(2*(c + d*x)) - #1] - 8*a*c*#1 - 4*b*c*#1 - 8*a*d*x*#1 - 4*b*d*x
*#1 + 4*a*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b*Log[E^(2*(c + d*x)) - #1]*#1 +
 2*a*c*#1^2 + 2*b*c*#1^2 + 2*a*d*x*#1^2 + 2*b*d*x*#1^2 - a*Log[E^(2*(c + d*
x)) - #1]*#1^2 - b*Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1
 + a*#1^2 - b*#1^2) & ])/(3*a^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.52, size = 173, normalized size = 0.80

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_\right.}\right.}{}}{d}$
default	$\frac{-\frac{\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_\right.}\right.}{}}{d}$
risch	$-\frac{4\left(3\,\mathrm{e}^{2dx+2c}-1\right)}{3ad\left(\mathrm{e}^{2dx+2c}-1\right)^3}-\frac{b\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{a^2d}+16\left(\sum_{\_R=\text{RootOf}\left(110592a^6d^3\_Z^3-6912a^4b\,d^2\_Z^2+144a^2b^2d\_Z+a^2b-\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a*(1/3*tanh(1/2*d*x+1/2*c)^3-3*tanh(1/2*d*x+1/2*c))-1/24/a/tanh(1
/2*d*x+1/2*c)^3+3/8/a/tanh(1/2*d*x+1/2*c)-b/a^2*ln(tanh(1/2*d*x+1/2*c))+1/3
*b/a^2*sum((_R^5*a+4*_R^2*b+3*_R*a)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh
(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 2*a*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a
 + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c
) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*b) - (d*x + c)/((a^3
- a^2*b)*d)) - 2*b^2*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d
*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)
*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*b) - (
d*x + c)/((a^3 - a^2*b)*d)) + 2*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d
*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b),
 x)/a + 2*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b
)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a^2 - 8*b*integr
ate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) +
3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a - 4*b^2*integrate(e^(2*d*x + 2*c)/
((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x +
 2*c) + a - b), x)/a^2 + 2/3*(3*b*d*x*e^(6*d*x + 6*c) - 9*b*d*x*e^(4*d*x +
4*c) - 3*b*d*x + 3*(3*b*d*x*e^(2*c) - 2*a*e^(2*c))*e^(2*d*x) + 2*a)/(a^2*d*
e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c) - a^2*d
) - b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - b*log((e^(d*x + c) - 1)*e^(-c
))/(a^2*d)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.50, size = 1954, normalized size = 9.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(1/3)*(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2
*d*e^(2*d*x + 2*c) - a^2*d)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3)
 - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^2
+ 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^
3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*a^2*b*d + 4*b^2)/(a^4*d^2))*arctan(-1/8*
sqrt(1/3)*((a^6 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a
^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*d^3*e^(2*d*x + 2*
c) - 4*(2*a^3*b + a^2*b^2 - a*b^3)*d*e^(2*d*x + 2*c) - 2*((a^5 - a^4*b - 2*
a^3*b^2)*d^2*e^(2*d*x + 2*c) + (a^5 + a^4*b)*d^2)*((1/2)^(1/3)*(I*sqrt(3) +
 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^
2*d)) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b
 - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^3 - 2*(a^3 - 2*a^2*b)*((1/2
)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d
^3))^(1/3) - 2*b/(a^2*d))*d^2 - 4*(2*a*b - b^2)*d)*sqrt(a^4 + 4*a^3*b + 5*a
^2*b^2 + 2*a*b^3 - 1/2*((a^6 + a^5*b)*d^2*e^(2*d*x + 2*c) - (a^6 + a^5*b)*d
^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^
3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2 + ((a^5 - a^4*b - 2*a^3*b^2)*d*e^(2*d*
x + 2*c) + (a^5 + 3*a^4*b + 2*a^3*b^2)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(
```

```
a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) +
(a^4 + 2*a^3*b + a^2*b^2)*e^(4*d*x + 4*c) + 2*(a^4 + a^3*b - a^2*b^2 - a*b^
3)*e^(2*d*x + 2*c)))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/
(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^2 + 4*((1
/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6
*d^3))^(1/3) - 2*b/(a^2*d))*a^2*b*d + 4*b^2)/(a^4*d^2))/(a^2*b + a*b^2)) -
2*(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c
) - a^2*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2
*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) +
 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^
2*d))^2*a^4*d^2 - (a^3 - 2*a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3)
 - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*d + a^2 -
3*a*b + 2*b^2 + (a^2 + a*b)*e^(2*d*x + 2*c)) - 48*a*e^(2*d*x + 2*c) + ((a^2
*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c) - a^
2*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b
^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) + 6*b*e^(6*d*x + 6*c) - 18*b*e^(4*d*x +
 4*c) + 18*b*e^(2*d*x + 2*c) - 6*b)*log(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
 - 1/2*((a^6 + a^5*b)*d^2*e^(2*d*x + 2*c) - (a^6 + a^5*b)*d^2)*((1/2)^(1/3)
*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1
/3) - 2*b/(a^2*d))^2 + ((a^5 - a^4*b - 2*a^3*b^2)*d*e^(2*d*x + 2*c) + (a^5
+ 3*a^4*b + 2*a^3*b^2)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(
a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) + (a^4 + 2*a^3*b +
 a^2*b^2)*e^(4*d*x + 4*c) + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(2*d*x + 2*
c)) - 12*(b*e^(6*d*x + 6*c) - 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) - b
)*log(e^(2*d*x + 2*c) - 1) + 16*a)/(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*
x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c) - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**3), x)
```

Giac [A]
time = 0.49, size = 180, normalized size = 0.84

$$\frac{\frac{2\,b\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a^2}-\frac{6\,b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^2}+\frac{11\,be^{(6\,dx+6\,c)}-33\,be^{(4\,dx+4\,c)}-24\,ae^{(2\,dx+2\,c)}+33\,be^{(2\,dx+2\,c)}+8\,a-11\,b}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c
) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b
))/a^2 - 6*b*log(abs(e^(2*d*x + 2*c) - 1))/a^2 + (11*b*e^(6*d*x + 6*c) - 33
*b*e^(4*d*x + 4*c) - 24*a*e^(2*d*x + 2*c) + 33*b*e^(2*d*x + 2*c) + 8*a - 11
*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B]
time = 3.22, size = 2500, normalized size = 11.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)),x)
```

```
[Out] 8/(3*(a*d - 3*a*d*exp(2*c + 2*d*x) + 3*a*d*exp(4*c + 4*d*x) - a*d*exp(6*c +
 6*d*x))) - 4/(a*d - 2*a*d*exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) + symsu
m(log((1507328*a*b^9 + 1572864*b^10 - 5242880*a^2*b^8 - 7479296*a^3*b^7 + 3
948544*a^4*b^6 + 5963776*a^5*b^5 - 278528*a^6*b^4 + 8192*a^7*b^3 - 1572864*
b^10*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b -
 b^3, z, k))*exp(2*d*x) - 1769472*a*b^9*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*
b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 42467328*root(
27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^
4*b^8*d^2 + 21626880*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z
 + a^2*b - b^3, z, k)^2*a^5*b^7*d^2 - 70189056*root(27*a^6*d^3*z^3 - 27*a^4
*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^6*b^6*d^2 + 18038784*ro
ot(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2
*a^7*b^5*d^2 - 11993088*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*
d*z + a^2*b - b^3, z, k)^2*a^8*b^4*d^2 + 147456*root(27*a^6*d^3*z^3 - 27*a^
4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^9*b^3*d^2 - 98304*root
(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a
^10*b^2*d^2 - 42467328*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d
*z + a^2*b - b^3, z, k)^3*a^6*b^7*d^3 - 12091392*root(27*a^6*d^3*z^3 - 27*a
^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^7*b^6*d^3 + 22708224*
root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)
^3*a^8*b^5*d^3 + 12386304*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^
2*d*z + a^2*b - b^3, z, k)^3*a^9*b^4*d^3 + 19759104*root(27*a^6*d^3*z^3 - 2
7*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^10*b^3*d^3 - 29491
2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z,
k)^3*a^11*b^2*d^3 - 14155776*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2
*b^2*d*z + a^2*b - b^3, z, k)*a^2*b^9*d - 10387456*root(27*a^6*d^3*z^3 - 27
*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^3*b^8*d + 32407552*ro
ot(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a
^4*b^7*d + 16187392*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z
+ a^2*b - b^3, z, k)*a^5*b^6*d - 29818880*root(27*a^6*d^3*z^3 - 27*a^4*b*d^
2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^6*b^5*d + 6135808*root(27*a^6*
```

d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^7*b^4*d - 376832*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^8*b^3*d + 8192*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^9*b^2*d - 3571712*a^2*b^8*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 30990336*a^3*b^7*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 43139072*a^4*b^6*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 8519680*a^5*b^5*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 245760*a^6*b^4*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 8192*a^7*b^3*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 42467328*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^4*b^8*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 22413312*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^5*b^7*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 54853632*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^6*b^6*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 67977216*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^7*b^5*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 60014592*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^8*b^4*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 2211840*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^9*b^3*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 147456*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^10*b^2*d^2*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 42467328*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^6*b^7*d^3*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 9732096*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^7*b^6*d^3*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))...

3.81 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=63

$$\frac{1}{8}(3a-b)x+\frac{(3a-b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

```
[Out] 1/8*(3*a-b)*x+1/8*(3*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)*cosh(d*x+c)^3
*sinh(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3756, 393, 205, 212}

$$\frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{(3a-b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(3a-b)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a - b)*x)/8 + ((3*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a + b)*
Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{(3a-b)\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,d$$
$$= \frac{(3a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh}{4d}$$
$$= \frac{1}{8}(3a-b)x + \frac{(3a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh}{}$$

Mathematica [A]
time = 0.11, size = 44, normalized size = 0.70

$$\frac{-4bdx+12a(c+dx)+8a\sinh(2(c+dx))+(a+b)\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
[Out] (-4*b*d*x + 12*a*(c + d*x) + 8*a*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(c + d*
x)])/(32*d)
```

Maple [A]
time = 2.38, size = 46, normalized size = 0.73

method	result	size
default	$\frac{\left(\frac{b}{8}+\frac{a}{8}\right)\sinh(4dx+4c)}{4d}+\frac{3ax}{8}-\frac{bx}{8}+\frac{a\sinh(2dx+2c)}{4d}$	46
risch	$-\frac{bx}{8}+\frac{3ax}{8}+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}+\frac{a\,e^{2dx+2c}}{8d}-\frac{a\,e^{-2dx-2c}}{8d}-\frac{e^{-4dx-4c}a}{64d}-\frac{e^{-4dx-4c}b}{64d}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1/8*b+1/8*a)/d*sinh(4*d*x+4*c)+3/8*a*x-1/8*b*x+1/4*a/d*sinh(2*d*x+2*c)
```

Maxima [A]
time = 0.26, size = 104, normalized size = 1.65

$$\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{64}b\left(\frac{8(dx+c)}{d}-\frac{e^{(4dx+4c)}}{d}+\frac{e^{(-4dx-4c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/64*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-
4*d*x - 4*c)/d)
```

Fricas [A]
time = 0.34, size = 63, normalized size = 1.00

$$\frac{(a+b)\cosh(dx+c)\sinh(dx+c)^3+(3a-b)dx+\left((a+b)\cosh(dx+c)^3+4a\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a - b)*d*x + ((a + b)*cosh(
d*x + c)^3 + 4*a*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^2(c+dx)\right)\cosh^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**4, x)
```

Giac [A]
time = 0.44, size = 104, normalized size = 1.65

$$\frac{8(dx+c)(3a-b)+ae^{(4dx+4c)}+be^{(4dx+4c)}+8ae^{(2dx+2c)}-\left(18ae^{(4dx+4c)}-6be^{(4dx+4c)}+8ae^{(2dx+2c)}+a+b\right)e^{(-4dx-4c)}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(8*(d*x + c)*(3*a - b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 8*a*e
^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) - 6*b*e^(4*d*x + 4*c) + 8*a*e^(2*d*x
 + 2*c) + a + b)*e^(-4*d*x - 4*c))/d
```

Mupad [B]
time = 0.21, size = 74, normalized size = 1.17

$$x\left(\frac{3\,a}{8}-\frac{b}{8}\right)-\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,\left(a+b\right)}{64\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,\left(a+b\right)}{64\,d}-\frac{a\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}+\frac{a\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*((3*a)/8 - b/8) - (exp(- 4*c - 4*d*x)*(a + b))/(64*d) + (exp(4*c + 4*d*x)
*(a + b))/(64*d) - (a*exp(- 2*c - 2*d*x))/(8*d) + (a*exp(2*c + 2*d*x))/(8*d
)
```

3.82 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}$$

```
[Out] a*sinh(d*x+c)/d+1/3*(a+b)*sinh(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3757}

$$\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d)
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 1.47

$$\frac{a\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}+\frac{b\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
[Out] (a*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^3)/(3*d)
```

Maple [A]
time = 2.21, size = 40, normalized size = 1.33

method	result	size
default	$\frac{\left(-\frac{b}{4}+\frac{3a}{4}\right)\sinh(dx+c)}{d}+\frac{\left(\frac{b}{4}+\frac{a}{4}\right)\sinh(3dx+3c)}{3d}$	40
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}+\frac{3a\,e^{dx+c}}{8d}-\frac{b\,e^{dx+c}}{8d}-\frac{3\,e^{-dx-c}a}{8d}+\frac{e^{-dx-c}b}{8d}-\frac{e^{-3dx-3c}a}{24d}-\frac{e^{-3dx-3c}b}{24d}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] (-1/4*b+3/4*a)/d*sinh(d*x+c)+1/3*(1/4*b+1/4*a)/d*sinh(3*d*x+3*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(28) = 56$.
time = 0.27, size = 83, normalized size = 2.77

$$\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{24\,d}+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] 1/24*b*(e^(d*x + c) - e^(-d*x - c))^3/d + 1/24*a*(e^(3*d*x + 3*c)/d + 9*e^(
d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Fricas [A]
time = 0.41, size = 45, normalized size = 1.50

$$\frac{(a+b)\sinh(dx+c)^3+3\left((a+b)\cosh(dx+c)^2+3\,a-b\right)\sinh(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
[Out] 1/12*((a + b)*sinh(d*x + c)^3 + 3*((a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(
d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^2(c+dx)\right)\cosh^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(28) = 56$.
time = 0.43, size = 84, normalized size = 2.80

$$\frac{ae^{(3\,dx+3\,c)} + be^{(3\,dx+3\,c)} + 9\,ae^{(dx+c)} - 3\,be^{(dx+c)} - \left(9\,ae^{(2\,dx+2\,c)} - 3\,be^{(2\,dx+2\,c)} + a + b\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*e^(3*d*x + 3*c) + b*e^(3*d*x + 3*c) + 9*a*e^(d*x + c) - 3*b*e^(d*x
+ c) - (9*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + a + b)*e^(-3*d*x - 3*c)
)/d
```

Mupad [B]
time = 0.21, size = 74, normalized size = 2.47

$$\frac{\mathrm{e}^{3\,c+3\,d\,x}\,\left(a+b\right)}{24\,d}-\frac{\mathrm{e}^{-3\,c-3\,d\,x}\,\left(a+b\right)}{24\,d}+\frac{\mathrm{e}^{c+d\,x}\,\left(3\,a-b\right)}{8\,d}-\frac{\mathrm{e}^{-c-d\,x}\,\left(3\,a-b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (exp(3*c + 3*d*x)*(a + b))/(24*d) - (exp(- 3*c - 3*d*x)*(a + b))/(24*d) + (
exp(c + d*x)*(3*a - b))/(8*d) - (exp(- c - d*x)*(3*a - b))/(8*d)
```

3.83 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}(a-b)x+\frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d}$$

[Out] 1/2*(a-b)*x+1/2*(a+b)*cosh(d*x+c)*sinh(d*x+c)/d

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3756, 393, 212}

$$\frac{(a+b)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}x(a-b)$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]

[Out] ((a - b)*x)/2 + ((a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x\right.}{2d}$$
$$= \frac{1}{2}(a-b)x + \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 0.97

$$\frac{2(a-b)(c+dx)+(a+b)\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (2*(a - b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 1.52, size = 54, normalized size = 1.64

method	result	size
derivativedivides	$\frac{a\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	54
default	$\frac{a\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	54
risch	$-\frac{bx}{2}+\frac{ax}{2}+\frac{a\,\mathrm{e}^{2dx+2c}}{8d}+\frac{\mathrm{e}^{2dx+2c}b}{8d}-\frac{a\,\mathrm{e}^{-2dx-2c}}{8d}-\frac{\mathrm{e}^{-2dx-2c}b}{8d}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sinh(d*x+c)*cosh(d*x+c)+1/2*d*x+1/2*c)+b*(1/2*sinh(d*x+c)*cosh(
d*x+c)-1/2*d*x-1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.
time = 0.27, size = 69, normalized size = 2.09

$$\frac{1}{8}\,a\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{8}\,b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*b*(4*x - e^(2*d*
x + 2*c)/d + e^(-2*d*x - 2*c)/d)
```

Fricas [A]
time = 0.40, size = 30, normalized size = 0.91

$$\frac{(a-b)dx + (a+b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*d*x + (a + b)*cosh(d*x + c)*sinh(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)\cosh^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(29) = 58.
time = 0.43, size = 78, normalized size = 2.36

$$\frac{4\left(dx+c\right)(a-b) + ae^{(2\,dx+2\,c)} + be^{(2\,dx+2\,c)} - \left(2\,ae^{(2\,dx+2\,c)} - 2\,be^{(2\,dx+2\,c)} + a + b\right)e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*(d*x + c)*(a - b) + a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - (2*a*e^(
2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x - 2*c))/d
```

Mupad [B]
time = 0.15, size = 27, normalized size = 0.82

$$x\left(\frac{a}{2}-\frac{b}{2}\right)+\frac{\sinh(2\,c+2\,d\,x)\,\left(a+b\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*(a/2 - b/2) + (sinh(2*c + 2*d*x)*(a + b))/(4*d)
```

3.84 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=27

$$-\frac{b\text{ArcTan}(\sinh(c+dx))}{d}+\frac{(a+b)\sinh(c+dx)}{d}$$

```
[Out] -b*arctan(sinh(d*x+c))/d+(a+b)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3757, 396, 209}

$$\frac{(a+b)\sinh(c+dx)}{d}-\frac{b\text{ArcTan}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/d) + ((a + b)*Sinh[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{(a+b)\sinh(c+dx)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= -\frac{b\tan^{-1}(\sinh(c+dx))}{d} + \frac{(a+b)\sinh(c+dx)}{d}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 1.74

$$-\frac{b\text{ArcTan}(\sinh(c+dx))}{d} + \frac{a\cosh(dx)\sinh(c)}{d} + \frac{a\cosh(c)\sinh(dx)}{d} + \frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/d) + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[
d*x])/d + (b*Sinh[c + d*x])/d
```

Maple [A]
time = 1.45, size = 32, normalized size = 1.19

method	result	size
derivativedivides	$\frac{a\sinh(dx+c)+b\left(\sinh(dx+c)-2\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	32
default	$\frac{a\sinh(dx+c)+b\left(\sinh(dx+c)-2\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	32
risch	$\frac{a\,\mathrm{e}^{dx+c}}{2d} + \frac{b\,\mathrm{e}^{dx+c}}{2d} - \frac{\mathrm{e}^{-dx-c}a}{2d} - \frac{\mathrm{e}^{-dx-c}b}{2d} + \frac{ib\ln\left(\mathrm{e}^{dx+c}-i\right)}{d} - \frac{ib\ln\left(\mathrm{e}^{dx+c}+i\right)}{d}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*sinh(d*x+c)+b*(sinh(d*x+c)-2*arctan(exp(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(27) = 54.
time = 0.47, size = 55, normalized size = 2.04

$$\frac{1}{2}b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right) + \frac{a\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a*sinh(
d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(27) = 54$.
time = 0.38, size = 102, normalized size = 3.78

$$\frac{(a+b)\cosh(dx+c)^2 + 2(a+b)\cosh(dx+c)\sinh(dx+c) + (a+b)\sinh(dx+c)^2 - 4(b\cosh(dx+c) + b\sinh(dx+c))\arctan(\cosh(dx+c) + \sinh(dx+c)) - a - b}{2(d\cosh(dx+c) + d\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a +
 b)*sinh(d*x + c)^2 - 4*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(cosh(d*x
 + c) + sinh(d*x + c)) - a - b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2(c + dx)\right)\cosh(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x), x)
```

Giac [A]
time = 0.43, size = 47, normalized size = 1.74

$$-\frac{4\,b\arctan\left(e^{(dx+c)}\right) - ae^{(dx+c)} - be^{(dx+c)} + (a+b)e^{(-dx-c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(4*b*arctan(e^(d*x + c)) - a*e^(d*x + c) - b*e^(d*x + c) + (a + b)*e^(
-d*x - c))/d
```

Mupad [B]
time = 0.14, size = 66, normalized size = 2.44

$$\frac{e^{c+d\,x}\,(a+b)}{2\,d} - \frac{2\,\mathrm{atan}\left(\frac{b\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}} - \frac{e^{-c-d\,x}\,(a+b)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (exp(c + d*x)*(a + b))/(2*d) - (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(
b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) - (exp(- c - d*x)*(a + b))/(2*d)
```

3.85 $\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$\frac{(2a+b)\text{ArcTan}(\sinh(c+dx))}{2d}-\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c))/d-1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3757, 393, 209}

$$\frac{(2a+b)\text{ArcTan}(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((2*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) - (b*Sech[c + d*x]*Tanh[c + d*x])/(
2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\operatorname{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= -\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{(2a+b)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin\right.}{2d}$$
$$= \frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 1.20

$$\frac{a\text{ArcTan}(\sinh(c+dx))}{d} + \frac{b\text{ArcTan}(\sinh(c+dx))}{2d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/d + (b*ArcTan[Sinh[c + d*x]])/(2*d) - (b*Sech[c +
 d*x]*Tanh[c + d*x])/(2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.46, size = 106, normalized size = 2.65

method	result	size
risch	$-\frac{b\,e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\left(1+e^{2dx+2c}\right)^2}+\frac{i\ln\left(e^{dx+c}+i\right)a}{d}+\frac{ib\ln\left(e^{dx+c}+i\right)}{2d}-\frac{i\ln\left(e^{dx+c}-i\right)a}{d}-\frac{ib\ln\left(e^{dx+c}-i\right)}{2d}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -b*exp(d*x+c)*(exp(2*d*x+2*c)-1)/d/(1+exp(2*d*x+2*c))^2+I/d*ln(exp(d*x+c)+I
)*a+1/2*I*b/d*ln(exp(d*x+c)+I)-I/d*ln(exp(d*x+c)-I)*a-1/2*I*b/d*ln(exp(d*x+
c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.
time = 0.48, size = 80, normalized size = 2.00

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{a\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(36) = 72$.
time = 0.39, size = 323, normalized size = 8.08

$$-\frac{b\cosh\left(dx+c\right)^{3}+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+b\sinh\left(dx+c\right)^{3}-\left(\left(2\,a+b\right)\cosh\left(dx+c\right)^{4}+4\left(2\,a+b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\left(2\,a+b\right)\sinh\left(dx+c\right)^{4}+2\left(2\,a+b\right)\cosh\left(dx+c\right)^{2}+2\left(3\left(2\,a+b\right)\cosh\left(dx+c\right)^{2}+2\,a+b\right)\sinh\left(dx+c\right)^{2}+4\left(\left(2\,a+b\right)\cosh\left(dx+c\right)^{3}+\left(2\,a+b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+2\,a+b\right)\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)-b\cosh\left(dx+c\right)+\left(3\,b\cosh\left(dx+c\right)^{2}-b\right)\sinh\left(dx+c\right)}{d\cosh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{4}+2\,d\cosh\left(dx+c\right)^{2}+2\left(3\,d\cosh\left(dx+c\right)^{2}+d\right)\sinh\left(dx+c\right)^{2}+4\left(d\cosh\left(dx+c\right)^{3}+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
 - ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*c
osh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^3 +
(2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c) + si
nh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(
d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 +
 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*c
osh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^{2}\left(c+dx\right)\right)\operatorname{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(36) = 72$.
time = 0.43, size = 84, normalized size = 2.10

$$\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{\left(2\,dx+2\,c\right)}-1\right)e^{\left(-dx-c\right)}\right)\right)\left(2\,a+b\right)-\frac{4\,b\left(e^{\left(dx+c\right)}-e^{\left(-dx-c\right)}\right)}{\left(e^{\left(dx+c\right)}-e^{\left(-dx-c\right)}\right)^{2}+4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(2*a + b) - 4*
b*(e^(d*x + c) - e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B]
time = 0.13, size = 125, normalized size = 3.12

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(2\,a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{4\,a^2+4\,a\,b+b^2}}\right)\,\sqrt{4\,a^2+4\,a\,b+b^2}}{\sqrt{d^2}}-\frac{b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{2\,b\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(2*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(4*a*b + 4*a^2
 + b^2)^(1/2)))*(4*a*b + 4*a^2 + b^2)^(1/2))/(d^2)^(1/2) - (b*exp(c + d*x))
/(d*(exp(2*c + 2*d*x) + 1)) + (2*b*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + e
xp(4*c + 4*d*x) + 1))
```

3.86 $\int \text{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] a*tanh(d*x+c)/d+1/3*b*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3756}

$$\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \text{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.00

$$\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.
time = 1.54, size = 60, normalized size = 2.14

method	result	size
risch	$-\frac{2\left(3 a \,{\mathrm e}^{4 d x +4 c}+3 b \,{\mathrm e}^{4 d x +4 c}+6 a \,{\mathrm e}^{2 d x +2 c}+3 a +b \right)}{3 d \left(1+{\mathrm e}^{2 d x +2 c}\right)^{3}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(3*a*exp(4*d*x+4*c)+3*b*exp(4*d*x+4*c)+6*a*exp(2*d*x+2*c)+3*a+b)/d/(1+
exp(2*d*x+2*c))^3
```

Maxima [**A**]
time = 0.27, size = 34, normalized size = 1.21

$$\frac{b \tanh\left(dx + c\right)^{3}}{3\,d} + \frac{2\,a}{d(e^{(-2\,dx - 2\,c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*b*tanh(d*x + c)^3/d + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(26) = 52$.
time = 0.35, size = 159, normalized size = 5.68

$$-\frac{4\left((3\,a + 2\,b)\cosh\left(dx + c\right)^{2} + 2\,b\cosh\left(dx + c\right)\sinh\left(dx + c\right) + (3\,a + 2\,b)\sinh\left(dx + c\right)^{2} + 3\,a\right)}{3\left(d\cosh\left(dx + c\right)^{4} + 4\,d\cosh\left(dx + c\right)\sinh\left(dx + c\right)^{3} + d\sinh\left(dx + c\right)^{4} + 4\,d\cosh\left(dx + c\right)^{2} + 2\left(3\,d\cosh\left(dx + c\right)^{2} + 2\,d\right)\sinh\left(dx + c\right)^{2} + 4\left(d\cosh\left(dx + c\right)^{3} + d\cosh\left(dx + c\right)\right)\sinh\left(dx + c\right) + 3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4/3*((3*a + 2*b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (3*a
+ 2*b)*sinh(d*x + c)^2 + 3*a)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d
*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^
2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c) + 3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^{2}\left(c + dx\right)\right)\operatorname{sech}^{2}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.
time = 0.44, size = 59, normalized size = 2.11

$$-\frac{2\left(3\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+3\,a+b\right)}{3\,d\left(e^{(2\,dx+2\,c)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 3*a
 + b)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

Mupad [B]
time = 1.22, size = 59, normalized size = 2.11

$$-\frac{2\left(3\,a+b+6\,a\,\mathrm{e}^{2\,c+2\,d\,x}+3\,a\,\mathrm{e}^{4\,c+4\,d\,x}+3\,b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{3\,d\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^2,x)
```

```
[Out] -(2*(3*a + b + 6*a*exp(2*c + 2*d*x) + 3*a*exp(4*c + 4*d*x) + 3*b*exp(4*c +
4*d*x)))/(3*d*(exp(2*c + 2*d*x) + 1)^3)
```

3.87 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=66

$$\frac{(4a+b)\mathrm{ArcTan}(\sinh(c+dx))}{8d}+\frac{(4a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d}-\frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

```
[Out] 1/8*(4*a+b)*arctan(sinh(d*x+c))/d+1/8*(4*a+b)*sech(d*x+c)*tanh(d*x+c)/d-1/4
*b*sech(d*x+c)^3*tanh(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3757, 393, 205, 209}

$$\frac{(4a+b)\mathrm{ArcTan}(\sinh(c+dx))}{8d}+\frac{(4a+b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}-\frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((4*a + b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((4*a + b)*Sech[c + d*x]*Tanh[c +
 d*x])/(8*d) - (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\operatorname{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{(4a+b)\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x,\right.}{4d}$$

$$= \frac{(4a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\operatorname{sech}^3(c+dx)\tanh(c+d}{4d}$$

$$= \frac{(4a+b)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{(4a+b)\operatorname{sech}(c+dx)\tanh(c+}{8d}$$

Mathematica [A]
time = 0.03, size = 93, normalized size = 1.41

$$\frac{a\operatorname{ArcTan}(\sinh(c+dx))}{2d} + \frac{b\operatorname{ArcTan}(\sinh(c+dx))}{8d} + \frac{a\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(8*d) + (a*Sech
[c + d*x]*Tanh[c + d*x])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(8*d) - (b
*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.81, size = 172, normalized size = 2.61

method	result
risch	$\frac{e^{dx+c}\left(4a\,e^{6dx+6c}+b\,e^{6dx+6c}+4a\,e^{4dx+4c}-7b\,e^{4dx+4c}-4a\,e^{2dx+2c}+7b\,e^{2dx+2c}-4a-b\right)}{4d\left(1+e^{2dx+2c}\right)^4} + \frac{i\ln\left(e^{dx+c}+i\right)a}{2d} + \frac{ib\ln\left(e^{dx+c}+i\right)}{8d} - \frac{i\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(d*x+c)*(4*a*exp(6*d*x+6*c)+b*exp(6*d*x+6*c)+4*a*exp(4*d*x+4*c)-7*b*
exp(4*d*x+4*c)-4*a*exp(2*d*x+2*c)+7*b*exp(2*d*x+2*c)-4*a-b)/d/(1+exp(2*d*x+
```

```
2*c))^4+1/2*I/d*ln(exp(d*x+c)+I)*a+1/8*I/d*ln(exp(d*x+c)+I)*b-1/2*I/d*ln(ex
p(d*x+c)-I)*a-1/8*I*b/d*ln(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(60) = 120$.
time = 0.48, size = 181, normalized size = 2.74

$$-\frac{1}{4}b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7e^{(-3dx-3c)}+7e^{(-5dx-5c)}-e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)-a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(
-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*
c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a*(arctan(e^(-d*x - c))
/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x -
 4*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(60) = 120$.
time = 0.39, size = 1046, normalized size = 15.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((4*a + b)*cosh(d*x + c)^7 + 7*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^6
+ (4*a + b)*sinh(d*x + c)^7 + (4*a - 7*b)*cosh(d*x + c)^5 + (21*(4*a + b)*c
osh(d*x + c)^2 + 4*a - 7*b)*sinh(d*x + c)^5 + 5*(7*(4*a + b)*cosh(d*x + c)^
3 + (4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a - 7*b)*cosh(d*x + c)^
3 + (35*(4*a + b)*cosh(d*x + c)^4 + 10*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a +
7*b)*sinh(d*x + c)^3 + (21*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a - 7*b)*cosh(
d*x + c)^3 - 3*(4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a + b)*cosh
(d*x + c)^8 + 8*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a + b)*sinh(d*
x + c)^8 + 4*(4*a + b)*cosh(d*x + c)^6 + 4*(7*(4*a + b)*cosh(d*x + c)^2 + 4
*a + b)*sinh(d*x + c)^6 + 8*(7*(4*a + b)*cosh(d*x + c)^3 + 3*(4*a + b)*cosh
(d*x + c))*sinh(d*x + c)^5 + 6*(4*a + b)*cosh(d*x + c)^4 + 2*(35*(4*a + b)*
cosh(d*x + c)^4 + 30*(4*a + b)*cosh(d*x + c)^2 + 12*a + 3*b)*sinh(d*x + c)^
4 + 8*(7*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a + b)*cosh(d*x + c)^3 + 3*(4*a
+ b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(4*a + b)*cosh(d*x + c)^2 + 4*(7*(4
*a + b)*cosh(d*x + c)^6 + 15*(4*a + b)*cosh(d*x + c)^4 + 9*(4*a + b)*cosh(d
*x + c)^2 + 4*a + b)*sinh(d*x + c)^2 + 8*((4*a + b)*cosh(d*x + c)^7 + 3*(4*
a + b)*cosh(d*x + c)^5 + 3*(4*a + b)*cosh(d*x + c)^3 + (4*a + b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*a + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (4*a
```

```
+ b)*cosh(d*x + c) + (7*(4*a + b)*cosh(d*x + c)^6 + 5*(4*a - 7*b)*cosh(d*x
+ c)^4 - 3*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c))/(d*cosh(d*
x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh
(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*
d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*co
sh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x
+ c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)\operatorname{sech}^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(60) = 120.
time = 0.44, size = 153, normalized size = 2.32

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)(4a+b) + \frac{4\left(4a\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 + b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 + 16a\left(e^{(dx+c)}-e^{(-dx-c)}\right) - 4b\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(4*a + b) + 4
*(4*a*(e^(d*x + c) - e^(-d*x - c))^3 + b*(e^(d*x + c) - e^(-d*x - c))^3 + 1
6*a*(e^(d*x + c) - e^(-d*x - c)) - 4*b*(e^(d*x + c) - e^(-d*x - c)))/((e^(d
*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B]
time = 1.25, size = 280, normalized size = 4.24

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(4a\sqrt{d^2}+b\sqrt{d^2}\right)}{d\sqrt{16a^2+8ab+b^2}}\right)\sqrt{16a^2+8ab+b^2}}{4\sqrt{d^2}} - \frac{\frac{e^{5c+5dx}(a+b)}{d} + \frac{2e^{3c+3dx}(a-b)}{d} + \frac{e^{c+dx}(a+b)}{d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1} - \frac{e^{c+dx}(2a+3b)}{2d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} + \frac{e^{c+dx}(4a+b)}{4d\left(e^{2c+2dx}+1\right)} + \frac{2b\,e^{c+dx}}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(4*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(8*a*b + 16*a^
2 + b^2)^(1/2)))*(8*a*b + 16*a^2 + b^2)^(1/2))/(4*(d^2)^(1/2)) - ((exp(5*c
+ 5*d*x)*(a + b))/d + (2*exp(3*c + 3*d*x)*(a - b))/d + (exp(c + d*x)*(a + b
))/d)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1) - (exp(c + d*x)*(2*a + 3*b))/(2*d*(2*exp(2*c + 2*d*x) + ex
p(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(4*a + b))/(4*d*(exp(2*c + 2*d*x) + 1)
) + (2*b*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) + 1))
```

3.88 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=48

$$\frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] a*tanh(d*x+c)/d-1/3*(a-b)*tanh(d*x+c)^3/d-1/5*b*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3756, 380}

$$-\frac{(a-b)\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}-\frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]^5)
/(5*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+bx^2\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a-(a-b)x^2-bx^4\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 86, normalized size = 1.79

$$\frac{a\tanh(c+dx)}{d}+\frac{2b\tanh(c+dx)}{15d}+\frac{b\,\text{sech}^2(c+dx)\tanh(c+dx)}{15d}-\frac{b\,\text{sech}^4(c+dx)\tanh(c+dx)}{5d}-\frac{a\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (2*b*Tanh[c + d*x])/(15*d) + (b*Sech[c + d*x]^2*Tanh[
c + d*x])/(15*d) - (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - (a*Tanh[c + d*
x]^3)/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.
time = 1.68, size = 96, normalized size = 2.00

method	result	size
risch	$-\frac{4\left(15a\,e^{6dx+6c}+15b\,e^{6dx+6c}+35a\,e^{4dx+4c}-5b\,e^{4dx+4c}+25a\,e^{2dx+2c}+5b\,e^{2dx+2c}+5a+b\right)}{15d\left(1+e^{2dx+2c}\right)^5}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/15*(15*a*exp(6*d*x+6*c)+15*b*exp(6*d*x+6*c)+35*a*exp(4*d*x+4*c)-5*b*exp(
4*d*x+4*c)+25*a*exp(2*d*x+2*c)+5*b*exp(2*d*x+2*c)+5*a+b)/d/(1+exp(2*d*x+2*c
))^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(44) = 88$.
time = 0.27, size = 371, normalized size = 7.73

$$\frac{4}{15}b\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}-\frac{5e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{15e^{(-6dx-6c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{1}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+\frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-
4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
```

```
- 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(44) = 88$.
time = 0.41, size = 345, normalized size = 7.19

$$-\frac{8\left(2\left(5a+4b\right)\cosh\left(dx+c\right)^{3}+6\left(5a+4b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+\left(5a+7b\right)\sinh\left(dx+c\right)^{3}+30a\cosh\left(dx+c\right)+\left(3\left(5a+7b\right)\cosh\left(dx+c\right)^{2}+5a-5b\right)\sinh\left(dx+c\right)\right)}{15\left(d\cosh\left(dx+c\right)^{7}+7d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{7}+5d\cosh\left(dx+c\right)^{5}+\left(21d\cosh\left(dx+c\right)^{2}+5d\right)\sinh\left(dx+c\right)^{5}+5\left(7d\cosh\left(dx+c\right)^{3}+5d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+11d\cosh\left(dx+c\right)^{3}+\left(35d\cosh\left(dx+c\right)^{4}+50d\cosh\left(dx+c\right)^{2}+9d\right)\sinh\left(dx+c\right)^{3}+\left(21d\cosh\left(dx+c\right)^{5}+50d\cosh\left(dx+c\right)^{3}+33d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+15d\cosh\left(dx+c\right)+\left(7d\cosh\left(dx+c\right)^{6}+25d\cosh\left(dx+c\right)^{4}+27d\cosh\left(dx+c\right)^{2}+5d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/15*(2*(5*a + 4*b)*cosh(d*x + c)^3 + 6*(5*a + 4*b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + (5*a + 7*b)*sinh(d*x + c)^3 + 30*a*cosh(d*x + c) + (3*(5*a + 7*b)
*cosh(d*x + c)^2 + 5*a - 5*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(
d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (21*d*
cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*cosh(
d*x + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 +
50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 50*d*
cosh(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d*x + c)
+ (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5*d)
*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^{2}\left(c + dx\right)\right)\operatorname{sech}^{4}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.
time = 0.44, size = 95, normalized size = 1.98

$$-\frac{4\left(15\,ae^{\left(6\,dx+6\,c\right)}+15\,be^{\left(6\,dx+6\,c\right)}+35\,ae^{\left(4\,dx+4\,c\right)}-5\,be^{\left(4\,dx+4\,c\right)}+25\,ae^{\left(2\,dx+2\,c\right)}+5\,be^{\left(2\,dx+2\,c\right)}+5\,a+b\right)}{15\,d\left(e^{\left(2\,dx+2\,c\right)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -4/15*(15*a*e^(6*d*x + 6*c) + 15*b*e^(6*d*x + 6*c) + 35*a*e^(4*d*x + 4*c) -
 5*b*e^(4*d*x + 4*c) + 25*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 5*a + b
)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

Mupad [B]
time = 0.17, size = 304, normalized size = 6.33

$$-\frac{\frac{8\,\left(a-b\right)}{15\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a+b\right)}{5\,d}+\frac{16\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a-b\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{\frac{2\,\left(a+b\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a+b\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a-b\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{2\,\left(a+b\right)}{5\,d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^4,x)
```

```
[Out] - ((8*(a - b))/(15*d) + (4*exp(2*c + 2*d*x)*(a + b))/(5*d))/(3*exp(2*c + 2*
d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*exp(2*c + 2*d*x)*(a
 + b))/(5*d) + (8*exp(6*c + 6*d*x)*(a + b))/(5*d) + (16*exp(4*c + 4*d*x)*(a
 - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(a + b))/(5*d) + (6
*exp(4*c + 4*d*x)*(a + b))/(5*d) + (8*exp(2*c + 2*d*x)*(a - b))/(5*d))/(4*e
xp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x
) + 1) - (2*(a + b))/(5*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.89 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=85

$$\frac{1}{8}\left(3a^2-2ab+3b^2\right)x+\frac{3(a^2-b^2)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)\left(a+b\tanh^2\right.}{4d}$$

```
[Out] 1/8*(3*a^2-2*a*b+3*b^2)*x+3/8*(a^2-b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)
*cosh(d*x+c)^3*sinh(d*x+c)*(a+b*tanh(d*x+c)^2)/d
```

Rubi [A]

time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3756, 424, 393, 212}

$$\frac{3(a^2-b^2)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x\left(3a^2-2ab+3b^2\right)+\frac{(a+b)\sinh(c+dx)\cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*x)/8 + (3*(a^2 - b^2)*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + ((a + b)*Cosh[c + d*x]^3*Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2))/(4
*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d} - \frac{\text{S}}{}$$

$$= \frac{3(a^2-b^2)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)}{}$$

$$= \frac{1}{8}\left(3a^2-2ab+3b^2\right)x + \frac{3(a^2-b^2)\cosh(c+dx)\sinh(c+dx)}{8d}$$

Mathematica [A]
time = 0.22, size = 63, normalized size = 0.74

$$\frac{4(3a^2-2ab+3b^2)(c+dx)+8(a^2-b^2)\sinh(2(c+dx))+(a+b)^2\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (4*(3*a^2 - 2*a*b + 3*b^2)*(c + d*x) + 8*(a^2 - b^2)*Sinh[2*(c + d*x)] + (a
 + b)^2*Sinh[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 1.76, size = 73, normalized size = 0.86

method	result
default	$\frac{\left(\frac{a^2}{2}-\frac{b^2}{2}\right)\sinh(2dx+2c)}{2d}+\frac{\left(\frac{1}{8}a^2+\frac{1}{4}ab+\frac{1}{8}b^2\right)\sinh(4dx+4c)}{4d}+\frac{3a^2x}{8}+\frac{3b^2x}{8}-\frac{abx}{4}$
risch	$\frac{3a^2x}{8}-\frac{abx}{4}+\frac{3b^2x}{8}+\frac{e^{4dx+4c}a^2}{64d}+\frac{e^{4dx+4c}ab}{32d}+\frac{e^{4dx+4c}b^2}{64d}+\frac{e^{2dx+2c}a^2}{8d}-\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}+\frac{e^{-2dx-2c}b^2}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1/2*a^2-1/2*b^2)/d*sinh(2*d*x+2*c)+1/4*(1/8*a^2+1/4*a*b+1/8*b^2)/d*sin
h(4*d*x+4*c)+3/8*a^2*x+3/8*b^2*x-1/4*a*b*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(79) = 158$.
time = 0.27, size = 171, normalized size = 2.01

$$\frac{1}{64}\,a^{2}\left(24\,x+\frac{e^{\left(4\,dx+4\,c\right)}}{d}+\frac{8\,e^{\left(2\,dx+2\,c\right)}}{d}-\frac{8\,e^{\left(-2\,dx-2\,c\right)}}{d}-\frac{e^{\left(-4\,dx-4\,c\right)}}{d}\right)+\frac{1}{64}\,b^{2}\left(24\,x+\frac{e^{\left(4\,dx+4\,c\right)}}{d}-\frac{8\,e^{\left(2\,dx+2\,c\right)}}{d}+\frac{8\,e^{\left(-2\,dx-2\,c\right)}}{d}-\frac{e^{\left(-4\,dx-4\,c\right)}}{d}\right)-\frac{1}{32}\,ab\left(\frac{8\,(dx+c)}{d}-\frac{e^{\left(4\,dx+4\,c\right)}}{d}+\frac{e^{\left(-4\,dx-4\,c\right)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*
x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/32*a*b*(8*(d*x
+ c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Fricas [A]
time = 0.35, size = 95, normalized size = 1.12

$$\frac{(a^{2}+2\,ab+b^{2})\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+(3\,a^{2}-2\,ab+3\,b^{2})dx+\left((a^{2}+2\,ab+b^{2})\cosh\left(dx+c\right)^{3}+4\,(a^{2}-b^{2})\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 - 2*a*b + 3
*b^2)*d*x + ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x +
 c))*sinh(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^{2}\left(c+dx\right)\right)^{2}\cosh^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(79) = 158$.
time = 0.49, size = 186, normalized size = 2.19

$$\frac{a^{2}e^{\left(4\,dx+4\,c\right)}+2\,abe^{\left(4\,dx+4\,c\right)}+b^{2}e^{\left(4\,dx+4\,c\right)}+8\,a^{2}e^{\left(2\,dx+2\,c\right)}-8\,b^{2}e^{\left(2\,dx+2\,c\right)}+8\,(3\,a^{2}-2\,ab+3\,b^{2})(dx+c)-\left(18\,a^{2}e^{\left(4\,dx+4\,c\right)}-12\,abe^{\left(4\,dx+4\,c\right)}+18\,b^{2}e^{\left(4\,dx+4\,c\right)}+8\,a^{2}e^{\left(2\,dx+2\,c\right)}-8\,b^{2}e^{\left(2\,dx+2\,c\right)}+a^{2}+2\,ab+b^{2}\right)e^{\left(-4\,dx-4\,c\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(a^2*e^(4*d*x + 4*c) + 2*a*b*e^(4*d*x + 4*c) + b^2*e^(4*d*x + 4*c) + 8
*a^2*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + 8*(3*a^2 - 2*a*b + 3*b^2)*(d
*x + c) - (18*a^2*e^(4*d*x + 4*c) - 12*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*
x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^
2)*e^(-4*d*x - 4*c))/d
```

Mupad [B]
time = 0.27, size = 102, normalized size = 1.20

$$x\left(\frac{3\,a^2}{8}-\frac{a\,b}{4}+\frac{3\,b^2}{8}\right)-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,\left(a^2-b^2\right)}{8\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{8\,d}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,{\left(a+b\right)}^2}{64\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^2}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*((3*a^2)/8 - (a*b)/4 + (3*b^2)/8) - (exp(- 2*c - 2*d*x)*(a^2 - b^2))/(8*d
) + (exp(2*c + 2*d*x)*(a^2 - b^2))/(8*d) - (exp(- 4*c - 4*d*x)*(a + b)^2)/(
64*d) + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d)
```

3.90 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=54

$$\frac{b^2\text{ArcTan}(\sinh(c+dx))}{d}+\frac{(a^2-b^2)\sinh(c+dx)}{d}+\frac{(a+b)^2\sinh^3(c+dx)}{3d}$$

```
[Out] b^2*arctan(sinh(d*x+c))/d+(a^2-b^2)*sinh(d*x+c)/d+1/3*(a+b)^2*sinh(d*x+c)^3
/d
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3757, 398, 209}

$$\frac{(a^2-b^2)\sinh(c+dx)}{d}+\frac{(a+b)^2\sinh^3(c+dx)}{3d}+\frac{b^2\text{ArcTan}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + ((a^2 - b^2)*Sinh[c + d*x])/d + ((a + b)^2*
Sinh[c + d*x]^3)/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2-b^2+(a+b)^2x^2+\frac{b^2}{1+x^2}\right)\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a^2-b^2)\sinh(c+dx)}{d} + \frac{(a+b)^2\sinh^3(c+dx)}{3d} + \frac{b^2\text{Subst}\left(\int\right.}{}$$

$$= \frac{b^2\tan^{-1}(\sinh(c+dx))}{d} + \frac{(a^2-b^2)\sinh(c+dx)}{d} + \frac{(a+b)^2\,\text{si}}{}$$

Mathematica [A]
time = 0.31, size = 71, normalized size = 1.31

$$\frac{\sinh(c+dx)\left((a+b)(5a-7b+(a+b)\cosh(2(c+dx)))+\frac{6b^2\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}}\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (Sinh[c + d*x]*((a + b)*(5*a - 7*b + (a + b)*Cosh[2*(c + d*x)]) + (6*b^2*Ar
cTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c + d*x]^2]))/(6*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.85, size = 231, normalized size = 4.28

method	result
risch	$\frac{e^{3dx+3c}a^2}{24d}+\frac{e^{3dx+3c}ab}{12d}+\frac{e^{3dx+3c}b^2}{24d}+\frac{3\,e^{dx+c}a^2}{8d}-\frac{ab\,e^{dx+c}}{4d}-\frac{5\,e^{dx+c}b^2}{8d}-\frac{3\,e^{-dx-c}a^2}{8d}+\frac{e^{-dx-c}ab}{4d}+\frac{5\,e^{-dx-c}b^2}{8d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*a^2+1/12/d*exp(3*d*x+3*c)*a*b+1/24/d*exp(3*d*x+3*c)*b
^2+3/8/d*exp(d*x+c)*a^2-1/4*a*b/d*exp(d*x+c)-5/8/d*exp(d*x+c)*b^2-3/8/d*exp
(-d*x-c)*a^2+1/4/d*exp(-d*x-c)*a*b+5/8/d*exp(-d*x-c)*b^2-1/24/d*exp(-3*d*x-
3*c)*a^2-1/12/d*exp(-3*d*x-3*c)*a*b-1/24/d*exp(-3*d*x-3*c)*b^2+I*b^2/d*ln(e
xp(d*x+c)+I)-I*b^2/d*ln(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(52) = 104$.
time = 0.48, size = 161, normalized size = 2.98

$$\frac{ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3}{12\,d} - \frac{1}{24}\,b^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)} - 1\right)e^{(3\,dx+3\,c)}}{d} - \frac{15\,e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d} + \frac{48\,\arctan\left(e^{(-dx-c)}\right)}{d}\right) + \frac{1}{24}\,a^2\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/24*b^2*((15*e^(-2*d*x - 2*c)
- 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arctan
(e^(-d*x - c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d
*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(52) = 104$.
time = 0.38, size = 519, normalized size = 9.61

$$\frac{(a^2+2ab+b^2)\cosh(dx+c)^6 + 6(a^2+2ab+b^2)\cosh(dx+c)\sinh(dx+c)^5 + (a^2+2ab+b^2)\sinh(dx+c)^6 + 3(3a^2-2ab-5b^2)\cosh(dx+c)^4 + 3\left(5(a^2+2ab+b^2)\cosh(dx+c)^2 + 3a^2-2ab-5b^2\right)\sinh(dx+c)^4 + 4\left(5(a^2+2ab+b^2)\cosh(dx+c)^3 + 3(3a^2-2ab-5b^2)\cosh(dx+c)\right)\sinh(dx+c)^3 - 3(3a^2-2ab-5b^2)\cosh(dx+c)^2 + 3\left(5(a^2+2ab+b^2)\cosh(dx+c)^4 + 6(3a^2-2ab-5b^2)\cosh(dx+c)^2 - 3a^2+2ab+5b^2\right)\sinh(dx+c)^2 - a^2-2ab-b^2 + 48\left(b^2\cosh(dx+c)^3 + 3b^2\cosh(dx+c)^2\sinh(dx+c) + 3b^2\cosh(dx+c)\sinh(dx+c)^2 + b^2\sinh(dx+c)^3\right)\arctan\left(\cosh(dx+c)+\sinh(dx+c)\right) + 6\left((a^2+2ab+b^2)\cosh(dx+c)^5 + 2(3a^2-2ab-5b^2)\cosh(dx+c)^3 - (3a^2-2ab-5b^2)\cosh(dx+c)\right)\sinh(dx+c)}{24\left(d\cosh(dx+c)^3 + 3d\cosh(dx+c)^2\sinh(dx+c) + 3d\cosh(dx+c)\sinh(dx+c)^2 + d\sinh(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(3*a^2 - 2*a
*b - 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*
a^2 - 2*a*b - 5*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^3 + 3*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a^2
- 2*a*b - 5*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4
 + 6*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^2 - 3*a^2 + 2*a*b + 5*b^2)*sinh(
d*x + c)^2 - a^2 - 2*a*b - b^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x +
 c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x +
c)^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*((a^2 + 2*a*b + b^2)*cosh(d
*x + c)^5 + 2*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^3 - (3*a^2 - 2*a*b - 5*
b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2
*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^2 \cosh^3\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**3, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(52) = 104.
time = 0.48, size = 152, normalized size = 2.81

$$\frac{48\,b^2\arctan\left(e^{(dx+c)}\right)+a^2e^{(3\,dx+3\,c)}+2\,abe^{(3\,dx+3\,c)}+b^2e^{(3\,dx+3\,c)}+9\,a^2e^{(dx+c)}-6\,abe^{(dx+c)}-15\,b^2e^{(dx+c)}-\left(9\,a^2e^{(2\,dx+2\,c)}-6\,abe^{(2\,dx+2\,c)}-15\,b^2e^{(2\,dx+2\,c)}+a^2+2\,ab+b^2\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/24*(48*b^2*arctan(e^(d*x + c)) + a^2*e^(3*d*x + 3*c) + 2*a*b*e^(3*d*x + 3*c) + b^2*e^(3*d*x + 3*c) + 9*a^2*e^(d*x + c) - 6*a*b*e^(d*x + c) - 15*b^2*e^(d*x + c) - (9*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) - 15*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-3*d*x - 3*c))/d

Mupad [B]
time = 0.26, size = 130, normalized size = 2.41

$$\frac{e^{3\,c+3\,d\,x}\,{\left(a+b\right)}^2}{24\,d}-\frac{e^{-3\,c-3\,d\,x}\,{\left(a+b\right)}^2}{24\,d}-\frac{e^{c+d\,x}\,\left(-3\,a^2+2\,a\,b+5\,b^2\right)}{8\,d}+\frac{2\,\mathrm{atan}\left(\frac{b^2\,e^{d\,x}\,e^c\,\sqrt{d^2}}{d\,\sqrt{b^4}}\right)\,\sqrt{b^4}}{\sqrt{d^2}}+\frac{e^{-c-d\,x}\,\left(-3\,a^2+2\,a\,b+5\,b^2\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)

[Out] (exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(- 3*c - 3*d*x)*(a + b)^2)/(24*d) - (exp(c + d*x)*(2*a*b - 3*a^2 + 5*b^2))/(8*d) + (2*atan((b^2*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^4)^(1/2)))*(b^4)^(1/2))/(d^2)^(1/2) + (exp(- c - d*x)*(2*a*b - 3*a^2 + 5*b^2))/(8*d)

3.91 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}(a-3b)(a+b)x+\frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] 1/2*(a-3*b)*(a+b)*x+1/2*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/d+b^2*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3756, 398, 393, 212}

$$\frac{(a+b)^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}x(a-3b)(a+b)+\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a - 3*b)*(a + b)*x)/2 + ((a + b)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (
b^2*Tanh[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2-b^2+2b(a+b)x^2}{(1-x^2)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{b^2\tanh(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx\right.}{d}$$
$$= \frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2\tanh(c+dx)}{d} + \frac{((a-}{}$$
$$= \frac{1}{2}(a-3b)(a+b)x + \frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2}{}$$

Mathematica [A]
time = 0.28, size = 54, normalized size = 1.06

$$\frac{(a-3b)(a+b)(c+dx)}{2d} + \frac{(a+b)^2\sinh(2(c+dx))}{4d} + \frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] ((a - 3*b)*(a + b)*(c + d*x))/(2*d) + ((a + b)^2*Sinh[2*(c + d*x)])/(4*d) +
 (b^2*Tanh[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(47) = 94$.
time = 1.45, size = 96, normalized size = 1.88

method	result
derivativedivides	$\frac{a^2\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$
default	$\frac{a^2\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$

risch	$\frac{a^2x}{2} - abx - \frac{3b^2x}{2} + \frac{e^{2dx+2c}a^2}{8d} + \frac{e^{2dx+2c}ab}{4d} + \frac{e^{2dx+2c}b^2}{8d} - \frac{e^{-2dx-2c}a^2}{8d} - \frac{e^{-2dx-2c}ab}{4d} - \frac{e^{-2dx-2c}b^2}{8d} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sinh(d*x+c)*cosh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(1/2*sinh(d*x+c)
*cosh(d*x+c)-1/2*d*x-1/2*c)+b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*
c+3/2*tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(47) = 94$.
time = 0.28, size = 140, normalized size = 2.75

$$\frac{1}{8}a^2\left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{4}ab\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{8}b^2\left(\frac{12(dx+c)}{d} + \frac{e^{(-2dx-2c)}}{d} - \frac{17e^{(-2dx-2c)} + 1}{d(e^{(-2dx-2c)} + e^{(-4dx-4c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/4*a*b*(4*x - e^(
2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b^2*(12*(d*x + c)/d + e^(-2*d*x
- 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c
))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.
time = 0.36, size = 105, normalized size = 2.06

$$\frac{(a^2 + 2ab + b^2)\sinh(dx+c)^3 + 4((a^2 - 2ab - 3b^2)dx - 2b^2)\cosh(dx+c) + (3(a^2 + 2ab + b^2)\cosh(dx+c)^2 + a^2 + 2ab + 9b^2)\sinh(dx+c)}{8d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^2 + 2*a*b + b^2)*sinh(d*x + c)^3 + 4*((a^2 - 2*a*b - 3*b^2)*d*x - 2
*b^2)*cosh(d*x + c) + (3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b
+ 9*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2(c+dx)\right)^2 \cosh^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(47) = 94.
time = 0.47, size = 167, normalized size = 3.27

$$\frac{a^2e^{(2\,dx+2\,c)} + 2\,abe^{(2\,dx+2\,c)} + b^2e^{(2\,dx+2\,c)} + 4\,(a^2 - 2\,ab - 3\,b^2)(dx + c) - \frac{a^2e^{(4\,dx+4\,c)} - 2\,abe^{(4\,dx+4\,c)} - 3\,b^2e^{(4\,dx+4\,c)} + 2\,a^2e^{(2\,dx+2\,c)} + 14\,b^2e^{(2\,dx+2\,c)} + a^2 + 2\,ab + b^2}{e^{(4\,dx+4\,c)} + e^{(2\,dx+2\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*e^(2*d*x + 2*c) + 2*a*b*e^(2*d*x + 2*c) + b^2*e^(2*d*x + 2*c) + 4*
(a^2 - 2*a*b - 3*b^2)*(d*x + c) - (a^2*e^(4*d*x + 4*c) - 2*a*b*e^(4*d*x + 4
*c) - 3*b^2*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x + 2*c
) + a^2 + 2*a*b + b^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

Mupad [B]
time = 1.29, size = 77, normalized size = 1.51

$$\frac{e^{2\,c+2\,d\,x}\,(a+b)^2}{8\,d} - \frac{2\,b^2}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{e^{-2\,c-2\,d\,x}\,(a+b)^2}{8\,d} - x\,\left(-\frac{a^2}{2}+a\,b+\frac{3\,b^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) - (
exp(- 2*c - 2*d*x)*(a + b)^2)/(8*d) - x*(a*b - a^2/2 + (3*b^2)/2)
```

3.92 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=60

$$-\frac{b(4a+3b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{(a+b)^2\sinh(c+dx)}{d}+\frac{b^2\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] -1/2*b*(4*a+3*b)*arctan(sinh(d*x+c))/d+(a+b)^2*sinh(d*x+c)/d+1/2*b^2*sech(d
*x+c)*tanh(d*x+c)/d
```

Rubi [A]

time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3757, 398, 393, 209}

$$-\frac{b(4a+3b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{(a+b)^2\sinh(c+dx)}{d}+\frac{b^2\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(b*(4*a + 3*b)*ArcTan[Sinh[c + d*x]])/d + ((a + b)^2*Sinh[c + d*x])/d
+ (b^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((a+b)^2-\frac{b(2a+b)+2b(a+b)x^2}{(1+x^2)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{(a+b)^2\sinh(c+dx)}{d} - \frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{(1+x^2)^2}\,dx, x, \sinh(c$$
$$= \frac{(a+b)^2\sinh(c+dx)}{d} + \frac{b^2\text{sech}(c+dx)\tanh(c+dx)}{2d} - \frac{(b(4a$$
$$= -\frac{b(4a+3b)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{(a+b)^2\sinh(c+dx)}{d} + \frac{b^2}{}$$

Mathematica [A]
time = 0.17, size = 58, normalized size = 0.97

$$\frac{2(a+b)^2\sinh(c+dx)+b\left(-2(4a+3b)\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+b\,\text{sech}(c+dx)\tanh(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*(a + b)^2*Sinh[c + d*x] + b*(-2*(4*a + 3*b)*ArcTan[Tanh[(c + d*x)/2]] +
b*Sech[c + d*x]*Tanh[c + d*x]))/(2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.83, size = 203, normalized size = 3.38

method	result
risch	$\frac{e^{dx+c}a^2}{2d}+\frac{ab\,e^{dx+c}}{d}+\frac{e^{dx+c}b^2}{2d}-\frac{e^{-dx-c}a^2}{2d}-\frac{e^{-dx-c}ab}{d}-\frac{e^{-dx-c}b^2}{2d}+\frac{b^2e^{dx+c}(e^{2dx+2c}-1)}{d(1+e^{2dx+2c})^2}+\frac{2ib\ln(e^{dx+c}-i)a}{d}+3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*exp(d*x+c)*a^2+a*b/d*exp(d*x+c)+1/2/d*exp(d*x+c)*b^2-1/2/d*exp(-d*x-c
)*a^2-1/d*exp(-d*x-c)*a*b-1/2/d*exp(-d*x-c)*b^2+b^2*exp(d*x+c)*(exp(2*d*x+2
*c)-1)/d/(1+exp(2*d*x+2*c))^2+2*I*b/d*ln(exp(d*x+c)-I)*a+3/2*I*b^2/d*ln(exp
(d*x+c)-I)-2*I*b*a/d*ln(exp(d*x+c)+I)-3/2*I*b^2/d*ln(exp(d*x+c)+I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(56) = 112$.
time = 0.47, size = 152, normalized size = 2.53

$$\frac{1}{2}b^2\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4e^{(-2dx-2c)}-e^{(-4dx-4c)}+1}{d\left(e^{(-dx-c)}+2e^{(-3dx-3c)}+e^{(-5dx-5c)}\right)}\right)+ab\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a^2\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5
*c)))) + a*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) +
a^2*sinh(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(56) = 112$.
time = 0.42, size = 774, normalized size = 12.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + (a^2 + 2*a*b +
3*b^2)*cosh(d*x + c)^4 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*
a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (
a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 + 2*a*b + 3*b^2)
*cosh(d*x + c)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b
 + 3*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^2 - a^2 - 2*
a*b - b^2 - 2*((4*a*b + 3*b^2)*cosh(d*x + c)^5 + 5*(4*a*b + 3*b^2)*cosh(d*x
 + c)*sinh(d*x + c)^4 + (4*a*b + 3*b^2)*sinh(d*x + c)^5 + 2*(4*a*b + 3*b^2)
*cosh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2)*si
nh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*co
sh(d*x + c))*sinh(d*x + c)^2 + (4*a*b + 3*b^2)*cosh(d*x + c) + (5*(4*a*b +
3*b^2)*cosh(d*x + c)^4 + 6*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2)
*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(3*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 - (a^2 + 2*
a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*
x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + 2*(5*d*c
```

```
osh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x
 + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^2 \cosh\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(56) = 112.
time = 0.48, size = 160, normalized size = 2.67

$$\frac{2\,a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 4\,ab\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2\,b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) - \left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(4\,ab + 3\,b^2\right) + \frac{4\,b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)}{\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^2*(e^(d*x + c) - e^(-d*x - c)) + 4*a*b*(e^(d*x + c) - e^(-d*x - c)
) + 2*b^2*(e^(d*x + c) - e^(-d*x - c)) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c
) - 1)*e^(-d*x - c)))*(4*a*b + 3*b^2) + 4*b^2*(e^(d*x + c) - e^(-d*x - c))/
((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B]
time = 0.24, size = 182, normalized size = 3.03

$$\frac{e^{c+d\,x}\,{\left(a+b\right)}^2}{2\,d} - \frac{e^{-c-d\,x}\,{\left(a+b\right)}^2}{2\,d} - \frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(3\,b^2\,\sqrt{d^2}+4\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{16\,a^2\,b^2+24\,a\,b^3+9\,b^4}}\right)\,\sqrt{16\,a^2\,b^2+24\,a\,b^3+9\,b^4}}{\sqrt{d^2}} + \frac{b^2\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{2\,b^2\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (exp(c + d*x)*(a + b)^2)/(2*d) - (exp(- c - d*x)*(a + b)^2)/(2*d) - (atan((
exp(d*x)*exp(c)*(3*b^2*(d^2)^(1/2) + 4*a*b*(d^2)^(1/2)))/(d*(24*a*b^3 + 9*b
^4 + 16*a^2*b^2)^(1/2)))*(24*a*b^3 + 9*b^4 + 16*a^2*b^2)^(1/2))/(d^2)^(1/2)
 + (b^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^2*exp(c + d*x))/(d*
(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.93 $\int \text{sech}(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal. Leaf size=91

$$\frac{(8a^2+8ab+3b^2)\,\text{ArcTan}(\sinh(c+dx))}{8d}-\frac{3b(2a+b)\text{sech}(c+dx)\tanh(c+dx)}{8d}-\frac{b\text{sech}^3(c+dx)\left(a+(a+b)\right.}{4}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c))/d-3/8*b*(2*a+b)*sech(d*x+c)*tan
h(d*x+c)/d-1/4*b*sech(d*x+c)^3*(a+(a+b)*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3757, 424, 393, 209}

$$\frac{(8a^2+8ab+3b^2)\,\text{ArcTan}(\sinh(c+dx))}{8d}-\frac{3b(2a+b)\tanh(c+dx)\text{sech}(c+dx)}{8d}-\frac{b\tanh(c+dx)\text{sech}^3(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) - (3*b*(2*a + b)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) - (b*Sech[c + d*x]^3*(a + (a + b)*Sinh[c + d
*x]^2)*Tanh[c + d*x])/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{4d} + $$

$$= -\frac{3b(2a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\operatorname{sech}^3(c+dx)\,(a+}{}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d} - \frac{3b(2a+b)\operatorname{sech}(c+d}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.97, size = 427, normalized size = 4.69

$$-\frac{\operatorname{csch}^3(c+dx)\left(128\,{}_5F_4\left(\tfrac{3}{2},2,2,2,2;1,1,1,\tfrac{9}{2};-\sinh^2(c+dx)\right)\sinh^6(c+dx)\left(a+a\sinh^2(c+dx)+b\sinh^2(c+dx)\right)^2+128\,{}_4F_3\left(\tfrac{3}{2},2,2,2;1,1,\tfrac{9}{2};-\sinh^2(c+dx)\right)\sinh^6(c+dx)\left(5b^2\sinh^4(c+dx)+2ab\sinh^2(c+dx)\left(6+5\sinh^2(c+dx)\right)+a^2\left(7+12\sinh^2(c+dx)+5\sinh^4(c+dx)\right)\right)+35\left(b^2\sinh^4(c+dx)\left(1947+485\sinh^2(c+dx)\right)+2ab\sinh^2(c+dx)\left(2625+2554\sinh^2(c+dx)+485\sinh^4(c+dx)\right)+a^2\left(3375+5907\sinh^2(c+dx)+3161\sinh^4(c+dx)+485\sinh^6(c+dx)\right)\right)-\frac{105\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)\left(b^2\sinh^4(c+dx)\left(649+378\sinh^2(c+dx)+9\sinh^4(c+dx)\right)+2ab\sinh^2(c+dx)\left(875+1143\sinh^2(c+dx)+389\sinh^4(c+dx)+9\sinh^6(c+dx)\right)+a^2\left(1125+2344\sinh^2(c+dx)+1674\sinh^4(c+dx)+400\sinh^6(c+dx)+9\sinh^8(c+dx)\right)\right)}{\sqrt{-\sinh^2(c+dx)}}\right)}{6720d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/6720*(Csch[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1
, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + a*Sinh[c + d*x]^2 + b*Sinh[c
 + d*x]^2)^2 + 128*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c +
 d*x]^2]*Sinh[c + d*x]^6*(5*b^2*Sinh[c + d*x]^4 + 2*a*b*Sinh[c + d*x]^2*(6
+ 5*Sinh[c + d*x]^2) + a^2*(7 + 12*Sinh[c + d*x]^2 + 5*Sinh[c + d*x]^4)) +
35*(b^2*Sinh[c + d*x]^4*(1947 + 485*Sinh[c + d*x]^2) + 2*a*b*Sinh[c + d*x]^
2*(2625 + 2554*Sinh[c + d*x]^2 + 485*Sinh[c + d*x]^4) + a^2*(3375 + 5907*Si
nh[c + d*x]^2 + 3161*Sinh[c + d*x]^4 + 485*Sinh[c + d*x]^6)) - (105*ArcTanh
[Sqrt[-Sinh[c + d*x]^2]]*(b^2*Sinh[c + d*x]^4*(649 + 378*Sinh[c + d*x]^2 +
9*Sinh[c + d*x]^4) + 2*a*b*Sinh[c + d*x]^2*(875 + 1143*Sinh[c + d*x]^2 + 38
9*Sinh[c + d*x]^4 + 9*Sinh[c + d*x]^6) + a^2*(1125 + 2344*Sinh[c + d*x]^2 +
 1674*Sinh[c + d*x]^4 + 400*Sinh[c + d*x]^6 + 9*Sinh[c + d*x]^8)))/Sqrt[-Si
nh[c + d*x]^2]))/d
```

Maple [C] Result contains complex when optimal does not.
time = 2.04, size = 218, normalized size = 2.40

method	result
risch	$-\frac{b\,\mathrm{e}^{dx+c}\left(8a\,\mathrm{e}^{6dx+6c}+5b\,\mathrm{e}^{6dx+6c}+8a\,\mathrm{e}^{4dx+4c}-3b\,\mathrm{e}^{4dx+4c}-8a\,\mathrm{e}^{2dx+2c}+3b\,\mathrm{e}^{2dx+2c}-8a-5b\right)}{4d\left(1+\mathrm{e}^{2dx+2c}\right)^{4}}+\frac{i\ln\left(\mathrm{e}^{dx+c}+i\right)a^{2}}{d}+\frac{iba\ln\left(\mathrm{e}^{dx+c}+i\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b*exp(d*x+c)*(8*a*exp(6*d*x+6*c)+5*b*exp(6*d*x+6*c)+8*a*exp(4*d*x+4*c)
-3*b*exp(4*d*x+4*c)-8*a*exp(2*d*x+2*c)+3*b*exp(2*d*x+2*c)-8*a-5*b)/d/(1+exp
(2*d*x+2*c))^4+I/d*ln(exp(d*x+c)+I)*a^2+I*b*a/d*ln(exp(d*x+c)+I)+3/8*I*b^2/
d*ln(exp(d*x+c)+I)-I/d*ln(exp(d*x+c)-I)*a^2-I*b*a/d*ln(exp(d*x+c)-I)-3/8*I/
d*ln(exp(d*x+c)-I)*b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(85) = 170$.
time = 0.47, size = 199, normalized size = 2.19

$$-\frac{1}{4}b^{2}\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5\,e^{(-dx-c)}-3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a^{2}\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*x - 3*c) +
 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 2*a*b*(arctan(e
^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) +
 e^(-4*d*x - 4*c) + 1))) + a^2*arctan(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(85) = 170$.
time = 0.36, size = 1373, normalized size = 15.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((8*a*b + 5*b^2)*cosh(d*x + c)^7 + 7*(8*a*b + 5*b^2)*cosh(d*x + c)*sin
h(d*x + c)^6 + (8*a*b + 5*b^2)*sinh(d*x + c)^7 + (8*a*b - 3*b^2)*cosh(d*x +
 c)^5 + (21*(8*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a*b - 3*b^2)*sinh(d*x + c)^
5 + 5*(7*(8*a*b + 5*b^2)*cosh(d*x + c)^3 + (8*a*b - 3*b^2)*cosh(d*x + c))*s
inh(d*x + c)^4 - (8*a*b - 3*b^2)*cosh(d*x + c)^3 + (35*(8*a*b + 5*b^2)*cosh
```

```
(d*x + c)^4 + 10*(8*a*b - 3*b^2)*cosh(d*x + c)^2 - 8*a*b + 3*b^2)*sinh(d*x
+ c)^3 + (21*(8*a*b + 5*b^2)*cosh(d*x + c)^5 + 10*(8*a*b - 3*b^2)*cosh(d*x
+ c)^3 - 3*(8*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^2 + 8*a*b
 + 3*b^2)*cosh(d*x + c)^8 + 8*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*
x + c)^7 + (8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(8*a^2 + 8*a*b + 3*b
^2)*cosh(d*x + c)^6 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2
+ 8*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(8*a^2
+ 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
 c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 24*a^2 + 24*a*b + 9*b^
2)*sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(8*a
^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*(7*(8*a
^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
 c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2)*
sinh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 8*((8*a^2 + 8*a*b + 3*b^2)*cosh(d
*x + c)^7 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(8*a^2 + 8*a*b +
3*b^2)*cosh(d*x + c)^3 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x +
c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (8*a*b + 5*b^2)*cosh(d*x + c) +
 (7*(8*a*b + 5*b^2)*cosh(d*x + c)^6 + 5*(8*a*b - 3*b^2)*cosh(d*x + c)^4 - 3
*(8*a*b - 3*b^2)*cosh(d*x + c)^2 - 8*a*b - 5*b^2)*sinh(d*x + c))/(d*cosh(d*
x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh
(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*
d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*co
sh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*
cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x
+ c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^2 \operatorname{sech}\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x), x)
```

Giac [A]
time = 0.46, size = 170, normalized size = 1.87

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)\left(8a^2 + 8ab + 3b^2\right) - \frac{4\left(8ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 5b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 32ab\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 12b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 8*a*
b + 3*b^2) - 4*(8*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 5*b^2*(e^(d*x + c) -
 e^(-d*x - c))^3 + 32*a*b*(e^(d*x + c) - e^(-d*x - c)) + 12*b^2*(e^(d*x + c
) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B]
time = 0.16, size = 303, normalized size = 3.33

$$\frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(8\,a^2\,\sqrt{d^2}+3\,b^2\,\sqrt{d^2}+8\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4+128\,a^3\,b+112\,a^2\,b^2+48\,a\,b^3+9\,b^4}}\right)\,\sqrt{64\,a^4+128\,a^3\,b+112\,a^2\,b^2+48\,a\,b^3+9\,b^4}}{4\,\sqrt{d^2}}-\frac{6\,b^2\,e^{c+d\,x}}{d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}+\frac{4\,b^2\,e^{c+d\,x}}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}-\frac{e^{c+d\,x}\,\left(5\,b^2+8\,a\,b\right)}{4\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(9\,b^2+8\,a\,b\right)}{2\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 8*a*b*(d^2)
^(1/2)))/(d*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2)))*(
48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2))/(4*(d^2)^(1/2))
 - (6*b^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6
*c + 6*d*x) + 1)) + (4*b^2*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c
 + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (exp(c + d*x)*(8*
a*b + 5*b^2))/(4*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(8*a*b + 9*b^2))
/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.94 $\int \mathrm{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] a^2*tanh(d*x+c)/d+2/3*a*b*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d
```

Rubi [A]

time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3756, 200}

$$\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^
5)/(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \mathrm{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{\mathrm{Subst}\left(\int \left(a+bx^2\right)^2\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \left(a^2+2abx^2+b^2x^4\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 49, normalized size = 1.00

$$\frac{a^2 \tanh(c+dx)}{d} + \frac{2ab \tanh^3(c+dx)}{3d} + \frac{b^2 \tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]^
5)/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(45) = 90.
time = 2.07, size = 170, normalized size = 3.47

method	result
risch	$-\frac{2\left(15a^2e^{8dx+8c}+30ab\,e^{8dx+8c}+15b^2e^{8dx+8c}+60a^2e^{6dx+6c}+60ab\,e^{6dx+6c}+90a^2e^{4dx+4c}+40ab\,e^{4dx+4c}+30b^2e^{4dx+4c}+60a^2e^{2dx+2c}\right.}{15d\left(1+e^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(15*a^2*exp(8*d*x+8*c)+30*a*b*exp(8*d*x+8*c)+15*b^2*exp(8*d*x+8*c)+60
*a^2*exp(6*d*x+6*c)+60*a*b*exp(6*d*x+6*c)+90*a^2*exp(4*d*x+4*c)+40*a*b*exp(
4*d*x+4*c)+30*b^2*exp(4*d*x+4*c)+60*a^2*exp(2*d*x+2*c)+20*a*b*exp(2*d*x+2*c
)+15*a^2+10*a*b+3*b^2)/d/(1+exp(2*d*x+2*c))^5
```

Maxima [A]
time = 0.28, size = 53, normalized size = 1.08

$$\frac{b^2 \tanh\left(dx+c\right)^5}{5\,d} + \frac{2\,ab\tanh\left(dx+c\right)^3}{3\,d} + \frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*tanh(d*x + c)^5/d + 2/3*a*b*tanh(d*x + c)^3/d + 2*a^2/(d*(e^(-2*d*x
 - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(45) = 90.
time = 0.43, size = 391, normalized size = 7.98

$$-\frac{4\left(\left(15\,a^2+20\,ab+9\,b^2\right)\cosh\left(dx+c\right)^4+8\left(5\,ab+3\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(15\,a^2+20\,ab+9\,b^2\right)\sinh\left(dx+c\right)^4+20\left(3\,a^2+2\,ab\right)\cosh\left(dx+c\right)^2+2\left(3\left(15\,a^2+20\,ab+9\,b^2\right)\cosh\left(dx+c\right)^2+30\,a^2+20\,ab\right)\sinh\left(dx+c\right)^2+45\,a^2+20\,ab+15\,b^2+8\left(\left(5\,ab+3\,b^2\right)\cosh\left(dx+c\right)^3+5\,ab\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}{15\left(d\cosh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)^4+3\left(5\,d\cosh\left(dx+c\right)^2+2\,d\right)\sinh\left(dx+c\right)^4+4\left(5\,d\cosh\left(dx+c\right)^3+4\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3+15\,d\cosh\left(dx+c\right)^2+3\left(5\,d\cosh\left(dx+c\right)^4+12\,d\cosh\left(dx+c\right)^2+5\,d\right)\sinh\left(dx+c\right)^2+2\left(3\,d\cosh\left(dx+c\right)^5+8\,d\cosh\left(dx+c\right)^3+5\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+10\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/15*((15*a^2 + 20*a*b + 9*b^2)*cosh(d*x + c)^4 + 8*(5*a*b + 3*b^2)*cosh(d
*x + c)*sinh(d*x + c)^3 + (15*a^2 + 20*a*b + 9*b^2)*sinh(d*x + c)^4 + 20*(3
*a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 20*a*b + 9*b^2)*cosh(d*x + c
)^2 + 30*a^2 + 20*a*b)*sinh(d*x + c)^2 + 45*a^2 + 20*a*b + 15*b^2 + 8*((5*a
*b + 3*b^2)*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d
*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 6*d*cos
h(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(
d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c)^2 + 3*
(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2 + 2*(3*d
*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c) +
 10*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{sech}^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(45) = 90$.
time = 0.46, size = 169, normalized size = 3.45

$$-\frac{2\left(15\,a^2e^{(8\,dx+8\,c)}+30\,abe^{(8\,dx+8\,c)}+15\,b^2e^{(8\,dx+8\,c)}+60\,a^2e^{(6\,dx+6\,c)}+60\,abe^{(6\,dx+6\,c)}+90\,a^2e^{(4\,dx+4\,c)}+40\,abe^{(4\,dx+4\,c)}+30\,b^2e^{(4\,dx+4\,c)}+60\,a^2e^{(2\,dx+2\,c)}+20\,abe^{(2\,dx+2\,c)}+15\,a^2+10\,ab+3\,b^2\right)}{15\,d(e^{(2\,dx+2\,c)}+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x +
8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x +
4*c) + 40*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x +
2*c) + 20*a*b*e^(2*d*x + 2*c) + 15*a^2 + 10*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c
) + 1)^5)
```

Mupad [B]
time = 1.24, size = 482, normalized size = 9.84

$$-\frac{\frac{2\,\left(a^2-b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,\left(a^2-b^2\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2-b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,{\left(a+b\right)}^2}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,{\left(a+b\right)}^2}{5\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a^2-b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{8\,c+8\,d\,x}\,{\left(a+b\right)}^2}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{\frac{2\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{15\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^2}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{2\,{\left(a+b\right)}^2}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^2,x)
```

```
[Out] - ((2*(a^2 - b^2))/(5*d) + (2*exp(2*c + 2*d*x)*(a + b)^2)/(5*d))/(2*exp(2*c
 + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(a^2 - b^2))/(5*d) + (6*exp(4*c + 4
*d*x)*(a^2 - b^2))/(5*d) + (2*exp(6*c + 6*d*x)*(a + b)^2)/(5*d) + (2*exp(2*
c + 2*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c
+ 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(a + b)^2)/(5*d
) + (8*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(a^2 - b^2
))/(5*d) + (2*exp(8*c + 8*d*x)*(a + b)^2)/(5*d) + (4*exp(4*c + 4*d*x)*(3*a^
2 - 2*a*b + 3*b^2))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(3*a^2
 - 2*a*b + 3*b^2))/(15*d) + (4*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d) + (2*exp
(4*c + 4*d*x)*(a + b)^2)/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) +
exp(6*c + 6*d*x) + 1) - (2*(a + b)^2)/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.95 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=125

$$\frac{(8a^2+4ab+b^2)\operatorname{ArcTan}(\sinh(c+dx))}{16d}+\frac{(8a^2+4ab+b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d}-\frac{b(8a+3b)\operatorname{sech}^3(c+}{24d}$$

```
[Out] 1/16*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c))/d+1/16*(8*a^2+4*a*b+b^2)*sech(d*
x+c)*tanh(d*x+c)/d-1/24*b*(8*a+3*b)*sech(d*x+c)^3*tanh(d*x+c)/d-1/6*b*sech(
d*x+c)^5*(a+(a+b)*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A]

time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3757, 424, 393, 205, 209}

$$\frac{(8a^2+4ab+b^2)\operatorname{ArcTan}(\sinh(c+dx))}{16d}+\frac{(8a^2+4ab+b^2)\tanh(c+dx)\operatorname{sech}(c+dx)}{16d}-\frac{b(8a+3b)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{24d}-\frac{b\tanh(c+dx)\operatorname{sech}^5(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 4*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((8*a^2 + 4*a*b + b^
2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) - (b*(8*a + 3*b)*Sech[c + d*x]^3*Tan
h[c + d*x])/(24*d) - (b*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[
c + d*x])/(6*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
+ p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \frac{\text{Su}}{}$$

$$= -\frac{b(8a+3b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} - \frac{b\operatorname{sech}^5(c+dx)\left(a+\right.}{}$$

$$= \frac{\left(8a^2+4ab+b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} - \frac{b(8a+3b)\operatorname{sech}^3(}{}$$

$$= \frac{\left(8a^2+4ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{\left(8a^2+4ab+b^2\right)\operatorname{sech}(}{16}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.02, size = 792, normalized size = 6.34

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x]*((-23555*(a + b))/a - (32970*(a + b)^2)/a^2 - 14980*Csch
[c + d*x]^2 - (91875*(a + b)*Csch[c + d*x]^2)/a - 65625*Csch[c + d*x]^4 - (
8855*(a + b)^2*Sinh[c + d*x]^2)/a^2 - 620*HypergeometricPFQ[{3/2, 2, 2, 2},
 {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 160*HypergeometricPFQ[{3/
2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 16*Hype
rgeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Si
nh[c + d*x]^2 - (968*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2},
 -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (288*(a + b)*HypergeometricPFQ[{3/2
, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (32*(
a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c +
 d*x]^2]*Sinh[c + d*x]^4)/a - (380*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2,
2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (128*(a + b)^2*Hy
pergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c
 + d*x]^6)/a^2 - (16*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1,
1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 + (65625*ArcTanh[Sqrt
[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(5/2) + (1680*ArcTanh[Sqrt[-Sinh[c
+ d*x]^2]]*Sinh[c + d*x]^4)/(-Sinh[c + d*x]^2)^(5/2) - (36855*ArcTanh[Sqrt[
-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(3/2) - (91875*(a + b)*ArcTanh[Sqrt[
-Sinh[c + d*x]^2]])/(a*(-Sinh[c + d*x]^2)^(3/2)) + (54180*(a + b)*ArcTanh[S
qrt[-Sinh[c + d*x]^2]])/(a*Sqrt[-Sinh[c + d*x]^2]) + (32970*(a + b)^2*ArcTa
nh[Sqrt[-Sinh[c + d*x]^2]])/(a^2*Sqrt[-Sinh[c + d*x]^2]) + (525*(a + b)^2*A
rcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4)/(a^2*Sqrt[-Sinh[c + d*x]^2]
) - (1365*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2])/a
 - (19845*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2])
/a^2))/(2520*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.30, size = 358, normalized size = 2.86

method	result
risch	$\frac{e^{dx+c}\left(24a^2 e^{10dx+10c}+12ab\, e^{10dx+10c}+3b^2 e^{10dx+10c}+72a^2 e^{8dx+8c}-60ab\, e^{8dx+8c}-47b^2 e^{8dx+8c}+48a^2 e^{6dx+6c}-72ab\, e^{6dx+6c}+78b\right.}{24d\left(1+e^{2dx+2c}\right)^{6}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*exp(d*x+c)*(24*a^2*exp(10*d*x+10*c)+12*a*b*exp(10*d*x+10*c)+3*b^2*exp(
10*d*x+10*c)+72*a^2*exp(8*d*x+8*c)-60*a*b*exp(8*d*x+8*c)-47*b^2*exp(8*d*x+8
*c)+48*a^2*exp(6*d*x+6*c)-72*a*b*exp(6*d*x+6*c)+78*b^2*exp(6*d*x+6*c)-48*a^
2*exp(4*d*x+4*c)+72*a*b*exp(4*d*x+4*c)-78*b^2*exp(4*d*x+4*c)-72*a^2*exp(2*d
*x+2*c)+60*a*b*exp(2*d*x+2*c)+47*b^2*exp(2*d*x+2*c)-24*a^2-12*a*b-3*b^2)/d/
(1+exp(2*d*x+2*c))^6+1/2*I/d*ln(exp(d*x+c)+I)*a^2+1/4*I*b*a/d*ln(exp(d*x+c)
+I)+1/16*I*b^2/d*ln(exp(d*x+c)+I)-1/2*I/d*ln(exp(d*x+c)-I)*a^2-1/4*I/d*ln(e
xp(d*x+c)-I)*a*b-1/16*I/d*ln(exp(d*x+c)-I)*b^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(117) = 234$.
time = 0.48, size = 345, normalized size = 2.76

$$-\frac{1}{24}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3e^{(-dx-c)}-47e^{(-3dx-3c)}+78e^{(-5dx-5c)}-78e^{(-7dx-7c)}+47e^{(-9dx-9c)}-3e^{(-11dx-11c)}}{d(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(-12dx-12c)}+1)}\right)-\frac{1}{2}ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7e^{(-3dx-3c)}+7e^{(-5dx-5c)}-e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)-a^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c)
 + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^(-
11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x
 - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) +
 1))) - 1/2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c
) + 7*e^(-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4
*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^2*(arctan(e^
(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) +
e^(-4*d*x - 4*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2824 vs. $2(117) = 234$.
time = 0.39, size = 2824, normalized size = 22.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 4*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^11 +
(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 4*a*b + b^2)*cos
h(d*x + c)^2 + 72*a^2 - 60*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(8*a^2 + 4
*a*b + b^2)*cosh(d*x + c)^3 + (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c))*sin
h(d*x + c)^8 + 6*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*(8*a^2
+ 4*a*b + b^2)*cosh(d*x + c)^4 + 6*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)
^2 + 8*a^2 - 12*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(8*a^2 + 4*a*b + b^2
)*cosh(d*x + c)^5 + 2*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^3 + (8*a^2 -
 12*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(8*a^2 - 12*a*b + 13*b
^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 21*(72
*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(8*a^2 - 12*a*b + 13*b^2)*cosh
(d*x + c)^2 - 8*a^2 + 12*a*b - 13*b^2)*sinh(d*x + c)^5 + 6*(165*(8*a^2 + 4*
a*b + b^2)*cosh(d*x + c)^7 + 21*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^5
+ 35*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(8*a^2 - 12*a*b + 13*b^2
)*cosh(d*x + c))*sinh(d*x + c)^4 - (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)
^3 + (495*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 84*(72*a^2 - 60*a*b - 47*
```

b^2)*cosh(d*x + c)^6 + 210*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^4 - 60*(
8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^2 - 72*a^2 + 60*a*b + 47*b^2)*sinh(d
x + c)^3 + 3(55*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^9 + 12*(72*a^2 - 60*a
*b - 47*b^2)*cosh(d*x + c)^7 + 42*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^5
 - 20*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^3 - (72*a^2 - 60*a*b - 47*b^2
)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^1
2 + 12*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (8*a^2 + 4*a*
b + b^2)*sinh(d*x + c)^12 + 6*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^10 + 6*(1
1*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c
)^10 + 20*(11*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 4*a*b + b^
2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^
8 + 15*(33*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 18*(8*a^2 + 4*a*b + b^2)
*cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^8 + 24*(33*(8*a^2 + 4
*a*b + b^2)*cosh(d*x + c)^5 + 30*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + 5*
(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(8*a^2 + 4*a*b +
b^2)*cosh(d*x + c)^6 + 4*(231*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 315*(
8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 105*(8*a^2 + 4*a*b + b^2)*cosh(d*x +
 c)^2 + 40*a^2 + 20*a*b + 5*b^2)*sinh(d*x + c)^6 + 24*(33*(8*a^2 + 4*a*b +
b^2)*cosh(d*x + c)^7 + 63*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^5 + 35*(8*a^2
 + 4*a*b + b^2)*cosh(d*x + c)^3 + 5*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*si
nh(d*x + c)^5 + 15*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 15*(33*(8*a^2 +
4*a*b + b^2)*cosh(d*x + c)^8 + 84*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 7
0*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 20*(8*a^2 + 4*a*b + b^2)*cosh(d*x
 + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(8*a^2 + 4*a*b + b^
2)*cosh(d*x + c)^9 + 36*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^7 + 42*(8*a^2 +
 4*a*b + b^2)*cosh(d*x + c)^5 + 20*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 +
3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 + 4*a*b +
 b^2)*cosh(d*x + c)^2 + 6*(11*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^10 + 45*(
8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 70*(8*a^2 + 4*a*b + b^2)*cosh(d*x +
c)^6 + 50*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 15*(8*a^2 + 4*a*b + b^2)*
cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^2 + 8*a^2 + 4*a*b + b^
2 + 12*((8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^11 + 5*(8*a^2 + 4*a*b + b^2)*co
sh(d*x + c)^9 + 10*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^7 + 10*(8*a^2 + 4*a*
b + b^2)*cosh(d*x + c)^5 + 5*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + (8*a^2
 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d
x + c)) - 3(8*a^2 + 4*a*b + b^2)*cosh(d*x + c) + 3*(11*(8*a^2 + 4*a*b + b
^2)*cosh(d*x + c)^10 + 3*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^8 + 14*(8
*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^6 - 10*(8*a^2 - 12*a*b + 13*b^2)*cosh
(d*x + c)^4 - (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^2 - 8*a^2 - 4*a*b -
b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^
11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 +
d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*
x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)
^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*c

```
osh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(
d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*
x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(
33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{sech}^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(117) = 234.
time = 0.49, size = 267, normalized size = 2.14

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)(8a^2+4ab+b^2)+\frac{4\left(24a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+12ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+3b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+192a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-32b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+384a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)-192ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)-48b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 4*
a*b + b^2) + 4*(24*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 12*a*b*(e^(d*x + c)
 - e^(-d*x - c))^5 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 192*a^2*(e^(d*x
 + c) - e^(-d*x - c))^3 - 32*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 384*a^2*(
e^(d*x + c) - e^(-d*x - c)) - 192*a*b*(e^(d*x + c) - e^(-d*x - c)) - 48*b^2
*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

Mupad [B]
time = 0.18, size = 572, normalized size = 4.58

$$\frac{\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\left(8a^2\sqrt{d^2}+b^2\sqrt{d^2}+4ab\sqrt{d^2}\right)}{d\sqrt{64a^4+64a^3b+32a^2b^2+8ab^3+b^4}}\right)\sqrt{64a^4+64a^3b+32a^2b^2+8ab^3+b^4}}{8\sqrt{d^2}}-\frac{\frac{2e^{c+dx}(a+b)^2}{3d}+\frac{8e^{3c+3dx}(a^2-b^2)}{3d}+\frac{8e^{7c+7dx}(a^2-b^2)}{3d}+\frac{2e^{9c+9dx}(a+b)^2}{3d}+\frac{4e^{5c+5dx}(3a^2-2ab+3b^2)}{3d}}{6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1}-\frac{2e^{c+dx}(15b^2+4ab)}{3d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}+\frac{16b^2e^{c+dx}}{3d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)}+\frac{e^{c+dx}(8a^2+4ab+b^2)}{8d\left(e^{2c+2dx}+1\right)}-\frac{e^{c+dx}(16a^2+44ab+23b^2)}{12d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}+\frac{e^{c+dx}(21b^2+20ab)}{3d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + b^2*(d^2)^(1/2) + 4*a*b*(d^2)^(
1/2)))/(d*(8*a*b^3 + 64*a^3*b + 64*a^4 + b^4 + 32*a^2*b^2)^(1/2)))*(8*a*b^3
 + 64*a^3*b + 64*a^4 + b^4 + 32*a^2*b^2)^(1/2))/(8*(d^2)^(1/2)) - ((2*exp(c
 + d*x)*(a + b)^2)/(3*d) + (8*exp(3*c + 3*d*x)*(a^2 - b^2))/(3*d) + (8*exp(
7*c + 7*d*x)*(a^2 - b^2))/(3*d) + (2*exp(9*c + 9*d*x)*(a + b)^2)/(3*d) + (4
```

```
*exp(5*c + 5*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(3*d))/(6*exp(2*c + 2*d*x) + 15*
exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c +
 10*d*x) + exp(12*c + 12*d*x) + 1) - (2*exp(c + d*x)*(4*a*b + 15*b^2))/(3*d
*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c +
8*d*x) + 1)) + (16*b^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c
+ 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) +
1)) + (exp(c + d*x)*(4*a*b + 8*a^2 + b^2))/(8*d*(exp(2*c + 2*d*x) + 1)) - (
exp(c + d*x)*(44*a*b + 16*a^2 + 23*b^2))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*
c + 4*d*x) + 1)) + (exp(c + d*x)*(20*a*b + 21*b^2))/(3*d*(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

3.96 $\int \text{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=76

$$\frac{a^2\tanh(c+dx)}{d}-\frac{a(a-2b)\tanh^3(c+dx)}{3d}-\frac{(2a-b)b\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] a^2*tanh(d*x+c)/d-1/3*a*(a-2*b)*tanh(d*x+c)^3/d-1/5*(2*a-b)*b*tanh(d*x+c)^5
/d-1/7*b^2*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3756, 380}

$$\frac{a^2\tanh(c+dx)}{d}-\frac{b(2a-b)\tanh^5(c+dx)}{5d}-\frac{a(a-2b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d - (a*(a - 2*b)*Tanh[c + d*x]^3)/(3*d) - ((2*a - b)*b*
Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int (1-x^2)\left(a+bx^2\right)^2\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(a^2-a(a-2b)x^2-(2a-b)bx^4-b^2x^6\right)\,dx, x, \tanh\right.}{d}$$

$$= \frac{a^2\tanh(c+dx)}{d} - \frac{a(a-2b)\tanh^3(c+dx)}{3d} - \frac{(2a-b)b\tanh^5}{5d}$$

Mathematica [A]
time = 0.39, size = 83, normalized size = 1.09

$$\frac{\left(70a^2+28ab+6b^2+\left(35a^2+14ab+3b^2\right)\operatorname{sech}^2(c+dx)-6b(7a+4b)\operatorname{sech}^4(c+dx)+15b^2\operatorname{sech}^6(c+dx)\right)\tanh(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((70*a^2 + 28*a*b + 6*b^2 + (35*a^2 + 14*a*b + 3*b^2)*Sech[c + d*x]^2 - 6*b
*(7*a + 4*b)*Sech[c + d*x]^4 + 15*b^2*Sech[c + d*x]^6)*Tanh[c + d*x])/(105*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(70) = 140.
time = 2.20, size = 239, normalized size = 3.14

method	result
risch	$-\frac{4\left(105a^2 e^{10dx+10c}+210ab\,e^{10dx+10c}+105b^2 e^{10dx+10c}+455a^2 e^{8dx+8c}+350ab\,e^{8dx+8c}-105b^2 e^{8dx+8c}+770a^2 e^{6dx+6c}+140ab\,e^{6dx}\right.}{105d\left(1+e^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/105*(105*a^2*exp(10*d*x+10*c)+210*a*b*exp(10*d*x+10*c)+105*b^2*exp(10*d*
x+10*c)+455*a^2*exp(8*d*x+8*c)+350*a*b*exp(8*d*x+8*c)-105*b^2*exp(8*d*x+8*c
)+770*a^2*exp(6*d*x+6*c)+140*a*b*exp(6*d*x+6*c)+210*b^2*exp(6*d*x+6*c)+630*
a^2*exp(4*d*x+4*c)+84*a*b*exp(4*d*x+4*c)-42*b^2*exp(4*d*x+4*c)+245*a^2*exp(
2*d*x+2*c)+98*a*b*exp(2*d*x+2*c)+21*b^2*exp(2*d*x+2*c)+35*a^2+14*a*b+3*b^2)
/d/(1+exp(2*d*x+2*c))^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(70) = 140.
time = 0.29, size = 928, normalized size = 12.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 4/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-
12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(d*(7*e^(-2
*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*
c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)
) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e
^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*
x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x
- 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) +
21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 3
5*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-
6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x -
 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
 - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
 + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 8/15*a*b*(5*e^(-2*d*x
 - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c)
+ 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
- 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2
*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(70) = 140$.
time = 0.34, size = 677, normalized size = 8.91

$$-\frac{8\,\left(2\,\left(35\,a^{2}+56\,ab+27\,b^{2}\right)\cosh\left(dx+c\right)^{5}+10\,\left(35\,a^{2}+56\,ab+27\,b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+\left(35\,a^{2}+98\,ab+51\,b^{2}\right)\sinh\left(dx+c\right)^{5}+14\,\left(25\,a^{2}+16\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)^{3}+\left(10\,\left(35\,a^{2}+98\,ab+51\,b^{2}\right)\cosh\left(dx+c\right)^{2}+105\,a^{2}+126\,ab-63\,b^{2}\right)\sinh\left(dx+c\right)^{3}+2\,\left(10\,\left(35\,a^{2}+56\,ab+27\,b^{2}\right)\cosh\left(dx+c\right)^{3}+21\,\left(25\,a^{2}+16\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+28\,\left(25\,a^{2}+4\,ab+3\,b^{2}\right)\cosh\left(dx+c\right)+\left(5\,\left(35\,a^{2}+98\,ab+51\,b^{2}\right)\cosh\left(dx+c\right)^{4}+63\,\left(5\,a^{2}+6\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)^{2}+70\,a^{2}+28\,ab+126\,b^{2}\right)\sinh\left(dx+c\right)\right)}{105\,\left(d\cosh\left(dx+c\right)^{9}+9\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{8}+d\sinh\left(dx+c\right)^{9}+7\,d\cosh\left(dx+c\right)^{7}+\left(36\,d\cosh\left(dx+c\right)^{2}+7\,d\right)\sinh\left(dx+c\right)^{7}+7\,\left(12\,d\cosh\left(dx+c\right)^{3}+7\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{6}+22\,d\cosh\left(dx+c\right)^{5}+\left(126\,d\cosh\left(dx+c\right)^{4}+147\,d\cosh\left(dx+c\right)^{2}+20\,d\right)\sinh\left(dx+c\right)^{5}+\left(126\,d\cosh\left(dx+c\right)^{5}+245\,d\cosh\left(dx+c\right)^{3}+110\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+42\,d\cosh\left(dx+c\right)^{3}+\left(84\,d\cosh\left(dx+c\right)^{6}+245\,d\cosh\left(dx+c\right)^{4}+200\,d\cosh\left(dx+c\right)^{2}+28\,d\right)\sinh\left(dx+c\right)^{3}+\left(36\,d\cosh\left(dx+c\right)^{7}+147\,d\cosh\left(dx+c\right)^{5}+220\,d\cosh\left(dx+c\right)^{3}+126\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+56\,d\cosh\left(dx+c\right)+\left(9\,d\cosh\left(dx+c\right)^{8}+49\,d\cosh\left(dx+c\right)^{6}+100\,d\cosh\left(dx+c\right)^{4}+84\,d\cosh\left(dx+c\right)^{2}+14\,d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/105*(2*(35*a^2 + 56*a*b + 27*b^2)*cosh(d*x + c)^5 + 10*(35*a^2 + 56*a*b
+ 27*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (35*a^2 + 98*a*b + 51*b^2)*sinh(d
*x + c)^5 + 14*(25*a^2 + 16*a*b - 3*b^2)*cosh(d*x + c)^3 + (10*(35*a^2 + 98
*a*b + 51*b^2)*cosh(d*x + c)^2 + 105*a^2 + 126*a*b - 63*b^2)*sinh(d*x + c)^
3 + 2*(10*(35*a^2 + 56*a*b + 27*b^2)*cosh(d*x + c)^3 + 21*(25*a^2 + 16*a*b
- 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 28*(25*a^2 + 4*a*b + 3*b^2)*cosh(
d*x + c) + (5*(35*a^2 + 98*a*b + 51*b^2)*cosh(d*x + c)^4 + 63*(5*a^2 + 6*a*
```

```
b - 3*b^2)*cosh(d*x + c)^2 + 70*a^2 + 28*a*b + 126*b^2)*sinh(d*x + c))/(d*c
osh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 7*
d*cosh(d*x + c)^7 + (36*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 + 7*(12*d*
cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 22*d*cosh(d*x + c)^5
 + (126*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 + 20*d)*sinh(d*x + c)^5 +
 (126*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 110*d*cosh(d*x + c))*sinh
(d*x + c)^4 + 42*d*cosh(d*x + c)^3 + (84*d*cosh(d*x + c)^6 + 245*d*cosh(d*x
 + c)^4 + 200*d*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^3 + (36*d*cosh(d*x +
c)^7 + 147*d*cosh(d*x + c)^5 + 220*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c))
*sinh(d*x + c)^2 + 56*d*cosh(d*x + c) + (9*d*cosh(d*x + c)^8 + 49*d*cosh(d*
x + c)^6 + 100*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x +
c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^2 \operatorname{sech}^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(70) = 140.
time = 0.47, size = 238, normalized size = 3.13

$$-\frac{4\,\left(105\,a^2e^{(10\,dx+10\,c)} + 210\,abe^{(10\,dx+10\,c)} + 105\,b^2e^{(10\,dx+10\,c)} + 455\,a^2e^{(8\,dx+8\,c)} + 350\,abe^{(8\,dx+8\,c)} - 105\,b^2e^{(8\,dx+8\,c)} + 770\,a^2e^{(6\,dx+6\,c)} + 140\,abe^{(6\,dx+6\,c)} + 210\,b^2e^{(6\,dx+6\,c)} + 630\,a^2e^{(4\,dx+4\,c)} + 84\,abe^{(4\,dx+4\,c)} - 42\,b^2e^{(4\,dx+4\,c)} + 245\,a^2e^{(2\,dx+2\,c)} + 98\,abe^{(2\,dx+2\,c)} + 21\,b^2e^{(2\,dx+2\,c)} + 35\,a^2 + 14\,ab + 3\,b^2\right)}{105\,d{\left(e^{(2\,dx+2\,c)} + 1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/105*(105*a^2*e^(10*d*x + 10*c) + 210*a*b*e^(10*d*x + 10*c) + 105*b^2*e^(
10*d*x + 10*c) + 455*a^2*e^(8*d*x + 8*c) + 350*a*b*e^(8*d*x + 8*c) - 105*b^
2*e^(8*d*x + 8*c) + 770*a^2*e^(6*d*x + 6*c) + 140*a*b*e^(6*d*x + 6*c) + 210
*b^2*e^(6*d*x + 6*c) + 630*a^2*e^(4*d*x + 4*c) + 84*a*b*e^(4*d*x + 4*c) - 4
2*b^2*e^(4*d*x + 4*c) + 245*a^2*e^(2*d*x + 2*c) + 98*a*b*e^(2*d*x + 2*c) +
21*b^2*e^(2*d*x + 2*c) + 35*a^2 + 14*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c) + 1)^
7)
```

Mupad [**B**]
time = 1.22, size = 732, normalized size = 9.63

$$-\frac{\frac{4\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{35\,d}+\frac{32\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{35\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^2}{7\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{32\,\left(a^2-b^2\right)}{105\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2}{21\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{\frac{32\,\left(a^2-b^2\right)}{105\,d}+\frac{64\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2-b^2\right)}{35\,d}+\frac{16\,{\mathrm{e}}^{6\,c+6\,d\,x}\,{\left(a+b\right)}^2}{21\,d}+\frac{16\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{35\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{\frac{32\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2-b^2\right)}{7\,d}+\frac{32\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(a^2-b^2\right)}{7\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2}{7\,d}+\frac{8\,{\mathrm{e}}^{10\,c+10\,d\,x}\,{\left(a+b\right)}^2}{7\,d}+\frac{16\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{7\,d}}{7\,{\mathrm{e}}^{2\,c+2\,d\,x}+21\,{\mathrm{e}}^{4\,c+4\,d\,x}+35\,{\mathrm{e}}^{6\,c+6\,d\,x}+35\,{\mathrm{e}}^{8\,c+8\,d\,x}+21\,{\mathrm{e}}^{10\,c+10\,d\,x}+7\,{\mathrm{e}}^{12\,c+12\,d\,x}+{\mathrm{e}}^{14\,c+14\,d\,x}+1}-\frac{\frac{4\,{\left(a+b\right)}^2}{21\,d}+\frac{32\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{21\,d}+\frac{64\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a^2-b^2\right)}{21\,d}+\frac{20\,{\mathrm{e}}^{8\,c+8\,d\,x}\,{\left(a+b\right)}^2}{21\,d}+\frac{8\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{7\,d}}{6\,{\mathrm{e}}^{2\,c+2\,d\,x}+15\,{\mathrm{e}}^{4\,c+4\,d\,x}+20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}+6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+12\,d\,x}+1}-\frac{4\,{\left(a+b\right)}^2}{21\,d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^4,x)

[Out] - ((4*(3*a^2 - 2*a*b + 3*b^2))/(35*d) + (32*exp(2*c + 2*d*x)*(a^2 - b^2))/(
35*d) + (4*exp(4*c + 4*d*x)*(a + b)^2)/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4
*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((32*(a^2 - b^2)
)/(105*d) + (8*exp(2*c + 2*d*x)*(a + b)^2)/(21*d))/(3*exp(2*c + 2*d*x) + 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((32*(a^2 - b^2))/(105*d) + (64*
exp(4*c + 4*d*x)*(a^2 - b^2))/(35*d) + (16*exp(6*c + 6*d*x)*(a + b)^2)/(21*
d) + (16*exp(2*c + 2*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(35*d))/(5*exp(2*c + 2*d
*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(
10*c + 10*d*x) + 1) - ((32*exp(4*c + 4*d*x)*(a^2 - b^2))/(7*d) + (32*exp(8*
c + 8*d*x)*(a^2 - b^2))/(7*d) + (8*exp(2*c + 2*d*x)*(a + b)^2)/(7*d) + (8*e
xp(10*c + 10*d*x)*(a + b)^2)/(7*d) + (16*exp(6*c + 6*d*x)*(3*a^2 - 2*a*b +
3*b^2))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d
*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) +
exp(14*c + 14*d*x) + 1) - ((4*(a + b)^2)/(21*d) + (32*exp(2*c + 2*d*x)*(a^2
 - b^2))/(21*d) + (64*exp(6*c + 6*d*x)*(a^2 - b^2))/(21*d) + (20*exp(8*c +
8*d*x)*(a + b)^2)/(21*d) + (8*exp(4*c + 4*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(7*
d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*ex
p(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - (4*(a + b
)^2)/(21*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.97 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=91

$$\frac{3}{8}(a+b)\left(a^2-2ab+5b^2\right)x+\frac{3(a-3b)(a+b)^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)^3\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

```
[Out] 3/8*(a+b)*(a^2-2*a*b+5*b^2)*x+3/8*(a-3*b)*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/d
+1/4*(a+b)^3*cosh(d*x+c)^3*sinh(d*x+c)/d-b^3*tanh(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3756, 398, 1171, 393, 212}

$$\frac{3}{8}x(a+b)\left(a^2-2ab+5b^2\right)+\frac{(a+b)^3\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3(a-3b)(a+b)^2\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{b^3\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a + b)*(a^2 - 2*a*b + 5*b^2)*x)/8 + (3*(a - 3*b)*(a + b)^2*Cosh[c + d*x
]*Sinh[c + d*x])/(8*d) + ((a + b)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) -
(b^3*Tanh[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^3+\frac{a^3+b^3+3b(a^2-b^2)x^2+3b^2(a+b)x^4}{(1-x^2)^3}\right)\,dx, x, \tanh(c+d$$
$$= -\frac{b^3\tanh(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{a^3+b^3+3b(a^2-b^2)x^2+3b^2(a+b)x^4}{(1-x^2)^3}\,dx, x,\right.}{d}$$
$$= \frac{(a+b)^3\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{b^3\tanh(c+dx)}{d}-\frac{\text{Subs}}{}$$
$$= \frac{3(a-3b)(a+b)^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)^3\cosh^3(c}{}$$
$$= \frac{3}{8}(a+b)\left(a^2-2ab+5b^2\right)x+\frac{3(a-3b)(a+b)^2\cosh(c+dx)\,\mathrm{s}}{8d}$$

Mathematica [A]
time = 0.50, size = 81, normalized size = 0.89

$$\frac{12(a^3-a^2b+3ab^2+5b^3)(c+dx)+8(a-2b)(a+b)^2\sinh(2(c+dx))+(a+b)^3\sinh(4(c+dx))-32b^3\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

[Out] (12*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*(c + d*x) + 8*(a - 2*b)*(a + b)^2*Sinh[
2*(c + d*x)] + (a + b)^3*Sinh[4*(c + d*x)] - 32*b^3*Tanh[c + d*x])/(32*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(85) = 170$.
time = 1.86, size = 293, normalized size = 3.22

method	result
risch	$\frac{3a^3x}{8} - \frac{3a^2bx}{8} + \frac{9a\,b^2x}{8} + \frac{15b^3x}{8} + \frac{\mathrm{e}^{4dx+4c}a^3}{64d} + \frac{3\,\mathrm{e}^{4dx+4c}a^2b}{64d} + \frac{3b^2\mathrm{e}^{4dx+4c}a}{64d} + \frac{b^3\mathrm{e}^{4dx+4c}}{64d} + \frac{\mathrm{e}^{2dx+2c}a^3}{8d} - \frac{3\,\mathrm{e}^{2dx+2c}a}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 3/8*a^3*x-3/8*a^2*b*x+9/8*a*b^2*x+15/8*b^3*x+1/64/d*exp(4*d*x+4*c)*a^3+3/64
/d*exp(4*d*x+4*c)*a^2*b+3/64*b^2/d*exp(4*d*x+4*c)*a+1/64*b^3/d*exp(4*d*x+4*
c)+1/8/d*exp(2*d*x+2*c)*a^3-3/8/d*exp(2*d*x+2*c)*a*b^2-1/4*b^3/d*exp(2*d*x+
2*c)-1/8/d*exp(-2*d*x-2*c)*a^3+3/8/d*exp(-2*d*x-2*c)*a*b^2+1/4*b^3/d*exp(-2
*d*x-2*c)-1/64/d*exp(-4*d*x-4*c)*a^3-3/64/d*exp(-4*d*x-4*c)*a^2*b-3/64*b^2/
d*exp(-4*d*x-4*c)*a-1/64*b^3/d*exp(-4*d*x-4*c)+2*b^3/d/(1+exp(2*d*x+2*c))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(85) = 170$.
time = 0.27, size = 267, normalized size = 2.93

$$\frac{1}{64}\,a^3\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} + \frac{8\,e^{(2\,dx+2\,c)}}{d} - \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) + \frac{3}{64}\,ab^2\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) + \frac{1}{64}\,b^3\left(\frac{120\,(dx+c)}{d} + \frac{16\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)}}{d} - \frac{15\,e^{(-2\,dx-2\,c)} + 144\,e^{(-4\,dx-4\,c)} - 1}{d(e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)})}\right) - \frac{3}{64}\,a^2b\left(\frac{8\,(dx+c)}{d} - \frac{e^{(4\,dx+4\,c)}}{d} + \frac{e^{(-4\,dx-4\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*
d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b^3*(120*(
d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2
*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
- 3/64*a^2*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(85) = 170$.
time = 0.37, size = 227, normalized size = 2.49

$$\frac{(a^3 + 3\,a^2b + 3\,ab^2 + b^3)\sinh(dx+c)^5 + \left(9\,a^3 + 3\,a^2b - 21\,ab^2 - 15\,b^3 + 10\,(a^3 + 3\,a^2b + 3\,ab^2 + b^3)\cosh(dx+c)^2\right)\sinh(dx+c)^3 + 8\,(8\,b^3 + 3\,(a^3 - a^2b + 3\,ab^2 + 5\,b^3)dx)\cosh(dx+c) + \left(5\,(a^3 + 3\,a^2b + 3\,ab^2 + b^3)\cosh(dx+c)^4 + 8\,a^3 - 24\,ab^2 - 80\,b^3 + 9\,(3\,a^3 + a^2b - 7\,ab^2 - 5\,b^3)\cosh(dx+c)^2\right)\sinh(dx+c)}{64\,d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

```
[Out] 1/64*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 + (9*a^3 + 3*a^2*b -
21*a*b^2 - 15*b^3 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^3 + 8*(8*b^3 + 3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*d*x)*cosh(d*x +
 c) + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^3 - 24*a*b^2
 - 80*b^3 + 9*(3*a^3 + a^2*b - 7*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c))/(d*cosh(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(85) = 170.
time = 0.62, size = 282, normalized size = 3.10

$$\frac{a^3e^{(4\,dx+4\,c)}+3\,a^2be^{(4\,dx+4\,c)}+3\,ab^2e^{(4\,dx+4\,c)}+b^3e^{(4\,dx+4\,c)}+8\,a^3e^{(2\,dx+2\,c)}-24\,ab^2e^{(2\,dx+2\,c)}-16\,b^3e^{(2\,dx+2\,c)}+24\,(a^3-a^2b+3\,ab^2+5\,b^3)(dx+c)+\frac{128\,b^3}{e^{(2\,dx+2\,c)}+1}-\left(18\,a^3e^{(4\,dx+4\,c)}-18\,a^2be^{(4\,dx+4\,c)}+54\,ab^2e^{(4\,dx+4\,c)}+90\,b^3e^{(4\,dx+4\,c)}+8\,a^3e^{(2\,dx+2\,c)}-24\,ab^2e^{(2\,dx+2\,c)}-16\,b^3e^{(2\,dx+2\,c)}+a^3+3\,a^2b+3\,ab^2+b^3\right)e^{(-4\,dx-4\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*e^(4*d*x + 4*c) + 3*a^2*b*e^(4*d*x + 4*c) + 3*a*b^2*e^(4*d*x + 4*
c) + b^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) - 24*a*b^2*e^(2*d*x + 2*c)
 - 16*b^3*e^(2*d*x + 2*c) + 24*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*(d*x + c) +
128*b^3/(e^(2*d*x + 2*c) + 1) - (18*a^3*e^(4*d*x + 4*c) - 18*a^2*b*e^(4*d*x
 + 4*c) + 54*a*b^2*e^(4*d*x + 4*c) + 90*b^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*
x + 2*c) - 24*a*b^2*e^(2*d*x + 2*c) - 16*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*e^(-4*d*x - 4*c))/d
```

Mupad [B]
time = 1.41, size = 133, normalized size = 1.46

$$x\,\left(\frac{3\,a^3}{8}-\frac{3\,a^2\,b}{8}+\frac{9\,a\,b^2}{8}+\frac{15\,b^3}{8}\right)+\frac{2\,b^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{{\mathrm{e}}^{-4\,c-4\,d\,x}\,{\left(a+b\right)}^3}{64\,d}+\frac{{\mathrm{e}}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^3}{64\,d}-\frac{{\mathrm{e}}^{-2\,c-2\,d\,x}\,{\left(a+b\right)}^2\,\left(a-2\,b\right)}{8\,d}+\frac{{\mathrm{e}}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2\,\left(a-2\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*((9*a*b^2)/8 - (3*a^2*b)/8 + (3*a^3)/8 + (15*b^3)/8) + (2*b^3)/(d*(exp(2*
c + 2*d*x) + 1)) - (exp(- 4*c - 4*d*x)*(a + b)^3)/(64*d) + (exp(4*c + 4*d*x
)*(a + b)^3)/(64*d) - (exp(- 2*c - 2*d*x)*(a + b)^2*(a - 2*b))/(8*d) + (exp
(2*c + 2*d*x)*(a + b)^2*(a - 2*b))/(8*d)
```

3.98 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=87

$$\frac{b^2(6a+5b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}+\frac{(a+b)^3\sinh^3(c+dx)}{3d}-\frac{b^3\text{sech}(c+dx)\,\text{t}}{2d}$$

```
[Out] 1/2*b^2*(6*a+5*b)*arctan(sinh(d*x+c))/d+(a-2*b)*(a+b)^2*sinh(d*x+c)/d+1/3*(
a+b)^3*sinh(d*x+c)^3/d-1/2*b^3*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3757, 398, 393, 209}

$$\frac{b^2(6a+5b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{(a+b)^3\sinh^3(c+dx)}{3d}+\frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}-\frac{b^3\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a - 2*b)*(a + b)^2*Sinh[c
 + d*x])/d + ((a + b)^3*Sinh[c + d*x]^3)/(3*d) - (b^3*Sech[c + d*x]*Tanh[c
+ d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^3}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left((a-2b)(a+b)^2+(a+b)^3x^2+\frac{b^2(3a+2b)+3b^2(a+b)x^2}{(1+x^2)^2}\right)\right.}{d}$$

$$= \frac{(a-2b)(a+b)^2\sinh(c+dx)}{d} + \frac{(a+b)^3\sinh^3(c+dx)}{3d} + \frac{\text{Subs}}{}$$

$$= \frac{(a-2b)(a+b)^2\sinh(c+dx)}{d} + \frac{(a+b)^3\sinh^3(c+dx)}{3d} - \frac{b^3\text{sec}}{}$$

$$= \frac{b^2(6a+5b)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{(a-2b)(a+b)^2\sinh(c+d}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.69, size = 494, normalized size = 5.68

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}
, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + a*Sinh[c + d*x]^2 + b*Sinh[c + d*x
]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(b^3*Sinh[c + d*x]^6*(2161 +
1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cosh[c
+ d*x]^6*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]
^6) + 3*a^2*b*(Sinh[c + d*x] + Sinh[c + d*x]^3)^2*(2401 + 1875*Sinh[c + d*x
]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]^4*(240
1 + 4180*Sinh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 244*Sinh[c + d*x]^6 + Sin
h[c + d*x]^8)))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*Sinh[c + d*x]^6*(32415 + 1
7320*Sinh[c + d*x]^2 + 753*Sinh[c + d*x]^4) + 3*a*b^2*Sinh[c + d*x]^4*(3601
5 + 50695*Sinh[c + d*x]^2 + 18073*Sinh[c + d*x]^4 + 753*Sinh[c + d*x]^6) +
3*a^2*b*Sinh[c + d*x]^2*(36015 + 88150*Sinh[c + d*x]^2 + 69728*Sinh[c + d*x
```

```
]^4 + 18826*Sinh[c + d*x]^6 + 753*Sinh[c + d*x]^8) + a^3*(36015 + 124165*Si
nh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 89514*Sinh[c + d*x]^6 + 19579*Sinh
[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(30240*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.00, size = 386, normalized size = 4.44

method	result
risch	$\frac{\mathrm{e}^{3dx+3c}a^3}{24d}+\frac{\mathrm{e}^{3dx+3c}a^2b}{8d}+\frac{a\,b^2\mathrm{e}^{3dx+3c}}{8d}+\frac{\mathrm{e}^{3dx+3c}b^3}{24d}+\frac{3\,\mathrm{e}^{dx+c}a^3}{8d}-\frac{3\,\mathrm{e}^{dx+c}a^2b}{8d}-\frac{15a\,\mathrm{e}^{dx+c}b^2}{8d}-\frac{9b^3\mathrm{e}^{dx+c}}{8d}-\frac{3\,\mathrm{e}^{-dx-c}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*a^3+1/8/d*exp(3*d*x+3*c)*a^2*b+1/8*a*b^2/d*exp(3*d*x+
3*c)+1/24/d*exp(3*d*x+3*c)*b^3+3/8/d*exp(d*x+c)*a^3-3/8/d*exp(d*x+c)*a^2*b-
15/8*a/d*exp(d*x+c)*b^2-9/8*b^3/d*exp(d*x+c)-3/8/d*exp(-d*x-c)*a^3+3/8/d*ex
p(-d*x-c)*a^2*b+15/8*a/d*exp(-d*x-c)*b^2+9/8/d*exp(-d*x-c)*b^3-1/24/d*exp(-
3*d*x-3*c)*a^3-1/8/d*exp(-3*d*x-3*c)*a^2*b-1/8*a*b^2/d*exp(-3*d*x-3*c)-1/24
/d*exp(-3*d*x-3*c)*b^3-b^3*exp(d*x+c)*(exp(2*d*x+2*c)-1)/d/(1+exp(2*d*x+2*c
))^2+5/2*I*b^3/d*ln(exp(d*x+c)+I)+3*I*b^2/d*ln(exp(d*x+c)+I)*a-3*I*b^2/d*ln
(exp(d*x+c)-I)*a-5/2*I*b^3/d*ln(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(81) = 162.
time = 0.47, size = 284, normalized size = 3.26

$$\frac{a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{8\,d}-\frac{1}{8}\,ab^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d}-\frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{48\,\arctan\left(e^{(-dx-c)}\right)}{d}\right)+\frac{1}{24}\,b^3\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)}\right)}\right)+\frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/8*a*b^2*((15*e^(-2*d*x - 2*c
) - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arct
an(e^(-d*x - c))/d) + 1/24*b^3*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 12
0*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e
^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x
- 7*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/
d - e^(-3*d*x - 3*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1840 vs. 2(81) = 162.
time = 0.38, size = 1840, normalized size = 21.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] 1/24*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*sinh(d*x + c)^10 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x +
c)^8 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3 + 45*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^3 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^7 + 2*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^6
 + 2*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^3 - 3*a^2*b
 - 21*a*b^2 - 25*b^3 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)^5 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^3 + 3*(5*a^3
- 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^3 -
3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^4 + 2*(105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*cosh(d*x + c)^6 + 35*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh
(d*x + c)^4 - 5*a^3 + 3*a^2*b + 21*a*b^2 + 25*b^3 + 15*(5*a^3 - 3*a^2*b - 2
1*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*
cosh(d*x + c)^5 + 5*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^3 -
 (5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3
 - 3*a^2*b - 3*a*b^2 - b^3 - (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*
x + c)^2 + (45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(11*a^3
 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^6 + 30*(5*a^3 - 3*a^2*b - 21*
a*b^2 - 25*b^3)*cosh(d*x + c)^4 - 11*a^3 + 3*a^2*b + 39*a*b^2 + 25*b^3 - 12
*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2
4*((6*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)*si
nh(d*x + c)^6 + (6*a*b^2 + 5*b^3)*sinh(d*x + c)^7 + 2*(6*a*b^2 + 5*b^3)*cos
h(d*x + c)^5 + (12*a*b^2 + 10*b^3 + 21*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^5 + 5*(7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 2*(6*a*b^2 + 5*b^
3)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (35
*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 6*a*b^2 + 5*b^3 + 20*(6*a*b^2 + 5*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^5 +
 20*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(6*a*b^2 + 5*b^3)*cosh(d*x + c))*
sinh(d*x + c)^2 + (7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(6*a*b^2 + 5*b^
3)*cosh(d*x + c)^4 + 3*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*ar
ctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^9 + 4*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^7
+ 6*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^5 - 4*(5*a^3 - 3*a^
2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^3 - (11*a^3 - 3*a^2*b - 39*a*b^2 - 2
5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)
*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x
 + c)^2 + 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c)
)*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 20*d*cosh(d
*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)
```

```
^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d*cosh(d*x + c)^6 + 10*d*cosh(
d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^3\cosh^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(81) = 162.
time = 0.59, size = 264, normalized size = 3.03

$$\frac{a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 3\,a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 3\,ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 12\,a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) - 36\,ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) - 24\,b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) - \frac{24\,b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)}{\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4} + 6\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(6\,ab^2 + 5\,b^3\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 3*a^2*b*(e^(d*x + c) - e^(-d*x -
 c))^3 + 3*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + b^3*(e^(d*x + c) - e^(-d*
x - c))^3 + 12*a^3*(e^(d*x + c) - e^(-d*x - c)) - 36*a*b^2*(e^(d*x + c) - e
^(-d*x - c)) - 24*b^3*(e^(d*x + c) - e^(-d*x - c)) - 24*b^3*(e^(d*x + c) -
e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4) + 6*(pi + 2*arctan(1/2*(
e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(6*a*b^2 + 5*b^3))/d
```

Mupad [B]
time = 0.36, size = 232, normalized size = 2.67

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(5\,b^3\,\sqrt{d^2}+6\,a\,b^2\,\sqrt{d^2}\right)}{d\,\sqrt{36\,a^2\,b^4+60\,a\,b^5+25\,b^6}}\right)\,\sqrt{36\,a^2\,b^4+60\,a\,b^5+25\,b^6}}{\sqrt{d^2}} - \frac{\mathrm{e}^{-3\,c-3\,d\,x}\,{\left(a+b\right)}^3}{24\,d} + \frac{\mathrm{e}^{3\,c+3\,d\,x}\,{\left(a+b\right)}^3}{24\,d} + \frac{3\,\mathrm{e}^{c+d\,x}\,{\left(a+b\right)}^2\,\left(a-3\,b\right)}{8\,d} - \frac{b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{3\,\mathrm{e}^{-c-d\,x}\,{\left(a+b\right)}^2\,\left(a-3\,b\right)}{8\,d} + \frac{2\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*b^3*(d^2)^(1/2) + 6*a*b^2*(d^2)^(1/2)))/(d*(60*a*
b^5 + 25*b^6 + 36*a^2*b^4)^(1/2)))*(60*a*b^5 + 25*b^6 + 36*a^2*b^4)^(1/2))/
(d^2)^(1/2) - (exp(- 3*c - 3*d*x)*(a + b)^3)/(24*d) + (exp(3*c + 3*d*x)*(a
+ b)^3)/(24*d) + (3*exp(c + d*x)*(a + b)^2*(a - 3*b))/(8*d) - (b^3*exp(c +
d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (3*exp(- c - d*x)*(a + b)^2*(a - 3*b))/(
8*d) + (2*b^3*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.99 $\int \cosh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal. Leaf size=78

$$\frac{1}{2}(a-5b)(a+b)^2x+\frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] 1/2*(a-5*b)*(a+b)^2*x+1/2*(a+b)^3*cosh(d*x+c)*sinh(d*x+c)/d+b^2*(3*a+2*b)*t
anh(d*x+c)/d+1/3*b^3*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3756, 398, 393, 212}

$$\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{(a+b)^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}x(a-5b)(a+b)^2+\frac{b^3\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a - 5*b)*(a + b)^2*x)/2 + ((a + b)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) +
 (b^2*(3*a + 2*b)*Tanh[c + d*x])/d + (b^3*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b^2(3a+2b)+b^3x^2+\frac{(a-2b)(a+b)^2+3b(a+b)^2x^2}{(1-x^2)^2}\right)\,dx, x,\right.}{d}$$

$$= \frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}+\frac{\text{Subst}\left(\int \frac{(a-2}{}\right.}{}$$

$$= \frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2(3a+2b)\tanh(c+dx}{d}$$

$$= \frac{1}{2}(a-5b)(a+b)^2x+\frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2}{}$$

Mathematica [A]
time = 0.60, size = 69, normalized size = 0.88

$$\frac{6(a-5b)(a+b)^2(c+dx)+3(a+b)^3\sinh(2(c+dx))+4b^2\left(9a+7b-b\,\text{sech}^2(c+dx)\right)\tanh(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
[Out] (6*(a - 5*b)*(a + b)^2*(c + d*x) + 3*(a + b)^3*Sinh[2*(c + d*x)] + 4*b^2*(9
*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(72) = 144$.
time = 1.53, size = 148, normalized size = 1.90

method	result
derivativedivides	$\frac{a^3\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a\,b^2\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right.}{d}$

default	$\frac{a^3\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a\,b^2\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)-}{d}$
risch	$\frac{a^3x}{2}-\frac{3a^2bx}{2}-\frac{9a\,b^2x}{2}-\frac{5b^3x}{2}+\frac{e^{2dx+2c}a^3}{8d}+\frac{3b\,e^{2dx+2c}a^2}{8d}+\frac{3\,e^{2dx+2c}a\,b^2}{8d}+\frac{b^3e^{2dx+2c}}{8d}-\frac{e^{-2dx-2c}a^3}{8d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sinh(d*x+c)*cosh(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(1/2*sinh(d*x+
c)*cosh(d*x+c)-1/2*d*x-1/2*c)+3*a*b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*
x-3/2*c+3/2*tanh(d*x+c))+b^3*(1/2*sinh(d*x+c)^5/cosh(d*x+c)^3-5/2*d*x-5/2*c
+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(72) = 144$.
time = 0.28, size = 256, normalized size = 3.28

$$\frac{1}{8}a^3\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{3}{8}a^2b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{24}b^3\left(\frac{60(dx+c)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx-2c)}+201e^{(-4dx-4c)}+147e^{(-6dx-6c)}+3}{d(e^{(-2dx-2c)}+3e^{(-4dx-4c)}+3e^{(-6dx-6c)}+e^{(-8dx-8c)})}\right)-\frac{3}{8}ab^2\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 3/8*a^2*b*(4*x - e
^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/24*b^3*(60*(d*x + c)/d + 3*e^(-2
*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x - 4*c) + 147*e^(-6*d*
x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c
) + e^(-8*d*x - 8*c)))) - 3/8*a*b^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d -
(17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(72) = 144$.
time = 0.37, size = 369, normalized size = 4.73

$$\frac{3(a^3+3a^2b+3ab^2+b^3)\sinh(dx+c)^5-4(18ab^2+14b^3-3(a^3-3a^2b-9ab^2-5b^3)dx)\cosh(dx+c)^3-12(18ab^2+14b^3-3(a^3-3a^2b-9ab^2-5b^3)dx)\cosh(dx+c)\sinh(dx+c)^2+(9a^3+27a^2b+99ab^2+65b^3+30(a^3+3a^2b+3ab^2+b^3)\cosh(dx+c)^2)\sinh(dx+c)^3-12(18ab^2+14b^3-3(a^3-3a^2b-9ab^2-5b^3)dx)\cosh(dx+c)+3(5(a^3+3a^2b+3ab^2+b^3)\cosh(dx+c)^4+2a^3+6a^2b+30ab^2+10b^3+(9a^3+27a^2b+99ab^2+65b^3)\cosh(dx+c)^2)\sinh(dx+c)}{24(d\cosh(dx+c)^3+3d\cosh(dx+c)\sinh(dx+c)^2+3d\cosh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 - 4*(18*a*b^2 + 14*
b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)^3 - 12*(18*a*b
^2 + 14*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)*sinh(d
*x + c)^2 + (9*a^3 + 27*a^2*b + 99*a*b^2 + 65*b^3 + 30*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 12*(18*a*b^2 + 14*b^3 - 3*(a
^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c) + 3*(5*(a^3 + 3*a^2*b +
```

```
3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*a^3 + 6*a^2*b + 30*a*b^2 + 10*b^3 + (9*a
^3 + 27*a^2*b + 99*a*b^2 + 65*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\cosh^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(72) = 144.
time = 0.56, size = 265, normalized size = 3.40

$$\frac{3\,a^3e^{(2\,dx+2\,c)}+9\,a^2be^{(2\,dx+2\,c)}+9\,ab^2e^{(2\,dx+2\,c)}+3\,b^3e^{(2\,dx+2\,c)}+12\,(a^3-3\,a^2b-9\,ab^2-5\,b^3)(dx+c)-3\left(2\,a^3e^{(2\,dx+2\,c)}-6\,a^2be^{(2\,dx+2\,c)}-18\,ab^2e^{(2\,dx+2\,c)}-10\,b^3e^{(2\,dx+2\,c)}+a^3+3\,a^2b+3\,ab^2+b^3\right)e^{(-2\,dx-2\,c)}-\frac{16\,(9\,ab^2e^{(4\,dx+4\,c)}+9\,b^3e^{(4\,dx+4\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+12\,b^3e^{(2\,dx+2\,c)}+9\,ab^2+7\,b^3)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^3*e^(2*d*x + 2*c) + 9*a^2*b*e^(2*d*x + 2*c) + 9*a*b^2*e^(2*d*x +
2*c) + 3*b^3*e^(2*d*x + 2*c) + 12*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*(d*x +
c) - 3*(2*a^3*e^(2*d*x + 2*c) - 6*a^2*b*e^(2*d*x + 2*c) - 18*a*b^2*e^(2*d*x
 + 2*c) - 10*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-2*d*x
 - 2*c) - 16*(9*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^
(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 7*b^3)/(e^(2*d*x + 2*c)
+ 1)^3)/d
```

Mupad [B]
time = 0.29, size = 243, normalized size = 3.12

$$\frac{\mathrm{e}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^3}{8\,d}-\frac{\frac{2\,\left(b^3+3\,a\,b^2\right)}{3\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(b^3+a\,b^2\right)}{d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}-\frac{2\,\left(b^3+a\,b^2\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,{\left(a+b\right)}^3}{8\,d}-\frac{\frac{2\,\left(b^3+a\,b^2\right)}{d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(b^3+3\,a\,b^2\right)}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(b^3+a\,b^2\right)}{d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}+\frac{x\,{\left(a+b\right)}^2\,\left(a-5\,b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - ((2*(3*a*b^2 + b^3))/(3*d) + (2*exp(2*
c + 2*d*x)*(a*b^2 + b^3))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) -
(2*(a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a + b)^
3)/(8*d) - ((2*(a*b^2 + b^3))/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + b^3))/(3*d
) + (2*exp(4*c + 4*d*x)*(a*b^2 + b^3))/d)/(3*exp(2*c + 2*d*x) + 3*exp(4*c +
 4*d*x) + exp(6*c + 6*d*x) + 1) + (x*(a + b)^2*(a - 5*b))/2
```

3.100 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal. Leaf size=99

$$-\frac{3b(4(a+b)^2+(2a+b)^2)\,\mathrm{ArcTan}(\sinh(c+dx))}{8d}+\frac{(a+b)^3\sinh(c+dx)}{d}+\frac{3b^2(4a+3b)\mathrm{sech}(c+dx)\tanh(c+\ldots}{8d}$$

```
[Out] -3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctan(sinh(d*x+c))/d+(a+b)^3*sinh(d*x+c)/d+3/
8*b^2*(4*a+3*b)*sech(d*x+c)*tanh(d*x+c)/d-1/4*b^3*sech(d*x+c)^3*tanh(d*x+c)
/d
```

Rubi [A]
time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3757, 398, 1171, 393, 209}

$$-\frac{3b(4(a+b)^2+(2a+b)^2)\,\mathrm{ArcTan}(\sinh(c+dx))}{8d}+\frac{3b^2(4a+3b)\tanh(c+dx)\mathrm{sech}(c+dx)}{8d}+\frac{(a+b)^3\sinh(c+dx)}{d}-\frac{b^3\tanh(c+dx)\mathrm{sech}^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[Sinh[c + d*x]])/(8*d) + ((a + b)^3
*Sinh[c + d*x])/d + (3*b^2*(4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) -
 (b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^3}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((a+b)^3 - \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(1+x^2)^3}\right) dx$$
$$= \frac{(a+b)^3\sinh(c+dx)}{d} - \frac{\text{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3}{(1+x^2)^3}\right.}{d}$$
$$= \frac{(a+b)^3\sinh(c+dx)}{d} - \frac{b^3\text{sech}^3(c+dx)\tanh(c+dx)}{4d} + \text{Subst}$$
$$= \frac{(a+b)^3\sinh(c+dx)}{d} + \frac{3b^2(4a+3b)\text{sech}(c+dx)\tanh(c+dx}{8d}$$
$$= -\frac{3b(4(a+b)^2+(2a+b)^2)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{(a+b)^3\sin}{d}$$

Mathematica [A]
time = 0.38, size = 93, normalized size = 0.94

$$\frac{-6b(8a^2+12ab+5b^2)\,\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+8(a+b)^3\sinh(c+dx)+3b^2(4a+3b)\text{sech}(c+dx)\tanh(c+dx)-2b^3\text{sech}^3(c+dx)\tanh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

[Out] (-6*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[Tanh[(c + d*x)/2]] + 8*(a + b)^3*Sinh
[c + d*x] + 3*b^2*(4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x] - 2*b^3*Sech[c +
d*x]^3*Tanh[c + d*x])/(8*d)

Maple [C] Result contains complex when optimal does not.
time = 2.11, size = 353, normalized size = 3.57

method	result
risch	$\frac{e^{dx+c}a^3}{2d}+\frac{3\,e^{dx+c}a^2b}{2d}+\frac{3a\,e^{dx+c}b^2}{2d}+\frac{b^3e^{dx+c}}{2d}-\frac{e^{-dx-c}a^3}{2d}-\frac{3\,e^{-dx-c}a^2b}{2d}-\frac{3a\,e^{-dx-c}b^2}{2d}-\frac{e^{-dx-c}b^3}{2d}+\frac{b^2e^{dx+c}(12a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*exp(d*x+c)*a^3+3/2/d*exp(d*x+c)*a^2*b+3/2*a/d*exp(d*x+c)*b^2+1/2*b^3/
d*exp(d*x+c)-1/2/d*exp(-d*x-c)*a^3-3/2/d*exp(-d*x-c)*a^2*b-3/2*a/d*exp(-d*x
-c)*b^2-1/2/d*exp(-d*x-c)*b^3+1/4*b^2*exp(d*x+c)*(12*a*exp(6*d*x+6*c)+9*b*e
xp(6*d*x+6*c)+12*a*exp(4*d*x+4*c)+b*exp(4*d*x+4*c)-12*a*exp(2*d*x+2*c)-b*ex
p(2*d*x+2*c)-12*a-9*b)/d/(1+exp(2*d*x+2*c))^4+3*I*b/d*ln(exp(d*x+c)-I)*a^2+
9/2*I*b^2/d*ln(exp(d*x+c)-I)*a+15/8*I*b^3/d*ln(exp(d*x+c)-I)-3*I*b/d*ln(exp
(d*x+c)+I)*a^2-9/2*I*b^2/d*ln(exp(d*x+c)+I)*a-15/8*I*b^3/d*ln(exp(d*x+c)+I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(93) = 186.
time = 0.49, size = 295, normalized size = 2.98

$$\frac{1}{4}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{2\,e^{(-dx-c)}}{d}+\frac{17\,e^{(-2dx-2c)}+13\,e^{(-4dx-4c)}+7\,e^{(-6dx-6c)}-7\,e^{(-8dx-8c)}+2}{d\left(e^{(-dx-c)}+4\,e^{(-3dx-3c)}+6\,e^{(-5dx-5c)}+4\,e^{(-7dx-7c)}+e^{(-9dx-9c)}\right)}\right)+\frac{3}{2}ab^2\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2dx-2c)}-e^{(-4dx-4c)}+1}{d\left(e^{(-dx-c)}+2\,e^{(-3dx-3c)}+e^{(-5dx-5c)}\right)}\right)+\frac{3}{2}a^2b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a^3\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*(15*arctan(e^(-d*x - c))/d - 2*e^(-d*x - c)/d + (17*e^(-2*d*x - 2*c
) + 13*e^(-4*d*x - 4*c) + 7*e^(-6*d*x - 6*c) - 7*e^(-8*d*x - 8*c) + 2)/(d*(
e^(-d*x - c) + 4*e^(-3*d*x - 3*c) + 6*e^(-5*d*x - 5*c) + 4*e^(-7*d*x - 7*c)
 + e^(-9*d*x - 9*c)))) + 3/2*a*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)
/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3
*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 3/2*a^2*b*(4*arctan(e^(-d*x - c))/d + e
^(d*x + c)/d - e^(-d*x - c)/d) + a^3*sinh(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2411 vs. 2(93) = 186.
time = 0.47, size = 2411, normalized size = 24.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/4*(2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 20*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 2*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*sinh(d*x + c)^10 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x
 + c)^8 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3 + 30*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 24*(10*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)^3 + (2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x +
c))*sinh(d*x + c)^7 + (4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^6
 + (420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^3 + 12*a^2*b
+ 24*a*b^2 + 5*b^3 + 84*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 6*(84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5
+ 28*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (4*a^3 + 12*a^2
*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (4*a^3 + 12*a^2*b +
 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + (420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^6 + 210*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^4 -
 4*a^3 - 12*a^2*b - 24*a*b^2 - 5*b^3 + 15*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(60*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(d*x + c)^7 + 42*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^
5 + 5*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - (4*a^3 + 12*a
^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*a^3 - 6*a^2*b -
 6*a*b^2 - 2*b^3 - 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^2 +
 3*(30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(2*a^3 + 6*a^2*
b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 5*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*
b^3)*cosh(d*x + c)^4 - 2*a^3 - 6*a^2*b - 10*a*b^2 - 5*b^3 - 2*(4*a^3 + 12*a
^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b + 1
2*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x
+ c)*sinh(d*x + c)^8 + (8*a^2*b + 12*a*b^2 + 5*b^3)*sinh(d*x + c)^9 + 4*(8*
a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 4*(8*a^2*b + 12*a*b^2 + 5*b^3 +
 9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8
*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (8*a^2*b + 12*a*b^2 + 5*b^3)*c
osh(d*x + c))*sinh(d*x + c)^6 + 6*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c
)^5 + 6*(21*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 12*a*b
^2 + 5*b^3 + 14*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^5 + 2*(63*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 70*(8*a^2*b + 12*
a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 15*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^4 + 4*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 4*(
21*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 35*(8*a^2*b + 12*a*b^2 +
5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 12*a*b^2 + 5*b^3 + 15*(8*a^2*b + 12*a*b^
2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(3*(8*a^2*b + 12*a*b^2 + 5
*b^3)*cosh(d*x + c)^7 + 7*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*
(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (8*a^2*b + 12*a*b^2 + 5*b^3)
*cosh(d*x + c))*sinh(d*x + c)^2 + (8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c
) + (9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 28*(8*a^2*b + 12*a*b^
2 + 5*b^3)*cosh(d*x + c)^6 + 30*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^
```

```
4 + 8*a^2*b + 12*a*b^2 + 5*b^3 + 12*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(10*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 12*(2*a^3 + 6*a^2*b + 10*a*b^2 +
 5*b^3)*cosh(d*x + c)^7 + 3*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x
+ c)^5 - 2*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - 3*(2*a^3
 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 4*d*cosh(d*x
 + c)^7 + 4*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x +
c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cos
h(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x
 + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*d*
cosh(d*x + c)^3 + 4*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cos
h(d*x + c)^2 + d)*sinh(d*x + c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x
+ c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*
x + c) + (9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4
 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\cosh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(93) = 186.
time = 0.56, size = 272, normalized size = 2.75

$$\frac{8\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+24\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+24\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+8\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)-3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(8\,a^2b+12\,ab^2+5\,b^3\right)+\frac{4\left(12\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+9\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+48\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+28\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^3*(e^(d*x + c) - e^(-d*x - c)) + 24*a^2*b*(e^(d*x + c) - e^(-d*x
- c)) + 24*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 8*b^3*(e^(d*x + c) - e^(-d*
x - c)) - 3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2*
b + 12*a*b^2 + 5*b^3) + 4*(12*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 9*b^3*
(e^(d*x + c) - e^(-d*x - c))^3 + 48*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 28
*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/
d
```

Mupad [B]
time = 0.31, size = 355, normalized size = 3.59

$$\frac{e^{c+dx}(a+b)^3}{2d}-\frac{e^{-c-dx}(a+b)^3}{2d}-\frac{3\,\mathrm{atan}\left(\frac{e^{dx}e^{c}\left(5b^3\sqrt{d^2}+12ab^2\sqrt{d^2}+8a^2b\sqrt{d^2}\right)}{d\sqrt{64a^4b^2+192a^3b^3+224a^2b^4+120ab^5+25b^6}}\right)\sqrt{64a^4b^2+192a^3b^3+224a^2b^4+120ab^5+25b^6}}{4\sqrt{d^2}}+\frac{3e^{c+dx}(3b^3+4ab^2)}{4d\left(e^{2c+2dx}+1\right)}+\frac{6b^3e^{c+dx}}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}-\frac{e^{c+dx}(13b^3+12ab^2)}{2d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}-\frac{4b^3e^{c+dx}}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (exp(c + d*x)*(a + b)^3)/(2*d) - (exp(- c - d*x)*(a + b)^3)/(2*d) - (3*atan
((exp(d*x)*exp(c)*(5*b^3*(d^2)^(1/2) + 12*a*b^2*(d^2)^(1/2) + 8*a^2*b*(d^2)
^(1/2)))/(d*(120*a*b^5 + 25*b^6 + 224*a^2*b^4 + 192*a^3*b^3 + 64*a^4*b^2)^(
1/2)))*(120*a*b^5 + 25*b^6 + 224*a^2*b^4 + 192*a^3*b^3 + 64*a^4*b^2)^(1/2))
/(4*(d^2)^(1/2)) + (3*exp(c + d*x)*(4*a*b^2 + 3*b^3))/(4*d*(exp(2*c + 2*d*x
) + 1)) + (6*b^3*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
+ exp(6*c + 6*d*x) + 1)) - (exp(c + d*x)*(12*a*b^2 + 13*b^3))/(2*d*(2*exp(2
*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (4*b^3*exp(c + d*x))/(d*(4*exp(2*c +
 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.101 $\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=149

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\operatorname{ArcTan}(\sinh(c+dx))}{16d}-\frac{b\left(44a^2+44ab+15b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d}-\frac{5b(2a\ -$$

```
[Out] 1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan(sinh(d*x+c))/d-1/48*b*(44*a^2+44*a*
b+15*b^2)*sech(d*x+c)*tanh(d*x+c)/d-5/24*b*(2*a+b)*sech(d*x+c)^3*(a+(a+b)*s
inh(d*x+c)^2)*tanh(d*x+c)/d-1/6*b*sech(d*x+c)^5*(a+(a+b)*sinh(d*x+c)^2)^2*t
anh(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3757, 424, 540, 393, 209}

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\operatorname{ArcTan}(\sinh(c+dx))}{16d}-\frac{b\left(44a^2+44ab+15b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{48d}-\frac{b\tanh(c+dx)\operatorname{sech}^5(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)^2}{6d}-\frac{5b(2a+b)\tanh(c+dx)\operatorname{sech}^3(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) - (b*(44*a
^2 + 44*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) - (5*b*(2*a + b)*
Sech[c + d*x]^3*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) - (b*Se
ch[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
```

```
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)^3}{(1+x^2)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d} + $$

$$= -\frac{5b(2a+b)\operatorname{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+}{24d}$$

$$= -\frac{b(44a^2+44ab+15b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d} - \frac{5b(2a+}{}$$

$$= \frac{(2a+b)(8a^2+8ab+5b^2)\tan^{-1}(\sinh(c+dx))}{16d} - \frac{b(44a^2+44}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.72, size = 1341, normalized size = 9.00

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (a^3*Sinh[c + d*x]*((9514449*(a + b))/a + (135323370*(a + b)^2)/a^2 + (5800
9455*(a + b)^3)/a^3 + 4093425*Csch[c + d*x]^2 + (168951510*(a + b)*Csch[c +
 d*x]^2)/a + (215549775*(a + b)^2*Csch[c + d*x]^2)/a^2 + 70189350*Csch[c +
d*x]^4 + (274542345*(a + b)*Csch[c + d*x]^4)/a + 117228825*Csch[c + d*x]^6
+ (7808535*(a + b)^2*Sinh[c + d*x]^2)/a^2 + (36772890*(a + b)^3*Sinh[c + d*
x]^2)/a^3 - 75520*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Si
nh[c + d*x]^2]*Sinh[c + d*x]^2 - 13824*HypergeometricPFQ[{3/2, 2, 2, 2, 2,
2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 1024*Hypergeome
tricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*S
inh[c + d*x]^2 + (2160711*(a + b)^3*Sinh[c + d*x]^4)/a^3 - (189696*(a + b)*
HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sin
h[c + d*x]^4)/a - (38400*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1
, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (3072*(a + b)*Hype
rgeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x
]^2]*Sinh[c + d*x]^4)/a - (158976*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2
, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (35328*(a +
 b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c +
 d*x]^2]*Sinh[c + d*x]^6)/a^2 - (3072*(a + b)^2*HypergeometricPFQ[{3/2, 2,
2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a
^2 - (44800*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2},
 -Sinh[c + d*x]^2]*Sinh[c + d*x]^8)/a^3 - (10752*(a + b)^3*HypergeometricPF
Q[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]
^8)/a^3 - (1024*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1,
 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8)/a^3 + (142065*ArcTanh[S
qrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8)/(-Sinh[c + d*x]^2)^(9/2) + (1172288
25*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(7/2) + (17069535*Ar
cTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4)/(-Sinh[c + d*x]^2)^(7/2) + (
33756345*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8)/(a^2*(-
Sinh[c + d*x]^2)^(7/2)) + (56109375*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2
]]*Sinh[c + d*x]^8)/(a^3*(-Sinh[c + d*x]^2)^(7/2)) - (109265625*ArcTanh[Sqr
t[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(5/2) - (274542345*(a + b)*ArcTanh
[Sqrt[-Sinh[c + d*x]^2]])/(a*(-Sinh[c + d*x]^2)^(5/2)) + (260465625*(a + b)
*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a*(-Sinh[c + d*x]^2)^(3/2)) + (215549775
*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a^2*(-Sinh[c + d*x]^2)^(3/2))
+ (174825*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6)/(a^2*(
-Sinh[c + d*x]^2)^(3/2)) + (9261945*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2
]]*Sinh[c + d*x]^6)/(a^3*(-Sinh[c + d*x]^2)^(3/2)) + (48825*(a + b)^3*ArcTa
nh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8)/(a^3*(-Sinh[c + d*x]^2)^(3/2))
- (41427855*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a*Sqrt[-Sinh[c + d*x]
^2]) - (207173295*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a^2*Sqrt[-Sin
h[c + d*x]^2]) - (58009455*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a^3*
Sqrt[-Sinh[c + d*x]^2]) + (210735*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*S
qrt[-Sinh[c + d*x]^2])/a))/(725760*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.17, size = 403, normalized size = 2.70

method	result
risch	$-\frac{b\,\mathrm{e}^{dx+c}\left(72a^2\mathrm{e}^{10dx+10c}+90ab\,\mathrm{e}^{10dx+10c}+33b^2\mathrm{e}^{10dx+10c}+216a^2\mathrm{e}^{8dx+8c}+126ab\,\mathrm{e}^{8dx+8c}-5b^2\mathrm{e}^{8dx+8c}+144a^2\mathrm{e}^{6dx+6c}+36ab\,\mathrm{e}^{6dx+}\right.}{24d(1+\mathrm{e}^{2c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*b*exp(d*x+c)*(72*a^2*exp(10*d*x+10*c)+90*a*b*exp(10*d*x+10*c)+33*b^2*
exp(10*d*x+10*c)+216*a^2*exp(8*d*x+8*c)+126*a*b*exp(8*d*x+8*c)-5*b^2*exp(8*
d*x+8*c)+144*a^2*exp(6*d*x+6*c)+36*a*b*exp(6*d*x+6*c)+90*b^2*exp(6*d*x+6*c)
-144*a^2*exp(4*d*x+4*c)-36*a*b*exp(4*d*x+4*c)-90*b^2*exp(4*d*x+4*c)-216*a^2
*exp(2*d*x+2*c)-126*a*b*exp(2*d*x+2*c)+5*b^2*exp(2*d*x+2*c)-72*a^2-90*a*b-3
3*b^2)/d/(1+exp(2*d*x+2*c))^6+I/d*ln(exp(d*x+c)+I)*a^3+3/2*I/d*ln(exp(d*x+c
)+I)*a^2*b+9/8*I/d*ln(exp(d*x+c)+I)*a*b^2+5/16*I/d*ln(exp(d*x+c)+I)*b^3-I/d
*ln(exp(d*x+c)-I)*a^3-3/2*I/d*ln(exp(d*x+c)-I)*a^2*b-9/8*I/d*ln(exp(d*x+c)-
I)*a*b^2-5/16*I/d*ln(exp(d*x+c)-I)*b^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(141) = 282.
time = 0.50, size = 362, normalized size = 2.43

$$-\frac{1}{24}\,b^3\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{33\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}+90\,e^{(-5\,dx-5\,c)}-90\,e^{(-7\,dx-7\,c)}+5\,e^{(-9\,dx-9\,c)}-33\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)-\frac{3}{4}\,ab^2\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5\,e^{(-dx-c)}-3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a^3\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*b^3*(15*arctan(e^(-d*x - c))/d + (33*e^(-d*x - c) - 5*e^(-3*d*x - 3*c
) + 90*e^(-5*d*x - 5*c) - 90*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) - 33*e^(
-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*
x - 3*c) + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c)
+ 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3*a^2
*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*arctan(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3465 vs. 2(141) = 282.
time = 0.39, size = 3465, normalized size = 23.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(3*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c)^11 + 33*(24*a^2*b + 3
0*a*b^2 + 11*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(24*a^2*b + 30*a*b^2 +
 11*b^3)*sinh(d*x + c)^11 + (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^9
 + (216*a^2*b + 126*a*b^2 - 5*b^3 + 165*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^9 + 9*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*
x + c)^3 + (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 +
 18*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 18*(55*(24*a^2*b + 30*a*b
^2 + 11*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 2*a*b^2 + 5*b^3 + 2*(216*a^2*b + 1
26*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(24*a^2*b + 30*
a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 2*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*
x + c)^3 + 3*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 1
8*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 18*(77*(24*a^2*b + 30*a*b^2
 + 11*b^3)*cosh(d*x + c)^6 + 7*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c
)^4 - 8*a^2*b - 2*a*b^2 - 5*b^3 + 21*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^5 + 18*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c
)^7 + 7*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 35*(8*a^2*b + 2*a
*b^2 + 5*b^3)*cosh(d*x + c)^3 - 5*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^4 - (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + (495*
(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c)^8 + 84*(216*a^2*b + 126*a*b^2
- 5*b^3)*cosh(d*x + c)^6 + 630*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^4
- 216*a^2*b - 126*a*b^2 + 5*b^3 - 180*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^3 + 3*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c
)^9 + 12*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 126*(8*a^2*b + 2
*a*b^2 + 5*b^3)*cosh(d*x + c)^5 - 60*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x +
 c)^3 - (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*
((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 12*(16*a^3 + 24*
a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (16*a^3 + 24*a^2
*b + 18*a*b^2 + 5*b^3)*sinh(d*x + c)^12 + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 +
 5*b^3)*cosh(d*x + c)^10 + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 11*(16
*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*
(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(16*a^3 + 24
*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(16*a^3 + 24
*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 15*(33*(16*a^3 + 24*a^2*b + 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 18
*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
24*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 30*(16*a^3
+ 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*
a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(16*a^3 + 24*a^2*b + 18*
a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 4*(231*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b
^3)*cosh(d*x + c)^6 + 315*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
 c)^4 + 80*a^3 + 120*a^2*b + 90*a*b^2 + 25*b^3 + 105*(16*a^3 + 24*a^2*b + 1
8*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(33*(16*a^3 + 24*a^2
*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 63*(16*a^3 + 24*a^2*b + 18*a*b^2 +
```

```
5*b^3)*cosh(d*x + c)^5 + 35*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*
x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 15*(3
3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 84*(16*a^3 + 24*
a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 70*(16*a^3 + 24*a^2*b + 18*a*b^
2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 20*(16*
a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 20*(1
1*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 36*(16*a^3 + 24*
a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 42*(16*a^3 + 24*a^2*b + 18*a*b^
2 + 5*b^3)*cosh(d*x + c)^5 + 20*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh
(d*x + c)^3 + 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(
d*x + c)^3 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 6*(16*a^3 + 24*a^2*b +
18*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 6*(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5
*b^3)*cosh(d*x + c)^10 + 45*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x
 + c)^8 + 70*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 50*(1
6*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b +
18*a*b^2 + 5*b^3 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 12*((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
c)^11 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 10*(16*a
^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 10*(16*a^3 + 24*a^2*b +
 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^
3)*cosh(d*x + c)^3 + (16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*
sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(24*a^2*b + 30*a*b
^2 + 11*b^3)*cosh(d*x + c) + 3*(11*(24*a^2*b + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(141) = 282.
time = 0.52, size = 310, normalized size = 2.08

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(16a^3+24a^2b+18ab^2+5b^3\right)-\frac{4\left(72a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+90ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+33b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+576a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+576ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+160b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+1152a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+864ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+240b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(16*a^3 + 2
4*a^2*b + 18*a*b^2 + 5*b^3) - 4*(72*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 +
```

```
90*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 33*b^3*(e^(d*x + c) - e^(-d*x - c
))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 + 576*a*b^2*(e^(d*x + c) -
e^(-d*x - c))^3 + 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 1152*a^2*b*(e^(d
*x + c) - e^(-d*x - c)) + 864*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 240*b^3*
(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

Mupad [B]
time = 1.39, size = 535, normalized size = 3.59

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(16\,a^3\,\sqrt{d^2}+5\,b^3\,\sqrt{d^2}+18\,a\,b^2\,\sqrt{d^2}+24\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{256\,a^6+768\,a^5\,b+1152\,a^4\,b^2+1024\,a^3\,b^3+564\,a^2\,b^4+180\,a\,b^5+25\,b^6}}\right)\,\sqrt{256\,a^6+768\,a^5\,b+1152\,a^4\,b^2+1024\,a^3\,b^3+564\,a^2\,b^4+180\,a\,b^5+25\,b^6}}{8\,\sqrt{d^2}}-\frac{e^{c+dx}\,\left(55\,b^3+54\,a\,b^2\right)}{3\,d\,\left(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1\right)}-\frac{80\,b^3\,e^{c+dx}}{3\,d\,\left(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1\right)}+\frac{6\,e^{c+dx}\,\left(5\,b^3+2\,a\,b^2\right)}{d\,\left(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}+\frac{32\,b^3\,e^{c+dx}}{3\,d\,\left(6\,e^{2c+2dx}+15\,e^{4c+4dx}+20\,e^{6c+6dx}+15\,e^{8c+8dx}+6\,e^{10c+10dx}+e^{12c+12dx}+1\right)}-\frac{e^{c+dx}\,\left(24\,a^2\,b+30\,a\,b^2+11\,b^3\right)}{8\,d\,\left(e^{2c+2dx}+1\right)}+\frac{e^{c+dx}\,\left(72\,a^2\,b+162\,a\,b^2+85\,b^3\right)}{12\,d\,\left(2\,e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(16*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 18*a*b^2*(
d^2)^(1/2) + 24*a^2*b*(d^2)^(1/2)))/(d*(180*a*b^5 + 768*a^5*b + 256*a^6 + 2
5*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2)))*(180*a*b^5 + 768
*a^5*b + 256*a^6 + 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2
))/(8*(d^2)^(1/2)) - (exp(c + d*x)*(54*a*b^2 + 55*b^3))/(3*d*(3*exp(2*c + 2
*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (80*b^3*exp(c + d*x))
/(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*e
xp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*exp(c + d*x)*(2*a*b^2 + 5*b
^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp
(8*c + 8*d*x) + 1)) + (32*b^3*exp(c + d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*e
xp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c +
10*d*x) + exp(12*c + 12*d*x) + 1)) - (exp(c + d*x)*(30*a*b^2 + 24*a^2*b + 1
1*b^3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(162*a*b^2 + 72*a^2*b
+ 85*b^3))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.102 $\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=67

$$\frac{a^3\tanh(c+dx)}{d}+\frac{a^2b\tanh^3(c+dx)}{d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

```
[Out] a^3*tanh(d*x+c)/d+a^2*b*tanh(d*x+c)^3/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/7*b^3*t
anh(d*x+c)^7/d
```

Rubi [A]

time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 200}

$$\frac{a^3\tanh(c+dx)}{d}+\frac{a^2b\tanh^3(c+dx)}{d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d + (a^2*b*Tanh[c + d*x]^3)/d + (3*a*b^2*Tanh[c + d*x]^
5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \left(a+bx^2\right)^3\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a^3\tanh(c+dx)}{d}+\frac{a^2b\tanh^3(c+dx)}{d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\end{aligned}$$

Mathematica [A]
time = 0.13, size = 67, normalized size = 1.00

$$\frac{a^3\tanh(c+dx)}{d}+\frac{a^2b\tanh^3(c+dx)}{d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d + (a^2*b*Tanh[c + d*x]^3)/d + (3*a*b^2*Tanh[c + d*x]^
5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(63) = 126.
time = 2.13, size = 348, normalized size = 5.19

method	result
risch	$-\frac{2(21a\,b^2+105a^2b\,e^{12dx+12c}+105a\,b^2e^{12dx+12c}+420a^2b\,e^{10dx+10c}+210a\,b^2e^{10dx+10c}+315a^2b\,e^{4dx+4c}+140a^2b\,e^{2dx+2c}+35a^2b+35}{}$…

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*(21*a*b^2+105*a^2*b*exp(12*d*x+12*c)+105*a*b^2*exp(12*d*x+12*c)+420*a
^2*b*exp(10*d*x+10*c)+210*a*b^2*exp(10*d*x+10*c)+315*a^2*b*exp(4*d*x+4*c)+1
40*a^2*b*exp(2*d*x+2*c)+35*a^2*b+35*a^3+5*b^3+315*a*b^2*exp(8*d*x+8*c)+420*
a*b^2*exp(6*d*x+6*c)+231*a*b^2*exp(4*d*x+4*c)+665*a^2*b*exp(8*d*x+8*c)+42*a
*b^2*exp(2*d*x+2*c)+560*a^2*b*exp(6*d*x+6*c)+210*a^3*exp(2*d*x+2*c)+35*a^3*
exp(12*d*x+12*c)+35*b^3*exp(12*d*x+12*c)+210*a^3*exp(10*d*x+10*c)+175*b^3*e
xp(8*d*x+8*c)+525*a^3*exp(4*d*x+4*c)+105*b^3*exp(4*d*x+4*c)+525*a^3*exp(8*d
*x+8*c)+700*a^3*exp(6*d*x+6*c))/d/(1+exp(2*d*x+2*c))^7
```

Maxima [A]
time = 0.26, size = 71, normalized size = 1.06

$$\frac{b^3\tanh\left(dx+c\right)^7}{7\,d}+\frac{3\,ab^2\tanh\left(dx+c\right)^5}{5\,d}+\frac{a^2b\tanh\left(dx+c\right)^3}{d}+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*tanh(d*x + c)^7/d + 3/5*a*b^2*tanh(d*x + c)^5/d + a^2*b*tanh(d*x +
c)^3/d + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 786 vs. 2(63) = 126.
time = 0.37, size = 786, normalized size = 11.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/35*((35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3)*cosh(d*x + c)^6 + 6*(35*a^2*
b + 42*a*b^2 + 15*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (35*a^3 + 70*a^2*b +
 63*a*b^2 + 20*b^3)*sinh(d*x + c)^6 + 14*(15*a^3 + 20*a^2*b + 9*a*b^2)*cosh
(d*x + c)^4 + (210*a^3 + 280*a^2*b + 126*a*b^2 + 15*(35*a^3 + 70*a^2*b + 63
*a*b^2 + 20*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(35*a^2*b + 42*a*b
^2 + 15*b^3)*cosh(d*x + c)^3 + 28*(5*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d
*x + c)^3 + 350*a^3 + 280*a^2*b + 210*a*b^2 + 7*(75*a^3 + 70*a^2*b + 39*a*b
^2 + 20*b^3)*cosh(d*x + c)^2 + (15*(35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3)*
cosh(d*x + c)^4 + 525*a^3 + 490*a^2*b + 273*a*b^2 + 140*b^3 + 84*(15*a^3 +
20*a^2*b + 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(35*a^2*b + 42*
a*b^2 + 15*b^3)*cosh(d*x + c)^5 + 56*(5*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 +
7*(25*a^2*b + 6*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 8*d*cosh(d*x
 + c)^6 + 4*(7*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^6 + 4*(14*d*cosh(d*x
+ c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 2*(35*
d*cosh(d*x + c)^4 + 60*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^4 + 8*(7*d*c
osh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^3
+ 56*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 42
*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 9*d*c
osh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c) +
35*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^{2}\left(c+dx\right)\right)^{3}\operatorname{sech}^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(63) = 126.
time = 0.56, size = 347, normalized size = 5.18

$$-\frac{2\,\left(35\,a^{3}e^{(12\,dx+12\,c)}+105\,a^{2}be^{(12\,dx+12\,c)}+105\,ab^{2}e^{(12\,dx+12\,c)}+35\,b^{3}e^{(12\,dx+12\,c)}+210\,a^{3}e^{(10\,dx+10\,c)}+420\,a^{2}be^{(10\,dx+10\,c)}+210\,ab^{2}e^{(10\,dx+10\,c)}+525\,a^{3}e^{(8\,dx+8\,c)}+665\,a^{2}be^{(8\,dx+8\,c)}+315\,ab^{2}e^{(8\,dx+8\,c)}+175\,b^{3}e^{(8\,dx+8\,c)}+700\,a^{3}e^{(6\,dx+6\,c)}+560\,a^{2}be^{(6\,dx+6\,c)}+420\,ab^{2}e^{(6\,dx+6\,c)}+525\,a^{3}e^{(4\,dx+4\,c)}+315\,a^{2}be^{(4\,dx+4\,c)}+231\,ab^{2}e^{(4\,dx+4\,c)}+105\,b^{3}e^{(4\,dx+4\,c)}+210\,a^{3}e^{(2\,dx+2\,c)}+140\,a^{2}be^{(2\,dx+2\,c)}+42\,ab^{2}e^{(2\,dx+2\,c)}+35\,a^{3}+35\,a^{2}b+21\,ab^{2}+5\,b^{3}\right)}{35\,d{\left(e^{(2\,dx+2\,c)}+1\right)}^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^3*e^(12*d*x + 12*c) + 105*a^2*b*e^(12*d*x + 12*c) + 105*a*b^2*e
^(12*d*x + 12*c) + 35*b^3*e^(12*d*x + 12*c) + 210*a^3*e^(10*d*x + 10*c) + 4
20*a^2*b*e^(10*d*x + 10*c) + 210*a*b^2*e^(10*d*x + 10*c) + 525*a^3*e^(8*d*x
 + 8*c) + 665*a^2*b*e^(8*d*x + 8*c) + 315*a*b^2*e^(8*d*x + 8*c) + 175*b^3*e
^(8*d*x + 8*c) + 700*a^3*e^(6*d*x + 6*c) + 560*a^2*b*e^(6*d*x + 6*c) + 420*
a*b^2*e^(6*d*x + 6*c) + 525*a^3*e^(4*d*x + 4*c) + 315*a^2*b*e^(4*d*x + 4*c)
 + 231*a*b^2*e^(4*d*x + 4*c) + 105*b^3*e^(4*d*x + 4*c) + 210*a^3*e^(2*d*x +
 2*c) + 140*a^2*b*e^(2*d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) + 35*a^3 + 35*
a^2*b + 21*a*b^2 + 5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

Mupad [B]
time = 1.33, size = 1050, normalized size = 15.67

$$-\frac{\frac{2\,\left(5\,a^3-3\,a^2\,b+3\,a\,b^2-5\,b^3\right)}{35\,d}+\frac{2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,{\left(a+b\right)}^3}{7\,d}-\frac{6\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(-5\,a^3+a^2\,b+a\,b^2-5\,b^3\right)}{35\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^2\,\left(a-b\right)}{7\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,{\left(a+b\right)}^3}{7\,d}+\frac{2\,{\mathrm{e}}^{12\,c+12\,d\,x}\,{\left(a+b\right)}^3}{7\,d}-\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(-5\,a^3+a^2\,b+a\,b^2-5\,b^3\right)}{7\,d}-\frac{6\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(-5\,a^3+a^2\,b+a\,b^2-5\,b^3\right)}{7\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(5\,a^3-3\,a^2\,b+3\,a\,b^2-5\,b^3\right)}{7\,d}+\frac{12\,{\mathrm{e}}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2\,\left(a-b\right)}{7\,d}+\frac{12\,{\mathrm{e}}^{10\,c+10\,d\,x}\,{\left(a+b\right)}^2\,\left(a-b\right)}{7\,d}}{7\,{\mathrm{e}}^{2\,c+2\,d\,x}+21\,{\mathrm{e}}^{4\,c+4\,d\,x}+35\,{\mathrm{e}}^{6\,c+6\,d\,x}+35\,{\mathrm{e}}^{8\,c+8\,d\,x}+21\,{\mathrm{e}}^{10\,c+10\,d\,x}+7\,{\mathrm{e}}^{12\,c+12\,d\,x}+{\mathrm{e}}^{14\,c+14\,d\,x}+1}-\frac{\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^3}{7\,d}-\frac{2\,\left(-5\,a^3+a^2\,b+a\,b^2-5\,b^3\right)}{35\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2\,\left(a-b\right)}{7\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{\frac{2\,{\left(a+b\right)}^2\,\left(a-b\right)}{7\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^3}{7\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,{\mathrm{e}}^{8\,c+8\,d\,x}\,{\left(a+b\right)}^3}{7\,d}-\frac{2\,\left(-5\,a^3+a^2\,b+a\,b^2-5\,b^3\right)}{35\,d}-\frac{12\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(-5\,a^3+a^2\,b+a\,b^2-5\,b^3\right)}{35\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(5\,a^3-3\,a^2\,b+3\,a\,b^2-5\,b^3\right)}{35\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,{\left(a+b\right)}^2\,\left(a-b\right)}{7\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{\frac{2\,{\left(a+b\right)}^2\,\left(a-b\right)}{7\,d}+\frac{2\,{\mathrm{e}}^{10\,c+10\,d\,x}\,{\left(a+b\right)}^3}{7\,d}-\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(-5\,a^3+a^2\,b+a\,b^2-5\,b^3\right)}{7\,d}-\frac{4\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(-5\,a^3+a^2\,b+a\,b^2-5\,b^3\right)}{7\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(5\,a^3-3\,a^2\,b+3\,a\,b^2-5\,b^3\right)}{7\,d}+\frac{10\,{\mathrm{e}}^{8\,c+8\,d\,x}\,{\left(a+b\right)}^2\,\left(a-b\right)}{7\,d}}{6\,{\mathrm{e}}^{2\,c+2\,d\,x}+15\,{\mathrm{e}}^{4\,c+4\,d\,x}+20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}+6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+12\,d\,x}+1}-\frac{2\,{\left(a+b\right)}^3}{7\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^2,x)
```

```
[Out] - ((2*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(35*d) + (2*exp(6*c + 6*d*x)*(a
+ b)^3)/(7*d) - (6*exp(2*c + 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d)
 + (6*exp(4*c + 4*d*x)*(a + b)^2*(a - b))/(7*d))/(4*exp(2*c + 2*d*x) + 6*ex
p(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(a + b)^3
)/(7*d) + (2*exp(12*c + 12*d*x)*(a + b)^3)/(7*d) - (6*exp(4*c + 4*d*x)*(a*b
^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d) - (6*exp(8*c + 8*d*x)*(a*b^2 + a^2*b - 5
*a^3 - 5*b^3))/(7*d) + (8*exp(6*c + 6*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b
^3))/(7*d) + (12*exp(2*c + 2*d*x)*(a + b)^2*(a - b))/(7*d) + (12*exp(10*c +
 10*d*x)*(a + b)^2*(a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x
) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*e
xp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((2*exp(4*c + 4*d*x)*(a + b)^
3)/(7*d) - (2*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d) + (4*exp(2*c + 2*d*x)
*(a + b)^2*(a - b))/(7*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6
*c + 6*d*x) + 1) - ((2*(a + b)^2*(a - b))/(7*d) + (2*exp(2*c + 2*d*x)*(a +
b)^3)/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*exp(8*c + 8*
d*x)*(a + b)^3)/(7*d) - (2*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d) - (12*ex
p(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d) + (8*exp(2*c + 2*d*x
)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(35*d) + (8*exp(6*c + 6*d*x)*(a + b)
^2*(a - b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c +
 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(a + b)^2*(a -
 b))/(7*d) + (2*exp(10*c + 10*d*x)*(a + b)^3)/(7*d) - (2*exp(2*c + 2*d*x)*(
a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d) - (4*exp(6*c + 6*d*x)*(a*b^2 + a^2*b
- 5*a^3 - 5*b^3))/(7*d) + (4*exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 -
5*b^3))/(7*d) + (10*exp(8*c + 8*d*x)*(a + b)^2*(a - b))/(7*d))/(6*exp(2*c +
 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) +
 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - (2*(a + b)^3)/(7*d*(exp(2
*c + 2*d*x) + 1))
```

3.103 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=198

$$\frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{ArcTan}(\sinh(c+dx))}{128d}+\frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d}$$

```
[Out] 1/128*(64*a^3+48*a^2*b+24*a*b^2+5*b^3)*arctan(sinh(d*x+c))/d+1/128*(64*a^3+
48*a^2*b+24*a*b^2+5*b^3)*sech(d*x+c)*tanh(d*x+c)/d-1/192*b*(72*a^2+52*a*b+1
5*b^2)*sech(d*x+c)^3*tanh(d*x+c)/d-1/48*b*(12*a+5*b)*sech(d*x+c)^5*(a+(a+b)
*sinh(d*x+c)^2)*tanh(d*x+c)/d-1/8*b*sech(d*x+c)^7*(a+(a+b)*sinh(d*x+c)^2)^2
*tanh(d*x+c)/d
```

Rubi [A]
time = 0.16, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3757, 424, 540, 393, 205, 209}

$$-\frac{b(72a^2+52ab+15b^2)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{192d}+\frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{ArcTan}(\sinh(c+dx))}{128d}+\frac{(64a^3+48a^2b+24ab^2+5b^3)\tanh(c+dx)\operatorname{sech}(c+dx)}{128d}-\frac{b\tanh(c+dx)\operatorname{sech}^7(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)^2}{8d}-\frac{b(12a+5b)\tanh(c+dx)\operatorname{sech}^5(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Sinh[c + d*x]])/(128*d) + ((
64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x])/(128*d)
- (b*(72*a^2 + 52*a*b + 15*b^2)*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d) - (b
*(12*a + 5*b)*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/
(48*d) - (b*Sech[c + d*x]^7*(a + (a + b)*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/
(8*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^3}{(1+x^2)^5}\,dx,x,\sinh(c+dx)\right)}{d}$$

$$=-\frac{b\operatorname{sech}^7(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d}+$$

$$=-\frac{b(12a+5b)\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c}{48d}$$

$$=-\frac{b(72a^2+52ab+15b^2)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d}-\frac{b(12a}{}$$

$$=\frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d}-\frac{b(}{}$$

$$=\frac{(64a^3+48a^2b+24ab^2+5b^3)\tan^{-1}(\sinh(c+dx))}{128d}+\frac{(64a^3+}{}$$

Mathematica [A]
time = 11.16, size = 158, normalized size = 0.80

$$\frac{6(64a^3+48a^2b+24ab^2+5b^3)\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+3(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)\tanh(c+dx)-2b(144a^2+168ab+59b^2)\operatorname{sech}^3(c+dx)\tanh(c+dx)+8b^2(24a+17b)\operatorname{sech}^5(c+dx)\tanh(c+dx)-48b^3\operatorname{sech}^7(c+dx)\tanh(c+dx)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (6*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]] + 3*(64
*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x] - 2*b*(144*
a^2 + 168*a*b + 59*b^2)*Sech[c + d*x]^3*Tanh[c + d*x] + 8*b^2*(24*a + 17*b)
*Sech[c + d*x]^5*Tanh[c + d*x] - 48*b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(384
*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.52, size = 611, normalized size = 3.09

method	result
risch	$\frac{e^{dx+c}\left(-72a\,b^2+72a\,b^2e^{14dx+14c}-432a^2b\,e^{12dx+12c}-984a\,b^2e^{12dx+12c}-2160a^2b\,e^{10dx+10c}-312a\,b^2e^{10dx+10c}+144a^2b\,e^{14dx+14c}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*exp(d*x+c)*(-72*a*b^2+72*a*b^2*exp(14*d*x+14*c)-432*a^2*b*exp(12*d*x+
12*c)-984*a*b^2*exp(12*d*x+12*c)-2160*a^2*b*exp(10*d*x+10*c)-312*a*b^2*exp(
```

```
10*d*x+10*c)+144*a^2*b*exp(14*d*x+14*c)+2160*a^2*b*exp(4*d*x+4*c)+432*a^2*b
*exp(2*d*x+2*c)-144*a^2*b-192*a^3-15*b^3+744*a*b^2*exp(8*d*x+8*c)-744*a*b^2
*exp(6*d*x+6*c)+312*a*b^2*exp(4*d*x+4*c)-1584*a^2*b*exp(8*d*x+8*c)+984*a*b^
2*exp(2*d*x+2*c)+1584*a^2*b*exp(6*d*x+6*c)+15*b^3*exp(14*d*x+14*c)-960*a^3*
exp(2*d*x+2*c)+960*a^3*exp(12*d*x+12*c)-397*b^3*exp(12*d*x+12*c)+1728*a^3*e
xp(10*d*x+10*c)+192*a^3*exp(14*d*x+14*c)+397*b^3*exp(2*d*x+2*c)-1765*b^3*ex
p(8*d*x+8*c)-1728*a^3*exp(4*d*x+4*c)-895*b^3*exp(4*d*x+4*c)+895*b^3*exp(10*
d*x+10*c)+960*a^3*exp(8*d*x+8*c)-960*a^3*exp(6*d*x+6*c)+1765*b^3*exp(6*d*x+
6*c))/d/(1+exp(2*d*x+2*c))^8+1/2*I/d*ln(exp(d*x+c)+I)*a^3+3/8*I/d*ln(exp(d*
x+c)+I)*a^2*b+3/16*I/d*ln(exp(d*x+c)+I)*a*b^2+5/128*I/d*ln(exp(d*x+c)+I)*b^
3-1/2*I/d*ln(exp(d*x+c)-I)*a^3-3/8*I/d*ln(exp(d*x+c)-I)*a^2*b-3/16*I/d*ln(e
xp(d*x+c)-I)*a*b^2-5/128*I/d*ln(exp(d*x+c)-I)*b^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(188) = 376$.
time = 0.49, size = 553, normalized size = 2.79

$$-\frac{1}{192}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15e^{(-dx-c)}-397e^{(-3dx-3c)}+895e^{(-5dx-5c)}-1765e^{(-7dx-7c)}+1765e^{(-9dx-9c)}-895e^{(-11dx-11c)}+397e^{(-13dx-13c)}-15e^{(-15dx-15c)}}{d(8e^{(-2dx-2c)}+28e^{(-4dx-4c)}+56e^{(-6dx-6c)}+70e^{(-8dx-8c)}+56e^{(-10dx-10c)}+28e^{(-12dx-12c)}+8e^{(-14dx-14c)}+e^{(-16dx-16c)}+1)}\right)-\frac{1}{8}ab^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3e^{(-dx-c)}-47e^{(-3dx-3c)}+78e^{(-5dx-5c)}-78e^{(-7dx-7c)}+47e^{(-9dx-9c)}-3e^{(-11dx-11c)}}{d(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(-12dx-12c)}+1)}\right)-\frac{3}{4}a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7e^{(-3dx-3c)}+7e^{(-5dx-5c)}-e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)-a^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) - 397*e^(-3*d*x -
3*c) + 895*e^(-5*d*x - 5*c) - 1765*e^(-7*d*x - 7*c) + 1765*e^(-9*d*x - 9*c)
 - 895*e^(-11*d*x - 11*c) + 397*e^(-13*d*x - 13*c) - 15*e^(-15*d*x - 15*c))
/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^
(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d
*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 1/8*a*b^2*(3*arctan(e^(-d*x - c))/
d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*
d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x -
2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*
e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 3/4*a^2*b*(arctan(e^(-d*x
- c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c) - e^(-7*d
*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c)
 + e^(-8*d*x - 8*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) -
e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6114 vs. $2(188) = 376$.
time = 0.41, size = 6114, normalized size = 30.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/192*(3*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^15 + 45*(64*a
^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^14 + 3*(64*a^
3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*sinh(d*x + c)^15 + (960*a^3 - 432*a^2*b -
984*a*b^2 - 397*b^3)*cosh(d*x + c)^13 + (960*a^3 - 432*a^2*b - 984*a*b^2 -
397*b^3 + 315*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^13 + 13*(105*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^
3 + (960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c))*sinh(d*x + c
)^12 + (1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^11 + (40
95*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 1728*a^3 - 2160
*a^2*b - 312*a*b^2 + 895*b^3 + 78*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 11*(819*(64*a^3 + 48*a^2*b + 24*a*b^
2 + 5*b^3)*cosh(d*x + c)^5 + 26*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)
*cosh(d*x + c)^3 + (1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^10 + (960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(
d*x + c)^9 + (15015*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^6
+ 715*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c)^4 + 960*a^3
 - 1584*a^2*b + 744*a*b^2 - 1765*b^3 + 55*(1728*a^3 - 2160*a^2*b - 312*a*b^
2 + 895*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 3*(6435*(64*a^3 + 48*a^2*b
+ 24*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 429*(960*a^3 - 432*a^2*b - 984*a*b^2
- 397*b^3)*cosh(d*x + c)^5 + 55*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^
3)*cosh(d*x + c)^3 + 3*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d
*x + c))*sinh(d*x + c)^8 - (960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*co
sh(d*x + c)^7 + (19305*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)
^8 + 1716*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c)^6 + 330
*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^4 - 960*a^3 +
1584*a^2*b - 744*a*b^2 + 1765*b^3 + 36*(960*a^3 - 1584*a^2*b + 744*a*b^2 -
1765*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + (15015*(64*a^3 + 48*a^2*b + 24
*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 1716*(960*a^3 - 432*a^2*b - 984*a*b^2 - 3
97*b^3)*cosh(d*x + c)^7 + 462*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)
*cosh(d*x + c)^5 + 84*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*
x + c)^3 - 7*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*x + c))*s
inh(d*x + c)^6 - (1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c
)^5 + (9009*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 1287*
(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c)^8 + 462*(1728*a^3
 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^6 + 126*(960*a^3 - 1584*
a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*x + c)^4 - 1728*a^3 + 2160*a^2*b + 312
*a*b^2 - 895*b^3 - 21*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^5 + (4095*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*co
sh(d*x + c)^11 + 715*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x +
 c)^9 + 330*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^7 +
 126*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*x + c)^5 - 35*(96
0*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d*x + c)^3 - 5*(1728*a^3 -
2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - (960*a^3
 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c)^3 + (1365*(64*a^3 + 48*a^
2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 286*(960*a^3 - 432*a^2*b - 984*a
```

```
*b^2 - 397*b^3)*cosh(d*x + c)^10 + 165*(1728*a^3 - 2160*a^2*b - 312*a*b^2 +
 895*b^3)*cosh(d*x + c)^8 + 84*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3
)*cosh(d*x + c)^6 - 35*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*cosh(d
*x + c)^4 - 960*a^3 + 432*a^2*b + 984*a*b^2 + 397*b^3 - 10*(1728*a^3 - 2160
*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (315*(64*a
^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^13 + 78*(960*a^3 - 432*a^2*
b - 984*a*b^2 - 397*b^3)*cosh(d*x + c)^11 + 55*(1728*a^3 - 2160*a^2*b - 312
*a*b^2 + 895*b^3)*cosh(d*x + c)^9 + 36*(960*a^3 - 1584*a^2*b + 744*a*b^2 -
1765*b^3)*cosh(d*x + c)^7 - 21*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3
)*cosh(d*x + c)^5 - 10*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*cosh(d
*x + c)^3 - 3*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*cosh(d*x + c))*si
nh(d*x + c)^2 + 3*((64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^16
+ 16*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^15
+ (64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*sinh(d*x + c)^16 + 8*(64*a^3 + 48*
a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^14 + 8*(64*a^3 + 48*a^2*b + 24*a*b^
2 + 5*b^3 + 15*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^14 + 112*(5*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^
3 + (64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^13
+ 28*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 28*(65*(64*a
^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b + 24*
a*b^2 + 5*b^3 + 26*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^12 + 112*(39*(64*a^3 + 48*a^2*b +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh^2 \left(c + dx\right)\right)^3 \operatorname{sech}^3 \left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 485 vs. 2(188) = 376.
time = 0.55, size = 485, normalized size = 2.45

$$\frac{3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)\left(64a^3 + 48a^2b + 24ab^2 + 5b^3\right) + \frac{4\left(192a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^7 + 144a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right)^7 + 72ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^7 + 15b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^7 + 2304a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 576a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 - 480ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 - 292b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 9216a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 - 2304a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 - 4224ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 - 880b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 12288a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) - 9216a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right) - 4608ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) - 960b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/768*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(64*a^3 +
48*a^2*b + 24*a*b^2 + 5*b^3) + 4*(192*a^3*(e^(d*x + c) - e^(-d*x - c))^7 +
144*a^2*b*(e^(d*x + c) - e^(-d*x - c))^7 + 72*a*b^2*(e^(d*x + c) - e^(-d*x
```

```
- c))^7 + 15*b^3*(e^(d*x + c) - e^(-d*x - c))^7 + 2304*a^3*(e^(d*x + c) - e
^(-d*x - c))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 - 480*a*b^2*(e^(d
*x + c) - e^(-d*x - c))^5 - 292*b^3*(e^(d*x + c) - e^(-d*x - c))^5 + 9216*a
^3*(e^(d*x + c) - e^(-d*x - c))^3 - 2304*a^2*b*(e^(d*x + c) - e^(-d*x - c))
^3 - 4224*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 - 880*b^3*(e^(d*x + c) - e^(
-d*x - c))^3 + 12288*a^3*(e^(d*x + c) - e^(-d*x - c)) - 9216*a^2*b*(e^(d*x
+ c) - e^(-d*x - c)) - 4608*a*b^2*(e^(d*x + c) - e^(-d*x - c)) - 960*b^3*(e
^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^4)/d
```

Mupad [B]
time = 1.36, size = 951, normalized size = 4.80

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(64\,a^3\sqrt{d^2}+5\,b^3\sqrt{d^2}+24\,a\,b^2\sqrt{d^2}+48\,a^2\,b\sqrt{d^2}\right)}{d\sqrt{4096\,a^6+6144\,a^5\,b+5376\,a^4\,b^2+2944\,a^3\,b^3+1056\,a^2\,b^4+240\,a\,b^5+25\,b^6}}\right)\sqrt{4096\,a^6+6144\,a^5\,b+5376\,a^4\,b^2+2944\,a^3\,b^3+1056\,a^2\,b^4+240\,a\,b^5+25\,b^6}}{64\sqrt{d^2}}-\text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(64*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 24*a*b^2*(
d^2)^(1/2) + 48*a^2*b*(d^2)^(1/2)))/(d*(240*a*b^5 + 6144*a^5*b + 4096*a^6 +
 25*b^6 + 1056*a^2*b^4 + 2944*a^3*b^3 + 5376*a^4*b^2)^(1/2)))*(240*a*b^5 +
6144*a^5*b + 4096*a^6 + 25*b^6 + 1056*a^2*b^4 + 2944*a^3*b^3 + 5376*a^4*b^2
)^(1/2))/(64*(d^2)^(1/2)) - ((exp(c + d*x)*(a + b)^3)/(2*d) + (exp(13*c + 1
3*d*x)*(a + b)^3)/(2*d) - (3*exp(5*c + 5*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^
3))/(2*d) - (3*exp(9*c + 9*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(2*d) + (2
*exp(7*c + 7*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/d + (3*exp(3*c + 3*d
*x)*(a + b)^2*(a - b))/d + (3*exp(11*c + 11*d*x)*(a + b)^2*(a - b))/d)/(8*e
xp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c +
8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*
x) + exp(16*c + 16*d*x) + 1) + (2*exp(c + d*x)*(48*a*b^2 + 85*b^3))/(3*d*(5
*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c +
 8*d*x) + exp(10*c + 10*d*x) + 1)) + (16*b^3*exp(c + d*x))/(d*(7*exp(2*c +
2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) +
21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) + (
exp(c + d*x)*(24*a*b^2 + 48*a^2*b + 64*a^3 + 5*b^3))/(64*d*(exp(2*c + 2*d*x
) + 1)) - (4*exp(c + d*x)*(6*a*b^2 + 35*b^3))/(3*d*(6*exp(2*c + 2*d*x) + 15
*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c
+ 10*d*x) + exp(12*c + 12*d*x) + 1)) - (exp(c + d*x)*(600*a*b^2 + 576*a^2*b
 + 144*a^3 + 203*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) +
 (exp(c + d*x)*(600*a*b^2 + 288*a^2*b + 305*b^3))/(24*d*(3*exp(2*c + 2*d*x)
 + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(c + d*x)*(168*a*b^2 +
 24*a^2*b + 145*b^3))/(4*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp
(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.104 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=102

$$\frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-3b)\tanh^3(c+dx)}{3d}-\frac{3a(a-b)b\tanh^5(c+dx)}{5d}-\frac{(3a-b)b^2\tanh^7(c+dx)}{7d}-\frac{b^3\tanh^9(c+dx)}{9d}$$

```
[Out] a^3*tanh(d*x+c)/d-1/3*a^2*(a-3*b)*tanh(d*x+c)^3/d-3/5*a*(a-b)*b*tanh(d*x+c)
^5/d-1/7*(3*a-b)*b^2*tanh(d*x+c)^7/d-1/9*b^3*tanh(d*x+c)^9/d
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3756, 380}

$$\frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-3b)\tanh^3(c+dx)}{3d}-\frac{b^2(3a-b)\tanh^7(c+dx)}{7d}-\frac{3ab(a-b)\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d - (a^2*(a - 3*b)*Tanh[c + d*x]^3)/(3*d) - (3*a*(a - b
)*b*Tanh[c + d*x]^5)/(5*d) - ((3*a - b)*b^2*Tanh[c + d*x]^7)/(7*d) - (b^3*T
anh[c + d*x]^9)/(9*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int(1-x^2)\left(a+bx^2\right)^3\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int\left(a^3-a^2(a-3b)x^2-3a(a-b)bx^4-(3a-b)b^2x^6-\right.\right.}{d}$$
$$= \frac{a^3\tanh(c+dx)}{d} - \frac{a^2(a-3b)\tanh^3(c+dx)}{3d} - \frac{3a(a-b)b\tan}{5d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 218 vs. $2(102) = 204$.
time = 0.62, size = 218, normalized size = 2.14

$$\frac{(5775a^3-1071a^2b+621ab^2-725b^3+10(903a^3-63a^2b-27ab^2+107b^3)\cosh(2(c+dx))+8(525a^3+126a^2b-81ab^2-50b^3)\cosh(4(c+dx))+1050a^3\cosh(6(c+dx))+630a^2b\cosh(6(c+dx))+270ab^2\cosh(6(c+dx))+50b^3\cosh(6(c+dx))+105a^3\cosh(8(c+dx))+63a^2b\cosh(8(c+dx))+27ab^2\cosh(8(c+dx))+5b^3\cosh(8(c+dx)))\operatorname{sech}^8(c+dx)\tanh(c+dx)}{20160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((5775*a^3 - 1071*a^2*b + 621*a*b^2 - 725*b^3 + 10*(903*a^3 - 63*a^2*b - 27
*a*b^2 + 107*b^3)*Cosh[2*(c + d*x)] + 8*(525*a^3 + 126*a^2*b - 81*a*b^2 - 5
0*b^3)*Cosh[4*(c + d*x)] + 1050*a^3*Cosh[6*(c + d*x)] + 630*a^2*b*Cosh[6*(c
 + d*x)] + 270*a*b^2*Cosh[6*(c + d*x)] + 50*b^3*Cosh[6*(c + d*x)] + 105*a^3
*Cosh[8*(c + d*x)] + 63*a^2*b*Cosh[8*(c + d*x)] + 27*a*b^2*Cosh[8*(c + d*x)
] + 5*b^3*Cosh[8*(c + d*x)])*Sech[c + d*x]^8*Tanh[c + d*x])/(20160*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(94) = 188$.
time = 2.38, size = 448, normalized size = 4.39

method	result
risch	$-\frac{4(27ab^2+945ab^2e^{14dx+14c}+3465a^2be^{12dx+12c}+945ab^2e^{12dx+12c}+4725a^2be^{10dx+10c}+945ab^2e^{10dx+10c}+945a^2be^{14dx+14c}+13}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/315*(27*a*b^2+945*a*b^2*exp(14*d*x+14*c)+3465*a^2*b*exp(12*d*x+12*c)+945
*a*b^2*exp(12*d*x+12*c)+4725*a^2*b*exp(10*d*x+10*c)+945*a*b^2*exp(10*d*x+10
*c)+945*a^2*b*exp(14*d*x+14*c)+1323*a^2*b*exp(4*d*x+4*c)+567*a^2*b*exp(2*d*
x+2*c)+63*a^2*b+105*a^3+5*b^3+2457*a*b^2*exp(8*d*x+8*c)+1323*a*b^2*exp(6*d*
x+6*c)+27*a*b^2*exp(4*d*x+4*c)+3213*a^2*b*exp(8*d*x+8*c)+243*a*b^2*exp(2*d*
x+2*c)+1827*a^2*b*exp(6*d*x+6*c)+315*b^3*exp(14*d*x+14*c)+945*a^3*exp(2*d*x
+2*c)+1995*a^3*exp(12*d*x+12*c)-525*b^3*exp(12*d*x+12*c)+5355*a^3*exp(10*d*
```

x+10*c)+315*a^3*exp(14*d*x+14*c)+45*b^3*exp(2*d*x+2*c)-945*b^3*exp(8*d*x+8*
c)+3465*a^3*exp(4*d*x+4*c)-135*b^3*exp(4*d*x+4*c)+1575*b^3*exp(10*d*x+10*c)
+7875*a^3*exp(8*d*x+8*c)+6825*a^3*exp(6*d*x+6*c)+945*b^3*exp(6*d*x+6*c))/d/
(1+exp(2*d*x+2*c))^9

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. $2(94) = 188$.
time = 0.29, size = 1847, normalized size = 18.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4/63*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) +
 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e
^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d
*x - 18*c) + 1)) - 27*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*
x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10
*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c)
+ e^(-18*d*x - 18*c) + 1)) + 189*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c) +
36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-
10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d
*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 189*e^(-8*d*x - 8*c)/(d*(9*e^(-2*d*
x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c)
 + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) +
 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 315*e^(-10*d*x - 10*c)/(
d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(
-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*
d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 105*e^(-12*
d*x - 12*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6
*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c)
 + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))
+ 63*e^(-14*d*x - 14*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e
^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12
*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x -
18*c) + 1)) + 1/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x
 - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 1
2*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) +
1))) + 12/35*a*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*
x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c
) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(
d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-
8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x -
14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4
```

```
*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7
*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*
e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
 - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c)
 + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^
(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21
*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d
*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 4/5*a^2*b*(
5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*
d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x -
4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^
(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x -
8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x -
 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))
) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(94) = 188$.
time = 0.36, size = 1185, normalized size = 11.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/315*(2*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^7 + 14*(
105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (
105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*sinh(d*x + c)^7 + 6*(245*a^3 + 3
36*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 483*a^2*b + 11
7*a*b^2 - 95*b^3 + 7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*c
osh(d*x + c)^3 + 3*(245*a^3 + 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c))
*sinh(d*x + c)^4 + 18*(245*a^3 + 168*a^2*b + 27*a*b^2 + 40*b^3)*cosh(d*x +
c)^3 + (35*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x + c)^4 + 94
5*a^3 + 1701*a^2*b + 459*a*b^2 + 855*b^3 + 30*(175*a^3 + 483*a^2*b + 117*a*
b^2 - 95*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 + 252*a^2*b
+ 243*a*b^2 + 80*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 336*a^2*b + 99*a*b^2
- 40*b^3)*cosh(d*x + c)^3 + 9*(245*a^3 + 168*a^2*b + 27*a*b^2 + 40*b^3)*cos
h(d*x + c))*sinh(d*x + c)^2 + 210*(35*a^3 + 12*a^2*b + 9*a*b^2)*cosh(d*x +
c) + (7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x + c)^6 + 15*(1
75*a^3 + 483*a^2*b + 117*a*b^2 - 95*b^3)*cosh(d*x + c)^4 + 525*a^3 + 693*a^
```

```
2*b + 567*a*b^2 - 945*b^3 + 27*(105*a^3 + 189*a^2*b + 51*a*b^2 + 95*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh
(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)^9 + (55*d*cosh(d*x +
c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^3 + 27*d*cosh(d*x + c))
*sinh(d*x + c)^8 + 37*d*cosh(d*x + c)^7 + (330*d*cosh(d*x + c)^4 + 324*d*co
sh(d*x + c)^2 + 35*d)*sinh(d*x + c)^7 + 7*(66*d*cosh(d*x + c)^5 + 108*d*cos
h(d*x + c)^3 + 37*d*cosh(d*x + c))*sinh(d*x + c)^6 + 93*d*cosh(d*x + c)^5 +
 3*(154*d*cosh(d*x + c)^6 + 378*d*cosh(d*x + c)^4 + 245*d*cosh(d*x + c)^2 +
 25*d)*sinh(d*x + c)^5 + (330*d*cosh(d*x + c)^7 + 1134*d*cosh(d*x + c)^5 +
1295*d*cosh(d*x + c)^3 + 465*d*cosh(d*x + c))*sinh(d*x + c)^4 + 162*d*cosh(
d*x + c)^3 + (165*d*cosh(d*x + c)^8 + 756*d*cosh(d*x + c)^6 + 1225*d*cosh(d
*x + c)^4 + 750*d*cosh(d*x + c)^2 + 90*d)*sinh(d*x + c)^3 + (55*d*cosh(d*x
+ c)^9 + 324*d*cosh(d*x + c)^7 + 777*d*cosh(d*x + c)^5 + 930*d*cosh(d*x + c
)^3 + 486*d*cosh(d*x + c))*sinh(d*x + c)^2 + 210*d*cosh(d*x + c) + (11*d*co
sh(d*x + c)^10 + 81*d*cosh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 375*d*cosh(
d*x + c)^4 + 270*d*cosh(d*x + c)^2 + 42*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{sech}^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(94) = 188.
time = 0.55, size = 447, normalized size = 4.38

$$-\frac{4\left(315a^3e^{(14dx+14c)}+945a^2be^{(14dx+14c)}+945ab^2e^{(14dx+14c)}+315b^3e^{(14dx+14c)}+1995a^3e^{(12dx+12c)}+3465a^2be^{(12dx+12c)}+945ab^2e^{(12dx+12c)}-525b^3e^{(12dx+12c)}+5355a^3e^{(10dx+10c)}+4725a^2be^{(10dx+10c)}+945ab^2e^{(10dx+10c)}+1575b^3e^{(10dx+10c)}+7875a^3e^{(8dx+8c)}+3213a^2be^{(8dx+8c)}+2457ab^2e^{(8dx+8c)}-945b^3e^{(8dx+8c)}+6825a^3e^{(6dx+6c)}+1827a^2be^{(6dx+6c)}+1323ab^2e^{(6dx+6c)}+945b^3e^{(6dx+6c)}+3465a^3e^{(4dx+4c)}+1323a^2be^{(4dx+4c)}+27ab^2e^{(4dx+4c)}-135b^3e^{(4dx+4c)}+945a^3e^{(2dx+2c)}+567a^2be^{(2dx+2c)}+243ab^2e^{(2dx+2c)}+45b^3e^{(2dx+2c)}+105a^3+63a^2b+27ab^2+5b^3\right)}{315d\left(e^{(2dx+2c)}+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -4/315*(315*a^3*e^(14*d*x + 14*c) + 945*a^2*b*e^(14*d*x + 14*c) + 945*a*b^2
*e^(14*d*x + 14*c) + 315*b^3*e^(14*d*x + 14*c) + 1995*a^3*e^(12*d*x + 12*c)
 + 3465*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) - 525*b^3*e^(
12*d*x + 12*c) + 5355*a^3*e^(10*d*x + 10*c) + 4725*a^2*b*e^(10*d*x + 10*c)
+ 945*a*b^2*e^(10*d*x + 10*c) + 1575*b^3*e^(10*d*x + 10*c) + 7875*a^3*e^(8*
d*x + 8*c) + 3213*a^2*b*e^(8*d*x + 8*c) + 2457*a*b^2*e^(8*d*x + 8*c) - 945*
b^3*e^(8*d*x + 8*c) + 6825*a^3*e^(6*d*x + 6*c) + 1827*a^2*b*e^(6*d*x + 6*c)
 + 1323*a*b^2*e^(6*d*x + 6*c) + 945*b^3*e^(6*d*x + 6*c) + 3465*a^3*e^(4*d*x
 + 4*c) + 1323*a^2*b*e^(4*d*x + 4*c) + 27*a*b^2*e^(4*d*x + 4*c) - 135*b^3*e
```

```
^(4*d*x + 4*c) + 945*a^3*e^(2*d*x + 2*c) + 567*a^2*b*e^(2*d*x + 2*c) + 243*
a*b^2*e^(2*d*x + 2*c) + 45*b^3*e^(2*d*x + 2*c) + 105*a^3 + 63*a^2*b + 27*a*
b^2 + 5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^9)
```

Mupad [B]
time = 1.33, size = 1424, normalized size = 13.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^4,x)
```

```
[Out] - ((4*(a + b)^2*(a - b))/(21*d) + (2*exp(2*c + 2*d*x)*(a + b)^3)/(9*d))/(3*
exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((5*exp(8*c
 + 8*d*x)*(a + b)^3)/(9*d) - (a*b^2 + a^2*b - 5*a^3 - 5*b^3)/(21*d) - (10*e
xp(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (16*exp(2*c + 2*d
*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(63*d) + (40*exp(6*c + 6*d*x)*(a +
 b)^2*(a - b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6
*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*
x) + 1) - ((4*(a + b)^2*(a - b))/(21*d) + (2*exp(10*c + 10*d*x)*(a + b)^3)/
(3*d) - (2*exp(2*c + 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d) - (20*ex
p(6*c + 6*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (16*exp(4*c + 4*d*
x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(21*d) + (20*exp(8*c + 8*d*x)*(a +
b)^2*(a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c
 + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d
*x) + exp(14*c + 14*d*x) + 1) - ((16*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(
315*d) + (4*exp(6*c + 6*d*x)*(a + b)^3)/(9*d) - (4*exp(2*c + 2*d*x)*(a*b^2
 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (8*exp(4*c + 4*d*x)*(a + b)^2*(a - b))/(
7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*e
xp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((8*exp(2*c + 2*d*x)*(a + b)^3)
/(9*d) + (8*exp(14*c + 14*d*x)*(a + b)^3)/(9*d) - (8*exp(6*c + 6*d*x)*(a*b^
2 + a^2*b - 5*a^3 - 5*b^3))/(3*d) - (8*exp(10*c + 10*d*x)*(a*b^2 + a^2*b -
5*a^3 - 5*b^3))/(3*d) + (32*exp(8*c + 8*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5
*b^3))/(9*d) + (16*exp(4*c + 4*d*x)*(a + b)^2*(a - b))/(3*d) + (16*exp(12*c
 + 12*d*x)*(a + b)^2*(a - b))/(3*d))/(9*exp(2*c + 2*d*x) + 36*exp(4*c + 4*d
*x) + 84*exp(6*c + 6*d*x) + 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) +
 84*exp(12*c + 12*d*x) + 36*exp(14*c + 14*d*x) + 9*exp(16*c + 16*d*x) + exp
(18*c + 18*d*x) + 1) - ((a + b)^3/(9*d) + (7*exp(12*c + 12*d*x)*(a + b)^3)/
(9*d) - (exp(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/d - (5*exp(8*c +
 8*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(3*d) + (16*exp(6*c + 6*d*x)*(3*a*
b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(9*d) + (4*exp(2*c + 2*d*x)*(a + b)^2*(a -
b))/(3*d) + (4*exp(10*c + 10*d*x)*(a + b)^2*(a - b))/d)/(8*exp(2*c + 2*d*x)
 + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp
(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c +
 16*d*x) + 1) - ((exp(4*c + 4*d*x)*(a + b)^3)/(3*d) - (a*b^2 + a^2*b - 5*a^
```

```
3 - 5*b^3)/(21*d) + (4*exp(2*c + 2*d*x)*(a + b)^2*(a - b))/(7*d))/(4*exp(2*
c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1
) - (a + b)^3/(9*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.105 $\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{(3a^2+10ab+15b^2)\,x}{8(a+b)^3}+\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3d}+\frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d}+\frac{\cosh^3(c+dx)\,\mathrm{s}}{4(a+}$$

[Out] 1/8*(3*a^2+10*a*b+15*b^2)*x/(a+b)^3+1/8*(3*a+7*b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d+b^(5/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/(a+b)^3/d/a^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3756, 425, 541, 536, 212, 211}

$$\frac{x(3a^2+10ab+15b^2)}{8(a+b)^3}+\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a+b)^3}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4d(a+b)}+\frac{(3a+7b)\sinh(c+dx)\cosh(c+dx)}{8d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]

[Out] ((3*a^2 + 10*a*b + 15*b^2)*x)/(8*(a + b)^3) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((3*a + 7*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*(a + b)^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c - a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a+bx^2)}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} + \frac{\text{Subst}\left(\int \frac{3a+4b+3bx^2}{(1-x^2)^2(a+bx^2)}\,dx,x,\tanh(c+dx)\right)}{4(a+b)d}$$

$$= \frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} + \frac{\text{Subst}\left(\int \frac{3a\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d} + \frac{b^3\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{(3a^2+10ab+15b^2)\,x}{8(a+b)^3} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3d} + \frac{(3a+7b)\cosh(c+dx)\,\text{si}\cdots}{8(a+b)^2d}$$

Mathematica [A]
time = 0.22, size = 115, normalized size = 0.96

$$\frac{(3a^2+10ab+15b^2)\,(c+dx)}{8(a+b)^3d}+\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3d}+\frac{(a+2b)\sinh(2(c+dx))}{4(a+b)^2d}+\frac{\sinh(4(c+dx))}{32(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 + 10*a*b + 15*b^2)*(c + d*x))/(8*(a + b)^3*d) + (b^(5/2)*ArcTan[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((a + 2*b)*Sinh[2*(c
 + d*x)])/(4*(a + b)^2*d) + Sinh[4*(c + d*x)]/(32*(a + b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(106) = 212$.
time = 2.74, size = 438, normalized size = 3.65

method	result
risch	$\frac{3a^2x}{8(a+b)^3}+\frac{5axb}{4(a+b)^3}+\frac{15x\,b^2}{8(a+b)^3}+\frac{e^{4dx+4c}}{64(a+b)d}+\frac{e^{2dx+2c}b}{4(a+b)^2d}+\frac{e^{2dx+2c}a}{8(a+b)^2d}-\frac{e^{-2dx-2c}b}{4(a^2+2ab+b^2)d}-\frac{e^{-2dx-2c}a}{8(a^2+2ab+b^2)d}\cdots$
derivativedivides	$-\frac{2b^3a\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(a+\sqrt{b(a+b)}+b\right)\operatorname{arct}\cdots}{2a\sqrt{b(a+b)}\,\cdots}\right.\cdots}{(a+b)^3}\cdots$
default	$-\frac{2b^3a\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(a+\sqrt{b(a+b)}+b\right)\operatorname{arct}\cdots}{2a\sqrt{b(a+b)}\,\cdots}\right.\cdots}{(a+b)^3}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^3/(a+b)^3*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b
*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(
1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)
+a+2*b)*a)^(1/2)))+1/2/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)-1)^4+2/(4*a+4*b)/(tan
h(1/2*d*x+1/2*c)-1)^3-1/8*(-7*a-11*b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/8
*(-5*a-9*b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)+1/8/(a+b)^3*(-3*a^2-10*a*b-15*b
^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)+1)^4+2/(4*
a+4*b)/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(-5*a-9*b)/(a+b)^2/(tanh(1/2*d*x+1/2*c
)+1)-1/8*(7*a+11*b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/8*(3*a^2+10*a*b+15*
b^2)/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(106) = 212$.
time = 0.56, size = 514, normalized size = 4.28

$$-\frac{(ab-b^2)(dx+c)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}+\frac{(8\,be^{(-2\,dx-2\,c)}+a+b)e^{(4\,dx+4\,c)}}{64\,(a^2+2\,ab+b^2)d}+\frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d}-\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d}-\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}\,d}-\frac{(a^2b-6\,ab^2+b^3)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}\,d}+\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}\,d}-\frac{3\,b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}\,(a+b)d}-\frac{8\,be^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}}{64\,(a^2+2\,ab+b^2)d}+\frac{3\,(dx+c)}{8\,(a+b)d}+\frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d}-\frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*b - b^2)*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*(8*b*
e^(-2*d*x - 2*c) + a + b)*e^(4*d*x + 4*c)/((a^2 + 2*a*b + b^2)*d) + 1/4*b*l
og((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a
*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4
*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2*((a + b)*
e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 1/8
*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt
(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*d) + 1/4*(a*b - b^2)*arct
an(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*s
qrt(a*b)*d) - 3/8*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/(sqrt(a*b)*(a + b)*d) - 1/64*(8*b*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
4*c))/((a^2 + 2*a*b + b^2)*d) + 3/8*(d*x + c)/((a + b)*d) + 1/8*e^(2*d*x +
2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(106) = 212$.
time = 0.49, size = 2180, normalized size = 18.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 + 10
```

*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^
6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + 6*a*b + 4*b^2)*sinh(
d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 6*(a^2 + 3*a*b + 2*
b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^4 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x + 60*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 4*(3*a
^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c) + 20*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
 + c)^3)*sinh(d*x + c)^3 - 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(7*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(
d*x + c)^2 + 30*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^4 - 2*a^2 - 6*a*b - 4*b
^2)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh(d*x + c)*sinh
(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b
^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*si
nh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a +
b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh
(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a
- b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*
sinh(d*x + c) + a + b)) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d
x + c)^7 + 4(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 + 6*(a^2 + 3*a*
b + 2*b^2)*cosh(d*x + c)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*
x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*sinh(d*x + c)^4), 1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)
^8 + 8*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 3*a*b + 2*b
^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + 6*
a*b + 4*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 6
*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x + 60*(a^2 + 3*a*b
+ 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d
x + c)^5 + 4(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c) + 20*(a^2 + 3*a*b
 + 2*b^2)*cosh(d*x + c)^3)*sinh(d*x + c)^3 - 8*(a^2 + 3*a*b + 2*b^2)*cosh(d
x + c)^2 + 4(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 + 10*a*b +
 15*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^4 - 2
*a^2 - 6*a*b - 4*b^2)*sinh(d*x + c)^2 + 64*(b^2*cosh(d*x + c)^4 + 4*b^2*cos
h(d*x + c)^3*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*
cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(b/a)*arctan(1/2*(
(a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*s

```
inh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^7 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 +
6*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
+ c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*sinh(d*x + c)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(106) = 212.
time = 2.01, size = 301, normalized size = 2.51

$$\frac{\frac{64\,b^3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}}+\frac{8\,(3\,a^2+10\,ab+15\,b^2)(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{(18\,a^2e^{(4\,dx+4\,c)}+60\,abe^{(4\,dx+4\,c)}+90\,b^2e^{(4\,dx+4\,c)}+8\,a^2e^{(2\,dx+2\,c)}+24\,abe^{(2\,dx+2\,c)}+16\,b^2e^{(2\,dx+2\,c)}+a^2+2\,ab+b^2)e^{(-4\,dx-4\,c)}}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+2\,c)}+16\,be^{(2\,dx+2\,c)}}{a^2+2\,ab+b^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(64*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqr
t(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) + 8*(3*a^2 + 10*a*b + 1
5*b^2)*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (18*a^2*e^(4*d*x + 4*c)
+ 60*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) +
 24*a*b*e^(2*d*x + 2*c) + 16*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4
*d*x - 4*c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (a*e^(4*d*x + 4*c) + b*e^(4*d
*x + 4*c) + 8*a*e^(2*d*x + 2*c) + 16*b*e^(2*d*x + 2*c))/(a^2 + 2*a*b + b^2)
)/d
```

Mupad [B]
time = 1.93, size = 967, normalized size = 8.06

$$\frac{x\,(3\,a^2+10\,a\,b+15\,b^2)}{8\,(a+b)^3}-\frac{e^{-4\,c-4\,d\,x}}{64\,d\,(a+b)}+\frac{e^{4\,c+4\,d\,x}}{64\,d\,(a+b)}+\text{[illegible]}-\frac{e^{-2\,c-2\,d\,x}\,(a+2\,b)}{8\,d\,(a+b)^2}+\frac{e^{2\,c+2\,d\,x}\,(a+2\,b)}{8\,d\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (x*(10*a*b + 3*a^2 + 15*b^2))/(8*(a + b)^3) - exp(- 4*c - 4*d*x)/(64*d*(a +
 b)) + exp(4*c + 4*d*x)/(64*d*(a + b)) + (atan((exp(2*c)*exp(2*d*x)*((4*b^3
)/(d*(a + b)^5*(b^5)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + ((a - b)*(a^4
*d*(b^5)^(1/2) - b^4*d*(b^5)^(1/2) - 2*a*b^3*d*(b^5)^(1/2) + 2*a^3*b*d*(b^5
)^(1/2)))/(b^3*(a + b)^2*(a*d^2*(a + b)^6)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 +
 b^3)*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 +
 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))) + ((a - b)*(a^4*d*(b^5)^(1/2) + b
^4*d*(b^5)^(1/2) + 4*a*b^3*d*(b^5)^(1/2) + 4*a^3*b*d*(b^5)^(1/2) + 6*a^2*b^
2*d*(b^5)^(1/2)))/(b^3*(a + b)^2*(a*d^2*(a + b)^6)^(1/2)*(3*a*b^2 + 3*a^2*b
 + a^3 + b^3)*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b
^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)))*((a^4*(a^7*d^2 + a*b^6*d^
2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*
b^2*d^2)^(1/2))/2 + (b^4*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2
 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/2 + 2*a*b^3*(a^
7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b
^3*d^2 + 15*a^5*b^2*d^2)^(1/2) + 2*a^3*b*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2
 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)
+ 3*a^2*b^2*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4
*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)))*(b^5)^(1/2))/(a^7*d^2 + a*b
^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15
*a^5*b^2*d^2)^(1/2) - (exp(- 2*c - 2*d*x)*(a + 2*b))/(8*d*(a + b)^2) + (exp
(2*c + 2*d*x)*(a + 2*b))/(8*d*(a + b)^2)
```

3.106 $\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}d}+\frac{(a+2b)\sinh(c+dx)}{(a+b)^2d}+\frac{\sinh^3(c+dx)}{3(a+b)d}$$

```
[Out] (a+2*b)*sinh(d*x+c)/(a+b)^2/d+1/3*sinh(d*x+c)^3/(a+b)/d+b^2*arctan(sinh(d*x
+c)*(a+b)^(1/2)/a^(1/2))/(a+b)^(5/2)/d/a^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3757, 398, 211}

$$\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a+b)^{5/2}}+\frac{\sinh^3(c+dx)}{3d(a+b)}+\frac{(a+2b)\sinh(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)*d)
 + ((a + 2*b)*Sinh[c + d*x])/((a + b)^2*d) + Sinh[c + d*x]^3/(3*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a+2b}{(a+b)^2}+\frac{x^2}{a+b}+\frac{b^2}{(a+b)^2(a+(a+b)x^2)}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a+2b)\sinh(c+dx)}{(a+b)^2 d}+\frac{\sinh^3(c+dx)}{3(a+b)d}+\frac{b^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+d$$

$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}d}+\frac{(a+2b)\sinh(c+dx)}{(a+b)^2 d}+\frac{\sinh^3(c+dx)}{3(a+b)d}$$

Mathematica [A]
time = 0.31, size = 79, normalized size = 0.99

$$\frac{-\frac{12b^2\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{3(3a+7b)\sinh(c+dx)}{(a+b)^2}+\frac{\sinh(3(c+dx))}{a+b}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((-12*b^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/
2)) + (3*(3*a + 7*b)*Sinh[c + d*x])/(a + b)^2 + Sinh[3*(c + d*x)]/(a + b))/
(12*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(70) = 140.
time = 2.75, size = 307, normalized size = 3.84

method	result
risch	$\frac{e^{3dx+3c}}{24(a+b)d}+\frac{3\,e^{dx+c}a}{8(a+b)^2 d}+\frac{7\,e^{dx+c}b}{8(a+b)^2 d}-\frac{3\,e^{-dx-c}a}{8(a+b)^2 d}-\frac{7\,e^{-dx-c}b}{8(a+b)^2 d}-\frac{e^{-3dx-3c}}{24(a+b)d}-\frac{b^2\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}\right.}{2\sqrt{-a^2-ab}\,(a+b)^2 a}$

derivativedivides	$-\frac{2}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}(2b+2a)}-\frac{1}{(2b+2a)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{2b+a}{(a+b)^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b^{2}a\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctan}}{2a\sqrt{b(a+b)}\,\sqrt{}}\right.}{}$
default	$-\frac{2}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}(2b+2a)}-\frac{1}{(2b+2a)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{2b+a}{(a+b)^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b^{2}a\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctan}}{2a\sqrt{b(a+b)}\,\sqrt{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/3/(tanh(1/2*d*x+1/2*c)-1)^3/(2*b+2*a)-1/(2*b+2*a)/(tanh(1/2*d*x+1/2
*c)-1)^2-(2*b+a)/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)+2*b^2/(a+b)^2*a*(-1/2*((b*
(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arcta
nh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*((b*(a+b)
)^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)))-2/3/(tanh(1/2*d*x+
1/2*c)+1)^3/(2*b+2*a)+1/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)+1)^2-(2*b+a)/(a+b)^2
/(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) + 3*(3*a*e^(4*c) + 7*b*e^(4*c))*e^(
4*d*x) - 3*(3*a*e^(2*c) + 7*b*e^(2*c))*e^(2*d*x) - a - b)*e^(-3*d*x)/(a^2*d
*e^(3*c) + 2*a*b*d*e^(3*c) + b^2*d*e^(3*c)) + 1/8*integrate(16*(b^2*e^(3*d*
x + 3*c) + b^2*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^(4*c) +
```

```
3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c
) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(70) = 140$.
time = 0.42, size = 1850, normalized size = 23.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*
cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^6 + 3
*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 + 10*a^2*b + 7*a*b
^2 + 5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3
 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d
*x + c))*sinh(d*x + c)^3 - a^3 - 2*a^2*b - a*b^2 - 3*(3*a^3 + 10*a^2*b + 7*
a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 3*a
^3 - 10*a^2*b - 7*a*b^2 + 6*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x +
 c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(-a^2
- a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*c
osh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3
*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)
*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
- cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a
+ b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*co
sh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*(
(a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) +
6*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 2*(3*a^3 + 10*a^2*b + 7*a*b^2)
*cosh(d*x + c)^3 - (3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c
))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + 3*(a^4 + 3*a^3*
b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*d*sinh(d*x + c)^3), 1/24*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
 c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*
a^2*b + a*b^2)*sinh(d*x + c)^6 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x +
c)^4 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2 + 5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 3*(
3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - 2*a^2*b
- a*b^2 - 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + 2*a^
2*b + a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 10*a^2*b - 7*a*b^2 + 6*(3*a^3 + 10*a
^2*b + 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*(b^2*cosh(d*x + c)^3
```

```
+ 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2
 + b^2*sinh(d*x + c)^3)*sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^3
 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a
 - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/
sqrt(a^2 + a*b)) + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x
 + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(a^2
 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) + 6*(
(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 2*(3*a^3 + 10*a^2*b + 7*a*b^2)*co
sh(d*x + c)^3 - (3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/
((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + 3*(a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^4 + 3*a^3*b + 3*
a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*d*sinh(d*x + c)^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 2.88, size = 2194, normalized size = 27.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

[Out] exp(3*c + 3*d*x)/(24*d*(a + b)) - exp(- 3*c - 3*d*x)/(24*d*(a + b)) + ((b^4
)^(1/2)*(2*atan((exp(d*x)*exp(c)*((4*(10*a^2*d*(b^4)^(5/2) + 12*a^6*d*(b^4)
^(3/2) + 2*a*b^9*d*(b^4)^(1/2) + 10*a^3*b^3*d*(b^4)^(3/2) + 2*a^2*b^8*d*(b^
4)^(1/2) + 20*a^3*b^7*d*(b^4)^(1/2) + 40*a^4*b^6*d*(b^4)^(1/2) + 30*a^5*b^5
d(b^4)^(1/2) + 2*a^7*b^3*d*(b^4)^(1/2)))/(a*b^5*(a + b)^5*(a*d^2*(a + b)^
5)^(1/2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5*a^4*b + a
^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*
a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)) - (2*(b^9*(a^6*d^2 +
a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^
(1/2) + 4*a*b^8*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3
*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + 6*a^2*b^7*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b
*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + 4*a^3*b^6*(
a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4
*b^2*d^2)^(1/2) + a^4*b^5*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^
2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)))/(a^2*b^3*d*(a + b)^7*(b^4)^(1/
2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5*a^4*b + a^5 + b
^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^
4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))) + (2*exp(3*c)*exp(3*d*x)*(
b^9*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 1
0*a^4*b^2*d^2)^(1/2) + 4*a*b^8*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b
^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + 6*a^2*b^7*(a^6*d^2 + a*b^
5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2
) + 4*a^3*b^6*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b
^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + a^4*b^5*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2
 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)))/(a^2*b^3*d*(a +
 b)^7*(b^4)^(1/2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5*
a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b
*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)))*((a^11*(a^6
*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^
2*d^2)^(1/2))/4 + (a*b^10*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^
2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/4 + (5*a^10*b*(a^6*d^2 + a*b^5*
d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))
/2 + (5*a^2*b^9*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3
*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/2 + (45*a^3*b^8*(a^6*d^2 + a*b^5*d^2 + 5*
a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/4 + 30*
a^4*b^7*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2
 + 10*a^4*b^2*d^2)^(1/2) + (105*a^5*b^6*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2
+ 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/2 + 63*a^6*b^5*(a
^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*
b^2*d^2)^(1/2) + (105*a^7*b^4*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^
4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/2 + 30*a^8*b^3*(a^6*d^2 + a
*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(
1/2) + (45*a^9*b^2*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*
a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))/4)) + 2*atan((b^2*exp(d*x)*exp(c)*(a*d
^2*(a + b)^5)^(1/2))/(2*a*d*(a + b)^2*(b^4)^(1/2)))))/(2*(a^6*d^2 + a*b^5*d

```
^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))
- (exp(- c - d*x)*(3*a + 7*b))/(8*d*(a + b)^2) + (exp(c + d*x)*(3*a + 7*b))
/(8*d*(a + b)^2)
```

3.107 $\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{(a+3b)x}{2(a+b)^2}+\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

```
[Out] 1/2*(a+3*b)*x/(a+b)^2+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d+b^(3/2)*arctan(b^
(1/2)*tanh(d*x+c)/a^(1/2))/(a+b)^2/d/a^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3756, 425, 536, 212, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a+b)^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)}+\frac{x(a+3b)}{2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + 3*b)*x)/(2*(a + b)^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[
a]])/(Sqrt[a]*(a + b)^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} + \frac{\text{Subst}\left(\int \frac{a+2b+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}\\ &= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d} + \frac{b^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d} + \frac{(a+3b)\text{Su}}{}\\ &= \frac{(a+3b)x}{2(a+b)^2} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 77, normalized size = 1.00

$$\frac{2\sqrt{a}(a+3b)(c+dx)+4b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}(a+b)\sinh(2(c+dx))}{4\sqrt{a}(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

[Out] (2*Sqrt[a]*(a + 3*b)*(c + d*x) + 4*b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/S
qrt[a]] + Sqrt[a]*(a + b)*Sinh[2*(c + d*x)])/(4*Sqrt[a]*(a + b)^2*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(65) = 130$.
time = 2.54, size = 311, normalized size = 4.04

method	result
risch	$\frac{ax}{2(a+b)^2}+\frac{3xb}{2(a+b)^2}+\frac{e^{2dx+2c}}{8(a+b)d}-\frac{e^{-2dx-2c}}{8(a+b)d}+\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a(a+b)^2d}-\frac{\sqrt{-ab}\,b\ln\left(e^{2\ldots\right.}{2\ldots}$
derivativedivides	$\frac{-\frac{2b^2a\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(a+\sqrt{b(a+b)}+b\right)\operatorname{arct}\ldots}{2a\sqrt{b(a+b)}\ldots}\right.}{(a+b)^2}\ldots}{\ldots}$
default	$\frac{-\frac{2b^2a\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(a+\sqrt{b(a+b)}+b\right)\operatorname{arct}\ldots}{2a\sqrt{b(a+b)}\ldots}\right.}{(a+b)^2}\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/d*(-2*b^2/(a+b)^2*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b
*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(
1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)
+a+2*b)*a)^(1/2)))+1/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)-1)^2+2/(4*a+4*b)/(tanh(
1/2*d*x+1/2*c)-1)+1/2/(a+b)^2*(-a-3*b)*ln(tanh(1/2*d*x+1/2*c)-1)-1/(2*b+2*a
)/(tanh(1/2*d*x+1/2*c)+1)^2+2/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)+1)+1/2*(a+3*b)
/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(65) = 130$.

time = 0.51, size = 316, normalized size = 4.10

$$\frac{b \log \left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d}-\frac{b \log \left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d}-\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}\,d}+\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}\,d}-\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}\,(a+b)d}+\frac{dx+c}{2\,(a+b)d}+\frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d}-\frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
2 + 2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*
d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2*((
a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d
) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 1/2*b*arctan(1/2*((a + b)*e^(-2*d*x -
 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) + 1/2*(d*x + c)/((a + b)*d)
 + 1/8*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(65) = 130.
time = 0.50, size = 948, normalized size = 12.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a + 3*b)*d*x*cosh(d*x + c)^2 + (a + b)*cosh(d*x + c)^4 + 4*(a + b)
*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(2*(a + 3*b)*d
*x + 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 + 2*
b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/a)*log(((a^2 + 2
*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^
2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 +
 a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*c
osh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b
)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sq
rt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)
*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(2*(a + 3*b)*d*x*cosh(d*x + c)
+ (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) - a - b)/((a^2 + 2*a*b + b^2)*d*co
sh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ 2*a*b + b^2)*d*sinh(d*x + c)^2), 1/8*(4*(a + 3*b)*d*x*cosh(d*x + c)^2 + (
a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*
sinh(d*x + c)^4 + 2*(2*(a + 3*b)*d*x + 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
```

```
x + c)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*
x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(2
*(a + 3*b)*d*x*cosh(d*x + c) + (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) - a -
 b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d
*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(65) = 130.
time = 1.05, size = 155, normalized size = 2.01

$$\frac{\frac{8\,b^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}}+\frac{4\,(dx+c)(a+3\,b)}{a^2+2\,ab+b^2}-\frac{\left(2\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-2\,dx-2\,c)}}{a^2+2\,ab+b^2}+\frac{e^{(2\,dx+2\,c)}}{a+b}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 4*(d*x + c)*(a + 3*b)/(a^2 + 2*a*b
+ b^2) - (2*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x - 2*
c)/(a^2 + 2*a*b + b^2) + e^(2*d*x + 2*c)/(a + b))/d
```

Mupad [B]
time = 1.93, size = 880, normalized size = 11.43

$$\frac{e^{2c+2dx}}{8d(a+b)}-\frac{e^{-2c-2dx}}{8d(a+b)}+\frac{x(a+3b)}{2(a+b)^2}+\frac{\mathrm{atan}\left(\left(e^{2c}e^{2dx}\left(\frac{2\left(2b^3\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}+2ab^2\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}\right)}{d(a+b)^5\sqrt{b^3}(a^3+3a^2b+3ab^2+b^3)\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}}-\frac{(a-b)\left(2ad(b^3)^{3/2}+bd(b^3)^{3/2}-a^4d\sqrt{b^3}-2a^3bd\sqrt{b^3}\right)}{b^2(a+b)^3\sqrt{ad^2(a+b)^4}(a^3+3a^2b+3ab^2+b^3)\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}}\right)+\frac{(a-b)\left(4ad(b^3)^{3/2}+bd(b^3)^{3/2}+a^4d\sqrt{b^3}+4a^3bd\sqrt{b^3}+6a^2b^2d\sqrt{b^3}\right)}{b^2(a+b)^3\sqrt{ad^2(a+b)^4}(a^3+3a^2b+3ab^2+b^3)\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}}\right)\left(\frac{a^4\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}}{2}+\frac{b^4\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}}{2}+3a^2b^2\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}+2ab^3\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}+2a^3b\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}\right)\right)\sqrt{b^3}}{\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/(8*d*(a + b)) + (x*(a +
 3*b))/(2*(a + b)^2) + (atan((exp(2*c)*exp(2*d*x)*((2*(2*b^3*(a^5*d^2 + a*b
^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 2*a*b^2*(a^5*
d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))/(d*(
a + b)^5*(b^3)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*d^2 +
```

```
 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)) - ((a - b)*(2*a*d*(b^3
)^(3/2) + b*d*(b^3)^(3/2) - a^4*d*(b^3)^(1/2) - 2*a^3*b*d*(b^3)^(1/2)))/(b^
2*(a + b)^3*(a*d^2*(a + b)^4)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^
2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))) + ((a
- b)*(4*a*d*(b^3)^(3/2) + b*d*(b^3)^(3/2) + a^4*d*(b^3)^(1/2) + 4*a^3*b*d*(
b^3)^(1/2) + 6*a^2*b^2*d*(b^3)^(1/2)))/(b^2*(a + b)^3*(a*d^2*(a + b)^4)^(1/
2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a
^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))*((a^4*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^
2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/2 + (b^4*(a^5*d^2 + a*b^4*d^2 + 4
*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/2 + 3*a^2*b^2*(a^5*d^2 +
 a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 2*a*b^3*(
a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) +
2*a^3*b*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)
^(1/2)))*(b^3)^(1/2))/(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 +
6*a^3*b^2*d^2)^(1/2)
```

3.108 $\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}d}+\frac{\sinh(c+dx)}{(a+b)d}$$

```
[Out] sinh(d*x+c)/(a+b)/d+b*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/(a+b)^(3/2)/d
/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3757, 396, 211}

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a+b)^{3/2}}+\frac{\sinh(c+dx)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)*d) +
 Sinh[c + d*x]/((a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\sinh(c+dx)}{(a+b)d} + \frac{b\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{(a+b)d}$$
$$= \frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{(a+b)d}$$

Mathematica [A]
time = 0.16, size = 54, normalized size = 1.02

$$-\frac{b\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
[Out] -((b*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(3/2)*d)
) + Sinh[c + d*x]/((a + b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(45) = 90.
time = 2.68, size = 208, normalized size = 3.92

method	result
risch	$\frac{e^{dx+c}}{2(a+b)d} - \frac{e^{-dx-c}}{2(a+b)d} - \frac{b\ln\left(e^{2dx+2c} - \frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}} - 1\right)}{2\sqrt{-a^2-ab}\,(a+b)d} + \frac{b\ln\left(e^{2dx+2c} + \frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}} - 1\right)}{2\sqrt{-a^2-ab}\,(a+b)d}$
derivativedivides	[formula cut off at the right edge of the page; visible part: numerator $-\frac{2}{(2b+2a)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$ + (term over $a+b$ with factor $2ba$ times a bracket beginning $-\frac{\left(\sqrt{b(a+b)}-b\right)\text{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}} + \frac{\left(\sqrt{b(\ldots}{2a\ldots}$ …), all over d]

default	$\frac{-\frac{2}{(2b+2a)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2ba\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(\sqrt{b\right.}\cdots}{\cdots}\right)}{a+b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)-1)+2*b/(a+b)*a*(-1/2*((b*(a+b))^(1/2
)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*((b*(a+b))^(1/2)+b)/
a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)))-2/(2*b+2*a)/(tanh(1/2*d*x+1/2
*c)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(b
*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a*b
*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)
), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(45) = 90$.
time = 0.39, size = 766, normalized size = 14.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
c) + (a^2 + a*b)*sinh(d*x + c)^2 - sqrt(-a^2 - a*b)*(b*cosh(d*x + c) + b*si
```

```
nh(d*x + c))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a
+ b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*
x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x
 + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 +
 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
 b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a +
b)) - a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^3 + 2*a^2*b
+ a*b^2)*d*sinh(d*x + c)), 1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)
*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + 2*sqrt(a^2 + a
*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x + c)^3
 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a
 - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/
sqrt(a^2 + a*b)) + 2*sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*ar
ctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - a^2 - a*b)/((
a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^3 + 2*a^2*b + a*b^2)*d*sinh(d*x
 + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh (c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.68, size = 154, normalized size = 2.91

$$\frac{e^{c+dx}}{2\,d\,(a+b)}-\frac{e^{-c-dx}}{2\,d\,(a+b)}-\frac{b\,\ln\left(\sqrt{-a}\,\sqrt{a+b}+2\,a\,e^{c+dx}-\sqrt{-a}\,e^{2\,c+2\,d\,x}\,\sqrt{a+b}\right)}{2\,\sqrt{-a}\,d\,(a+b)^{3/2}}+\frac{b\,\ln\left(2\,a\,e^{c+dx}-\sqrt{-a}\,\sqrt{a+b}+\sqrt{-a}\,e^{2\,c+2\,d\,x}\,\sqrt{a+b}\right)}{2\,\sqrt{-a}\,d\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] exp(c + d*x)/(2*d*(a + b)) - exp(- c - d*x)/(2*d*(a + b)) - (b*log((-a)^(1/
2)*(a + b)^(1/2) + 2*a*exp(c + d*x) - (-a)^(1/2)*exp(2*c + 2*d*x)*(a + b)^(
1/2)))/(2*(-a)^(1/2)*d*(a + b)^(3/2)) + (b*log(2*a*exp(c + d*x) - (-a)^(1/2
)*(a + b)^(1/2) + (-a)^(1/2)*exp(2*c + 2*d*x)*(a + b)^(1/2)))/(2*(-a)^(1/2)
*d*(a + b)^(3/2))
```

3.109 $\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}\,d}$$

```
[Out] arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/d/a^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3757, 211}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(28) = 56$.
time = 2.10, size = 152, normalized size = 4.22

method	result
risch	$-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}$
derivativedivides	$\frac{2a\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\,\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(\sqrt{b(a+b)}+b\right)\arctan\left(\cdots\right)}{2a\sqrt{b(a+b)}\,\sqrt{(2\sqrt{\cdots}}}\cdots\right)}{d}$
default	$\frac{2a\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\,\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(\sqrt{b(a+b)}+b\right)\arctan\left(\cdots\right)}{2a\sqrt{b(a+b)}\,\sqrt{(2\sqrt{\cdots}}}\cdots\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*a*(-1/2*((b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2
```

```
))+1/2*((b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^
(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/(b*tanh(d*x + c)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(28) = 56.
time = 0.42, size = 511, normalized size = 14.19

$$\left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4-2\,(3\,a+b)\cosh(dx+c)^2+2\,\left(3\,(a+b)\cosh(dx+c)^2-3\,a-b\right)\sinh(dx+c)^2+4\,\left((a+b)\cosh(dx+c)^3-(3\,a+b)\cosh(dx+c)\right)\sinh(dx+c)-4\,\left(\cosh(dx+c)^3+3\cosh(dx+c)\sinh(dx+c)^2+\sinh(dx+c)^3+\left(3\cosh(dx+c)^2-1\right)\sinh(dx+c)-\cosh(dx+c)\right)\sqrt{-a^2-ab}+a+b}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2\,(a-b)\cosh(dx+c)^2+2\,\left(3\,(a+b)\cosh(dx+c)^2+a-b\right)\sinh(dx+c)^2+4\,\left((a+b)\cosh(dx+c)^3+(a-b)\cosh(dx+c)\right)\sinh(dx+c)+a+b}\right)}{2\,(a^2+ab)d},\ \frac{\sqrt{a^2+ab}\arctan\left(\frac{(a+b)\cosh(dx+c)^3+3\,(a+b)\cosh(dx+c)\sinh(dx+c)^2+(a+b)\sinh(dx+c)^3+(3\,a-b)\cosh(dx+c)+\left(3\,(a+b)\cosh(dx+c)^2+3\,a-b\right)\sinh(dx+c)}{2\sqrt{a^2+ab}}\right)+\sqrt{a^2+ab}\arctan\left(\frac{\sqrt{a^2+ab}(\cosh(dx+c)+\sinh(dx+c))}{2\,a}\right)}{(a^2+ab)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(
d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
 c) + a + b))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*
x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^
3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*
x + c))/sqrt(a^2 + a*b)) + sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh
(d*x + c) + sinh(d*x + c))/a))/((a^2 + a*b)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 0.34, size = 147, normalized size = 4.08

$$\frac{\operatorname{atan}\left(\frac{4\,a^2\,d^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{a^2\,d^2+b\,a\,d^2}\,\sqrt{a\,d^2\,\left(a+b\right)}+\mathrm{e}^{3\,c}\,\mathrm{e}^{3\,d\,x}\,\sqrt{a^2\,d^2+b\,a\,d^2}\,\sqrt{a\,d^2\,\left(a+b\right)}}{2\,a\,d\,\sqrt{a\,d^2\,\left(a+b\right)}}\right)+\operatorname{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{a\,d^2\,\left(a+b\right)}}{2\,a\,d}\right)}{\sqrt{a^2\,d^2+b\,a\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] (atan((4*a^2*d^2*exp(d*x)*exp(c) - exp(d*x)*exp(c)*(a^2*d^2 + a*b*d^2)^(1/2
)*(a*d^2*(a + b))^(1/2) + exp(3*c)*exp(3*d*x)*(a^2*d^2 + a*b*d^2)^(1/2)*(a*
d^2*(a + b))^(1/2))/(2*a*d*(a*d^2*(a + b))^(1/2))) + atan((exp(d*x)*exp(c)*
(a*d^2*(a + b))^(1/2))/(2*a*d)))/(a^2*d^2 + a*b*d^2)^(1/2)
```

3.110 $\int \frac{\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

```
[Out] arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3756, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 32, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(24) = 48.
time = 2.30, size = 160, normalized size = 5.00

method	result
risch	$-\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}$
derivativedivides	$\frac{2a\left(-\frac{\left(a-\sqrt{b(a+b)}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(-a-\sqrt{b(a+b)}-b\right)\arctan\left(\ldots\right.}{2a\sqrt{b(a+b)}\sqrt{(\ldots}}\right.}{d}$
default	$\frac{2a\left(-\frac{\left(a-\sqrt{b(a+b)}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(-a-\sqrt{b(a+b)}-b\right)\arctan\left(\ldots\right.}{2a\sqrt{b(a+b)}\sqrt{(\ldots}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*a*(-1/2*(a-(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1
/2))+1/2*(-a-(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b
```

)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)
))

Maxima [A]
time = 0.51, size = 36, normalized size = 1.12

$$-\frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(24) = 48.
time = 0.34, size = 455, normalized size = 14.22

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)\sinh(dx+c)^4+2\,(a^2-b^2)\cosh(dx+c)^2+2\,\left(3\,(a^2+2\,ab+b^2)\cosh(dx+c)^2+a^2-b^2\right)\sinh(dx+c)^2+a^2-6\,ab+b^2+4\,\left((a^2+2\,ab+b^2)\cosh(dx+c)^3+(a^2-b^2)\cosh(dx+c)\right)\sinh(dx+c)-4\,\left((a+b)\cosh(dx+c)^2+2\,(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a-b\right)\sqrt{-ab}}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2\,(a-b)\cosh(dx+c)^2+2\,\left(3\,(a+b)\cosh(dx+c)^2+a-b\right)\sinh(dx+c)^2+4\,\left((a+b)\cosh(dx+c)^3+(a-b)\cosh(dx+c)\right)\sinh(dx+c)+a+b}\right)}{2\,abd},\ \frac{\sqrt{ab}\arctan\left(\frac{\left((a+b)\cosh(dx+c)^2+2\,(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a-b\right)\sqrt{ab}}{2\,ab}\right)}{abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4
+ 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh
(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)
^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(
3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x +
c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b))/(a*b*d), sqrt(a*b)*ar
ctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) +
 (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b))/(a*b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [A]
time = 0.56, size = 44, normalized size = 1.38

$$\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt
(a*b)*d)
```

Mupad [B]
time = 1.46, size = 81, normalized size = 2.53

$$\frac{\operatorname{atan}\left(\frac{a\,\sqrt{a\,b\,d^2}-b\,\sqrt{a\,b\,d^2}+a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{a\,b\,d^2}+b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{a\,b\,d^2}}{2\,a\,b\,d}\right)}{\sqrt{a\,b\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] atan((a*(a*b*d^2)^(1/2) - b*(a*b*d^2)^(1/2) + a*exp(2*c)*exp(2*d*x)*(a*b*d^
2)^(1/2) + b*exp(2*c)*exp(2*d*x)*(a*b*d^2)^(1/2))/(2*a*b*d))/(a*b*d^2)^(1/2
)
```

3.111 $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\text{ArcTan}(\sinh(c+dx))}{bd}+\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

```
[Out] -arctan(sinh(d*x+c))/b/d+arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2
)/b/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3757, 400, 209, 211}

$$\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}-\frac{\text{ArcTan}(\sinh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(ArcTan[Sinh[c + d*x]]/(b*d)) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Sinh[c +
d*x])/Sqrt[a]])/(Sqrt[a]*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{bd} + \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+\right.}{bd}\\ &= -\frac{\tan^{-1}(\sinh(c+dx))}{bd} + \frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 55, normalized size = 1.00

$$-\frac{\frac{\sqrt{a+b}\,\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}} + 2\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -(((Sqrt[a + b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a] + 2*Ar
cTan[Tanh[(c + d*x)/2]])/(b*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(47) = 94$.
time = 2.50, size = 175, normalized size = 3.18

method	result
risch	$\frac{i\ln(e^{dx+c}-i)}{db} - \frac{i\ln(e^{dx+c}+i)}{db} + \frac{\sqrt{-a(a+b)}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-a(a+b)}\,e^{dx+c}}{a+b}-1\right)}{2adb} - \frac{\sqrt{-a(a}{}$

derivativedivides	$\frac{2a(a+b)\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(\sqrt{b(a+b)}+b\right)\arctan\left(\cdots\right.}{2a\sqrt{b(a+b)}\sqrt{\left(2\cdots\right.}}\right.}{b}$ over d
default	$\frac{2a(a+b)\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(\sqrt{b(a+b)}+b\right)\arctan\left(\cdots\right.}{2a\sqrt{b(a+b)}\sqrt{\left(2\cdots\right.}}\right.}{b}$ over d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b*a*(a+b)*(-1/2*((b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^
(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2))+1/2*((b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)
+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)
^(1/2)))-2/b*arctan(tanh(1/2*d*x+1/2*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(d*x + c))/(b*d) + 8*integrate(1/4*((a*e^(3*c) + b*e^(3*c))*e^(
3*d*x) + (a*e^c + b*e^c)*e^(d*x))/(a*b + b^2 + (a*b*e^(4*c) + b^2*e^(4*c))*
e^(4*d*x) + 2*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(47) = 94$.
time = 0.45, size = 540, normalized size = 9.82

$$\left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4-2(3a+b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2-3a-b\right)\sinh(dx+c)^2+4\left((a+b)\cosh(dx+c)^3-(3a+b)\cosh(dx+c)\right)\sinh(dx+c)+4\left(a\cosh(dx+c)^3+3a\cosh(dx+c)\sinh(dx+c)^2+a\sinh(dx+c)^3-a\cosh(dx+c)+\left(3a\cosh(dx+c)^2-a\right)\sinh(dx+c)\right)\sqrt{-\frac{a+b}{a}}+a+b}{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2(a-b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2+a-b\right)\sinh(dx+c)^2+4\left((a+b)\cosh(dx+c)^3+(a-b)\cosh(dx+c)\right)\sinh(dx+c)+a+b}\right)-4\arctan\left(\cosh(dx+c)+\sinh(dx+c)\right)}{2bd},\ \frac{\sqrt{\frac{a+b}{a}}\arctan\left(\frac{1}{2}\sqrt{\frac{a+b}{a}}\left(\cosh(dx+c)+\sinh(dx+c)\right)\right)+\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((a+b)\cosh(dx+c)^3+3(a+b)\cosh(dx+c)\sinh(dx+c)^2+(a+b)\sinh(dx+c)^3+(3a-b)\cosh(dx+c)+\left(3(a+b)\cosh(dx+c)^2+3a-b\right)\sinh(dx+c)\right)\sqrt{\frac{a+b}{a}}}{2(a+b)}\right)-2\arctan\left(\cosh(dx+c)+\sinh(dx+c)\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3
 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c)
+ (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-(a + b)/a) + a + b)/((a +
b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh
(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a
- b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*
sinh(d*x + c) + a + b)) - 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d), (
sqrt((a + b)/a)*arctan(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c)))
 + sqrt((a + b)/a)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
 + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) +
 (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a + b)/a)/(a +
b)) - 2*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.70, size = 449, normalized size = 8.16

$$\frac{\sqrt{a+b}\left(2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{a+b}\,\sqrt{a\,b^2\,d^2}}{2\,a\,b\,d}\right)-2\,\mathrm{atan}\left(\frac{\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(\frac{64\,\left(2\,a\,b^2\,d\,\sqrt{a+b}-6\,a^2\,b\,d\,\sqrt{a+b}\right)}{a^3\,b^3\,d^2\,{\left(a+b\right)}^2\,\left(a^2+2\,a\,b+b^2\right)}+\frac{32\,\left(3\,a^2\,\sqrt{a\,b^2\,d^2}-b^2\,\sqrt{a\,b^2\,d^2}+2\,a\,b\,\sqrt{a\,b^2\,d^2}\right)}{a^3\,b^2\,d\,{\left(a+b\right)}^{3/2}\,\left(a^2+2\,a\,b+b^2\right)\,\sqrt{a\,b^2\,d^2}}\right)-\frac{32\,\mathrm{e}^{3\,c}\,\mathrm{e}^{3\,d\,x}\,\left(3\,a^2\,\sqrt{a\,b^2\,d^2}-b^2\,\sqrt{a\,b^2\,d^2}+2\,a\,b\,\sqrt{a\,b^2\,d^2}\right)}{a^3\,b^2\,d\,{\left(a+b\right)}^{3/2}\,\left(a^2+2\,a\,b+b^2\right)\,\sqrt{a\,b^2\,d^2}}\right)\,\left(a^4\,b\,\left(a+b\right)\,\sqrt{a\,b^2\,d^2}+a^2\,b^3\,\left(a+b\right)\,\sqrt{a\,b^2\,d^2}+2\,a^3\,b^2\,\left(a+b\right)\,\sqrt{a\,b^2\,d^2}\right)}{192\,a-64\,b}\right)\right)}{2\,\sqrt{a\,b^2\,d^2}}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(9\,a^2\,\sqrt{b^2\,d^2}+b^2\,\sqrt{b^2\,d^2}-6\,a\,b\,\sqrt{b^2\,d^2}\right)}{9\,d\,a^2\,b-6\,d\,a\,b^2+d\,b^3}\right)}{\sqrt{b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] ((a + b)^(1/2)*(2*atan((exp(d*x)*exp(c)*(a + b)^(1/2)*(a*b^2*d^2)^(1/2))/(2
*a*b*d)) - 2*atan(((exp(d*x)*exp(c)*((64*(2*a*b^2*d*(a + b)^(1/2) - 6*a^2*b
*d*(a + b)^(1/2)))/(a^3*b^3*d^2*(a + b)^2*(2*a*b + a^2 + b^2)) + (32*(3*a^2
*(a*b^2*d^2)^(1/2) - b^2*(a*b^2*d^2)^(1/2) + 2*a*b*(a*b^2*d^2)^(1/2)))/(a^3
*b^2*d*(a + b)^(3/2)*(2*a*b + a^2 + b^2)*(a*b^2*d^2)^(1/2))) - (32*exp(3*c)
*exp(3*d*x)*(3*a^2*(a*b^2*d^2)^(1/2) - b^2*(a*b^2*d^2)^(1/2) + 2*a*b*(a*b^2
*d^2)^(1/2)))/(a^3*b^2*d*(a + b)^(3/2)*(2*a*b + a^2 + b^2)*(a*b^2*d^2)^(1/2
)))*(a^4*b*(a + b)*(a*b^2*d^2)^(1/2) + a^2*b^3*(a + b)*(a*b^2*d^2)^(1/2) +
2*a^3*b^2*(a + b)*(a*b^2*d^2)^(1/2)))/(192*a - 64*b))))/(2*(a*b^2*d^2)^(1/2
)) - (2*atan((exp(d*x)*exp(c)*(9*a^2*(b^2*d^2)^(1/2) + b^2*(b^2*d^2)^(1/2)
- 6*a*b*(b^2*d^2)^(1/2)))/(b^3*d - 6*a*b^2*d + 9*a^2*b*d)))/(b^2*d^2)^(1/2)
```

3.112 $\int \frac{\text{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}-\frac{\tanh(c+dx)}{bd}$$

```
[Out] (a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/d/a^(1/2)-tanh(d*x+c)/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3756, 396, 211}

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}-\frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Tan
h[c + d*x]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{\tanh(c+dx)}{bd}+\frac{(a+b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{bd}\\ &= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}-\frac{\tanh(c+dx)}{bd}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 50, normalized size = 1.00

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}-\frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Tan
h[c + d*x]/(b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(42) = 84.
time = 2.18, size = 197, normalized size = 3.94

method	result
derivativedivides	$$\frac{\frac{2a(a+b)\left(-\frac{\left(a-\sqrt{b(a+b)}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(-a-\sqrt{b(a+b)}-b\right)\text{arcta}}{2a\sqrt{b(a+b)}\sqrt{}}\right.}{b}}{d}$$

default	$\frac{2a(a+b)\left(-\frac{\left(a-\sqrt{b(a+b)}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(-a-\sqrt{b(a+b)}-b\right)\operatorname{arc}}{2a\sqrt{b(a+b)}}\right.}{b}$ / d
risch	$\frac{2}{bd\left(1+e^{2dx+2c}\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)a}{2\sqrt{-ab}\,db}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b*a*(a+b)*(-1/2*(a-(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b)
)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a
-2*b)*a)^(1/2))+1/2*(-a-(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^
(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*
b)*a)^(1/2)))-2/b*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1))
```

Maxima [A]
time = 0.52, size = 63, normalized size = 1.26

$$-\frac{(a+b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,bd}-\frac{2}{(be^{(-2\,dx-2\,c)}+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(a + b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b
)*b*d) - 2/((b*e^(-2*d*x - 2*c) + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(42) = 84.
time = 0.39, size = 649, normalized size = 12.98

$$\left[-\frac{((a+b)\cosh(dx+c)^2+2(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a+b)\sqrt{-ab}\log\left([\text{illegible}]\right)-4ab}{2\left(ab^2d\cosh(dx+c)^2+2ab^2d\cosh(dx+c)\sinh(dx+c)+ab^2d\sinh(dx+c)^2+ab^2d\right)},\ \frac{((a+b)\cosh(dx+c)^2+2(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a+b)\sqrt{ab}\arctan\left([\text{illegible}]\right)+2ab}{ab^2d\cosh(dx+c)^2+2ab^2d\cosh(dx+c)\sinh(dx+c)+ab^2d\sinh(dx+c)^2+ab^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (
a + b)*sinh(d*x + c)^2 + a + b)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*
x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a
*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2
+ 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(
d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x +
c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4
+ 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a
- b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^
2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a +
 b)) - 4*a*b)/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x +
 c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d), (((a + b)*cosh(d*x + c)^2 + 2*(a
+ b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*sqrt(a*
b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x +
 c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 2*a*b)/(a*b^2*d*c
osh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2*d*sinh(d*x +
 c)^2 + a*b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Giac [A]
time = 0.59, size = 70, normalized size = 1.40

$$\frac{\frac{(a+b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2}{b\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] ((a + b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*
b))/(sqrt(a*b)*b) + 2/(b*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B]
time = 1.61, size = 176, normalized size = 3.52

$$\frac{2}{b\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{\ln\left(-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}-\frac{2\,\left(a\,d+b\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{3/2}\,d}\right)\,\left(a+b\right)}{2\,\sqrt{-a}\,b^{3/2}\,d}-\frac{\ln\left(\frac{2\,\left(a\,d+b\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{3/2}\,d}-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}\right)\,\left(a+b\right)}{2\,\sqrt{-a}\,b^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] 2/(b*d*(exp(2*c + 2*d*x) + 1)) + (log(- (4*exp(2*c + 2*d*x))/b - (2*(a*d +
b*d + a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(3/2)*d))
*(a + b))/(2*(-a)^(1/2)*b^(3/2)*d) - (log((2*(a*d + b*d + a*d*exp(2*c + 2*d
*x) - b*d*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(3/2)*d) - (4*exp(2*c + 2*d*x))/
b)*(a + b))/(2*(-a)^(1/2)*b^(3/2)*d)
```

3.113 $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=86

$$-\frac{(2a+3b)\operatorname{ArcTan}(\sinh(c+dx))}{2b^2d}+\frac{(a+b)^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}-\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a+3*b)*arctan(sinh(d*x+c))/b^2/d+(a+b)^(3/2)*arctan(sinh(d*x+c)*(a+
b)^(1/2)/a^(1/2))/b^2/d/a^(1/2)-1/2*sech(d*x+c)*tanh(d*x+c)/b/d
```

Rubi [A]
time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3757, 425, 536, 209, 211}

$$\frac{(a+b)^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}-\frac{(2a+3b)\operatorname{ArcTan}(\sinh(c+dx))}{2b^2d}-\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((2*a + 3*b)*ArcTan[Sinh[c + d*x]])/(b^2*d) + ((a + b)^(3/2)*ArcTan[(S
qrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^2*d) - (Sech[c + d*x]*Tanh[c
 + d*x])/(2*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+(a+b)x^2)}\,dx,x,\sinh(c+dx)\right)}{d}\\
&= -\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd}+\frac{\text{Subst}\left(\int \frac{a+2b+(-a-b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx,x,\sinh(c+dx)\right)}{2bd}\\
&= -\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd}+\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\sinh(c+dx)\right)}{b^2d}\\
&= -\frac{(2a+3b)\tan^{-1}(\sinh(c+dx))}{2b^2d}+\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}-\text{sec}
\end{aligned}
$$

Mathematica [A]
time = 0.46, size = 79, normalized size = 0.92

$$
-\frac{\frac{2(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}}+2(2a+3b)\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+b\operatorname{sech}(c+dx)\tanh(c+dx)}{2b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((2*(a + b)^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a]
 + 2*(2*a + 3*b)*ArcTan[Tanh[(c + d*x)/2]] + b*Sech[c + d*x]*Tanh[c + d*x])
/(b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(74) = 148$.

time = 2.50, size = 236, normalized size = 2.74

method	result
derivativedivides	$-\frac{2\left(-\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{(2a+3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^2}+\frac{2\left(a^2+2ab+b^2\right)a\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctan}}{2a\sqrt{b(a+b)}\sqrt{}}\right.}{}$
default	$-\frac{2\left(-\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{(2a+3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^2}+\frac{2\left(a^2+2ab+b^2\right)a\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctan}}{2a\sqrt{b(a+b)}\sqrt{}}\right.}{}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{db\left(1+e^{2dx+2c}\right)^2}+\frac{i\ln\left(e^{dx+c}-i\right)a}{d\,b^2}+\frac{3i\ln\left(e^{dx+c}-i\right)}{2db}-\frac{i\ln\left(e^{dx+c}+i\right)a}{d\,b^2}-\frac{3i\ln\left(e^{dx+c}+i\right)}{2db}+\frac{\sqrt{-a(a+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^2*((-1/2*b*tanh(1/2*d*x+1/2*c)^3+1/2*b*tanh(1/2*d*x+1/2*c))/(tanh
(1/2*d*x+1/2*c)^2+1)^2+1/2*(2*a+3*b)*arctan(tanh(1/2*d*x+1/2*c)))+2/b^2*(a^
2+2*a*b+b^2)*a*(-1/2*((b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1
/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b
)*a)^(1/2))+1/2*((b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(
1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(e^(3*d*x + 3*c) - e^(d*x + c))/(b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*
c) + b*d) - (2*a*e^c + 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(b^2*d) + 32*int
egrate(1/16*((a^2*e^(3*c) + 2*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (a^2*e
^c + 2*a*b*e^c + b^2*e^c)*e^(d*x))/(a*b^2 + b^3 + (a*b^2*e^(4*c) + b^3*e^(4
*c))*e^(4*d*x) + 2*(a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(74) = 148$.
time = 0.42, size = 1584, normalized size = 18.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*b*cosh(d*x + c)^3 + 6*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*b*sinh(d
*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*co
sh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a +
b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt(-(a + b)/a)*log(((a + b)*cosh
(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x +
c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b
)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*s
inh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a
*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c
))*sqrt(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d
*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 2*((2*a + 3*b)
*cosh(d*x + c)^4 + 4*(2*a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a + 3*b
)*sinh(d*x + c)^4 + 2*(2*a + 3*b)*cosh(d*x + c)^2 + 2*(3*(2*a + 3*b)*cosh(d
*x + c)^2 + 2*a + 3*b)*sinh(d*x + c)^2 + 4*((2*a + 3*b)*cosh(d*x + c)^3 + (
2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 3*b)*arctan(cosh(d*x + c) +
 sinh(d*x + c)) - 2*b*cosh(d*x + c) + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*
d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x +
 c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x
+ c))*sinh(d*x + c)), -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)
^2 + b*sinh(d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)
*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*
(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x +
 c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt((a + b)/a)*arcta
```

```
n(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c))) - ((a + b)*cosh(d*x
+ c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
+ 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*
x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c
) + a + b)*sqrt((a + b)/a)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*
cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*
x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a + b)/
a)/(a + b)) + ((2*a + 3*b)*cosh(d*x + c)^4 + 4*(2*a + 3*b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (2*a + 3*b)*sinh(d*x + c)^4 + 2*(2*a + 3*b)*cosh(d*x + c)^2
 + 2*(3*(2*a + 3*b)*cosh(d*x + c)^2 + 2*a + 3*b)*sinh(d*x + c)^2 + 4*((2*a
+ 3*b)*cosh(d*x + c)^3 + (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 3
*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x
 + c)^2 - b)*sinh(d*x + c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*
sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d +
 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x +
c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 2.10, size = 1012, normalized size = 11.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)),x)

[Out] ((2*atan(((exp(d*x)*exp(c)*((64*(12*a^2*b^4*d*(3*a*b^2 + 3*a^2*b + a^3 + b^
3)^(1/2) - 2*a*b^5*d*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^(1/2) + 18*a^3*b^3*d*(
3*a*b^2 + 3*a^2*b + a^3 + b^3)^(1/2) + 6*a^4*b^2*d*(3*a*b^2 + 3*a^2*b + a^3
 + b^3)^(1/2)))/(a^3*b^9*d^2*(a + b)^2*(2*a*b + a^2 + b^2)) - (32*(3*a^5*(a
*b^4*d^2)^(1/2) - b^5*(a*b^4*d^2)^(1/2) + 4*a*b^4*(a*b^4*d^2)^(1/2) + 15*a^
4*b*(a*b^4*d^2)^(1/2) + 20*a^2*b^3*(a*b^4*d^2)^(1/2) + 27*a^3*b^2*(a*b^4*d^
2)^(1/2)))/(a^3*b^7*d*((a + b)^3)^(1/2)*(2*a*b + a^2 + b^2)*(a*b^4*d^2)^(1/
2))) + (32*exp(3*c)*exp(3*d*x)*(3*a^5*(a*b^4*d^2)^(1/2) - b^5*(a*b^4*d^2)^(
1/2) + 4*a*b^4*(a*b^4*d^2)^(1/2) + 15*a^4*b*(a*b^4*d^2)^(1/2) + 20*a^2*b^3*
(a*b^4*d^2)^(1/2) + 27*a^3*b^2*(a*b^4*d^2)^(1/2)))/(a^3*b^7*d*((a + b)^3)^(
1/2)*(2*a*b + a^2 + b^2)*(a*b^4*d^2)^(1/2)))*(a^2*b^7*(a*b^4*d^2)^(1/2) + 2
*a^3*b^6*(a*b^4*d^2)^(1/2) + a^4*b^5*(a*b^4*d^2)^(1/2)))/(384*a*b^2 + 576*a
^2*b + 192*a^3 - 64*b^3)) + 2*atan((exp(d*x)*exp(c)*(a + b)^2*(a*b^4*d^2)^(
1/2))/(2*a*b^2*d*((a + b)^3)^(1/2))))*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^(1/2)
)/(2*(a*b^4*d^2)^(1/2)) - (atan((exp(d*x)*exp(c)*(18*a^7*(b^4*d^2)^(1/2) +
3*b^7*(b^4*d^2)^(1/2) + 30*a^2*b^5*(b^4*d^2)^(1/2) + 342*a^3*b^4*(b^4*d^2)^
(1/2) + 555*a^4*b^3*(b^4*d^2)^(1/2) + 396*a^5*b^2*(b^4*d^2)^(1/2) - 34*a*b^
6*(b^4*d^2)^(1/2) + 135*a^6*b*(b^4*d^2)^(1/2)))/(b^8*d*(12*a*b + 4*a^2 + 9*
b^2)^(1/2) - 12*a*b^7*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2) + 18*a^2*b^6*d*(12*a
*b + 4*a^2 + 9*b^2)^(1/2) + 102*a^3*b^5*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2) +
117*a^4*b^4*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2) + 54*a^5*b^3*d*(12*a*b + 4*a^2
 + 9*b^2)^(1/2) + 9*a^6*b^2*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2)))*(12*a*b + 4*
a^2 + 9*b^2)^(1/2))/(b^4*d^2)^(1/2) - exp(c + d*x)/(b*d*(exp(2*c + 2*d*x) +
 1)) + (2*exp(c + d*x))/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.114 $\int \frac{\text{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{(a+b)^2\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{\tanh^3(c+dx)}{3bd}$$

```
[Out] (a+b)^2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(5/2)/d/a^(1/2)-(a+2*b)*tanh(
d*x+c)/b^2/d+1/3*tanh(d*x+c)^3/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3756, 398, 211}

$$\frac{(a+b)^2\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{\tanh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(a + 2*b)*Tanh[c + d*x])/(b^2*d) + Tanh[c + d*x]^3/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
```

```
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{a+2b}{b^2}+\frac{x^2}{b}+\frac{a^2+2ab+b^2}{b^2(a+bx^2)}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{\tanh^3(c+dx)}{3bd}+\frac{(a+b)^2\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(\right.}{b^2d}\\
&= \frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a+2b)\tanh(c+dx)}{b^2d}+\frac{\tanh^3(c+dx)}{3bd}
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 71, normalized size = 0.95

$$\frac{(a+b)^2\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{\left(3a+5b+b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(3*a + 5*b + b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(65) = 130$.
time = 2.49, size = 252, normalized size = 3.36

method	result

derivativedivides	$\frac{2a\left(a^2+2ab+b^2\right)\left(-\frac{\left(a-\sqrt{b(a+b)}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(-a-\sqrt{b(a+b)}\right.}{2a\sqrt{b(a+}}\right.}{b^2}$
default	$\frac{2a\left(a^2+2ab+b^2\right)\left(-\frac{\left(a-\sqrt{b(a+b)}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(-a-\sqrt{b(a+b)}\right.}{2a\sqrt{b(a+}}\right.}{b^2}$
risch	$\frac{2a\,e^{4dx+4c}+2b\,e^{4dx+4c}+4a\,e^{2dx+2c}+8b\,e^{2dx+2c}+2a+\frac{10b}{3}}{d\,b^2\left(1+e^{2dx+2c}\right)^3}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)a^2}{2\sqrt{-ab}\,d\,b^2}-\frac{\ln\left(e^{2dx}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*a*(a^2+2*a*b+b^2)*(-1/2*(a-(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/
((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(-a-(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((
2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2)))+2/b^2*((-2*b-a)*tanh(1/2*d*x+1/2*c)^5+(-8/3*b-2*a
)*tanh(1/2*d*x+1/2*c)^3+(-2*b-a)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^
2+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(65) = 130$.
time = 0.51, size = 140, normalized size = 1.87

$$-\frac{2\left(6(a+2b)e^{(-2dx-2c)}+3(a+b)e^{(-4dx-4c)}+3a+5b\right)}{3\left(3b^2e^{(-2dx-2c)}+3b^2e^{(-4dx-4c)}+b^2e^{(-6dx-6c)}+b^2\right)d}-\frac{\left(a^2+2ab+b^2\right)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -2/3*(6*(a + 2*b)*e^(-2*d*x - 2*c) + 3*(a + b)*e^(-4*d*x - 4*c) + 3*a + 5*b
)/((3*b^2*e^(-2*d*x - 2*c) + 3*b^2*e^(-4*d*x - 4*c) + b^2*e^(-6*d*x - 6*c)
+ b^2)*d) - (a^2 + 2*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a -
b)/sqrt(a*b))/(sqrt(a*b)*b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(65) = 130$.
time = 0.38, size = 2032, normalized size = 27.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 20*a*b^2
 + 24*(a^2*b + 2*a*b^2)*cosh(d*x + c)^2 + 24*(a^2*b + 2*a*b^2 + 3*(a^2*b +
a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b
+ b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(5*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a
*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*c
osh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b
+ b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4
*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x
 + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c)
+ (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4
*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b
)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)
) + 48*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b + 2*a*b^2)*cosh(d*x + c))*
sinh(d*x + c))/(a*b^3*d*cosh(d*x + c)^6 + 6*a*b^3*d*cosh(d*x + c)*sinh(d*x
+ c)^5 + a*b^3*d*sinh(d*x + c)^6 + 3*a*b^3*d*cosh(d*x + c)^4 + 3*a*b^3*d*co
sh(d*x + c)^2 + a*b^3*d + 3*(5*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)*sinh(d*x
+ c)^4 + 4*(5*a*b^3*d*cosh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c))*sinh(d*x +
 c)^3 + 3*(5*a*b^3*d*cosh(d*x + c)^4 + 6*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)
*sinh(d*x + c)^2 + 6*(a*b^3*d*cosh(d*x + c)^5 + 2*a*b^3*d*cosh(d*x + c)^3 +
 a*b^3*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b + a*b^2)*cosh(d*x + c
```

```
)^4 + 24*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b + a*b^2)*
sinh(d*x + c)^4 + 6*a^2*b + 10*a*b^2 + 12*(a^2*b + 2*a*b^2)*cosh(d*x + c)^2
 + 12*(a^2*b + 2*a*b^2 + 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 +
2*a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3
*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b
 + b^2 + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*
arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c)
 + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 24*((a^2*b + a*b^2)*
cosh(d*x + c)^3 + (a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*
cosh(d*x + c)^6 + 6*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^3*d*sinh(d*
x + c)^6 + 3*a*b^3*d*cosh(d*x + c)^4 + 3*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d
+ 3*(5*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)*sinh(d*x + c)^4 + 4*(5*a*b^3*d*co
sh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*b^3*d*cos
h(d*x + c)^4 + 6*a*b^3*d*cosh(d*x + c)^2 + a*b^3*d)*sinh(d*x + c)^2 + 6*(a*
b^3*d*cosh(d*x + c)^5 + 2*a*b^3*d*cosh(d*x + c)^3 + a*b^3*d*cosh(d*x + c))*
sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(65) = 130.
time = 0.63, size = 135, normalized size = 1.80

$$\frac{\frac{3\,(a^2+2\,ab+b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\,\left(3\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+12\,be^{(2\,dx+2\,c)}+3\,a+5\,b\right)}{b^2\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^2 + 2*a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c
) + a - b)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2*(3*a*e^(4*d*x + 4*c) + 3*b*e^(4*d
*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 12*b*e^(2*d*x + 2*c) + 3*a + 5*b)/(b^2*(e
^(2*d*x + 2*c) + 1)^3))/d
```

Mupad [B]
time = 1.65, size = 252, normalized size = 3.36

$$\frac{4}{b\,d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{8}{3\,b\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}+\frac{2\,\left(a+b\right)}{b^2\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{\ln\left(-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)}{b^2}-\frac{2\,\left(a+b\right)\,\left(a+b+a\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{5/2}}\right)\,{\left(a+b\right)}^2}{2\,\sqrt{-a}\,b^{5/2}\,d}-\frac{\ln\left(\frac{2\,\left(a+b\right)\,\left(a+b+a\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{5/2}}-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)}{b^2}\right)\,{\left(a+b\right)}^2}{2\,\sqrt{-a}\,b^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)),x)
```

```
[Out] 4/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - 8/(3*b*d*(3*exp(2*c +
 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (2*(a + b))/(b^2*d*
(exp(2*c + 2*d*x) + 1)) + (log(- (4*exp(2*c + 2*d*x)*(a + b))/b^2 - (2*(a +
 b)*(a + b + a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(5/2))
)*(a + b)^2)/(2*(-a)^(1/2)*b^(5/2)*d) - (log((2*(a + b)*(a + b + a*exp(2*c
+ 2*d*x) - b*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(5/2)) - (4*exp(2*c + 2*d*x)*
(a + b))/b^2)*(a + b)^2)/(2*(-a)^(1/2)*b^(5/2)*d)
```

3.115 $\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=128

$$\frac{b^2(6a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}d}+\frac{(a+3b)\sinh(c+dx)}{(a+b)^3d}+\frac{\sinh^3(c+dx)}{3(a+b)^2d}+\frac{b^3\sinh(c+dx)}{2a(a+b)^3d\left(a+(a+b)\sinh\right.}$$

```
[Out] 1/2*b^2*(6*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(7/2)
/d+(a+3*b)*sinh(d*x+c)/(a+b)^3/d+1/3*sinh(d*x+c)^3/(a+b)^2/d+1/2*b^3*sinh(d
*x+c)/a/(a+b)^3/d/(a+(a+b)*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3757, 398, 393, 211}

$$\frac{b^2(6a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{2ad(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh^3(c+dx)}{3d(a+b)^2}+\frac{(a+3b)\sinh(c+dx)}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a +
 b)^(7/2)*d) + ((a + 3*b)*Sinh[c + d*x])/((a + b)^3*d) + Sinh[c + d*x]^3/(3
*(a + b)^2*d) + (b^3*Sinh[c + d*x])/(2*a*(a + b)^3*d*(a + (a + b)*Sinh[c +
d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a+3b}{(a+b)^3}+\frac{x^2}{(a+b)^2}+\frac{b^2(3a+b)+3b^2(a+b)x^2}{(a+b)^3(a+(a+b)x^2)^2}\right)\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a+3b)\sinh(c+dx)}{(a+b)^3d}+\frac{\sinh^3(c+dx)}{3(a+b)^2d}+\frac{\text{Subst}\left(\int \frac{b^2(3a+b)+3b^2(a+b)x^2}{(a+(a+b)x^2)^2}\,dx, x, \text{s}\right.}{(a+b)^3d}$$

$$= \frac{(a+3b)\sinh(c+dx)}{(a+b)^3d}+\frac{\sinh^3(c+dx)}{3(a+b)^2d}+\frac{b^3\sinh(c+dx)}{2a(a+b)^3d\left(a+(a+b)\sinh^2(c+\right.}$$

$$= \frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}d}+\frac{(a+3b)\sinh(c+dx)}{(a+b)^3d}+\frac{\sinh^3(c+}{3(a+b)}$$

Mathematica [A]
time = 0.80, size = 111, normalized size = 0.87

$$\frac{-\frac{6b^2(6a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}(a+b)^{7/2}}+\frac{3\left(3a+11b+\frac{4b^3}{a(a-b+(a+b)\cosh(2(c+dx)))}\right)\sinh(c+dx)}{(a+b)^3}+\frac{\sinh(3(c+dx))}{(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((-6*b^2*(6*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(3/2)*(a
 + b)^(7/2)) + (3*(3*a + 11*b + (4*b^3)/(a*(a - b + (a + b)*Cosh[2*(c + d*x
)]))))*Sinh[c + d*x])/(a + b)^3 + Sinh[3*(c + d*x)]/(a + b)^2)/(12*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(114) = 228$.

time = 3.00, size = 378, normalized size = 2.95

method	result
derivativedivides	$\frac{-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{a+3b}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{}{2(a+b)^2}}{}$
default	$\frac{-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{a+3b}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{}{2(a+b)^2}}{}$
risch	$\frac{e^{3dx+3c}}{24(a^2+2ab+b^2)d}+\frac{3\,e^{dx+c}a}{8(a^2+2ab+b^2)(a+b)d}+\frac{11\,e^{dx+c}b}{8(a^2+2ab+b^2)(a+b)d}-\frac{3\,e^{-dx-c}a}{8(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{11\,e^{-dx-c}b}{8(a^3+3a^2b+3a\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/3/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c
)+1)^2-(a+3*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)-1/3/(a+b)^2/(tanh(1/2*d*x+1/
2*c)-1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-(a+3*b)/(a+b)^3/(tanh(1/2*d
*x+1/2*c)-1)+2*b^2/(a+b)^3*((-1/2/a*b*tanh(1/2*d*x+1/2*c)^3+1/2/a*b*tanh(1/
2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1
/2*d*x+1/2*c)^2+a)+1/2*(6*a+b)*(-1/2*((b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/
((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*((b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(
b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(
1/2)+a+2*b)*a)^(1/2)))))

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -1/24*(a^3 + 2*a^2*b + a*b^2 - (a^3*e^(10*c) + 2*a^2*b*e^(10*c) + a*b^2*e^(
10*c))*e^(10*d*x) - (11*a^3*e^(8*c) + 42*a^2*b*e^(8*c) + 31*a*b^2*e^(8*c))*
e^(8*d*x) - 2*(5*a^3*e^(6*c) + 4*a^2*b*e^(6*c) - 49*a*b^2*e^(6*c) + 12*b^3*
e^(6*c))*e^(6*d*x) + 2*(5*a^3*e^(4*c) + 4*a^2*b*e^(4*c) - 49*a*b^2*e^(4*c)
+ 12*b^3*e^(4*c))*e^(4*d*x) + (11*a^3*e^(2*c) + 42*a^2*b*e^(2*c) + 31*a*b^2
*e^(2*c))*e^(2*d*x))/((a^5*d*e^(7*c) + 4*a^4*b*d*e^(7*c) + 6*a^3*b^2*d*e^(7
*c) + 4*a^2*b^3*d*e^(7*c) + a*b^4*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^(5*c) +
 2*a^4*b*d*e^(5*c) - 2*a^2*b^3*d*e^(5*c) - a*b^4*d*e^(5*c))*e^(5*d*x) + (a^
5*d*e^(3*c) + 4*a^4*b*d*e^(3*c) + 6*a^3*b^2*d*e^(3*c) + 4*a^2*b^3*d*e^(3*c)
 + a*b^4*d*e^(3*c))*e^(3*d*x)) + 1/8*integrate(8*((6*a*b^2*e^(3*c) + b^3*e^
(3*c))*e^(3*d*x) + (6*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4 + (a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^3*b^2*e^(4*c
) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c))*e^(4*d*x) + 2*(a^5*e^(2*c) + 2*a^4*b
*e^(2*c) - 2*a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3642 vs. $2(114) = 228$.
time = 0.44, size = 6934, normalized size = 54.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/24*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^10 + 10*(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*sinh(d*x + c)^10 + (11*a^5 + 53*a^4*b + 73*a^3*b^2
 + 31*a^2*b^3)*cosh(d*x + c)^8 + (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b
^3 + 45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^3 + (11*a^5
 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(
5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^6 + 2*(
5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4 + 105*(a^5 + 3*a^4*b +
 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^4 + 14*(11*a^5 + 53*a^4*b + 73*a^3*b^2
+ 31*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a
^3*b^2 + a^2*b^3)*cosh(d*x + c)^5 + 14*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31
*a^2*b^3)*cosh(d*x + c)^3 + 3*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 +
12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*
b^3 - 2*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c
)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^6 - 5*a^5
- 9*a^4*b + 45*a^3*b^2 + 37*a^2*b^3 - 12*a*b^4 + 35*(11*a^5 + 53*a^4*b + 73
*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^4 + 15*(5*a^5 + 9*a^4*b - 45*a^3*b^2 -
 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(a^5 + 3*a
^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^7 + 7*(11*a^5 + 53*a^4*b + 73*a^3
*b^2 + 31*a^2*b^3)*cosh(d*x + c)^5 + 5*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a
^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3 - (5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2
*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (11*a^5 + 53*a^4*b + 73*a
^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^2 + (45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*cosh(d*x + c)^8 + 28*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*co
sh(d*x + c)^6 - 11*a^5 - 53*a^4*b - 73*a^3*b^2 - 31*a^2*b^3 + 30*(5*a^5 + 9
*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 - 12*(5*a^5 +
9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - 6*((6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^7 + 7*(6*a^2*b^2 + 7*a*b
^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 + 7*a*b^3 + b^4)*sinh(
d*x + c)^7 + 2*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^5 + (12*a^2*b^2 -
10*a*b^3 - 2*b^4 + 21*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 5*(7*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^3 + 2*(6*a^2*b^2 -
 5*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 7*a*b^3 + b^4
)*cosh(d*x + c)^3 + (35*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^4 + 6*a^2
*b^2 + 7*a*b^3 + b^4 + 20*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^3 + (21*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^5 + 20*(6*a^2*b
^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*
x + c))*sinh(d*x + c)^2 + (7*(6*a^2*b^2 + 7*a*b^3 + b^4)*cosh(d*x + c)^6 +
10*(6*a^2*b^2 - 5*a*b^3 - b^4)*cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 7*a*b^3 + b
^4)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x
+ c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
- 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sin
```

```
h(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d
*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x +
 c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 -
a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*
cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a
- b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*cosh(d*x + c)^9 + 4*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*
b^3)*cosh(d*x + c)^7 + 6*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*
b^4)*cosh(d*x + c)^5 - 4*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*
b^4)*cosh(d*x + c)^3 - (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d
*x + c))*sinh(d*x + c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b
^4 + a^2*b^5)*d*cosh(d*x + c)^7 + 7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^
3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^6 + (a^7 + 5*a^6*b +
 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*sinh(d*x + c)^7 + 2*(a^7
+ 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5
+ (21*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cos
h(d*x + c)^2 + 2*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b
^5)*d)*sinh(d*x + c)^5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b
^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 5*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4
*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 2*(a^7 + 3*a^6*b + 2*a^5*b^
2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35
*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x
 + c)^4 + 20*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(cosh(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

3.116 $\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal. Leaf size=140

$$\frac{(a+5b)x}{2(a+b)^3}+\frac{b^{3/2}(5a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}-\frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] 1/2*(a+5*b)*x/(a+b)^3+1/2*b^(3/2)*(5*a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2
))/a^(3/2)/(a+b)^3/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2
)-1/2*(a-b)*b*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3756, 425, 541, 536, 212, 211}

$$\frac{b^{3/2}(5a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^3}-\frac{b(a-b)\tanh(c+dx)}{2ad(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x(a+5b)}{2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + 5*b)*x)/(2*(a + b)^3) + (b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(
a + b)*d*(a + b*Tanh[c + d*x]^2)) - ((a - b)*b*Tanh[c + d*x])/(2*a*(a + b)^
2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b+3bx^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx\right.}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Su}}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{(b^2}{}$$

$$= \frac{(a+5b)x}{2(a+b)^3} + \frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3d} + \frac{\cosh(c+dx)\sinh(c-}{2(a+b)d\left(a+b\tanh^2(c\right.}$$

Mathematica [A]
time = 0.59, size = 110, normalized size = 0.79

$$\frac{2(a+5b)(c+dx) + \frac{2b^{3/2}(5a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + (a+b)\sinh(2(c+dx)) + \frac{2b^2(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}}{4(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*(a + 5*b)*(c + d*x) + (2*b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a]])/a^(3/2) + (a + b)*Sinh[2*(c + d*x)] + (2*b^2*(a + b)*Sinh[2*(c
+ d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(4*(a + b)^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(124) = 248.
time = 2.76, size = 386, normalized size = 2.76

method	result

derivativedivides	$\frac{\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-a-5b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{2(a+b)^2}}{}$
default	$\frac{\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-a-5b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{2(a+b)^2}}{}$
risch	$\frac{xa}{2(a+b)(a^2+2ab+b^2)}+\frac{5xb}{2(a+b)(a^2+2ab+b^2)}+\frac{e^{2dx+2c}}{8(a^2+2ab+b^2)d}-\frac{e^{-2dx-2c}}{8(a^2+2ab+b^2)d}-\frac{b^2\left(a\,e^{2dx+2c}\right.}{da(a+b)^3\left(a\,e^{4dx+4c}+b\,e^{4dx+}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)
```

```
-1)+1/2/(a+b)^3*(-a-5*b)*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/(a+b)^2/(tanh(1/2*d*
x+1/2*c)+1)^2+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)+1/2*(a+5*b)/(a+b)^3*ln(ta
nh(1/2*d*x+1/2*c)+1)-2*b^2/(a+b)^3*((-1/2/a*(a+b)*tanh(1/2*d*x+1/2*c)^3-1/2
/a*(a+b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2
*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(5*a+b)*(-1/2*(-a+(b*(a+b))^(1/2)-b)
/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/
(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(124) = 248$.
time = 0.61, size = 840, normalized size = 6.00

$$\frac{b\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{2(a^3+3a^2b+3ab^2+b^3)d}-\frac{b\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a^3+3a^2b+3ab^2+b^3)d}-\frac{(3a^2b-6ab^2-b^3)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab}\,d}+\frac{(3a^2b-6ab^2-b^3)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab}\,d}-\frac{(3ab+b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{4(a^3+2a^2b+ab^2)\sqrt{ab}\,d}+\frac{a^2b-b^3+(a^2b-6ab^2+b^3)e^{(2dx+2c)}}{4(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4+(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)e^{(4dx+4c)}+2(a^5+2a^4b-2a^2b^3-ab^4)e^{(2dx+2c)})d}-\frac{a^2b-b^3+(a^2b-6ab^2+b^3)e^{(-2dx-2c)}}{4(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4+2(a^5+2a^4b-2a^2b^3-ab^4)e^{(-2dx-2c)}+(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)e^{(-4dx-4c)})d}+\frac{ab+b^2+(ab-b^2)e^{(-2dx-2c)}}{2(a^4+3a^3b+3a^2b^2+ab^3+2(a^4+a^3b-a^2b^2-ab^3)e^{(-2dx-2c)}+(a^4+3a^3b+3a^2b^2+ab^3)e^{(-4dx-4c)})d}+\frac{dx+c}{2(a^2+2ab+b^2)d}+\frac{e^{(2dx+2c)}}{8(a^2+2ab+b^2)d}-\frac{e^{(-2dx-2c)}}{8(a^2+2ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a
 + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/8*(
3*a^2*b - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/8*(3*a^2*b - 6*
a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/4*(3*a*b + b^2)*arctan(1/2
*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqr
t(a*b)*d) + 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x + 2*c))/((a
^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 +
 4*a^2*b^3 + a*b^4)*e^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)
*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(-2*d*x
 - 2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(a^5 + 2*a^4*b
 - 2*a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
*b^3 + a*b^4)*e^(-4*d*x - 4*c))*d) + 1/2*(a*b + b^2 + (a*b - b^2)*e^(-2*d*x
 - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a
*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*
c))*d) + 1/2*(d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(2*d*x + 2*c)/((a^2
+ 2*a*b + b^2)*d) - 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(124) = 248$.
time = 0.40, size = 4324, normalized size = 30.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/8*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*c
osh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^8 + 2*
(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 -
a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3
+ 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
 + c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 +
 2*(35*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 4*a*b^2 + 4*b^3 + 4*(a^3 +
 4*a^2*b - 5*a*b^2)*d*x + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x
+ c)^5 + 5*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3
- 4*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x +
 c)^3 - a^3 - 2*a^2*b - a*b^2 - 2*(a^3 + 3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b
 + 5*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
 c)^6 + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^4
- a^3 - 3*a*b^2 - 4*b^3 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x - 24*(a*b^2 - b^3
 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*((5*
a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^6 + 6*(5*a^2*b + 6*a*b^2 + b^3)*cosh(d
*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 6*a*b^2 + b^3)*sinh(d*x + c)^6 + 2*(5*
a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^4 + (10*a^2*b - 8*a*b^2 - 2*b^3 + 15*(
5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b +
 6*a*b^2 + b^3)*cosh(d*x + c)^3 + 2*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)
)*sinh(d*x + c)^3 + (5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(5*a^2*
b + 6*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^2*b + 6*a*b^2 + b^3 + 12*(5*a^2*b
- 4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 6*a*b^2
 + b^3)*cosh(d*x + c)^5 + 4*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^3 + (5*
a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 +
 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c
)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2
 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)
*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a
*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*
sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*co
sh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a -
b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*co
sh(d*x + c)^7 + 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x
+ c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3 -
(a^3 + 3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*si
nh(d*x + c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x +
c)^6 + 6*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*si
nh(d*x + c)^5 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*sinh(d*x
```

```
+ c)^6 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + (15*(a^5
 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*
a^4*b - 2*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^4 + (a^5 + 4*a^4*b + 6*a^3*b^2
+ 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 4*a^4*b + 6*a^3*b^2 +
4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4
)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 12*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*
cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d)*sinh(d
*x + c)^2 + 2*(3*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x
 + c)^5 + 4*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3 + (a^5 +
4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1
/8*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cos
h(d*x + c)*sinh(d*x + c)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^8 + 2*(a
^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 - a*
b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 +
3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x +
 c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 + 2
*(35*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 4*a*b^2 + 4*b^3 + 4*(a^3 + 4
*a^2*b - 5*a*b^2)*d*x + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
c)^5 + 5*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3 -
4*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^3 - a^3 - 2*a^2*b - a*b^2 - 2*(a^3 + 3*a*b^2 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(cosh(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(124) = 248.
time = 1.30, size = 416, normalized size = 2.97

$$\frac{\frac{12\,(dx+c)(a+5\,b)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{12\,\left(5\,ab^2+b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\sqrt{ab}}+\frac{3\,e^{(2\,dx+2\,c)}}{a^2+2\,ab+b^2}-\frac{2\,a^3e^{(6\,dx+6\,c)}+12\,a^2be^{(6\,dx+6\,c)}+10\,ab^2e^{(6\,dx+6\,c)}+7\,a^3e^{(4\,dx+4\,c)}+22\,a^2be^{(4\,dx+4\,c)}+7\,ab^2e^{(4\,dx+4\,c)}-24\,b^3e^{(4\,dx+4\,c)}+8\,a^3e^{(2\,dx+2\,c)}+12\,a^2be^{(2\,dx+2\,c)}+28\,ab^2e^{(2\,dx+2\,c)}+24\,b^3e^{(2\,dx+2\,c)}+3\,a^3+6\,a^2b+3\,ab^2}{\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\left(ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+2\,ae^{(4\,dx+4\,c)}-2\,be^{(4\,dx+4\,c)}+ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}\right)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(12*(d*x + c)*(a + 5*b)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 12*(5*a*b^2
+ b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)
)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) + 3*e^(2*d*x + 2*c)/(a^2
+ 2*a*b + b^2) - (2*a^3*e^(6*d*x + 6*c) + 12*a^2*b*e^(6*d*x + 6*c) + 10*a*b
^2*e^(6*d*x + 6*c) + 7*a^3*e^(4*d*x + 4*c) + 22*a^2*b*e^(4*d*x + 4*c) + 7*a
*b^2*e^(4*d*x + 4*c) - 24*b^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) + 12*
a^2*b*e^(2*d*x + 2*c) + 28*a*b^2*e^(2*d*x + 2*c) + 24*b^3*e^(2*d*x + 2*c) +
 3*a^3 + 6*a^2*b + 3*a*b^2)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*e^(6*d*
x + 6*c) + b*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x + 4*c) +
a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

3.117 $\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{b(4a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}d}+\frac{\sinh(c+dx)}{(a+b)^2d}+\frac{b^2\sinh(c+dx)}{2a(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*b*(4*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(5/2)/d
+sinh(d*x+c)/(a+b)^2/d+1/2*b^2*sinh(d*x+c)/a/(a+b)^2/d/(a+(a+b)*sinh(d*x+c)
^2)
```

Rubi [A]
time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3757, 398, 393, 211}

$$\frac{b(4a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{5/2}}+\frac{b^2\sinh(c+dx)}{2ad(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b
)^(5/2)*d) + Sinh[c + d*x]/((a + b)^2*d) + (b^2*Sinh[c + d*x])/(2*a*(a + b)
^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
 :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2}+\frac{b(2a+b)+2b(a+b)x^2}{(a+b)^2(a+(a+b)x^2)^2}\right)\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{(a+b)^2d}+\frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{(a+b)^2d}$$

$$= \frac{\sinh(c+dx)}{(a+b)^2d}+\frac{b^2\sinh(c+dx)}{2a(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}+\frac{(b(4a+b))\text{Subst}\big(}{}$$

$$= \frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}d}+\frac{\sinh(c+dx)}{(a+b)^2d}+\frac{b^2\sinh(c}{2a(a+b)^2d\,(a+(a+}$$

Mathematica [A]
time = 0.60, size = 89, normalized size = 0.88

$$\frac{-\frac{b(4a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}(a+b)^{5/2}}+\frac{2\left(1+\frac{b^2}{a(a-b+(a+b)\cosh(2(c+dx)))}\right)\sinh(c+dx)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-((b*(4*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(3/2)*(a +
b)^(5/2))) + (2*(1 + b^2/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))*Sinh[c +
d*x])/(a + b)^2)/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(89) = 178$.

time = 2.88, size = 286, normalized size = 2.83

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b\left(\frac{-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(4a+b)\left(-\frac{\left(\sqrt{b\,(a+b)}\right.}{2a\sqrt{b\,(}}\right.}{}\right.}{}}{}$
default	$\frac{-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b\left(\frac{-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(4a+b)\left(-\frac{\left(\sqrt{b\,(a+b)}\right.}{2a\sqrt{b\,(}}\right.}{}\right.}{}}{}$
risch	$\frac{e^{dx+c}}{2(a^2+2ab+b^2)d}-\frac{e^{-dx-c}}{2(a^2+2ab+b^2)d}+\frac{b^2 e^{dx+c}\left(e^{2dx+2c}-1\right)}{d(a+b)^2 a\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}-\frac{b\ln\left(e^{2dx}\right.}{\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)+2*b/(a+b)^2*((-1/2/a*b*tanh(1/2*d*x+1/2*c)^3+1/2/a*b*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(4*a+b)*(-1/2*((b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*((b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))-1/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1))

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -1/2*(a^2 + a*b - (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) - (a^2*e^(4*c) - 3*a*b*e^(4*c) + 2*b^2*e^(4*c))*e^(4*d*x) + (a^2*e^(2*c) - 3*a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(5*c) + 3*a^3*b*d*e^(5*c) + 3*a^2*b^2*d*e^(5*c) + a*b^3*d*e^(5*c))*e^(5*d*x) + 2*(a^4*d*e^(3*c) + a^3*b*d*e^(3*c) - a^2*b^2*d*e^(3*c) - a*b^3*d*e^(3*c))*e^(3*d*x) + (a^4*d*e^c + 3*a^3*b*d*e^c + 3*a^2*b^2*d*e^c + a*b^3*d*e^c)*e^(d*x)) + 1/2*integrate(2*((4*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (4*a*b*e^c + b^2*e^c)*e^(d*x))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + a^3*b*e^(2*c) - a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1815 vs. $2(89) = 178$.
time = 0.43, size = 3502, normalized size = 34.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(2*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 12*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x + c)^6 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + 2*(a^4 - 2

*a^3*b - a^2*b^2 + 2*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 - 2*a^4 - 4*a^3*b - 2*a^2*b^2 + 8*(5*(a^4 + 2*a^3*b + a^2*b
^2)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 - 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2 + 2*(
15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 + 2*a^3*b + a^2*b^2 - 2*
a*b^3 + 6*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - ((4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a*b^2 + b
^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*sinh(d*x + c)
^5 + 2*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a*b^2 - b
^3 + 5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4
*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(
d*x + c))*sinh(d*x + c)^2 + (4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c) + (5*(4
*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 + 6*(4*a^
2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((
(a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)
*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)
^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh
(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x +
c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x +
c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(3*(a^4 +
2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*
cosh(d*x + c)^3 - (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d
*x + c))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)
^5 + 5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)*si
nh(d*x + c)^4 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*sinh(d*
x + c)^5 + 2*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + 2*(5
*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (a^6
+ 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^3 + (a^6 + 4*a^5*b + 6*a
^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c) + 2*(5*(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + 3*(a^6 + 2*a^5*b - 2*a^3*b^
3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^6 + 4*a^5*b + 6*a^4*b
^2 + 4*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 + 6*(a^6 + 2*a^5*b - 2*a^3*b^3
- a^2*b^4)*d*cosh(d*x + c)^2 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2
*b^4)*d)*sinh(d*x + c)), 1/2*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 6
*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3*b +
a^2*b^2)*sinh(d*x + c)^6 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x +
c)^4 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 - 2*a^3*b - a^2*b^2 + 4*(5*(a^4 + 2*a^
3*b + a^2*b^2)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d
*x + c))*sinh(d*x + c)^3 - (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c
)^2 + (15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 + 2*a^3*b + a^2*b
^2 - 2*a*b^3 + 6*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(

```
d*x + c)^2 + ((4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a*
b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*sinh(d
*x + c)^5 + 2*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a*
b^2 - b^3 + 5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
2*(5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3
)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)
+ (5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 +
6*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)
*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c
)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d
*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((4*a^2*b + 5*a*b^2
+ b^3)*cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x
 + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*sinh(d*x + c)^5 + 2*(4*a^2*b - 3*a*b^2
- b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a*b^2 - b^3 + 5*(4*a^2*b + 5*a*b^2
+ b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 5*a*b^2 + b^3)*co
sh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^2
+ (4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c) + (5*(4*a^2*b + 5*a*b^2 + b^3)*co
sh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 + 6*(4*a^2*b - 3*a*b^2 - b^3)*cosh(
d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)*arct...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
[Out] Integral(cosh(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

3.118 $\int \frac{\text{sech}(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$\frac{(2a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d}+\frac{b\sinh(c+dx)}{2a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(3/2)/d+1
/2*b*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3757, 393, 211}

$$\frac{(2a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}+\frac{b\sinh(c+dx)}{2ad(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^
(3/2)*d) + (b*Sinh[c + d*x])/(2*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{(2a+b)\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \text{s}\right.}{2a(a+b)d}$$

$$= \frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d} + \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+(a+b)\sinh^2(c+dx\right.}$$

Mathematica [A]
time = 0.26, size = 78, normalized size = 0.94

$$\frac{\frac{(2a+b)\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{b\sinh(c+dx)}{a\left(a+(a+b)\sinh^2(c+dx)\right)}}{2(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a +
b]) + (b*Sinh[c + d*x])/(a*(a + (a + b)*Sinh[c + d*x]^2)))/(2*(a + b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(71) = 142$.
time = 2.49, size = 252, normalized size = 3.04

method	result

derivativedivides	$\frac{\frac{-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(2a+b)\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh}{\sqrt{\left(2\sqrt{b(a+}\right.}}\right.}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}\right.}}\right.}{}}{d}$
default	$\frac{\frac{-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(2a+b)\left(-\frac{\left(\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh}{\sqrt{\left(2\sqrt{b(a+}\right.}}\right.}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}\right.}}\right.}{}}{d}$
risch	$\frac{b\,e^{dx+c}\left(e^{2dx+2c}-1\right)}{da(a+b)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,(a+b)d}-\frac{\ln\left(e^{2dx+2c}-\right.}{4\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/2*b/a/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/a/(a+b)*tanh(1/2*d*x+1/2
*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/
2*c)^2+a)+(2*a+b)/(a+b)*(-1/2*((b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*
(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1
/2)-a-2*b)*a)^(1/2))+1/2*((b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e
^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c)
```

```
- a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/2*((2*a*e^(3*c) + b*e^(3*c))*
e^(3*d*x) + (2*a*e^c + b*e^c)*e^(d*x))/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^(4*c
) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a*b^2*e^(
2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(71) = 142$.
time = 0.43, size = 2041, normalized size = 24.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b + a*b^2)*cosh(d*x + c)*
sinh(d*x + c)^2 + 4*(a^2*b + a*b^2)*sinh(d*x + c)^3 - ((2*a^2 + 3*a*b + b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - a*b - b^2)*cosh(d*x + c
)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - a*b - b^2)*sinh(
d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^3
 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((
(a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)
*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)
^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh
(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x +
c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x +
 c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh
(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*(a^2*b + a
*b^2)*cosh(d*x + c) - 4*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 +
4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^5 + a^4*b - a
^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*d*cosh(d*x + c)^2 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c
)^2 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 4*((a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d*cosh(
d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 6*(a^2*b
 + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2*b + a*b^2)*sinh(d*x + c)^3
 + ((2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 -
 a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 +
2*a^2 - a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*
b + b^2)*cosh(d*x + c)^3 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)
```

```
)*sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) +
(3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((2
*a^2 + 3*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + (2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - a*b -
 b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2
- a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*b + b^
2)*cosh(d*x + c)^3 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt
(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) -
 2*(a^2*b + a*b^2)*cosh(d*x + c) - 2*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cos
h(d*x + c)^2)*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(
d*x + c)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^
5 + a^4*b - a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 3*a^4*b + 3*
a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d)
*sinh(d*x + c)^2 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 4*((a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 + a^4*b - a^3*b^2 - a^2
*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)\,\left(b\tanh(c+d\,x)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)
```

3.119 $\int \frac{\text{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] 1/2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/d/b^(1/2)+1/2*tanh(d*x+c)/a
/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3756, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Tanh[c + d*
x]/(2*a*d*(a + b*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
```

```
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}+\frac{\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2ad}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 63, normalized size = 0.95

$$
\frac{\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}\,\tanh(c+dx)}{a+b\tanh^2(c+dx)}}{2a^{3/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[b] + (Sqrt[a]*Tanh[c + d*x])/
(a + b*Tanh[c + d*x]^2))/(2*a^(3/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(54) = 108$.
time = 2.45, size = 232, normalized size = 3.52

method	result
risch	$-\frac{a\,e^{2dx+2c}-b\,e^{2dx+2c}+a+b}{(a+b)ad\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,da}+$

derivativedivides	$\frac{-\frac{2\left(-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{\left(a+\sqrt{b\left(a+b\right)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\right.}{\sqrt{\left(2\sqrt{b\left(a+b\right)}-\right.}}\right.}{2a\sqrt{b\left(a+b\right)}\sqrt{\left(2\sqrt{b\left(a+b\right)}+a-\right.}}}{d}$
default	$\frac{-\frac{2\left(-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{\left(a+\sqrt{b\left(a+b\right)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\right.}{\sqrt{\left(2\sqrt{b\left(a+b\right)}-\right.}}\right.}{2a\sqrt{b\left(a+b\right)}\sqrt{\left(2\sqrt{b\left(a+b\right)}+a-\right.}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2/a*tanh(1/2*d*x+1/2*c)^3-1/2/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/
2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)-1/2*(
a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2*(-a+(
b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(54) = 108$.
time = 0.53, size = 125, normalized size = 1.89

$$\frac{(a-b)e^{(-2\,dx-2\,c)}+a+b}{(a^3+2\,a^2b+ab^2+2\,(a^3-ab^2)e^{(-2\,dx-2\,c)}+(a^3+2\,a^2b+ab^2)e^{(-4\,dx-4\,c)})d}-\frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^3 + 2*a^2*b + a*b^2 + 2*(a^3 - a*b^2
)*e^(-2*d*x - 2*c) + (a^3 + 2*a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) - 1/2*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(54) = 108$.
time = 0.38, size = 1515, normalized size = 22.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 + ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^4*b - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b + 2*a*b^2 + 2*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b - a*b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^4*b - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(54) = 108.
time = 0.70, size = 138, normalized size = 2.09

$$\frac{\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2\,\left(ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a+b\right)}{(a^2+ab)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
(sqrt(a*b)*a) - 2*(a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)/((a^2 + a
*b)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2
*d*x + 2*c) + a + b)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cosh(c+d\,x)^2\,\left(b\tanh(c+d\,x)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2), x)
```

3.120 $\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=72

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}\,d}+\frac{\sinh(c+dx)}{2ad\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*sinh(d*x+c)/a/d/(a+(a+b)*sinh(d*x+c)^2)+1/2*arctan(sinh(d*x+c)*(a+b)^(1
/2)/a^(1/2))/a^(3/2)/d/(a+b)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3757, 205, 211}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a+b}}+\frac{\sinh(c+dx)}{2ad\left((a+b)\sinh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]*d) + Sin
h[c + d*x]/(2*a*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\sinh(c+dx)}{2ad\left(a+(a+b)\sinh^2(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \sinh(c+dx)\right)}{2ad}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}\,d}+\frac{\sinh(c+dx)}{2ad\left(a+(a+b)\sinh^2(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 69, normalized size = 0.96

$$\frac{\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+\frac{\sqrt{a}\,\sinh(c+dx)}{a+(a+b)\sinh^2(c+dx)}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/Sqrt[a + b] + (Sqrt[a]*Sinh[c
+ d*x])/(a + (a + b)*Sinh[c + d*x]^2))/(2*a^(3/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(60) = 120.
time = 2.54, size = 228, normalized size = 3.17

method	result
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{ad\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{4\sqrt{-a^2-ab}\,da}+\frac{\ln\left(e^{2dx+2c}+\frac{2\cdots}{\sqrt{-\cdots}}\right)}{4\sqrt{-a^2-\cdots}}$

derivativedivides	$\frac{\frac{-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\left(\sqrt{b(a+b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}+\right.}}\right.}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}+a+\right.}}}{d}$
default	$\frac{\frac{-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\left(\sqrt{b(a+b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}+\right.}}\right.}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}+a+\right.}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/2/a*tanh(1/2*d*x+1/2*c)^3+1/2/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2
*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*((
b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2*((b*(a+b
))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a
*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^3*d + 2*a^
2*b*d + a*b^2*d + (a^3*d*e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(
4*d*x) + 2*(a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/8*(
e^(3*d*x + 3*c) + e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*c) + a*b*e^(4*c))*e^(
4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(60) = 120$.
time = 0.36, size = 1555, normalized size = 21.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a +
 b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cos
h(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((
a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt
(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a
+ b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*
x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x
 + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 +
 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
 b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a +
b)) - 4*(a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2
- a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4
 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^
2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4
+ 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2
 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x +
c)^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*
cosh(d*x + c)^3 + 6*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*
b)*sinh(d*x + c)^3 + ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(
a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt(a^2 + a*b)*arctan(1/
2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a +
 b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2
+ 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((a + b)*cosh(d*x + c)^4 + 4*(
a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4
*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*s
qrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a
) - 2*(a^2 + a*b)*cosh(d*x + c) + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 -
a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 +
 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*
b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 +
2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 +
 (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)
^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)^3\,\left(b\tanh(c+d\,x)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)
```

3.121 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=77

$$-\frac{(a-b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}+\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] -1/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/b^(3/2)/d+1/2*(a+b)*
tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3756, 393, 211}

$$\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)}-\frac{(a-b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*b^(3/2)*d)
+ ((a + b)*Tanh[c + d*x])/(2*a*b*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
```

```
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{2abd}\\ &= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 83, normalized size = 1.08

$$\frac{(-a+b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)+\frac{\sqrt{a}\,\sqrt{b}\,(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2a^{3/2}b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + (Sqrt[a]*Sqrt[b]*(a + b
)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*a^(3/2)*b^(3/2
)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(65) = 130$.
time = 2.41, size = 259, normalized size = 3.36

method	result

derivativedivides	$\frac{-\frac{2\left(-\frac{(a+b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ab}-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ab}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{(a-b)\left(-\frac{\left(a-\sqrt{b(a+b)}+b\right)\operatorname{arctanh}\left(\frac{a}{\sqrt{\left(2\sqrt{b(a}}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}}}\right.}{}}{d}$
default	$\frac{-\frac{2\left(-\frac{(a+b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ab}-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ab}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{(a-b)\left(-\frac{\left(a-\sqrt{b(a+b)}+b\right)\operatorname{arctanh}\left(\frac{a}{\sqrt{\left(2\sqrt{b(a}}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}}}\right.}{}}{d}$
risch	$-\frac{a\,e^{2dx+2c}-b\,e^{2dx+2c}+a+b}{bda\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2ab}{(a+b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,db}+\frac{\ln\left(e^{2dx}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(a+b)/a/b*tanh(1/2*d*x+1/2*c)^3-1/2*(a+b)/a/b*tanh(1/2*d*x+1/
2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1
/2*c)^2+a)-(a-b)/b*(-1/2*(a-(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+
b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)
-a-2*b)*a)^(1/2))+1/2*(-a-(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2))))
```

Maxima [A]

time = 0.53, size = 127, normalized size = 1.65

$$\frac{(a-b)e^{(-2\,dx-2\,c)}+a+b}{\left(a^2b+ab^2+2\left(a^2b-ab^2\right)e^{(-2\,dx-2\,c)}+\left(a^2b+ab^2\right)e^{(-4\,dx-4\,c)}\right)d}+\frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}\,abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2*b + a*b^2 + 2*(a^2*b - a*b^2)*e^(-
2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/2*(a - b)*arctan(1/
2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(65) = 130$.
time = 0.39, size = 1443, normalized size = 18.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b - a
*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 - ((a
^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^2 - b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(
a^2 - b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2
 + 4*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh
(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x +
c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2
+ 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d
*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)))/((a^3*b^2 + a^2
*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 - a^2*b^3)*d*cosh
(d*x + c)^2 + 2*(3*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^3*b^2 - a^2*b
^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 + a^2*b^3)*d + 4*((a^3*b^2 + a^2*b^3)*d*c
osh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*
(2*a^2*b + 2*a*b^2 + 2*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(a^2*b - a*b^2)*
cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b - a*b^2)*sinh(d*x + c)^2 + ((a^2 - b
^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 -
b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 -
b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*(
(a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
 c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c
)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)))/((a^3*
b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 - a^2*b^
3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^3*b^
```

```
2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 + a^2*b^3)*d + 4*((a^3*b^2 + a^2
*b^3)*d*cosh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c
))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(65) = 130.
time = 0.71, size = 143, normalized size = 1.86

$$-\frac{\frac{(a-b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,ab}+\frac{2\left(ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a+b\right)}{\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)ab}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a - b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sq
rt(a*b))/(sqrt(a*b)*a*b) + 2*(a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b
)/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*
d*x + 2*c) + a + b)*a*b))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)^4\,\left(b\tanh(c+d\,x)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)
```

3.122 $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=102

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{b^2d}-\frac{(2a-b)\sqrt{a+b}\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}+\frac{(a+b)\sinh(c+dx)}{2abd\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] arctan(sinh(d*x+c))/b^2/d+1/2*(a+b)*sinh(d*x+c)/a/b/d/(a+(a+b)*sinh(d*x+c)^
2)-1/2*(2*a-b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2)/a^(3/2)/
b^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3757, 425, 536, 209, 211}

$$-\frac{(2a-b)\sqrt{a+b}\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}+\frac{(a+b)\sinh(c+dx)}{2abd\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\operatorname{ArcTan}(\sinh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b^2*d) - ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a + b]*
Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) + ((a + b)*Sinh[c + d*x])/(2*a*b
*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+(a+b)x^2)^2}\,dx,x,\sinh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\sinh(c+dx)}{2abd\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{a-b+(-a-b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx,x,\sinh(c+\right.}{2abd}$$

$$= \frac{(a+b)\sinh(c+dx)}{2abd\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{b^2d} - \frac{((2a}{}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{b^2d} - \frac{(2a-b)\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} + \frac{}{2abd\,(}$$

Mathematica [A]
time = 0.38, size = 203, normalized size = 1.99

$$\frac{(a-b)\left(\left(2a^2+ab-b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+4a^{3/2}\sqrt{a+b}\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a+b)\left(\left(2a^2+ab-b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+4a^{3/2}\sqrt{a+b}\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)\cosh(2(c+dx))+2\sqrt{a}\,b(a+b)^{3/2}\sinh(c+dx)}{2a^{3/2}b^2\sqrt{a+b}\,d(a-b+(a+b)\cosh(2(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)*((2*a^2 + a*b - b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] +
 4*a^(3/2)*Sqrt[a + b]*ArcTan[Tanh[(c + d*x)/2]]) + (a + b)*((2*a^2 + a*b -
 b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] + 4*a^(3/2)*Sqrt[a + b]*A
rcTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)] + 2*Sqrt[a]*b*(a + b)^(3/2)*Sin
h[c + d*x])/(2*a^(3/2)*b^2*Sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c + d*x)]
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(90) = 180$.

time = 2.45, size = 273, normalized size = 2.68

method	result
derivativedivides	$\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2} - \frac{2\left(\frac{\frac{b(a+b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a} - \frac{b(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a} + \frac{\left(2a^2+ab-b^2\right)\left(-\frac{\left(\sqrt{b\,(a+}\right.}{2a\sqrt{b}}\right.}{}\right.}{}$

default	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2\left(\frac{\frac{b(a+b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\left(2a^2+ab-b^2\right)\left(-\frac{\left(\sqrt{b\left(a+b\right)}\right.}{2a\sqrt{b\left(\right.}}\right.}{}\right.}{}}{}$
risch	$\frac{e^{dx+c}(a+b)\left(e^{2dx+2c}-1\right)}{bda\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{i\ln\left(e^{dx+c}+i\right)}{d\,b^2}-\frac{i\ln\left(e^{dx+c}-i\right)}{d\,b^2}-\frac{\sqrt{-a\left(a+b\right)}\,\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*arctan(tanh(1/2*d*x+1/2*c))-2/b^2*((1/2*b*(a+b)/a*tanh(1/2*d*x+1
/2*c)^3-1/2*b*(a+b)/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tan
h(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(2*a^2+a*b-b^2)*(-1/2*(
(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*ar
ctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*((b*(a
+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(
a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^2*b*d + a*
b^2*d + (a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^2*b*d*e^(2*c)
- a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integrat
e(1/32*((2*a^2*e^(3*c) + a*b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (2*a^2*e^c
```

```
+ a*b*e^c - b^2*e^c)*e^(d*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*b^3*e
^(4*c))*e^(4*d*x) + 2*(a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(90) = 180$.
time = 0.39, size = 2140, normalized size = 20.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b + b^2)*cosh(d*x + c)^3 + 12*(a*b + b^2)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a*b + b^2)*sinh(d*x + c)^3 - ((2*a^2 + a*b - b^2)*cosh(d*x + c
)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b -
b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^
2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2 + 2*a
^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b +
b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x +
 c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 -
 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh
(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*
x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(
d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqr
t(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3
*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c
)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*((a^2 + a*b)*cosh(
d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh
(d*x + c)^4 + 2*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c
)^2 + a^2 - a*b)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)
^3 + (a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(
d*x + c)) - 4*(a*b + b^2)*cosh(d*x + c) + 4*(3*(a*b + b^2)*cosh(d*x + c)^2
- a*b - b^2)*sinh(d*x + c))/((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^2*b
^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 + a*b^3)*d*sinh(d*x
+ c)^4 + 2*(a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*c
osh(d*x + c)^2 + (a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d
 + 4*((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^2*b^2 - a*b^3)*d*cosh(d*x +
c))*sinh(d*x + c)), 1/2*(2*(a*b + b^2)*cosh(d*x + c)^3 + 6*(a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^2 + 2*(a*b + b^2)*sinh(d*x + c)^3 - ((2*a^2 + a*b -
 b^2)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3
 + (2*a^2 + a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x +
 c)^2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sin
h(d*x + c)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 +
 (2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/a)*arctan
```

```
(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c))) - ((2*a^2 + a*b - b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (
2*a^2 + a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^
2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*
x + c)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + (2*
a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/a)*arctan(1/2
*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a +
b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 +
 3*a - b)*sinh(d*x + c))*sqrt((a + b)/a)/(a + b)) + 4*((a^2 + a*b)*cosh(d*x
 + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*
x + c)^4 + 2*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2
 + a^2 - a*b)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)^3
+ (a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x
 + c)) - 2*(a*b + b^2)*cosh(d*x + c) + 2*(3*(a*b + b^2)*cosh(d*x + c)^2 - a
*b - b^2)*sinh(d*x + c))/((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^2*b^2
+ a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 + a*b^3)*d*sinh(d*x + c
)^4 + 2*(a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh
(d*x + c)^2 + (a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d +
4*((a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^2*b^2 - a*b^3)*d*cosh(d*x + c))
*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^5\,{\left(b\,\tanh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2), x)
```

3.123 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{(3a-b)(a+b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{\tanh(c+dx)}{b^2d}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] -1/2*(3*a-b)*(a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/b^(5/2)/d+ta
nh(d*x+c)/b^2/d+1/2*(a+b)^2*tanh(d*x+c)/a/b^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3756, 398, 393, 211}

$$-\frac{(3a-b)(a+b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2d\left(a+b\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*b
^(5/2)*d) + Tanh[c + d*x]/(b^2*d) + ((a + b)^2*Tanh[c + d*x])/(2*a*b^2*d*(a
 + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{(a+bx^2)^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a^2-b^2+2b(a+b)x^2}{b^2(a+bx^2)^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\tanh(c+dx)}{b^2d}-\frac{\operatorname{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{(a+bx^2)^2}\,dx,x,\tanh(c+dx)\right)}{b^2d}\\ &= \frac{\tanh(c+dx)}{b^2d}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2d\left(a+b\tanh^2(c+dx)\right)}-\frac{((3a-b)(a+b))\operatorname{Subst}\left(\int \frac{}{a\cdots}\right.}{2ab^2\cdots}\\ &= -\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{\tanh(c+dx)}{b^2d}+\frac{(a+b)^2\tan\cdots}{2ab^2d\left(a+b\tan\cdots\right.}\end{aligned}$$

Mathematica [A]
time = 0.46, size = 102, normalized size = 1.05

$$\frac{-\frac{(3a-b)(a+b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{b}(a+b)^2\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}+2\sqrt{b}\tanh(c+dx)}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-(((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) + (
Sqrt[b]*(a + b)^2*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])
) + 2*Sqrt[b]*Tanh[c + d*x])/(2*b^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(85) = 170$.

time = 2.33, size = 306, normalized size = 3.15

method	result
derivativedivides	$\frac{\frac{2\left(\frac{(a^2+2ab+b^2)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{(a^2+2ab+b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+(3a^2+2ab-b^2)\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{}{\sqrt{(}}\right.}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b}\right.}}\right.}{b^2}$
default	$\frac{\frac{2\left(\frac{(a^2+2ab+b^2)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{(a^2+2ab+b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+(3a^2+2ab-b^2)\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{}{\sqrt{(}}\right.}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b}\right.}}\right.}{b^2}$
risch	$-\frac{3a^2e^{4dx+4c}+2ab\,e^{4dx+4c}-b^2e^{4dx+4c}+6a^2e^{2dx+2c}-2ab\,e^{2dx+2c}+3a^2+4ab+b^2}{a\,b^2d\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)\left(1+e^{2dx+2c}\right)}-\frac{3a\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-}}{(a+b)\sqrt{-a}}\right.}{4\sqrt{-ab}\,d\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*((1/2*(a^2+2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^3+1/2*(a^2+2*a*b+b^2
)/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2
+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(3*a^2+2*a*b-b^2)*(-1/2*(-a+(b*(a+b))^(1/
2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(
1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+
b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*
d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))+2/b^2*tanh(1/2*d*x+1/2*c)
/(tanh(1/2*d*x+1/2*c)^2+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(85) = 170$.

time = 0.59, size = 209, normalized size = 2.15

$$\frac{3 a^2+4 a b+b^2+2 (3 a^2-a b) e^{(-2 d x-2 c)}+(3 a^2+2 a b-b^2) e^{(-4 d x-4 c)}}{(a^2 b^2+a b^3+(3 a^2 b^2-a b^3) e^{(-2 d x-2 c)}+(3 a^2 b^2-a b^3) e^{(-4 d x-4 c)}+(a^2 b^2+a b^3) e^{(-6 d x-6 c)}) d}+\frac{(3 a^2+2 a b-b^2) \arctan\left(\frac{(a+b) e^{(-2 d x-2 c)}+a-b}{2 \sqrt{a b}}\right)}{2 \sqrt{a b}\, a b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (3*a^2 + 4*a*b + b^2 + 2*(3*a^2 - a*b)*e^(-2*d*x - 2*c) + (3*a^2 + 2*a*b -
b^2)*e^(-4*d*x - 4*c))/((a^2*b^2 + a*b^3 + (3*a^2*b^2 - a*b^3)*e^(-2*d*x -
2*c) + (3*a^2*b^2 - a*b^3)*e^(-4*d*x - 4*c) + (a^2*b^2 + a*b^3)*e^(-6*d*x -
 6*c))*d) + 1/2*(3*a^2 + 2*a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c)
+ a - b)/sqrt(a*b))/(sqrt(a*b)*a*b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1282 vs. $2(85) = 170$.
time = 0.50, size = 2869, normalized size = 29.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 2*a^
2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 2*a^2*b^2 - a*b
^3)*sinh(d*x + c)^4 + 12*a^3*b + 16*a^2*b^2 + 4*a*b^3 + 8*(3*a^3*b - a^2*b^
2)*cosh(d*x + c)^2 + 8*(3*a^3*b - a^2*b^2 + 3*(3*a^3*b + 2*a^2*b^2 - a*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d
*x + c)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^5
 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(d*x + c)^6 + (9*a^3 + 3*a^2*b - 5*a
*b^2 + b^3)*cosh(d*x + c)^4 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 15*(3*a^3
+ 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^3 + 5
*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 5*a^2*b + a*b^2 - b^3 + (9*a^3 + 3*
a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(3*a^3 + 5*a^2*b + a*b^2 - b^3
)*cosh(d*x + c)^4 + 9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 6*(9*a^3 + 3*a^2*b -
5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^3 + 5*a^2*b + a
*b^2 - b^3)*cosh(d*x + c)^5 + 2*(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x
+ c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2
 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 -
b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c
)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a -
```

b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 16*((3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^3 + (3*a^3*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3*b^3 + a^2*b^4)*d*sinh(d*x + c)^6 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^4 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 + 6*(3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^3 + a^2*b^4)*d + 2*(3*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^5 + 2*(3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(3*a^3*b + 2*a^2*b^2 - a*b^3)*sinh(d*x + c)^4 + 6*a^3*b + 8*a^2*b^2 + 2*a*b^3 + 4*(3*a^3*b - a^2*b^2)*cosh(d*x + c)^2 + 4*(3*a^3*b - a^2*b^2 + 3*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(d*x + c)^6 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^4 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 15*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 5*a^2*b + a*b^2 - b^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^4 + 9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 6*(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 8*((3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^3 + (3*a^3*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3*b^3 + a^2*b^4)*d*sinh(d*x + c)^6 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^4 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 + 6*(3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^3 + a^2*b^4)*d + 2*(3*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^5 + 2*(3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(85) = 170.
time = 0.73, size = 232, normalized size = 2.39

$$-\frac{\frac{(3a^2+2ab-b^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}+\frac{2\left(3a^2e^{(4dx+4c)}+2abe^{(4dx+4c)}-b^2e^{(4dx+4c)}+6a^2e^{(2dx+2c)}-2abe^{(2dx+2c)}+3a^2+4ab+b^2\right)}{\left(ae^{(6dx+6c)}+be^{(6dx+6c)}+3ae^{(4dx+4c)}-be^{(4dx+4c)}+3ae^{(2dx+2c)}-be^{(2dx+2c)}+a+b\right)ab^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a^2 + 2*a*b - b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*
c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 2*(3*a^2*e^(4*d*x + 4*c) + 2*a*b
*e^(4*d*x + 4*c) - b^2*e^(4*d*x + 4*c) + 6*a^2*e^(2*d*x + 2*c) - 2*a*b*e^(2
*d*x + 2*c) + 3*a^2 + 4*a*b + b^2)/((a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c)
+ 3*a*e^(4*d*x + 4*c) - b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) - b*e^(2*d*
x + 2*c) + a + b)*a*b^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^6\,\left(b\,\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2), x)
```

3.124 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=155

$$\frac{(4a+5b)\operatorname{ArcTan}(\sinh(c+dx))}{2b^3d}-\frac{(4a-b)(a+b)^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(a+b)(2a+b)\sinh(c+}{2ab^2d\left(a+(a+b)\sinh^2(c\right.}$$

```
[Out] 1/2*(4*a+5*b)*arctan(sinh(d*x+c))/b^3/d-1/2*(4*a-b)*(a+b)^(3/2)*arctan(sinh
(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/b^3/d+1/2*(a+b)*(2*a+b)*sinh(d*x+c)/a/
b^2/d/(a+(a+b)*sinh(d*x+c)^2)-1/2*sech(d*x+c)*tanh(d*x+c)/b/d/(a+(a+b)*sinh
(d*x+c)^2)
```

Rubi [A]
time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3757, 425, 541, 536, 209, 211}

$$-\frac{(4a-b)(a+b)^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(4a+5b)\operatorname{ArcTan}(\sinh(c+dx))}{2b^3d}+\frac{(2a+b)(a+b)\sinh(c+dx)}{2ab^2d\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2bd\left((a+b)\sinh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((4*a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*b^3*d) - ((4*a - b)*(a + b)^(3/2)*Ar
cTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + ((a + b)*(2*
a + b)*Sinh[c + d*x])/(2*a*b^2*d*(a + (a + b)*Sinh[c + d*x]^2)) - (Sech[c +
 d*x]*Tanh[c + d*x])/(2*b*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b-3(a+b)x^2}{(1+x^2)(a+(a+b)x^2)^2}\,dx, x, \sinh(c+\right.}{2bd}$$

$$= \frac{(a+b)(2a+b)\sinh(c+dx)}{2ab^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\text{Subst}}{}$$

$$= \frac{(a+b)(2a+b)\sinh(c+dx)}{2ab^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{((4a}{}$$

$$= \frac{(4a+5b)\tan^{-1}(\sinh(c+dx))}{2b^3d} - \frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right.}{2a^{3/2}b^3d}$$

Mathematica [A]
time = 0.97, size = 265, normalized size = 1.71

$$\frac{2\sqrt{a}\,b(a+b)^{5/2}\sinh(c+dx)+(a-b)\left((4a-b)(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+2a^{3/2}\sqrt{a+b}\,(4a+5b)\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a^{3/2}b\sqrt{a+b}\operatorname{sech}(c+dx)\tanh(c+dx)\right)+(a+b)\cosh(2(c+dx))\left((4a-b)(a+b)^2\text{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+2a^{3/2}\sqrt{a+b}\,(4a+5b)\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a^{3/2}b\sqrt{a+b}\operatorname{sech}(c+dx)\tanh(c+dx)\right)}{2a^{3/2}b^3\sqrt{a+b}\,d(a-b+(a+b)\cosh(2(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*Sqrt[a]*b*(a + b)^(5/2)*Sinh[c + d*x] + (a - b)*((4*a - b)*(a + b)^2*Arc
Tan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] + 2*a^(3/2)*Sqrt[a + b]*(4*a + 5*b
)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*b*Sqrt[a + b]*Sech[c + d*x]*Tanh[c +
d*x]) + (a + b)*Cosh[2*(c + d*x)]*((4*a - b)*(a + b)^2*ArcTan[(Sqrt[a]*Csch
[c + d*x])/Sqrt[a + b]] + 2*a^(3/2)*Sqrt[a + b]*(4*a + 5*b)*ArcTan[Tanh[(c
+ d*x)/2]] + a^(3/2)*b*Sqrt[a + b]*Sech[c + d*x]*Tanh[c + d*x]))/(2*a^(3/2)
*b^3*Sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c + d*x)]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(139) = 278.
time = 2.73, size = 351, normalized size = 2.26

method	result

derivativedivides	$\frac{\frac{2\left(-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+(5b+4a)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}-\frac{2\left(\frac{\frac{b\left(a^2+2ab+b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b\left(a^2+2ab+b\right.}{}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\mathrm{ta}\right.}\right.}{}$
default	$\frac{\frac{2\left(-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+(5b+4a)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}-\frac{2\left(\frac{\frac{b\left(a^2+2ab+b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b\left(a^2+2ab+b\right.}{}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\mathrm{ta}\right.}\right.}{}$
risch	$\frac{e^{dx+c}\left(2a^2e^{6dx+6c}+3ab\,e^{6dx+6c}+b^2e^{6dx+6c}+2a^2e^{4dx+4c}-ab\,e^{4dx+4c}+b^2e^{4dx+4c}-2a^2e^{2dx+2c}+ab\,e^{2dx+2c}-b^2e^{2dx+2c}\right.}{d\,b^2\left(1+e^{2dx+2c}\right)^2a\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^3*((-1/2*b*tanh(1/2*d*x+1/2*c)^3+1/2*b*tanh(1/2*d*x+1/2*c))/(tanh(
1/2*d*x+1/2*c)^2+1)^2+1/2*(5*b+4*a)*arctan(tanh(1/2*d*x+1/2*c)))-2/b^3*((1/
```

```
2*b*(a^2+2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^3-1/2*b*(a^2+2*a*b+b^2)/a*tanh(1/
2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1
/2*d*x+1/2*c)^2+a)+1/2*(4*a^3+7*a^2*b+2*a*b^2-b^3)*(-1/2*((b*(a+b))^(1/2)-b
)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*
d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*((b*(a+b))^(1/2)+b)/a/(
b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((2*a^2*e^(7*c) + 3*a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (2*a^2*e^(5*c) -
 a*b*e^(5*c) + b^2*e^(5*c))*e^(5*d*x) - (2*a^2*e^(3*c) - a*b*e^(3*c) + b^2*
e^(3*c))*e^(3*d*x) - (2*a^2*e^c + 3*a*b*e^c + b^2*e^c)*e^(d*x))/(4*a^2*b^2*
d*e^(6*d*x + 6*c) + 4*a^2*b^2*d*e^(2*d*x + 2*c) + a^2*b^2*d + a*b^3*d + (a^
2*b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x) + 2*(3*a^2*b^2*d*e^(4*c) - a*b
^3*d*e^(4*c))*e^(4*d*x)) + (4*a*e^c + 5*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(
b^3*d) - 128*integrate(1/128*((4*a^3*e^(3*c) + 7*a^2*b*e^(3*c) + 2*a*b^2*e^
(3*c) - b^3*e^(3*c))*e^(3*d*x) + (4*a^3*e^c + 7*a^2*b*e^c + 2*a*b^2*e^c - b
^3*e^c)*e^(d*x))/(a^2*b^3 + a*b^4 + (a^2*b^3*e^(4*c) + a*b^4*e^(4*c))*e^(4*
d*x) + 2*(a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3490 vs. $2(139) = 278$.
time = 0.44, size = 6396, normalized size = 41.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 28*(2*a^2*b + 3*a*b^2 +
 b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(2*a^2*b + 3*a*b^2 + b^3)*sinh(d*x
+ c)^7 + 4*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^5 + 4*(2*a^2*b - a*b^2 + b
^3 + 21*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*
(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^2*b - a*b^2 + b^3)*cosh(d*
x + c))*sinh(d*x + c)^4 - 4*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^3 + 4*(35
*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 2*a^2*b + a*b^2 - b^3 + 10*(2*
a^2*b - a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(2*a^2*b + 3*
a*b^2 + b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^3 -
 3*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((4*a^3 + 7*a^2
```

```
*b + 2*a*b^2 - b^3)*cosh(d*x + c)^8 + 8*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*c
osh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*sinh(d*x +
 c)^8 + 4*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b -
a*b^2 + 7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(4*a^3 + 3*a
^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 5*a^2*b - 6*a*b^
2 + b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x
 + c)^4 + 12*a^3 + 5*a^2*b - 6*a*b^2 + b^3 + 30*(4*a^3 + 3*a^2*b - a*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*co
sh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 5*
a^2*b - 6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 7*a^2*b + 2
*a*b^2 - b^3 + 4*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3 +
7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^6 + 15*(4*a^3 + 3*a^2*b - a*b^2)*cos
h(d*x + c)^4 + 4*a^3 + 3*a^2*b - a*b^2 + 3*(12*a^3 + 5*a^2*b - 6*a*b^2 + b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*
cosh(d*x + c)^7 + 3*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^5 + (12*a^3 + 5
*a^2*b - 6*a*b^2 + b^3)*cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - a*b^2)*cosh(d*
x + c))*sinh(d*x + c))*sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x + c)^4 + 4*(a
 + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)
*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(
a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 -
 a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-(a + b)/a
) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cos
h(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b
)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*((4*a^3 + 9*a^2*b + 5*a*b^2)*c
osh(d*x + c)^8 + 8*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^
7 + (4*a^3 + 9*a^2*b + 5*a*b^2)*sinh(d*x + c)^8 + 4*(4*a^3 + 5*a^2*b)*cosh(
d*x + c)^6 + 4*(4*a^3 + 5*a^2*b + 7*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^3 +
3*(4*a^3 + 5*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 11*a^2*b -
 5*a*b^2)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)
^4 + 12*a^3 + 11*a^2*b - 5*a*b^2 + 30*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^2)*si
nh(d*x + c)^4 + 8*(7*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^5 + 10*(4*a^
3 + 5*a^2*b)*cosh(d*x + c)^3 + (12*a^3 + 11*a^2*b - 5*a*b^2)*cosh(d*x + c))
*sinh(d*x + c)^3 + 4*a^3 + 9*a^2*b + 5*a*b^2 + 4*(4*a^3 + 5*a^2*b)*cosh(d*x
 + c)^2 + 4*(7*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^6 + 15*(4*a^3 + 5*
a^2*b)*cosh(d*x + c)^4 + 4*a^3 + 5*a^2*b + 3*(12*a^3 + 11*a^2*b - 5*a*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x
+ c)^7 + 3*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^5 + (12*a^3 + 11*a^2*b - 5*a*b^2
)*cosh(d*x + c)^3 + (4*a^3 + 5*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(
cosh(d*x + c) + sinh(d*x + c)) - 4*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)
+ 4*(7*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 5*(2*a^2*b - a*b^2 + b^3
)*cosh(d*x + c)^4 - 2*a^2*b - 3*a*b^2 - b^3 - 3*(2*a^2*b - a*b^2 + b^3)*cos
```

```
h(d*x + c)^2)*sinh(d*x + c))/(4*a^2*b^3*d*cosh(d*x + c)^6 + (a^2*b^3 + a*b^
4)*d*cosh(d*x + c)^8 + 8*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7
+ (a^2*b^3 + a*b^4)*d*sinh(d*x + c)^8 + 4*a^2*b^3*d*cosh(d*x + c)^2 + 4*(a^
2*b^3*d + 7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(3*a^2
*b^3 - a*b^4)*d*cosh(d*x + c)^4 + 8*(3*a^2*b^3*d*cosh(d*x + c) + 7*(a^2*b^3
 + a*b^4)*d*cosh(d*x + c)^3)*sinh(d*x + c)^5 + 2*(30*a^2*b^3*d*cosh(d*x + c
)^2 + 35*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + (3*a^2*b^3 - a*b^4)*d)*sinh(
d*x + c)^4 + 8*(10*a^2*b^3*d*cosh(d*x + c)^3 + 7*(a^2*b^3 + a*b^4)*d*cosh(d
*x + c)^5 + (3*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^
2*b^3*d*cosh(d*x + c)^4 + 7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + a^2*b^3*d
 + 3*(3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + (a^2*b^3 + a*
b^4)*d + 8*(3*a^2*b^3*d*cosh(d*x + c)^5 + (a^2*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**7/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)^7\,\left(b\tanh(c+d\,x)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2), x)
```

3.125 $\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=198

$$\frac{(a+7b)x}{2(a+b)^4}+\frac{b^{3/2}(35a^2+14ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^4d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}-\frac{(2a}{4a(a+b)}$$

```
[Out] 1/2*(a+7*b)*x/(a+b)^4+1/8*b^(3/2)*(35*a^2+14*a*b+3*b^2)*arctan(b^(1/2)*tanh
(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^4/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b
*tanh(d*x+c)^2)^2-1/4*(2*a-b)*b*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
^2-1/8*(a-3*b)*b*(4*a+b)*tanh(d*x+c)/a^2/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.22, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3756, 425, 541, 536, 212, 211}

$$-\frac{b(a-3b)(4a+b)\tanh(c+dx)}{8a^2d(a+b)^3\left(a+b\tanh^2(c+dx)\right)}+\frac{b^{3/2}(35a^2+14ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^4}-\frac{b(2a-b)\tanh(c+dx)}{4ad(a+b)^2\left(a+b\tanh^2(c+dx)\right)^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{x(a+7b)}{2(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + 7*b)*x)/(2*(a + b)^4) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^4*d) + (Cosh[c + d*x]*Sin
h[c + d*x])/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - ((2*a - b)*b*Tanh[c +
 d*x])/(4*a*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b)*b*(4*a + b)
*Tanh[c + d*x])/(8*a^2*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+2b+5bx^2}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+d\right.}{2(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{S}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{}{8}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{}{8}$$

$$= \frac{(a+7b)x}{2(a+b)^4} + \frac{b^{3/2}(35a^2+14ab+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^4d} + \frac{\cosh(c+}{2(a+b)d\,(a}$$

Mathematica [A]
time = 0.94, size = 164, normalized size = 0.83

$$\frac{4(a+7b)(c+dx) + \frac{b^{3/2}(35a^2+14ab+3b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + 2(a+b)\sinh(2(c+dx)) + \frac{4b^3(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{b^2(a+b)(13a+3b)\sinh(2(c+dx))}{a^2(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (4*(a + 7*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sqrt[b]
*Tanh[c + d*x])/Sqrt[a]])/a^(5/2) + 2*(a + b)*Sinh[2*(c + d*x)] + (4*b^3*(a
 + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (b^2*(
a + b)*(13*a + 3*b)*Sinh[2*(c + d*x)])/(a^2*(a - b + (a + b)*Cosh[2*(c + d*
x)])))/(8*(a + b)^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(180) = 360$.
time = 3.14, size = 501, normalized size = 2.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/(a+b)^4*((-1/8*(13*a^2+18*a*b+5*b^2)/a*tanh(1/2*d*x+1/2*c)^7-1/
8*(39*a^3+98*a^2*b+71*a*b^2+12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^5-1/8*(39*a^3+9
8*a^2*b+71*a*b^2+12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^3-1/8*(13*a^2+18*a*b+5*b^2
)/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2
+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8/a*(35*a^2+14*a*b+3*b^2)*(-1/2*(-a+(b*(a
+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b)
)^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))+1/2/(a+b)^3/(tanh
(1/2*d*x+1/2*c)-1)^2+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+1/2/(a+b)^4*(-a-7*
b)*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/(a+b
)^3/(tanh(1/2*d*x+1/2*c)+1)+1/2*(a+7*b)/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. $2(180) = 360$.
time = 0.84, size = 1806, normalized size = 9.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^
4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/4*b*log(2*(a - b)*e^(-2*d*x
 - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*(
(a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4
*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b
^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*
b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 1/16*(15*a^2*b + 10*a*b
^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/16*(9*a^4*b + 4*a^3*b^2 -
 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a
*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*
b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b
^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15
*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 +
 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a
^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c) + 2*(3*a^8 + 10*a
^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(4*
d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6
)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 -
 3*b^5 + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^(-2*d*x
- 2*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^(-4*d*
x - 4*c) + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a*b^4 + b^5)*e^(-6*d*x
```

```
- 6*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5
+ a^2*b^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)
*e^(-2*d*x - 2*c) + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*
b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(-6*d*x - 6*c) + (a^8 + 6*a^7*b + 1
5*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(-8*d*x - 8*c)
)*d) + 1/8*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^
2 - 23*a*b^3 - 9*b^4)*e^(-2*d*x - 2*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 +
 3*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x
 - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 +
4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x -
 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)
*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 -
a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^
3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + 1/2*(d*x + c)/((a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d) + 1/8*e^(2*d*x + 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) -
1/8*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6780 vs. $2(180) = 360$.
time = 0.51, size = 13887, normalized size = 70.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^12 + 24*(a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(d*x + c)^12 + 8*(a^5 + a^4*b - a^3*b^2 -
 a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^10 +
 4*(2*a^5 + 2*a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + 2*(a^5 + 9*a^4*b + 15*a^3*b^2
 + 7*a^2*b^3)*d*x + 33*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^10 + 40*(11*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x
 + c)^3 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2
+ 7*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(5*a^5 - a^4*b - 27*a^
3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*
b^3)*d*x)*cosh(d*x + c)^8 + 2*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*
a*b^4 + 6*b^5 + 495*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^4 +
 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x + 180*(a^5 + a^4*b - a^3*b^2
- a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^2)*
sinh(d*x + c)^8 + 16*(99*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c
)^5 + 60*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7
*a^2*b^3)*d*x)*cosh(d*x + c)^3 + (5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 +
34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x +
```

c))*sinh(d*x + c)^7 - 4*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3
*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^6 + 4*(462*(a
^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^6 - 39*a^3*b^2 + 17*a^2*b
^3 - 33*a*b^4 - 9*b^5 + 420*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4
*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 4*(3*a^5 + 19*a^4*b - 1
1*a^3*b^2 + 21*a^2*b^3)*d*x + 14*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 +
34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x +
c)^2)*sinh(d*x + c)^6 + 8*(198*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(
d*x + c)^7 + 252*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3
*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^5 + 14*(5*a^5 - a^4*b - 27*a^3*b^2 + 7
*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)
*cosh(d*x + c)^3 - 3*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5
+ 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5
- 2*a^5 - 6*a^4*b - 6*a^3*b^2 - 2*a^2*b^3 - 2*(5*a^5 - a^4*b + 77*a^3*b^2 +
31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*
d*x)*cosh(d*x + c)^4 + 2*(495*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*
x + c)^8 + 840*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b
^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^6 - 5*a^5 + a^4*b - 77*a^3*b^2 - 31*a^2*
b^3 + 70*a*b^4 + 18*b^5 + 70*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a
*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^
4 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x - 30*(39*a^3*b^2 - 17*a^2*
b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(55*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*cosh(d*x + c)^9 + 120*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4
*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^7 + 14*(5*a^5 - a^4*b - 27*
a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^
2*b^3)*d*x)*cosh(d*x + c)^5 - 10*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^
5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^3 - (
5*a^5 - a^4*b + 77*a^3*b^2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a
^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(2*a^5
+ 2*a^4*b + 11*a^3*b^2 + 27*a^2*b^3 + 19*a*b^4 + 3*b^5 - 2*(a^5 + 9*a^4*b +
15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^5 + 3*a^4*b + 3*a^
3*b^2 + a^2*b^3)*cosh(d*x + c)^10 + 90*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (
a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^8 + 14*(5*a^5 -
a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3
*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^6 - 2*a^5 - 2*a^4*b - 11*a^3*b^2 - 27*
a^2*b^3 - 19*a*b^4 - 3*b^5 - 15*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5
- 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 2*
(a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x - 3*(5*a^5 - a^4*b + 77*a^3*b^
2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^
3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 84*a^3*b^2 + 66*a^2
*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^10 + 10*(35*a^4*b + 84*a^3*b^2 + 66*
a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)*sinh(d*x + c)^9 + (35*a^4*b + 84*
a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*sinh(d*x + c)^10 + 4*(35*a^4*b + 1
4*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*cosh(d*x + c)^8 + (140*a^4*b + 5

```
6*a^3*b^2 - 128*a^2*b^3 - 56*a*b^4 - 12*b^5 + 45*(35*a^4*b + 84*a^3*b^2 + 6
6*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*
a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*cosh(d*x + c)^3 + 4*(35
*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(180) = 360.
time = 1.46, size = 546, normalized size = 2.76

$$\frac{\frac{4\,(dx+c)(a+7\,b)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{\left(2\,ae^{(2\,dx+2\,c)}+14\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-2\,dx-2\,c)}}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{\left(35\,a^2b^2+14\,ab^3+3\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4\right)\sqrt{ab}}+\frac{e^{(2\,dx+2\,c)}}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,\left(13\,a^3b^2e^{(6\,dx+6\,c)}-a^2b^3e^{(6\,dx+6\,c)}-17\,ab^4e^{(6\,dx+6\,c)}-3\,b^5e^{(6\,dx+6\,c)}+39\,a^3b^2e^{(4\,dx+4\,c)}-17\,a^2b^3e^{(4\,dx+4\,c)}+33\,ab^4e^{(4\,dx+4\,c)}+9\,b^5e^{(4\,dx+4\,c)}+39\,a^3b^2e^{(2\,dx+2\,c)}+13\,a^2b^3e^{(2\,dx+2\,c)}-35\,ab^4e^{(2\,dx+2\,c)}-9\,b^5e^{(2\,dx+2\,c)}+13\,a^3b^2+29\,a^2b^3+19\,ab^4+3\,b^5\right)}{\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*(d*x + c)*(a + 7*b)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (2
*a*e^(2*d*x + 2*c) + 14*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x - 2*c)/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (35*a^2*b^2 + 14*a*b^3 + 3*b^4)*arct
an(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 4
*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)) + e^(2*d*x + 2*c)/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(13*a^3*b^2*e^(6*d*x + 6*c) - a^2*b^3*e^(6*
d*x + 6*c) - 17*a*b^4*e^(6*d*x + 6*c) - 3*b^5*e^(6*d*x + 6*c) + 39*a^3*b^2*
e^(4*d*x + 4*c) - 17*a^2*b^3*e^(4*d*x + 4*c) + 33*a*b^4*e^(4*d*x + 4*c) + 9
*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) + 13*a^2*b^3*e^(2*d*x + 2
*c) - 35*a*b^4*e^(2*d*x + 2*c) - 9*b^5*e^(2*d*x + 2*c) + 13*a^3*b^2 + 29*a^
2*b^3 + 19*a*b^4 + 3*b^5)/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4
)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d
*x + 2*c) + a + b)^2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(c+d\,x\right)}^2}{{\left(b\,{\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

3.126 $\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{3b(8a^2+4ab+b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}d}+\frac{\sinh(c+dx)}{(a+b)^3d}+\frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)^2}+\frac{}{8a}$$

[Out] 3/8*b*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(7/2)/d+sinh(d*x+c)/(a+b)^3/d+1/4*b^3*sinh(d*x+c)/a/(a+b)^3/d/(a+(a+b)*sinh(d*x+c)^2)^2+3/8*b^2*(4*a+b)*sinh(d*x+c)/a^2/(a+b)^3/d/(a+(a+b)*sinh(d*x+c)^2)

Rubi [A]
time = 0.16, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3757, 398, 1171, 393, 211}

$$\frac{3b^2(4a+b)\sinh(c+dx)}{8a^2d(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{3b(8a^2+4ab+b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{4ad(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)^2}+\frac{\sinh(c+dx)}{d(a+b)^3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^(7/2)*d) + Sinh[c + d*x]/((a + b)^3*d) + (b^3*Sinh[c + d*x])/(4*a*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + b)*Sinh[c + d*x])/(8*a^2*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(a+(a+b)x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^3} + \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(a+b)^3(a+(a+b)x^2)^3}\right)\,dx, x, \sinh(c+dx\right.}{d}$$

$$= \frac{\sinh(c+dx)}{(a+b)^3d} + \frac{\text{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(a+(a+b)x^2)^3}\,dx, x, \sinh(c-\right.}{(a+b)^3d}$$

$$= \frac{\sinh(c+dx)}{(a+b)^3d} + \frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-3b(2a+}{(a}\right.}{}$$

$$= \frac{\sinh(c+dx)}{(a+b)^3d} + \frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3b^2(4a}{8a^2(a+b)^3d\,(a}$$

$$= \frac{3b(8a^2+4ab+b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}d} + \frac{\sinh(c+dx)}{(a+b)^3d} + \frac{}{4a(a+b)^3}$$

Mathematica [A]
time = 1.98, size = 145, normalized size = 0.94

$$\frac{-\frac{3b(8a^2+4ab+b^2)\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}(a+b)^{7/2}}+\frac{2\left(4+\frac{3b^3}{a^2(a-b+(a+b)\cosh(2(c+dx)))}+\frac{4b^2(3a-2b+3(a+b)\cosh(2(c+dx)))}{a(a-b+(a+b)\cosh(2(c+dx)))^2}\right)\sinh(c+dx)}{(a+b)^3}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((-3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(
a^(5/2)*(a + b)^(7/2)) + (2*(4 + (3*b^3)/(a^2*(a - b + (a + b)*Cosh[2*(c +
d*x)])) + (4*b^2*(3*a - 2*b + 3*(a + b)*Cosh[2*(c + d*x)]))/(a*(a - b + (a
+ b)*Cosh[2*(c + d*x)])^2))*Sinh[c + d*x])/(a + b)^3)/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(140) = 280.
time = 3.26, size = 375, normalized size = 2.44

method	result
derivativedivides	$-\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{b(12a+5b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{3\left(4a^2+15ab+4b^2\right)b\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}+\frac{3\left(4a^2+15ab+4b^2\right)b\left(\tanh^3\right.}{8a^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}\right.}{}$

default	$-\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{b(12a+5b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{3\left(4a^2+15ab+4b^2\right)b\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}+\frac{3\left(4a^2+15ab+4b^2\right)b\left(\cdots\right)}{8a^2}\cdots}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.\right.}\cdots\right.}{\cdots}$
risch	$\frac{e^{dx+c}}{2(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{e^{-dx-c}}{2(a^3+3a^2b+3a\,b^2+b^3)d}+\frac{(12a^2e^{6dx+6c}+15ab\,e^{6dx+6c}+3b^2e^{6dx+6c}+12a^2e^{4dx+4c}-\cdots}{4(a\,e^{4dx+4c}+b\,e^{4dx+4c}+\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+2*b/(a+b)^3*((-1/8*b*(12*a+5*b)/a*t
anh(1/2*d*x+1/2*c)^7-3/8*(4*a^2+15*a*b+4*b^2)/a^2*b*tanh(1/2*d*x+1/2*c)^5+3
/8*(4*a^2+15*a*b+4*b^2)/a^2*b*tanh(1/2*d*x+1/2*c)^3+1/8*b*(12*a+5*b)/a*tanh
(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tan
h(1/2*d*x+1/2*c)^2+a)^2+3/8/a*(8*a^2+4*a*b+b^2)*(-1/2*((b*(a+b))^(1/2)-b)/a
/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*((b*(a+b))^(1/2)+b)/a/(b*(
a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))-1/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - 2*(a^4*e^(10*c) + 2*a^3*b*e^(10*c) + a^
2*b^2*e^(10*c))*e^(10*d*x) - (6*a^4*e^(8*c) - 4*a^3*b*e^(8*c) + 2*a^2*b^2*e
```

```
^(8*c) + 15*a*b^3*e^(8*c) + 3*b^4*e^(8*c))*e^(8*d*x) - (4*a^4*e^(6*c) - 8*a
^3*b*e^(6*c) + 32*a^2*b^2*e^(6*c) - 25*a*b^3*e^(6*c) - 9*b^4*e^(6*c))*e^(6*
d*x) + (4*a^4*e^(4*c) - 8*a^3*b*e^(4*c) + 32*a^2*b^2*e^(4*c) - 25*a*b^3*e^(
4*c) - 9*b^4*e^(4*c))*e^(4*d*x) + (6*a^4*e^(2*c) - 4*a^3*b*e^(2*c) + 2*a^2*
b^2*e^(2*c) + 15*a*b^3*e^(2*c) + 3*b^4*e^(2*c))*e^(2*d*x))/((a^7*d*e^(9*c)
+ 5*a^6*b*d*e^(9*c) + 10*a^5*b^2*d*e^(9*c) + 10*a^4*b^3*d*e^(9*c) + 5*a^3*b
^4*d*e^(9*c) + a^2*b^5*d*e^(9*c))*e^(9*d*x) + 4*(a^7*d*e^(7*c) + 3*a^6*b*d*
e^(7*c) + 2*a^5*b^2*d*e^(7*c) - 2*a^4*b^3*d*e^(7*c) - 3*a^3*b^4*d*e^(7*c) -
 a^2*b^5*d*e^(7*c))*e^(7*d*x) + 2*(3*a^7*d*e^(5*c) + 7*a^6*b*d*e^(5*c) + 6*
a^5*b^2*d*e^(5*c) + 6*a^4*b^3*d*e^(5*c) + 7*a^3*b^4*d*e^(5*c) + 3*a^2*b^5*d
*e^(5*c))*e^(5*d*x) + 4*(a^7*d*e^(3*c) + 3*a^6*b*d*e^(3*c) + 2*a^5*b^2*d*e^
(3*c) - 2*a^4*b^3*d*e^(3*c) - 3*a^3*b^4*d*e^(3*c) - a^2*b^5*d*e^(3*c))*e^(3
*d*x) + (a^7*d*e^c + 5*a^6*b*d*e^c + 10*a^5*b^2*d*e^c + 10*a^4*b^3*d*e^c +
5*a^3*b^4*d*e^c + a^2*b^5*d*e^c)*e^(d*x)) + 1/2*integrate(3/2*((8*a^2*b*e^(
3*c) + 4*a*b^2*e^(3*c) + b^3*e^(3*c))*e^(3*d*x) + (8*a^2*b*e^c + 4*a*b^2*e^
c + b^3*e^c)*e^(d*x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a
^6*e^(4*c) + 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*
b^4*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + 2*a^5*b*e^(2*c) - 2*a^3*b^3*e^(2*
c) - a^2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6260 vs. $2(140) = 280$.
time = 0.50, size = 11392, normalized size = 73.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^10 + 80*(a^6 +
 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 8*(a^6 + 3*
a^5*b + 3*a^4*b^2 + a^3*b^3)*sinh(d*x + c)^10 + 4*(6*a^6 + 2*a^5*b - 2*a^4*
b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c)^8 + 4*(6*a^6 + 2*a^5
*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5 + 90*(a^6 + 3*a^5*b + 3*
a^4*b^2 + a^3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 32*(30*(a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^3 + (6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17
*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(4*a^6
- 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c)^6
+ 4*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5 + 420*
(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^4 + 28*(6*a^6 + 2*a^5*b
 - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c)^2)*sinh(d*x
 + c)^6 - 8*a^6 - 24*a^5*b - 24*a^4*b^2 - 8*a^3*b^3 + 8*(252*(a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^5 + 28*(6*a^6 + 2*a^5*b - 2*a^4*b^2 +
 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c)^3 + 3*(4*a^6 - 4*a^5*b +
24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c))*sinh(d*x + c)
```

^5 - 4*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*co
sh(d*x + c)^4 + 4*(420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^
6 - 4*a^6 + 4*a^5*b - 24*a^4*b^2 - 7*a^3*b^3 + 34*a^2*b^4 + 9*a*b^5 + 70*(6
*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x +
c)^4 + 15*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(60*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3
*b^3)*cosh(d*x + c)^7 + 14*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a
^2*b^4 + 3*a*b^5)*cosh(d*x + c)^5 + 5*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3
*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c)^3 - (4*a^6 - 4*a^5*b + 24*a^4*b^
2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(6
*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x +
c)^2 + 4*(90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^8 + 28*(6*
a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5)*cosh(d*x + c
)^6 - 6*a^6 - 2*a^5*b + 2*a^4*b^2 - 17*a^3*b^3 - 18*a^2*b^4 - 3*a*b^5 + 15*
(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5)*cosh(d*x
+ c)^4 - 6*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3
+ 6*a*b^4 + b^5)*cosh(d*x + c)^9 + 9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6
*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^8 + (8*a^4*b + 20*a^3*b^2 + 17*a^
2*b^3 + 6*a*b^4 + b^5)*sinh(d*x + c)^9 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3
 - 4*a*b^4 - b^5)*cosh(d*x + c)^7 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*
a*b^4 - b^5 + 9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*cosh(d*
x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*
b^4 + b^5)*cosh(d*x + c)^3 + (8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b
^5)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 +
 10*a*b^4 + 3*b^5)*cosh(d*x + c)^5 + 2*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 +
 10*a*b^4 + 3*b^5 + 63*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5)*
cosh(d*x + c)^4 + 42*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh
(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6
*a*b^4 + b^5)*cosh(d*x + c)^5 + 70*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b
^4 - b^5)*cosh(d*x + c)^3 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4
 + 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b
^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 20*a^3*b^2 + 17*a^2*
b^3 + 6*a*b^4 + b^5)*cosh(d*x + c)^6 + 8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*
a*b^4 - b^5 + 35*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x
 + c)^4 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5)*cosh(d*x
 + c)^2)*sinh(d*x + c)^3 + 4*(9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^
4 + b^5)*cosh(d*x + c)^7 + 21*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 -
b^5)*cosh(d*x + c)^5 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*
b^5)*cosh(d*x + c)^3 + 3*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*
cosh(d*x + c))*sinh(d*x + c)^2 + (8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b
^4 + b^5)*cosh(d*x + c) + (9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 +
 b^5)*cosh(d*x + c)^8 + 28*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5
)*cosh(d*x + c)^6 + 8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5 + 10*
(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5)*cosh(d*x + c)^4 + 12

```
*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*
x + c))*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*co
sh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3
 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x +...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

3.127 $\int \frac{\mathbf{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{(8a^2+8ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d}+\frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2}+\frac{3b(2a+}{8a^2(a+b)^2d\,(a+}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+
b)^(5/2)/d+1/4*b*cosh(d*x+c)^2*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2
)^2+3/8*b*(2*a+b)*sinh(d*x+c)/a^2/(a+b)^2/d/(a+(a+b)*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3757, 424, 393, 211}

$$\frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{(8a^2+8ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}}+\frac{b\sinh(c+dx)\cosh^2(c+dx)}{4ad(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^
(5/2)*(a + b)^(5/2)*d) + (b*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*(a + b)*d*(
a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b*(2*a + b)*Sinh[c + d*x])/(8*a^2*(a +
 b)^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+(a+b)x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{4a+3b+(4a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \sinh(\right.}{4a(a+b)d}$$

$$= \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3b(2a+b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+(a+b)\sinh^2(c-\right.}$$

$$= \frac{(8a^2+8ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d} + \frac{b\cosh^2(c+dx)\sinh(c-}{4a(a+b)d\left(a+(a+b)\sinh^2\right.}$$

Mathematica [A]
time = 0.81, size = 134, normalized size = 0.93

$$\frac{-\frac{(8a^2+8ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{2\sqrt{a}\,b(8a^2-ab-3b^2+(8a^2+11ab+3b^2)\cosh(2(c+dx)))\sinh(c+dx)}{(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-(((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a
 + b)^(5/2)) + (2*Sqrt[a]*b*(8*a^2 - a*b - 3*b^2 + (8*a^2 + 11*a*b + 3*b^2)
```

```
*Cosh[2*(c + d*x)])*Sinh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*(c +
d*x)])^2))/(8*a^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(130) = 260$.
time = 2.74, size = 394, normalized size = 2.74

method	result
derivativedivides	$\frac{-\frac{b(8a+5b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a\left(a^2+2ab+b^2\right)}-\frac{\left(8a^2+29ab+12b^2\right)b\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2\left(a^2+2ab+b^2\right)}+\frac{\left(8a^2+29ab+12b^2\right)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2\left(a^2+2ab+b^2\right)}+\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\right.}{4a\left(a^2+2ab+b^2\right)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}$
default	$\frac{-\frac{b(8a+5b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a\left(a^2+2ab+b^2\right)}-\frac{\left(8a^2+29ab+12b^2\right)b\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2\left(a^2+2ab+b^2\right)}+\frac{\left(8a^2+29ab+12b^2\right)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2\left(a^2+2ab+b^2\right)}+\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\right.}{4a\left(a^2+2ab+b^2\right)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}$
risch	$\frac{(8a^2e^{6dx+6c}+11ab\,e^{6dx+6c}+3b^2e^{6dx+6c}+8a^2e^{4dx+4c}-13ab\,e^{4dx+4c}-9b^2e^{4dx+4c}-8a^2e^{2dx+2c}+13ab\,e^{2dx+2c}+9b^2e^{2dx}}{4(a^2+2ab+b^2)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)^2a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(8*a^2
+29*a*b+12*b^2)/a^2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/8*(8*a^2+29*a
*b+12*b^2)/a^2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/8*b*(8*a+5*b)/a/(a
^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*
x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/4/a*(8*a^2+8*a*b+3*b^2)/(a^2+2*
a*b+b^2)*(-1/2*((b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-
2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(
1/2))+1/2*((b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*
a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((8*a^2*b*e^(7*c) + 11*a*b^2*e^(7*c) + 3*b^3*e^(7*c))*e^(7*d*x) + (8*a^
2*b*e^(5*c) - 13*a*b^2*e^(5*c) - 9*b^3*e^(5*c))*e^(5*d*x) - (8*a^2*b*e^(3*c
) - 13*a*b^2*e^(3*c) - 9*b^3*e^(3*c))*e^(3*d*x) - (8*a^2*b*e^c + 11*a*b^2*e
^c + 3*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3*d + a
^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c) + 4*a^3
*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 2*a^5*b*
d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d
*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*e^(4*c) +
3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^(2*c) - 2*a
^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/8*((8*a^2*
e^(3*c) + 8*a*b*e^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (8*a^2*e^c + 8*a*b*e^c
 + 3*b^2*e^c)*e^(d*x))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5*e^(4*c)
+ 3*a^4*b*e^(4*c) + 3*a^3*b^2*e^(4*c) + a^2*b^3*e^(4*c))*e^(4*d*x) + 2*(a^5
*e^(2*c) + a^4*b*e^(2*c) - a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c))*e^(2*d*x)), x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4377 vs. $2(130) = 260$.
time = 0.43, size = 7909, normalized size = 54.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^7 + 28
*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^
6 + 4*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^7 + 4*(8*
a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^5 + 4*(8*a^4*b - 5*
a^3*b^2 - 22*a^2*b^3 - 9*a*b^4 + 21*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*
a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(8*a^4*b + 19*a^3*b^2 + 14*
a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + (8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*
a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3
 - 9*a*b^4)*cosh(d*x + c)^3 - 4*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4
 - 35*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 - 10*(8
*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3
 + 4*(21*(8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 + 10
```

```
*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^3 - 3*(8*a^4*b
- 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^
4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^4 +
24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (
8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(8*a^
4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^4 + 8*a
^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4 + 7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*
a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^4 + 24*a^3*b +
27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(8*a^4 + 8*a^3*b - 5*a^2
*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 + 8*a^3*
b + 17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 24*a^3*
b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 + 8*a^3*b + 17*
a^2*b^2 + 18*a*b^3 + 9*b^4 + 30*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*
a*b^3 + 3*b^4 + 8*(7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cos
h(d*x + c)^5 + 10*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x
+ c)^3 + (24*a^4 + 8*a^3*b + 17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*
x + c)^2 + 4*(7*(8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x
 + c)^6 + 15*(8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^
4 + 8*a^4 + 8*a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4 + 3*(24*a^4 + 8*a^3*b + 1
7*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^4
+ 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^4 + 8*
a^3*b - 5*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 + 8*a^3*b +
17*a^2*b^2 + 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (8*a^4 + 8*a^3*b - 5*a^2*b
^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log(((
a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*
sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^
2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(
d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c
)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x +
c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2
 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(
d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*(8*a^4*b +
19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c) + 4*(7*(8*a^4*b + 19*a^3*b
^2 + 14*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 - 8*a^4*b - 19*a^3*b^2 - 14*a^2*
b^3 - 3*a*b^4 + 5*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c
)^4 - 3*(8*a^4*b - 5*a^3*b^2 - 22*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*sinh(
d*x + c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*
d*cosh(d*x + c)^8 + 8*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4
+ a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^8 + 5*a^7*b + 10*a^6*b^2 +
10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^8 + 3*a^7*b + 2*
a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^8 +
5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2
```

```
+ (a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d)*sinh(d*x
 + c)^6 + 2*(3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^
5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^
4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b
^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 5
*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 +
 30*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4 - a^3*b^5)*d*cosh(d*
x + c)^2 + (3*a^8 + 7*a^7*b + 6*a^6*b^2 + 6*a^5*b^3 + 7*a^4*b^4 + 3*a^3*b^5
)*d)*sinh(d*x + c)^4 + 4*(a^8 + 3*a^7*b + 2*a^6*b^2 - 2*a^5*b^3 - 3*a^4*b^4
 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^8 + 5*a...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)\,\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)
```

3.128 $\int \frac{\text{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=96

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2}+\frac{3\tanh(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] 3/8*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/d/b^(1/2)+1/4*tanh(d*x+c)/a
/d/(a+b*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/a^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3756, 205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{3\tanh(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d) + Tanh[c
+ d*x]/(4*a*d*(a + b*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])/(8*a^2*d*(a +
b*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= \frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{3\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,\right.}{8}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d} + \frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+d}{8a^2d\left(a+b\tanh^2(\right.}$$

Mathematica [A]
time = 0.59, size = 77, normalized size = 0.80

$$\frac{\frac{3\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}} + \frac{\tanh(c+dx)\left(5a+3b\tanh^2(c+dx)\right)}{a^2\left(a+b\tanh^2(c+dx)\right)^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[b]) + (Tanh[c +
d*x]*(5*a + 3*b*Tanh[c + d*x]^2))/(a^2*(a + b*Tanh[c + d*x]^2)^2))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(82) = 164.
time = 2.54, size = 284, normalized size = 2.96

method	result

derivativedivides	$-\frac{2\left(-\frac{5\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{3(5a+4b)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{3(5a+4b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}-\frac{3\left(-\frac{\left(-a+\sqrt{b\,(a+}\right.}{2a\sqrt{b}}\right.}{}$
default	$-\frac{2\left(-\frac{5\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{3(5a+4b)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{3(5a+4b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}-\frac{3\left(-\frac{\left(-a+\sqrt{b\,(a+}\right.}{2a\sqrt{b}}\right.}{}$
risch	$-\frac{5a^3 e^{6dx+6c}-a^2 b\,e^{6dx+6c}-9a\,b^2 e^{6dx+6c}-3b^3 e^{6dx+6c}+15a^3 e^{4dx+4c}-a^2 b\,e^{4dx+4c}+9a\,b^2 e^{4dx+4c}+9b^3 e^{4dx+4c}+15a^3 e}{4(a^2+2ab+b^2)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-5/8/a*tanh(1/2*d*x+1/2*c)^7-3/8*(5*a+4*b)/a^2*tanh(1/2*d*x+1/2*c)
^5-3/8*(5*a+4*b)/a^2*tanh(1/2*d*x+1/2*c)^3-5/8/a*tanh(1/2*d*x+1/2*c))/(a*ta
nh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^
2-3/4/a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-
a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)
^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/
2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(82) = 164$.
time = 0.63, size = 366, normalized size = 3.81

$$\frac{5\,a^3+13\,a^2b+11\,ab^2+3\,b^3+(15\,a^3+13\,a^2b-11\,ab^2-9\,b^3)e^{(-2\,dx-2\,c)}+(15\,a^3-a^2b+9\,ab^2+9\,b^3)e^{(-4\,dx-4\,c)}+(5\,a^3-a^2b-9\,ab^2-3\,b^3)e^{(-6\,dx-6\,c)}}{4\,(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+4\,(a^6+2\,a^5b-2\,a^3b^3-a^2b^4)e^{(-2\,dx-2\,c)}+2\,(3\,a^6+4\,a^5b+2\,a^4b^2+4\,a^3b^3+3\,a^2b^4)e^{(-4\,dx-4\,c)}+4\,(a^6+2\,a^5b-2\,a^3b^3-a^2b^4)e^{(-6\,dx-6\,c)}+(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)e^{(-8\,dx-8\,c)})d}-\frac{3\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(5*a^3 + 13*a^2*b + 11*a*b^2 + 3*b^3 + (15*a^3 + 13*a^2*b - 11*a*b^2 -
9*b^3)*e^(-2*d*x - 2*c) + (15*a^3 - a^2*b + 9*a*b^2 + 9*b^3)*e^(-4*d*x - 4*
c) + (5*a^3 - a^2*b - 9*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^6 + 4*a^5*b +
6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + 4*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*e
^(-2*d*x - 2*c) + 2*(3*a^6 + 4*a^5*b + 2*a^4*b^2 + 4*a^3*b^3 + 3*a^2*b^4)*e
^(-4*d*x - 4*c) + 4*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*e^(-6*d*x - 6*c)
+ (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) -
3/8*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2
*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2768 vs. $2(82) = 164$.
time = 0.41, size = 5840, normalized size = 60.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 24*(5
*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(
5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 20*a^4*b + 52*a^
3*b^2 + 44*a^2*b^3 + 12*a*b^4 + 4*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4
)*cosh(d*x + c)^4 + 4*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4 + 15*(5*a^4
*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(
5*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (15*a^4*b - a
^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4*b
+ 13*a^3*b^2 - 11*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 + 4*(15*a^4*b + 13*a^3
*b^2 - 11*a^2*b^3 - 9*a*b^4 + 15*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*
cosh(d*x + c)^4 + 6*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*cosh(d*x + c
)^2)*sinh(d*x + c)^2 + 3*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(
d*x + c)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)*si
nh(d*x + c)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x + c)^8
 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(a^4 + 2*a^3*b - 2
*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(
d*x + c)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*
(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a^4 + 4
*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^5 + 10*(a
^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^2*b^
2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b - 2*
a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
```

```
b^4)*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^4 +
 a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 +
 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x
 + c)^5 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^3 +
 (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*l
og(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*co
sh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(
d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (
a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a
 + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-
a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
 + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cos
h(d*x + c))*sinh(d*x + c) + a + b)) + 8*(3*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 -
 3*a*b^4)*cosh(d*x + c)^5 + 2*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*co
sh(d*x + c)^3 + (15*a^4*b + 13*a^3*b^2 - 11*a^2*b^3 - 9*a*b^4)*cosh(d*x + c
))*sinh(d*x + c))/((a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c)^8 + 8*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4
 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)
*d*cosh(d*x + c)^6 + 4*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*
b^5)*d*cosh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d)*sinh(
d*x + c)^6 + 2*(3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d*
cosh(d*x + c)^4 + 8*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5
)*d*cosh(d*x + c)^3 + 3*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh(d*
x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4
+ a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)
*d*cosh(d*x + c)^2 + (3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b
^5)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh
(d*x + c)^2 + 8*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c)^5 + 10*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cosh(d*x +
 c)^3 + (3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 +
 a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*
d*cosh(d*x + c)^4 + 3*(3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*
b^5)*d*cosh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d)*sinh(
d*x + c)^2 + (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d + 8*((
a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*
(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*cos...
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(82) = 164.
time = 1.01, size = 320, normalized size = 3.33

$$\frac{\frac{3\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}-\frac{2\left(5a^3e^{(6dx+6c)}-a^2be^{(6dx+6c)}-9ab^2e^{(6dx+6c)}-3b^3e^{(6dx+6c)}+15a^3e^{(4dx+4c)}-a^2be^{(4dx+4c)}+9ab^2e^{(4dx+4c)}+9b^3e^{(4dx+4c)}+15a^3e^{(2dx+2c)}+13a^2be^{(2dx+2c)}-11ab^2e^{(2dx+2c)}-9b^3e^{(2dx+2c)}+5a^3+13a^2b+11ab^2+3b^3\right)}{\left(a^4+2a^3b+a^2b^2\right)\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)
)/(sqrt(a*b)*a^2) - 2*(5*a^3*e^(6*d*x + 6*c) - a^2*b*e^(6*d*x + 6*c) - 9*a*
b^2*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 15*a^3*e^(4*d*x + 4*c) - a^2*
b*e^(4*d*x + 4*c) + 9*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 15*a^
3*e^(2*d*x + 2*c) + 13*a^2*b*e^(2*d*x + 2*c) - 11*a*b^2*e^(2*d*x + 2*c) - 9
*b^3*e^(2*d*x + 2*c) + 5*a^3 + 13*a^2*b + 11*a*b^2 + 3*b^3)/((a^4 + 2*a^3*b
 + a^2*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)^2\,\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

3.129 $\int \frac{\text{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=129

$$\frac{(4a+3b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{3/2}d}+\frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2}+\frac{(4a+3b)\sinh(c+dx)}{8a^2(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/8*(4*a+3*b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(3/2)/d
+1/4*b*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)^2+1/8*(4*a+3*b)*sinh(d
*x+c)/a^2/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3757, 393, 205, 211}

$$\frac{(4a+3b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{3/2}}+\frac{(4a+3b)\sinh(c+dx)}{8a^2d(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{b\sinh(c+dx)}{4ad(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((4*a + 3*b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b
)^(3/2)*d) + (b*Sinh[c + d*x])/(4*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2)
^2) + ((4*a + 3*b)*Sinh[c + d*x])/(8*a^2*(a + b)*d*(a + (a + b)*Sinh[c + d*
x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+(a+b)x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{\left(\frac{3}{a}+\frac{1}{a+b}\right)\text{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^2}\,dx,\right.}{4d}$$

$$= \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{(4a+3b)\sinh(c+dx)}{8a^2(a+b)d\left(a+(a+b)\sinh^2(c+\right.}$$

$$= \frac{(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{3/2}d} + \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx\right.}$$

Mathematica [A]
time = 0.56, size = 123, normalized size = 0.95

$$\frac{-\frac{8\sinh(c+dx)}{\left(a+(a+b)\sinh^2(c+dx)\right)^2} + (4a+3b)\left(\frac{3\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}} + \frac{5a\sinh(c+dx)+3(a+b)\sinh^3(c+dx)}{a^2\left(a+(a+b)\sinh^2(c+dx)\right)^2}\right)}{24(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((-8*Sinh[c + d*x])/(a + (a + b)*Sinh[c + d*x]^2)^2 + (4*a + 3*b)*((3*ArcTa
n[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]) + (5*a*Sinh[c
```

```
 + d*x] + 3*(a + b)*Sinh[c + d*x]^3)/(a^2*(a + (a + b)*Sinh[c + d*x]^2)^2))
)/(24*(a + b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(115) = 230$.
time = 2.95, size = 342, normalized size = 2.65

method	result
derivativedivides	$\frac{-\frac{(5b+4a)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a(a+b)}-\frac{\left(4a^2+13ab+12b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2(a+b)}+\frac{\left(4a^2+13ab+12b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2(a+b)}+\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}$
default	$\frac{-\frac{(5b+4a)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a(a+b)}-\frac{\left(4a^2+13ab+12b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2(a+b)}+\frac{\left(4a^2+13ab+12b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2(a+b)}+\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}$
risch	$\frac{\left(4a^2e^{6dx+6c}+7ab\,e^{6dx+6c}+3b^2e^{6dx+6c}+4a^2e^{4dx+4c}-ab\,e^{4dx+4c}-9b^2e^{4dx+4c}-4a^2e^{2dx+2c}+ab\,e^{2dx+2c}+9b^2e^{2dx+2c}-\right.}{4(a+b)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)^2a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/8*(5*b+4*a)/a/(a+b)*tanh(1/2*d*x+1/2*c)^7-1/8*(4*a^2+13*a*b+12*b
^2)/a^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+1/8*(4*a^2+13*a*b+12*b^2)/a^2/(a+b)*tan
h(1/2*d*x+1/2*c)^3+1/8*(5*b+4*a)/a/(a+b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d
*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/4/a*
(4*a+3*b)/(a+b)*(-1/2*((b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(
1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*
b)*a)^(1/2))+1/2*((b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^
(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*a^2*e^(7*c) + 7*a*b*e^(7*c) + 3*b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(
5*c) - a*b*e^(5*c) - 9*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) - a*b*e^(3*c
) - 9*b^2*e^(3*c))*e^(3*d*x) - (4*a^2*e^c + 7*a*b*e^c + 3*b^2*e^c)*e^(d*x))
/(a^5*d + 3*a^4*b*d + 3*a^3*b^2*d + a^2*b^3*d + (a^5*d*e^(8*c) + 3*a^4*b*d*
e^(8*c) + 3*a^3*b^2*d*e^(8*c) + a^2*b^3*d*e^(8*c))*e^(8*d*x) + 4*(a^5*d*e^(
6*c) + a^4*b*d*e^(6*c) - a^3*b^2*d*e^(6*c) - a^2*b^3*d*e^(6*c))*e^(6*d*x) +
 2*(3*a^5*d*e^(4*c) + a^4*b*d*e^(4*c) + a^3*b^2*d*e^(4*c) + 3*a^2*b^3*d*e^(
4*c))*e^(4*d*x) + 4*(a^5*d*e^(2*c) + a^4*b*d*e^(2*c) - a^3*b^2*d*e^(2*c) -
a^2*b^3*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/32*((4*a*e^(3*c) + 3*b*e^(3*c
))*e^(3*d*x) + (4*a*e^c + 3*b*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4
*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) -
a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3627 vs. $2(115) = 230$.
time = 0.47, size = 6614, normalized size = 51.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^7 + 28*(4*
a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4
*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^7 + 4*(4*a^4 + 3*a^3*
b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^5 + 4*(4*a^4 + 3*a^3*b - 10*a^2*b^2
 - 9*a*b^3 + 21*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^5 + 20*(7*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x
+ c)^3 + (4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^4 - 4*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^3 + 4*(35*
(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 - 4*a^4 - 3*a^3*b
 + 10*a^2*b^2 + 9*a*b^3 + 10*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^
3)*cosh(d*x + c)^5 + 10*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x +
 c)^3 - 3*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^2 - ((4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^8 + 8*(4*a^3
+ 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 11*
```

a^2*b + 10*a*b^2 + 3*b^3)*sinh(d*x + c)^8 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3 + 7*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 3*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 12*a^3 + a^2*b + 6*a*b^2 + 9*b^3 + 30*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^3 + (12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^6 + 15*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^4 + 4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3 + 3*(12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^7 + 3*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + (12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c) + 4*(7*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^6 + 5*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4 - 4*a^4 - 11*a^3*b - 10*a^2*b^2 - 3*a*b^3 - 3*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^8 + 8*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*sinh(d*x + c)^8 + 4*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^4 + 30*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^5 + 10*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^3 + (3*a^7 + 4*a^6*b + 2

```
*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^6 + 15*(a^
7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^7 + 4*a^6*b +
 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^6*b - 2*
a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*
b^3 + a^3*b^4)*d + 8*((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*c
osh(d*x + c)^7 + 3*(a^7 + 2*a^6*b - 2*a^4*b^3 -...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(b\,\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)
```

3.130 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=115

$$-\frac{(a-3b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}+\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2}-\frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] -1/8*(a-3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/b^(3/2)/d+1/4*(a+b
)*tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)^2-1/8*(a-3*b)*tanh(d*x+c)/a^2/b/d/(
a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3756, 393, 205, 211}

$$-\frac{(a-3b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}-\frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)}+\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*((a - 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*b^(3/2)*d
) + ((a + b)*Tanh[c + d*x])/(4*a*b*d*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b
)*Tanh[c + d*x])/(8*a^2*b*d*(a + b*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4abd}$$

$$= \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-3b)\operatorname{Subst}}{}$$

$$= -\frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-}{8a^2bd\,(a}$$

Mathematica [A]
time = 0.72, size = 115, normalized size = 1.00

$$\frac{\frac{(-a+3b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}} + \frac{\sqrt{a}\,\left(a^2+6ab-3b^2+\left(a^2+4ab+3b^2\right)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (((-a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/b^(3/2) + (Sqrt[a]*(a
^2 + 6*a*b - 3*b^2 + (a^2 + 4*a*b + 3*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d
*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d*x)])^2))/(8*a^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(101) = 202$.
time = 2.64, size = 334, normalized size = 2.90

method	result
derivativedivides	$-\frac{2\left(-\frac{(a+5b)\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ab}-\frac{\left(3a^{2}+11ab+12b^{2}\right)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}b}-\frac{\left(3a^{2}+11ab+12b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}b}-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{8ab}}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}$
default	$-\frac{2\left(-\frac{(a+5b)\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ab}-\frac{\left(3a^{2}+11ab+12b^{2}\right)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}b}-\frac{\left(3a^{2}+11ab+12b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}b}-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{8ab}}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}$
risch	$-\frac{a^{3}e^{6dx+6c}-a^{2}b\,e^{6dx+6c}-5a\,b^{2}e^{6dx+6c}-3b^{3}e^{6dx+6c}+3a^{3}e^{4dx+4c}+7a^{2}b\,e^{4dx+4c}-3a\,b^{2}e^{4dx+4c}+9b^{3}e^{4dx+4c}+3a^{3}e^{2d}}{4(a+b)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/8*(a+5*b)/a/b*tanh(1/2*d*x+1/2*c)^7-1/8*(3*a^2+11*a*b+12*b^2)/a
^2/b*tanh(1/2*d*x+1/2*c)^5-1/8*(3*a^2+11*a*b+12*b^2)/a^2/b*tanh(1/2*d*x+1/2
*c)^3-1/8*(a+5*b)/a/b*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tan
h(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1/4/a*(a-3*b)/b*(-1/2*(a-
(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*ar
ctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(-a-(b
*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arct
an(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(101) = 202$.
time = 0.65, size = 360, normalized size = 3.13

$$\frac{a^3+5a^2b+7ab^2+3b^3+(3a^3+13a^2b+ab^2-9b^3)e^{(-2dx-2c)}+(3a^3+7a^2b-3ab^2+9b^3)e^{(-4dx-4c)}+(a^3-a^2b-5ab^2-3b^3)e^{(-6dx-6c)}}{4(a^5b+3a^4b^2+3a^3b^3+a^2b^4+4(a^5b+a^4b^2-a^3b^3-a^2b^4)e^{(-2dx-2c)}+2(3a^5b+a^4b^2+a^3b^3+3a^2b^4)e^{(-4dx-4c)}+4(a^5b+a^4b^2-a^3b^3-a^2b^4)e^{(-6dx-6c)}+(a^5b+3a^4b^2+3a^3b^3+a^2b^4)e^{(-8dx-8c)})d}+\frac{(a-3b)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3 + (3*a^3 + 13*a^2*b + a*b^2 - 9*b^3)*e
^(-2*d*x - 2*c) + (3*a^3 + 7*a^2*b - 3*a*b^2 + 9*b^3)*e^(-4*d*x - 4*c) + (a
^3 - a^2*b - 5*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^5*b + 3*a^4*b^2 + 3*a^3
*b^3 + a^2*b^4 + 4*(a^5*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-2*d*x - 2*c) +
 2*(3*a^5*b + a^4*b^2 + a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^5*b +
a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 3*a^4*b^2 + 3*a^3*
b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) + 1/8*(a - 3*b)*arctan(1/2*((a + b)*e^(
-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2677 vs. $2(101) = 202$.
time = 0.41, size = 5659, normalized size = 49.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 24*(a^4
*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^4*
b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 4*a^4*b + 20*a^3*b^2 +
 28*a^2*b^3 + 12*a*b^4 + 4*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh
(d*x + c)^4 + 4*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4 + 15*(a^4*b - a^
3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(a^4*
b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (3*a^4*b + 7*a^3*b^2 -
 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4*b + 13*a^3*
b^2 + a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 + 4*(3*a^4*b + 13*a^3*b^2 + a^2*b^
3 - 9*a*b^4 + 15*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 +
6*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - ((a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^8 + 8*(a^4 - 6*a^2
*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 - 6*a^2*b^2 -
8*a*b^3 - 3*b^4)*sinh(d*x + c)^8 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 +
 3*b^4)*cosh(d*x + c)^6 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4 +
7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*
(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + 3*(a^4 - 2*a^3*b -
 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 - 8
*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 - 6*a^2*b^2 - 8*a*
b^3 - 3*b^4)*cosh(d*x + c)^4 + 3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4 + 30*(a^
4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4
 + a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4 + 8*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*
b^4)*cosh(d*x + c)^5 + 10*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cos
h(d*x + c)^3 + (3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x + c))*sinh(d*
```

```
x + c)^3 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2
+ 4*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 15*(a^4 - 2*a^
3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + a^4 - 2*a^3*b - 4*a^2*
b^2 + 2*a*b^3 + 3*b^4 + 3*(3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 8*((a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c
)^7 + 3*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + (3*
a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 4*a^2
*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2
+ 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x +
c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^
2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2
)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cos
h(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a
 + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*s
inh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 +
 a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c
))*sinh(d*x + c) + a + b)) + 8*(3*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*c
osh(d*x + c)^5 + 2*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c
)^3 + (3*a^4*b + 13*a^3*b^2 + a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x +
c))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^6
*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (
a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^6*b^2 +
 a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + 3*a^5*b^3
 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (a^6*b^2 + a^5*b^3 - a^4*b^4 -
a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)
*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cos
h(d*x + c)^3 + 3*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*s
inh(d*x + c)^5 + 2*(35*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d
*x + c)^4 + 30*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 +
(3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6*b^2
 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + 3*a^5*b
^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + 10*(a^6*b^2 + a^5*b^3 - a^4*b
^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^
5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*
d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d*cosh(d*
x + c)^2 + (a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + (a^
6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d + 8*((a^6*b^2 + 3*a^5*b^3 + 3*a^
4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b
^5)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d*cosh(
d*x + c)^3 + (a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(
d*x + c)), -1/8*(2*(a^4*b - a^3*b^2 - 5*a^2*b^3...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(101) = 202.
time = 0.88, size = 319, normalized size = 2.77

$$-\frac{\frac{(a-3b)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b}+\frac{2\left(a^3e^{(6dx+6c)}-a^2be^{(6dx+6c)}-5ab^2e^{(6dx+6c)}-3b^3e^{(6dx+6c)}+3a^3e^{(4dx+4c)}+7a^2be^{(4dx+4c)}-3ab^2e^{(4dx+4c)}+9b^3e^{(4dx+4c)}+3a^3e^{(2dx+2c)}+13a^2be^{(2dx+2c)}+ab^2e^{(2dx+2c)}-9b^3e^{(2dx+2c)}+a^3+5a^2b+7ab^2+3b^3\right)}{\left(a^3b+a^2b^2\right)\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((a - 3*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/
sqrt(a*b))/(sqrt(a*b)*a^2*b) + 2*(a^3*e^(6*d*x + 6*c) - a^2*b*e^(6*d*x + 6*
c) - 5*a*b^2*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 3*a^3*e^(4*d*x + 4*c
) + 7*a^2*b*e^(4*d*x + 4*c) - 3*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*
c) + 3*a^3*e^(2*d*x + 2*c) + 13*a^2*b*e^(2*d*x + 2*c) + a*b^2*e^(2*d*x + 2*
c) - 9*b^3*e^(2*d*x + 2*c) + a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)/((a^3*b + a^2
*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^
(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)^4\,\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)
```

3.131 $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=104

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a+b}\,d}+\frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2}+\frac{3\sinh(c+dx)}{8a^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

```
[Out] 1/4*sinh(d*x+c)/a/d/(a+(a+b)*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+(a+b
)*sinh(d*x+c)^2)+3/8*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/d/(a+b
)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3757, 205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a+b}}+\frac{3\sinh(c+dx)}{8a^2d\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh(c+dx)}{4ad\left((a+b)\sinh^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a + b]*d) +
 Sinh[c + d*x]/(4*a*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/
(8*a^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
```

```
Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^2}\,dx, x, \sinh(c+dx)\right)}{4ad}$$

$$= \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+(a+b)\sinh^2(c+dx)\right)} + \frac{3\text{Sub}}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a+b}\,d} + \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{}{8a^2d\,(a+}$$

Mathematica [A]
time = 0.21, size = 88, normalized size = 0.85

$$\frac{\frac{3\text{ArcTan}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{5a\sinh(c+dx)+3(a+b)\sinh^3(c+dx)}{a\left(a+(a+b)\sinh^2(c+dx)\right)^2}}{8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b]) + (5
*a*Sinh[c + d*x] + 3*(a + b)*Sinh[c + d*x]^3)/(a*(a + (a + b)*Sinh[c + d*x]
^2)^2))/(8*a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(90) = 180.
time = 2.70, size = 276, normalized size = 2.65

method	result

risch	$\frac{e^{dx+c}(3a\,e^{6dx+6c}+3b\,e^{6dx+6c}+11a\,e^{4dx+4c}-9b\,e^{4dx+4c}-11a\,e^{2dx+2c}+9b\,e^{2dx+2c}-3a-3b)}{4(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b)^2 a^2 d} - \frac{3\ln\left(e^{2dx+2c}-\frac{2a\,e}{\sqrt{-a}}\right.}{16\sqrt{-a^2-a}}$
derivativedivides	$\frac{\frac{-\frac{5\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}+\frac{3(a-4b)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}-\frac{3(a-4b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}+\frac{-\frac{3\left(\sqrt{b(a+b)}-b\right)\operatorname{arctan}}{8a\sqrt{b(a+b)}\,\sqrt{}}}{}}{}$
default	$\frac{\frac{-\frac{5\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}+\frac{3(a-4b)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}-\frac{3(a-4b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}+\frac{-\frac{3\left(\sqrt{b(a+b)}-b\right)\operatorname{arctan}}{8a\sqrt{b(a+b)}\,\sqrt{}}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-5/8/a*tanh(1/2*d*x+1/2*c)^7+3/8*(a-4*b)/a^2*tanh(1/2*d*x+1/2*c)^5-
3/8*(a-4*b)/a^2*tanh(1/2*d*x+1/2*c)^3+5/8/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/
2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+3/4
/a*(-1/2*((b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a
)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+
1/2*((b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*(a*e^(7*c) + b*e^(7*c))*e^(7*d*x) + (11*a*e^(5*c) - 9*b*e^(5*c))*e^(
5*d*x) - (11*a*e^(3*c) - 9*b*e^(3*c))*e^(3*d*x) - 3*(a*e^c + b*e^c)*e^(d*x)
)/(a^4*d + 2*a^3*b*d + a^2*b^2*d + (a^4*d*e^(8*c) + 2*a^3*b*d*e^(8*c) + a^2
*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^4*d*e^(6*c) - a^2*b^2*d*e^(6*c))*e^(6*d*x)
 + 2*(3*a^4*d*e^(4*c) - 2*a^3*b*d*e^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x)
```

```
+ 4*(a^4*d*e^(2*c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 32*integrate(3/128*(e^
(3*d*x + 3*c) + e^(d*x + c))/(a^3 + a^2*b + (a^3*e^(4*c) + a^2*b*e^(4*c))*e
^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2712 vs. $2(90) = 180$.
time = 0.42, size = 5077, normalized size = 48.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^7 + 84*(a^3 + 2*a^2*b + a*b
^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 12*(a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c
)^7 + 4*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^5 + 4*(11*a^3 + 2*a^2*b
- 9*a*b^2 + 63*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2
0*(21*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + (11*a^3 + 2*a^2*b - 9*a*b^2
)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x
+ c)^3 + 4*(105*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 11*a^3 - 2*a^2*b
+ 9*a*b^2 + 10*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
3 + 4*(63*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 10*(11*a^3 + 2*a^2*b -
9*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c))*si
nh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4
*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a
^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a
^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d
*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh
(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3
*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 +
2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d
*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x
+ c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a +
 b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*c
osh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 +
4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(co
sh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh
(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/(
(a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)
*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2
 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x +
```

```
 c))*sinh(d*x + c) + a + b)) - 12*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c) + 4
*(21*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 5*(11*a^3 + 2*a^2*b - 9*a*b^
2)*cosh(d*x + c)^4 - 3*a^3 - 6*a^2*b - 3*a*b^2 - 3*(11*a^3 + 2*a^2*b - 9*a*
b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)
*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c
)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^8
 + 4*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^
5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + a^5*b - a^4*b^2 - a^3
*b^3)*d)*sinh(d*x + c)^6 + 2*(3*a^6 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d
*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 +
3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
5*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 + 30*(a^6 + a^5*b
 - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 + (3*a^6 + a^5*b + a^4*b^2 + 3*a^3*
b^3)*d)*sinh(d*x + c)^4 + 4*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x +
c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 + 10*(a
^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^3 + (3*a^6 + a^5*b + a^4*b^
2 + 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a
^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 15*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*
d*cosh(d*x + c)^4 + 3*(3*a^6 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)
^2 + (a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(
d*x + c)^7 + 3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^5 + (3*a^6
 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^3 + (a^6 + a^5*b - a^4*b^2
- a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(6*(a^3 + 2*a^2*b + a*b^2)*
cosh(d*x + c)^7 + 42*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^6
+ 6*(a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^7 + 2*(11*a^3 + 2*a^2*b - 9*a*b^2
)*cosh(d*x + c)^5 + 2*(11*a^3 + 2*a^2*b - 9*a*b^2 + 63*(a^3 + 2*a^2*b + a*b
^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(21*(a^3 + 2*a^2*b + a*b^2)*cosh(
d*x + c)^3 + (11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 -
2*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 + 2*(105*(a^3 + 2*a^2*b + a*
b^2)*cosh(d*x + c)^4 - 11*a^3 - 2*a^2*b + 9*a*b^2 + 10*(11*a^3 + 2*a^2*b -
9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*(a^3 + 2*a^2*b + a*b^2)*c
osh(d*x + c)^5 + 10*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^
3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((a^2 + 2*a*b + b
^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cos...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^5\,{\left(b\,{\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3), x)
```

3.132 $\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{(3a^2-2ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d}+\frac{(a+b)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2}+\frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(c+dx)}{8d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] 1/8*(3*a^2-2*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/b^(5/2)
/d+1/4*(a+b)*sech(d*x+c)^2*tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)^2+3/8*(1/a
^2-1/b^2)*tanh(d*x+c)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3756, 424, 393, 211}

$$\frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(c+dx)}{8d\left(a+b\tanh^2(c+dx)\right)}+\frac{(3a^2-2ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d}+\frac{(a+b)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2
)*b^(5/2)*d) + ((a + b)*Sech[c + d*x]^2*Tanh[c + d*x])/(4*a*b*d*(a + b*Tanh
[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Tanh[c + d*x])/(8*d*(a + b*Tanh[c +
d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-a+3b+(3a-b)x^2}{(a+bx^2)^2}\,dx, x, \tanh(c+\right.}{4abd}$$

$$= \frac{(a+b)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3(a^2-b^2)\tanh(c+dx)}{8a^2b^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{(3a^2-}{}$$

$$= \frac{(3a^2-2ab+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a+b)\operatorname{sech}^2(c+dx)\tanh(c+d}{4abd\left(a+b\tanh^2(c+dx)\right)^2}$$

Mathematica [A]
time = 0.68, size = 128, normalized size = 0.98

$$\frac{(3a^2-2ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - \frac{\sqrt{a}\sqrt{b}(a+b)(3a^2-10ab+3b^2+3(a^2-b^2)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - (Sqrt[a]
*Sqrt[b]*(a + b)*(3*a^2 - 10*a*b + 3*b^2 + 3*(a^2 - b^2)*Cosh[2*(c + d*x)])
*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(8*a^(5/2)*b^(5/
2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(117) = 234$.

time = 2.82, size = 376, normalized size = 2.87

method	result
derivativedivides	$-\frac{2\left(\frac{\left(3a^2-2ab-5b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a\,b^2}+\frac{\left(9a^3+14a^2b-7a\,b^2-12b^3\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2b^2}+\frac{\left(9a^3+14a^2b-7a\,b^2-12b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2b^2}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}$
default	$-\frac{2\left(\frac{\left(3a^2-2ab-5b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a\,b^2}+\frac{\left(9a^3+14a^2b-7a\,b^2-12b^3\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2b^2}+\frac{\left(9a^3+14a^2b-7a\,b^2-12b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2b^2}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}$
risch	$\frac{3a^3e^{6dx+6c}+a^2b\,e^{6dx+6c}+a\,b^2e^{6dx+6c}+3b^3e^{6dx+6c}+9a^3e^{4dx+4c}-15a^2b\,e^{4dx+4c}+15a\,b^2e^{4dx+4c}-9b^3e^{4dx+4c}+9a^3e^{2d}}{4\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(1/8*(3*a^2-2*a*b-5*b^2)/a/b^2*tanh(1/2*d*x+1/2*c)^7+1/8*(9*a^3+14*
a^2*b-7*a*b^2-12*b^3)/a^2/b^2*tanh(1/2*d*x+1/2*c)^5+1/8*(9*a^3+14*a^2*b-7*a
*b^2-12*b^3)/a^2/b^2*tanh(1/2*d*x+1/2*c)^3+1/8*(3*a^2-2*a*b-5*b^2)/a/b^2*ta
nh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*t
anh(1/2*d*x+1/2*c)^2+a)^2-1/4/a*(3*a^2-2*a*b+3*b^2)/b^2*(-1/2*(-a+(b*(a+b))
^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1
/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(117) = 234$.

time = 0.66, size = 332, normalized size = 2.53

$$-\frac{3\,a^{3}+3\,a^{2}b-3\,ab^{2}-3\,b^{3}+(9\,a^{3}-13\,a^{2}b-13\,ab^{2}+9\,b^{3})e^{(-2\,dx-2\,c)}+3\,(3\,a^{3}-5\,a^{2}b+5\,ab^{2}-3\,b^{3})e^{(-4\,dx-4\,c)}+(3\,a^{3}+a^{2}b+ab^{2}+3\,b^{3})e^{(-6\,dx-6\,c)}}{4\,(a^{4}b^{2}+2\,a^{3}b^{3}+a^{2}b^{4}+4\,(a^{4}b^{2}-a^{2}b^{4})e^{(-2\,dx-2\,c)}+2\,(3\,a^{4}b^{2}-2\,a^{3}b^{3}+3\,a^{2}b^{4})e^{(-4\,dx-4\,c)}+4\,(a^{4}b^{2}-a^{2}b^{4})e^{(-6\,dx-6\,c)}+(a^{4}b^{2}+2\,a^{3}b^{3}+a^{2}b^{4})e^{(-8\,dx-8\,c)})d}-\frac{(3\,a^{2}-2\,ab+3\,b^{2})\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{2}b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + (9*a^3 - 13*a^2*b - 13*a*b^2 + 9*
b^3)*e^(-2*d*x - 2*c) + 3*(3*a^3 - 5*a^2*b + 5*a*b^2 - 3*b^3)*e^(-4*d*x - 4
*c) + (3*a^3 + a^2*b + a*b^2 + 3*b^3)*e^(-6*d*x - 6*c))/((a^4*b^2 + 2*a^3*b
^3 + a^2*b^4 + 4*(a^4*b^2 - a^2*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^4*b^2 - 2*a^
3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^4*b^2 - a^2*b^4)*e^(-6*d*x - 6*c
) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) - 1/8*(3*a^2 - 2*a
*b + 3*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(
a*b)*a^2*b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2464 vs. $2(117) = 234$.
time = 0.46, size = 5233, normalized size = 39.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 24*(3*a^
4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(3*a^4
*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^6 + 12*a^4*b + 12*a^3*b^2 -
 12*a^2*b^3 - 12*a*b^4 + 12*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cos
h(d*x + c)^4 + 12*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4 + 5*(3*a^4*b +
 a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(3*a
^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + 3*(3*a^4*b - 5*a^3*b^
2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(9*a^4*b - 13*a
^3*b^2 - 13*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2 + 4*(9*a^4*b - 13*a^3*b^2 -
13*a^2*b^3 + 9*a*b^4 + 15*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x
+ c)^4 + 18*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 - ((3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x +
 c)^8 + 8*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)*sin
h(d*x + c)^7 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*sinh(d*x + c
)^8 + 4*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(3*a^4 - 2*
a^3*b + 2*a*b^3 - 3*b^4 + 7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)
*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a
*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(
d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9
*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^
```

4)*cosh(d*x + c)^4 + 9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4 + 30*
(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^
4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 8*(7*(3*a^4 + 4*a^3*b + 2*a^2*b
^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b
^4)*cosh(d*x + c)^3 + (9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4)*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d
x + c)^2 + 4(7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x +
c)^6 + 15*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 3*a^4 - 2*
a^3*b + 2*a*b^3 - 3*b^4 + 3*(9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b
^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*
a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh
(d*x + c)^5 + (9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x +
c)^3 + (3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*s
qrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^
2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 -
b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
- b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a +
b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 +
(a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(3*(3*a^4*b + a^3*b^2 +
a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 + 6*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3
*a*b^4)*cosh(d*x + c)^3 + (9*a^4*b - 13*a^3*b^2 - 13*a^2*b^3 + 9*a*b^4)*cos
h(d*x + c))*sinh(d*x + c))/((a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)
^8 + 8*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^5*b^3 - a^3*b^5)*d*c
osh(d*x + c)^6 + 4*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (
a^5*b^3 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^
5)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)
^3 + 3*(a^5*b^3 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^
3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^5*b^3 - a^3*b^5)*d*cosh(
d*x + c)^2 + (3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^
5*b^3 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^5 + 10*(a^5*b^3 - a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^5*b^3 -
2*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 + 2
*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^5*b^3 - a^3*b^5)*d*cosh(d*x +
c)^4 + 3*(3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (a^5*b^3
- a^3*b^5)*d)*sinh(d*x + c)^2 + (a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d + 8*((a^5
*b^3 + 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^5*b^3 - a^3*b^5)*d*cos
h(d*x + c)^5 + (3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (a^5
*b^3 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(2*(3*a^4*b + a^3*b^2
+ a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 12*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*
a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a

```
*b^4)*sinh(d*x + c)^6 + 6*a^4*b + 6*a^3*b^2 - 6*a^2*b^3 - 6*a*b^4 + 6*(3*a^
4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{6}(c+dx)}{\left(a+b\tanh^{2}(c+dx)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(117) = 234.
time = 0.91, size = 322, normalized size = 2.46

$$\frac{\frac{(3a^2-2ab+3b^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b^2}+\frac{2\left(3a^3e^{(6dx+6c)}+a^2be^{(6dx+6c)}+ab^2e^{(6dx+6c)}+3b^3e^{(6dx+6c)}+9a^3e^{(4dx+4c)}-15a^2be^{(4dx+4c)}+15ab^2e^{(4dx+4c)}-9b^3e^{(4dx+4c)}+9a^3e^{(2dx+2c)}-13a^2be^{(2dx+2c)}-13ab^2e^{(2dx+2c)}+9b^3e^{(2dx+2c)}+3a^3+3a^2b-3ab^2-3b^3\right)}{\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)^2a^2b^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 - 2*a*b + 3*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2
*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2) + 2*(3*a^3*e^(6*d*x + 6*c) + a^
2*b*e^(6*d*x + 6*c) + a*b^2*e^(6*d*x + 6*c) + 3*b^3*e^(6*d*x + 6*c) + 9*a^3
*e^(4*d*x + 4*c) - 15*a^2*b*e^(4*d*x + 4*c) + 15*a*b^2*e^(4*d*x + 4*c) - 9*
b^3*e^(4*d*x + 4*c) + 9*a^3*e^(2*d*x + 2*c) - 13*a^2*b*e^(2*d*x + 2*c) - 13
*a*b^2*e^(2*d*x + 2*c) + 9*b^3*e^(2*d*x + 2*c) + 3*a^3 + 3*a^2*b - 3*a*b^2
- 3*b^3)/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*
b*e^(2*d*x + 2*c) + a + b)^2*a^2*b^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cosh\left(c+d\,x\right)}^6\,{\left(b\,{\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3), x)
```

3.133 $\int \frac{\text{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=156

$$-\frac{\text{ArcTan}(\sinh(c+dx))}{b^3d}+\frac{\sqrt{a+b}\,(8a^2-4ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}+\frac{(a+b)\sinh(c+a}{4abd\,(a+(a+b)\sinh^2(}$$

```
[Out] -arctan(sinh(d*x+c))/b^3/d+1/4*(a+b)*sinh(d*x+c)/a/b/d/(a+(a+b)*sinh(d*x+c)
^2)^2-1/8*(4*a-3*b)*(a+b)*sinh(d*x+c)/a^2/b^2/d/(a+(a+b)*sinh(d*x+c)^2)+1/8
*(8*a^2-4*a*b+3*b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2)/a^
(5/2)/b^3/d
```

Rubi [A]

time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3757, 425, 541, 536, 209, 211}

$$-\frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sqrt{a+b}\,(8a^2-4ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}+\frac{(a+b)\sinh(c+dx)}{4abd\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\text{ArcTan}(\sinh(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -(ArcTan[Sinh[c + d*x]]/(b^3*d)) + (Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Arc
Tan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^3*d) + ((a + b)*Sinh
[c + d*x])/(4*a*b*d*(a + (a + b)*Sinh[c + d*x]^2)^2) - ((4*a - 3*b)*(a + b)
*Sinh[c + d*x])/(8*a^2*b^2*d*(a + (a + b)*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+(a+b)x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a-3b-3(a+b)x^2}{(1+x^2)(a+(a+b)x^2)^2}\,dx, x, \sinh(c\right.}{4abd}$$

$$= \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left(a+(a+b)\sinh^2(c+dx)\right)} + \text{S}$$

$$= \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \text{S}$$

$$= -\frac{\tan^{-1}(\sinh(c+dx))}{b^3d} + \frac{\sqrt{a+b}\,(8a^2-4ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\,\sinh(c+dx}{\sqrt{a}}\right.}{8a^{5/2}b^3d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.21, size = 317, normalized size = 2.03

$$-\frac{\frac{2\sqrt{a+b}\,(8a^2-4ab+3b^2)\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}} + \frac{2(8a^3+4a^2b-ab^2+3b^3)\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}} + 64\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \frac{i\sqrt{a+b}\,(8a^2-4ab+3b^2)\log(a-b+(a+b)\cosh(2(c+dx)))}{a^{5/2}} - \frac{i(8a^3+4a^2b-ab^2+3b^3)\log(a-b+(a+b)\cosh(2(c+dx)))}{a^{5/2}\sqrt{a+b}} - \frac{32b^2(a+b)\sinh(c+dx)}{a(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{8b(4a^2+ab-3b^2)\sinh(c+dx)}{a^2(a-b+(a+b)\cosh(2(c+dx)))}}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/32*((2*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x]
)/Sqrt[a + b]])/a^(5/2) + (2*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*ArcTan[(Sqrt
[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + 64*ArcTan[Tanh[(c
+ d*x)/2]] + (I*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Log[a - b + (a + b)*Cos
h[2*(c + d*x)]])/a^(5/2) - (I*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*Log[a - b +
 (a + b)*Cosh[2*(c + d*x)]])/(a^(5/2)*Sqrt[a + b]) - (32*b^2*(a + b)*Sinh[c
 + d*x])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (8*b*(4*a^2 + a*b - 3*
b^2)*Sinh[c + d*x])/(a^2*(a - b + (a + b)*Cosh[2*(c + d*x)]))))/(b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(142) = 284$.
time = 2.72, size = 389, normalized size = 2.49

method	result

derivativedivides	$\frac{2\left(\frac{\frac{b\left(4a^2-ab-5b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{\left(4a^3+23a^2b+7a\,b^2-12b^3\right)b\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{\left(4a^3+23a^2b+7a\,b^2-12b^3\right)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}\right.}{}$
default	$\frac{2\left(\frac{\frac{b\left(4a^2-ab-5b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{\left(4a^3+23a^2b+7a\,b^2-12b^3\right)b\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{\left(4a^3+23a^2b+7a\,b^2-12b^3\right)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}\right.}{}$
risch	$-\frac{\left(4a^3e^{6dx+6c}+5a^2b\,e^{6dx+6c}-2a\,b^2e^{6dx+6c}-3b^3e^{6dx+6c}+4a^3e^{4dx+4c}-19a^2b\,e^{4dx+4c}-14a\,b^2e^{4dx+4c}+9b^3e^{4dx+4c}-4a^3\right.}{4\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^3*((1/8*b*(4*a^2-a*b-5*b^2)/a*tanh(1/2*d*x+1/2*c)^7+1/8*(4*a^3+23*
a^2*b+7*a*b^2-12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^5-1/8*(4*a^3+23*a^2*b+7*a*b
^2-12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^3-1/8*b*(4*a^2-a*b-5*b^2)/a*tanh(1/2*d
*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*
d*x+1/2*c)^2+a)^2+1/8/a*(8*a^3+4*a^2*b-a*b^2+3*b^3)*(-1/2*((b*(a+b))^(1/2)-
b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*((b*(a+b))^(1/2)+b)/a/
(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))-2/b^3*arctan(tanh(1/2*d*x+1/2*
c)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((4*a^3*e^(7*c) + 5*a^2*b*e^(7*c) - 2*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*e
^(7*d*x) + (4*a^3*e^(5*c) - 19*a^2*b*e^(5*c) - 14*a*b^2*e^(5*c) + 9*b^3*e^(
5*c))*e^(5*d*x) - (4*a^3*e^(3*c) - 19*a^2*b*e^(3*c) - 14*a*b^2*e^(3*c) + 9*
b^3*e^(3*c))*e^(3*d*x) - (4*a^3*e^c + 5*a^2*b*e^c - 2*a*b^2*e^c - 3*b^3*e^c
)*e^(d*x))/(a^4*b^2*d + 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) + 2*a^
3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^4*b^2*d*e^(6*c) - a^2
*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^4*b^2*d*e^(4*c) - 2*a^3*b^3*d*e^(4*c) +
3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^4*b^2*d*e^(2*c) - a^2*b^4*d*e^(2*c))*
e^(2*d*x)) - 2*arctan(e^(d*x + c))/(b^3*d) + 128*integrate(1/512*((8*a^3*e^
(3*c) + 4*a^2*b*e^(3*c) - a*b^2*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) + (8*a^3
*e^c + 4*a^2*b*e^c - a*b^2*e^c + 3*b^3*e^c)*e^(d*x))/(a^3*b^3 + a^2*b^4 + (
a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c))*e^(4*d*x) + 2*(a^3*b^3*e^(2*c) - a^2*b^4
*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4457 vs. $2(142) = 284$.
time = 0.49, size = 8070, normalized size = 51.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^7 + 28*(4*a
^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4*a^
3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*sinh(d*x + c)^7 + 4*(4*a^3*b - 19*a^2*b^
2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^5 + 4*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3
+ 9*b^4 + 21*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^5 + 20*(7*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^3
+ (4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^4
- 4*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + 4*(35*(4*a^
3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^4 - 4*a^3*b + 19*a^2*b^2 +
 14*a*b^3 - 9*b^4 + 10*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x +
 c)^2)*sinh(d*x + c)^3 + 4*(21*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh
(d*x + c)^5 + 10*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^3
- 3*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^
2 - ((8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(
8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)
^7 + (8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(
8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^4 -
 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 +
2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^4 + 12*a^3*b
+ 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(8*a^4 - 4*a^3*b - 5*a^2
*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 - 28*a^3
```

```
*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 12*a^3
*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 - 28*a^3*b + 41*
a^2*b^2 - 18*a*b^3 + 9*b^4 + 30*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*
b^3 + 3*b^4 + 8*(7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*
x + c)^5 + 10*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)
^3 + (24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x +
 c)^2 + 4*(7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)
^6 + 15*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 8
*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4 + 3*(24*a^4 - 28*a^3*b + 41*a^
2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^4 + 12
*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^4 - 4*a^3*b
- 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 - 28*a^3*b + 41*a^
2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (8*a^4 - 4*a^3*b - 5*a^2*b^2 +
4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/a)*log(((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3
*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)
^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d
*x + c))*sqrt(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x +
 c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*
cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 32*((a^4
 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x + c)^8 + 4*(a^4
 - a^2*b^2)*cosh(d*x + c)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*
x + c)^3 + 3*(a^4 - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 - 2*
a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d
*x + c)^4 + 3*a^4 - 2*a^3*b + 3*a^2*b^2 + 30*(a^4 - a^2*b^2)*cosh(d*x + c)^
2)*sinh(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^
2)*cosh(d*x + c)^5 + 10*(a^4 - a^2*b^2)*cosh(d*x + c)^3 + (3*a^4 - 2*a^3*b
+ 3*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 - a^2*b^2)*cosh(d*x +
c)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 15*(a^4 - a^2*b^2)*
cosh(d*x + c)^4 + a^4 - a^2*b^2 + 3*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 3*
(a^4 - a^2*b^2)*cosh(d*x + c)^5 + (3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(d*x +
c)^3 + (a^4 - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) +
 sinh(d*x + c)) - 4*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c) +
 4*(7*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 5*(4*a^3*b
- 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^4 - 4*a^3*b - 5*a^2*b^2 + 2*
a*b^3 + 3*b^4 - 3*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^
```

```
4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*si...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^7\,{\left(b\,{\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3), x)
```

3.134 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=54

$$(a+b)x-\frac{(a+b)\tanh(c+dx)}{d}-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)*x-(a+b)*tanh(d*x+c)/d-1/3*(a+b)*tanh(d*x+c)^3/d-1/5*b*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3712, 3554, 8}

$$-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{(a+b)\tanh(c+dx)}{d}+x(a+b)-\frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - ((a + b)*Tanh[c + d*x])/d - ((a + b)*Tanh[c + d*x]^3)/(3*d) - (
b*Tanh[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{b\tanh^5(c+dx)}{5d}+(a+b)\int \tanh^4(c+dx)\,dx$$
$$= -\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}+(a+b)\int \tanh^2($$
$$= -\frac{(a+b)\tanh(c+dx)}{d}-\frac{(a+b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+}{5d}$$
$$= (a+b)x-\frac{(a+b)\tanh(c+dx)}{d}-\frac{(a+b)\tanh^3(c+dx)}{3d}-b$$

Mathematica [A]
time = 0.04, size = 97, normalized size = 1.80

$$\frac{a\tanh^{-1}(\tanh(c+dx))}{d}+\frac{b\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a\tanh(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}-\frac{a\tanh^3(c+dx)}{3d}-\frac{b\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d
*x])/d - (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]
^3)/(3*d) - (b*Tanh[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.38, size = 85, normalized size = 1.57

method	result
derivativedivides	$\frac{-\frac{b\left(\tanh^5(dx+c)\right)}{5}-\frac{\left(\tanh^3(dx+c)\right)a}{3}-\frac{b\left(\tanh^3(dx+c)\right)}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(1+}{}}{d}$
default	$\frac{-\frac{b\left(\tanh^5(dx+c)\right)}{5}-\frac{\left(\tanh^3(dx+c)\right)a}{3}-\frac{b\left(\tanh^3(dx+c)\right)}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(1+}{}}{d}$
risch	$ax+bx+\frac{4a\,e^{8dx+8c}+6b\,e^{8dx+8c}+12a\,e^{6dx+6c}+12b\,e^{6dx+6c}+\frac{44a\,e^{4dx+4c}}{3}+\frac{56b\,e^{4dx+4c}}{3}+\frac{28a\,e^{2dx+2c}}{3}+\frac{28b\,e^{2dx+2c}}{3}}{d\left(1+e^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*b*tanh(d*x+c)^5-1/3*tanh(d*x+c)^3*a-1/3*b*tanh(d*x+c)^3-a*tanh(d*
x+c)-b*tanh(d*x+c)-1/2*(a+b)*ln(tanh(d*x+c)-1)+1/2*(a+b)*ln(1+tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(50) = 100.
time = 0.29, size = 199, normalized size = 3.69

$$\frac{1}{15}b\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)+\frac{1}{3}a\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/15*b*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*
e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1))) + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c
) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))
)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(50) = 100$.
time = 0.39, size = 339, normalized size = 6.28

$$\frac{(15(a+b)dx+20a+23b)\cosh(dx+c)^5+5(15(a+b)dx+20a+23b)\cosh(dx+c)\sinh(dx+c)^4-(20a+23b)\sinh(dx+c)^5+5(15(a+b)dx+20a+23b)\cosh(dx+c)^3-5\left(2(20a+23b)\cosh(dx+c)^2+8a+5b\right)\sinh(dx+c)^3+5\left(2(15(a+b)dx+20a+23b)\cosh(dx+c)^3+3(15(a+b)dx+20a+23b)\cosh(dx+c)\right)\sinh(dx+c)^2+10(15(a+b)dx+20a+23b)\cosh(dx+c)-5\left((20a+23b)\cosh(dx+c)^4+3(8a+5b)\cosh(dx+c)^2+4a+10b\right)\sinh(dx+c)}{15\left(d\cosh(dx+c)^5+5d\cosh(dx+c)\sinh(dx+c)^4+5d\cosh(dx+c)^3+5\left(2d\cosh(dx+c)^3+3d\cosh(dx+c)\right)\sinh(dx+c)^2+10d\cosh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*((15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^5 + 5*(15*(a + b)*d*x +
20*a + 23*b)*cosh(d*x + c)*sinh(d*x + c)^4 - (20*a + 23*b)*sinh(d*x + c)^5
+ 5*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^3 - 5*(2*(20*a + 23*b)*cos
h(d*x + c)^2 + 8*a + 5*b)*sinh(d*x + c)^3 + 5*(2*(15*(a + b)*d*x + 20*a + 2
3*b)*cosh(d*x + c)^3 + 3*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c))*sinh
(d*x + c)^2 + 10*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c) - 5*((20*a +
23*b)*cosh(d*x + c)^4 + 3*(8*a + 5*b)*cosh(d*x + c)^2 + 4*a + 10*b)*sinh(d*
x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d
*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 1
0*d*cosh(d*x + c))
```

Sympy [**A**]
time = 0.15, size = 82, normalized size = 1.52

$$\begin{cases} ax - \frac{a\tanh^3(c+dx)}{3d} - \frac{a\tanh(c+dx)}{d} + bx - \frac{b\tanh^5(c+dx)}{5d} - \frac{b\tanh^3(c+dx)}{3d} - \frac{b\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x - a*tanh(c + d*x)**3/(3*d) - a*tanh(c + d*x)/d + b*x - b*tan
h(c + d*x)**5/(5*d) - b*tanh(c + d*x)**3/(3*d) - b*tanh(c + d*x)/d, Ne(d, 0
)), (x*(a + b*tanh(c)**2)*tanh(c)**4, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(50) = 100$.

time = 0.46, size = 134, normalized size = 2.48

$$\frac{15\,(dx+c)(a+b)+\frac{2\left(30\,ae^{(8\,dx+8\,c)}+45\,be^{(8\,dx+8\,c)}+90\,ae^{(6\,dx+6\,c)}+90\,be^{(6\,dx+6\,c)}+110\,ae^{(4\,dx+4\,c)}+140\,be^{(4\,dx+4\,c)}+70\,ae^{(2\,dx+2\,c)}+70\,be^{(2\,dx+2\,c)}+20\,a+23\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*(a + b) + 2*(30*a*e^(8*d*x + 8*c) + 45*b*e^(8*d*x + 8*c)
 + 90*a*e^(6*d*x + 6*c) + 90*b*e^(6*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) + 14
0*b*e^(4*d*x + 4*c) + 70*a*e^(2*d*x + 2*c) + 70*b*e^(2*d*x + 2*c) + 20*a +
23*b)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.22, size = 50, normalized size = 0.93

$$x\,(a+b)-\frac{\tanh(c+d\,x)^3\,(a+b)}{3\,d}-\frac{b\tanh(c+d\,x)^5}{5\,d}-\frac{\tanh(c+d\,x)\,(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*(a + b) - (tanh(c + d*x)^3*(a + b))/(3*d) - (b*tanh(c + d*x)^5)/(5*d) - (
tanh(c + d*x)*(a + b))/d
```

3.135 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=49

$$\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{(a+b)\tanh^2(c+dx)}{2d}-\frac{b\tanh^4(c+dx)}{4d}$$

```
[Out] (a+b)*ln(cosh(d*x+c))/d-1/2*(a+b)*tanh(d*x+c)^2/d-1/4*b*tanh(d*x+c)^4/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3712, 3554, 3556}

$$-\frac{(a+b)\tanh^2(c+dx)}{2d}+\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] ((a + b)*Log[Cosh[c + d*x]])/d - ((a + b)*Tanh[c + d*x]^2)/(2*d) - (b*Tanh[
c + d*x]^4)/(4*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{b\tanh^4(c+dx)}{4d} + (a+b)\int \tanh^3(c+dx)\,dx$$
$$= -\frac{(a+b)\tanh^2(c+dx)}{2d} - \frac{b\tanh^4(c+dx)}{4d} + (a+b)\int \tanh(c$$
$$= \frac{(a+b)\log(\cosh(c+dx))}{d} - \frac{(a+b)\tanh^2(c+dx)}{2d} - \frac{b\tanh^4(c}{4d}$$

Mathematica [A]
time = 0.19, size = 43, normalized size = 0.88

$$-\frac{-4(a+b)\log(\cosh(c+dx)) + 2(a+b)\tanh^2(c+dx) + b\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/4*(-4*(a + b)*Log[Cosh[c + d*x]] + 2*(a + b)*Tanh[c + d*x]^2 + b*Tanh[c
+ d*x]^4)/d
```

Maple [A]
time = 0.37, size = 71, normalized size = 1.45

method	result
derivativedivides	$\frac{-\frac{b\left(\tanh^4(dx+c)\right)}{4}-\frac{\left(\tanh^2(dx+c)\right)a}{2}-\frac{b\left(\tanh^2(dx+c)\right)}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(1+\tanh(dx+c))}{2}}{d}$
default	$\frac{-\frac{b\left(\tanh^4(dx+c)\right)}{4}-\frac{\left(\tanh^2(dx+c)\right)a}{2}-\frac{b\left(\tanh^2(dx+c)\right)}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(1+\tanh(dx+c))}{2}}{d}$
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}+\frac{2\,e^{2dx+2c}\left(a\,e^{4dx+4c}+2b\,e^{4dx+4c}+2a\,e^{2dx+2c}+2b\,e^{2dx+2c}+a+2b\right)}{d\left(1+e^{2dx+2c}\right)^4}+\frac{\ln\left(1+e^{2dx+2c}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*b*tanh(d*x+c)^4-1/2*tanh(d*x+c)^2*a-1/2*b*tanh(d*x+c)^2-1/2*(a+b)
*ln(tanh(d*x+c)-1)+1/2*(-a-b)*ln(1+tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(45) = 90$.
time = 0.48, size = 168, normalized size = 3.43

$$b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d\left(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1\right)}\right)+a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x
- 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*
e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + a*(x + c/d + log(e^(-2*d*x - 2
*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. $2(45) = 90$.
time = 0.34, size = 1205, normalized size = 24.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^7
 + (a + b)*d*x*sinh(d*x + c)^8 + 2*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^
6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 + 2*(a + b)*d*x - a - 2*b)*sinh(d*x +
 c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 + 3*(2*(a + b)*d*x - a - 2*b)*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^4
 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x + 15*(2*(a + b)*d*x -
a - 2*b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*d*x*co
sh(d*x + c)^5 + 5*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^3 + (3*(a + b)*d*
x - 2*a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a + b)*d*x + 2*(2*(a + b)*
d*x - a - 2*b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^6 + 15*(2*
(a + b)*d*x - a - 2*b)*cosh(d*x + c)^4 + 2*(a + b)*d*x + 6*(3*(a + b)*d*x -
 2*a - 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x
+ c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sinh(d*x + c)^8
+ 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*
x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*x + c)^4 + 30*(a
 + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x
+ c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x + c)^6 + 15*(a + b)
*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 8*(
(a + b)*cosh(d*x + c)^7 + 3*(a + b)*cosh(d*x + c)^5 + 3*(a + b)*cosh(d*x +
c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*cosh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*d*x*cosh(d*x + c)^7 + 3*(2*(a
 + b)*d*x - a - 2*b)*cosh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d
*x + c)^3 + (2*(a + b)*d*x - a - 2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh
(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*c
osh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(
```

```
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(
35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d
*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9
*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d
*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(42) = 84$.
time = 0.12, size = 88, normalized size = 1.80

$$\begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} - \frac{a\tanh^2(c+dx)}{2d} + bx - \frac{b\log(\tanh(c+dx)+1)}{d} - \frac{b\tanh^4(c+dx)}{4d} - \frac{b\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)\tanh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - a*tanh(c + d*x)**2/(2*d) + b*
x - b*log(tanh(c + d*x) + 1)/d - b*tanh(c + d*x)**4/(4*d) - b*tanh(c + d*x)
**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.
time = 0.44, size = 92, normalized size = 1.88

$$-\frac{(dx+c)(a+b) - (a+b)\log\left(e^{(2\,dx+2\,c)}+1\right) - \frac{2\left((a+2\,b)e^{(6\,dx+6\,c)} + 2\,(a+b)e^{(4\,dx+4\,c)} + (a+2\,b)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*(a + b) - (a + b)*log(e^(2*d*x + 2*c) + 1) - 2*((a + 2*b)*e^(6*
d*x + 6*c) + 2*(a + b)*e^(4*d*x + 4*c) + (a + 2*b)*e^(2*d*x + 2*c))/(e^(2*d
*x + 2*c) + 1)^4)/d
```

Mupad [B]
time = 0.12, size = 53, normalized size = 1.08

$$x\,(a+b) - \frac{\tanh(c+d\,x)^2\,(a+b)}{2\,d} - \frac{b\tanh(c+d\,x)^4}{4\,d} - \frac{\ln\left(\tanh(c+d\,x)+1\right)\,(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*(a + b) - (tanh(c + d*x)^2*(a + b))/(2*d) - (b*tanh(c + d*x)^4)/(4*d) - (
log(tanh(c + d*x) + 1)*(a + b))/d
```

3.136 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=36

$$(a+b)x-\frac{(a+b)\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] (a+b)*x-(a+b)*tanh(d*x+c)/d-1/3*b*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3712, 3554, 8}

$$-\frac{(a+b)\tanh(c+dx)}{d}+x(a+b)-\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - ((a + b)*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= -\frac{b\tanh^3(c+dx)}{3d}-(-a-b)\int \tanh^2(c+dx)\,dx\\
&= -\frac{(a+b)\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}-(-a-b)\int 1\,dx\\
&= (a+b)x-\frac{(a+b)\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 1.81

$$\frac{a\tanh^{-1}(\tanh(c+dx))}{d}+\frac{b\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a\tanh(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d
*x])/d - (b*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.37, size = 63, normalized size = 1.75

method	result	size
derivativedivides	$\frac{-\frac{b\left(\tanh^3(dx+c)\right)}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(1+\tanh(dx+c))}{2}}{d}$	63
default	$\frac{-\frac{b\left(\tanh^3(dx+c)\right)}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(1+\tanh(dx+c))}{2}}{d}$	63
risch	$ax+bx+\frac{2a\,\mathrm{e}^{4dx+4c}+4b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}+4b\,\mathrm{e}^{2dx+2c}+2a+\frac{8b}{3}}{d\left(1+\mathrm{e}^{2dx+2c}\right)^3}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*b*tanh(d*x+c)^3-a*tanh(d*x+c)-b*tanh(d*x+c)-1/2*(a+b)*ln(tanh(d*x
+c)-1)+1/2*(a+b)*ln(1+tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(34) = 68.
time = 0.29, size = 105, normalized size = 2.92

$$\frac{1}{3}\,b\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)+a\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*
e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a*(x + c/
d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(34) = 68.
time = 0.33, size = 160, normalized size = 4.44

$$\frac{(3\,(a+b)dx+3\,a+4\,b)\cosh\left(dx+c\right)^3+3\,(3\,(a+b)dx+3\,a+4\,b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2-(3\,a+4\,b)\sinh\left(dx+c\right)^3+3\,(3\,(a+b)dx+3\,a+4\,b)\cosh\left(dx+c\right)-3\,\left((3\,a+4\,b)\cosh\left(dx+c\right)^2+a\right)\sinh\left(dx+c\right)}{3\,\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+3\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*(a + b)*d*x + 3*a + 4*b)*cosh(d*x + c)^3 + 3*(3*(a + b)*d*x + 3*a +
 4*b)*cosh(d*x + c)*sinh(d*x + c)^2 - (3*a + 4*b)*sinh(d*x + c)^3 + 3*(3*(a
 + b)*d*x + 3*a + 4*b)*cosh(d*x + c) - 3*((3*a + 4*b)*cosh(d*x + c)^2 + a)*
sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d
*cosh(d*x + c))
```

Sympy [A]
time = 0.10, size = 54, normalized size = 1.50

$$\begin{cases} ax - \frac{a\tanh{(c+dx)}}{d} + bx - \frac{b\tanh^3{(c+dx)}}{3d} - \frac{b\tanh{(c+dx)}}{d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2{(c)}\right)\tanh^2{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x - a*tanh(c + d*x)/d + b*x - b*tanh(c + d*x)**3/(3*d) - b*tan
h(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(34) = 68.
time = 0.43, size = 86, normalized size = 2.39

$$\frac{3\,(dx+c)(a+b) + \frac{2\,\left(3\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+3\,a+4\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*(a + b) + 2*(3*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) + 6
*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 3*a + 4*b)/(e^(2*d*x + 2*c) + 1)
^3)/d
```

Mupad [B]
time = 1.17, size = 34, normalized size = 0.94

$$x\,(a+b) - \frac{b\tanh(c+d\,x)^3}{3\,d} - \frac{\tanh(c+d\,x)\,(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*(a + b) - (b*tanh(c + d*x)^3)/(3*d) - (tanh(c + d*x)*(a + b))/d
```

3.137 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

```
[Out] (a+b)*ln(cosh(d*x+c))/d-1/2*b*tanh(d*x+c)^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3712, 3556}

$$\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
[Out] ((a + b)*Log[Cosh[c + d*x]])/d - (b*Tanh[c + d*x]^2)/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= -\frac{b\tanh^2(c+dx)}{2d}-(-a-b)\int \tanh(c+dx)\,dx\\ &= \frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.32

$$\frac{a\log(\cosh(c+dx))}{d}+\frac{b\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d + (b*Log[Cosh[c + d*x]])/d - (b*Tanh[c + d*x]^2)/(
2*d)
```

Maple [A]
time = 0.37, size = 49, normalized size = 1.58

method	result	size
derivativedivides	$\frac{-\frac{b\left(\tanh^2(dx+c)\right)}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(1+\tanh(dx+c))}{2}}{d}$	49
default	$\frac{-\frac{b\left(\tanh^2(dx+c)\right)}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(1+\tanh(dx+c))}{2}}{d}$	49
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}+\frac{2b\,\mathrm{e}^{2dx+2c}}{d\left(1+\mathrm{e}^{2dx+2c}\right)^2}+\frac{\ln\left(1+\mathrm{e}^{2dx+2c}\right)a}{d}+\frac{\ln\left(1+\mathrm{e}^{2dx+2c}\right)b}{d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b*tanh(d*x+c)^2-1/2*(a+b)*ln(tanh(d*x+c)-1)+1/2*(-a-b)*ln(1+tanh(
d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(29) = 58.
time = 0.51, size = 76, normalized size = 2.45

$$b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a\log\left(\cosh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d
*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*log(cosh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(29) = 58.
time = 0.37, size = 399, normalized size = 12.87

$$-\frac{(a+b)dx\cosh\left(dx+c\right)^4+4\,(a+b)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(a+b)dx\sinh\left(dx+c\right)^4+(a+b)dx+2\,((a+b)dx-b)\cosh\left(dx+c\right)^2+2\,(3\,(a+b)dx\cosh\left(dx+c\right)^2+(a+b)dx-b)\sinh\left(dx+c\right)^2-((a+b)\cosh\left(dx+c\right)^4+4\,(a+b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(a+b)\sinh\left(dx+c\right)^4+2\,(a+b)\cosh\left(dx+c\right)^2+2\,(3\,(a+b)\cosh\left(dx+c\right)^2+a+b)\sinh\left(dx+c\right)^2+4\,((a+b)\cosh\left(dx+c\right)^3+(a+b)\cosh\left(dx+c\right))\sinh\left(dx+c\right)+a+b)\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)+4\,((a+b)dx\cosh\left(dx+c\right)^3+((a+b)dx-b)\cosh\left(dx+c\right))\sinh\left(dx+c\right)}{d\cosh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4+2\,d\cosh\left(dx+c\right)^2+2\,(3\,d\cosh\left(dx+c\right)^2+d)\sinh\left(dx+c\right)^2+4\,(d\cosh\left(dx+c\right)^3+d\cosh\left(dx+c\right))\sinh\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^4 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + b)*d*x*sinh(d*x + c)^4 + (a + b)*d*x + 2*((a + b)*d*x - b)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a + b)*d*x - b)*sinh(d*x + c)
^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (
a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
+ c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + b)*cosh
(d*x + c))*sinh(d*x + c) + a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh
(d*x + c))) + 4*((a + b)*d*x*cosh(d*x + c)^3 + ((a + b)*d*x - b)*cosh(d*x +
 c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3
+ d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sin
h(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.
time = 0.08, size = 60, normalized size = 1.94

$$\begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} + bx - \frac{b\log(\tanh(c+dx)+1)}{d} - \frac{b\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a + b\tanh^2(c))\tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d + b*x - b*log(tanh(c + d*x) + 1
)/d - b*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c), T
rue))
```

Giac [A]
time = 0.43, size = 57, normalized size = 1.84

$$-\frac{(dx+c)(a+b) - (a+b)\log\left(e^{(2\,dx+2\,c)}+1\right) - \frac{2\,be^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*(a + b) - (a + b)*log(e^(2*d*x + 2*c) + 1) - 2*b*e^(2*d*x + 2*c
)/(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B]
time = 1.17, size = 37, normalized size = 1.19

$$x\,(a+b) - \frac{b\tanh(c+d\,x)^2}{2\,d} - \frac{\ln\left(\tanh(c+d\,x)+1\right)\,(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*(a + b) - (b*tanh(c + d*x)^2)/(2*d) - (log(tanh(c + d*x) + 1)*(a + b))/d
```

3.138 $\int \left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=19

$$ax+bx-\frac{b\tanh(c+dx)}{d}$$

```
[Out] a*x+b*x-b*tanh(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3554, 8}

$$ax-\frac{b\tanh(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tanh[c + d*x]^2,x]
```

```
[Out] a*x + b*x - (b*Tanh[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^2(c+dx)\right)\,dx &= ax+b\int \tanh^2(c+dx)\,dx\\ &= ax-\frac{b\tanh(c+dx)}{d}+b\int 1\,dx\\ &= ax+bx-\frac{b\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.47

$$ax+\frac{b\tanh^{-1}(\tanh(c+dx))}{d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tanh[c + d*x]^2,x]
```

```
[Out] a*x + (b*ArcTanh[Tanh[c + d*x]])/d - (b*Tanh[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 0.28, size = 47, normalized size = 2.47

method	result	size
risch	$ax + bx + \frac{2b}{d(1+e^{2dx+2c})}$	27
default	$ax - \frac{b\tanh(dx+c)}{d} - \frac{b\ln(\tanh(dx+c)-1)}{2d} + \frac{b\ln(1+\tanh(dx+c))}{2d}$	47
derivativedivides	$\frac{-b\tanh(dx+c)+\frac{(-a-b)\ln(\tanh(dx+c)-1)}{2}-\frac{(-a-b)\ln(1+\tanh(dx+c))}{2}}{d}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tanh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*x-b*tanh(d*x+c)/d-1/2*b/d*ln(tanh(d*x+c)-1)+1/2*b/d*ln(1+tanh(d*x+c))
```

Maxima [A]
time = 0.29, size = 31, normalized size = 1.63

$$b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)} + 1)}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a*x
```

Fricas [A]
time = 0.35, size = 37, normalized size = 1.95

$$\frac{((a+b)dx + b)\cosh{(dx+c)} - b\sinh{(dx+c)}}{d\cosh{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (((a + b)*d*x + b)*cosh(d*x + c) - b*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [A]
time = 0.06, size = 20, normalized size = 1.05

$$ax + b\left(\begin{cases} x - \frac{\tanh{(c+dx)}}{d} & \text{for}\: d \neq 0 \\ x\tanh^2{(c)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tanh(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((x - tanh(c + d*x)/d, Ne(d, 0)), (x*tanh(c)**2, True))
```

Giac [A]
time = 0.42, size = 29, normalized size = 1.53

$$ax + \frac{\left(dx + c + \frac{2}{e^{(2\,dx+2\,c)}+1}\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] a*x + (d*x + c + 2/(e^(2*d*x + 2*c) + 1))*b/d
```

Mupad [B]
time = 0.07, size = 18, normalized size = 0.95

$$x\,(a+b) - \frac{b\tanh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tanh(c + d*x)^2,x)
```

```
[Out] x*(a + b) - (b*tanh(c + d*x))/d
```

3.139 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=25

$$\frac{b\log(\cosh(c+dx))}{d}+\frac{a\log(\sinh(c+dx))}{d}$$

```
[Out] b*ln(cosh(d*x+c))/d+a*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3706, 3556}

$$\frac{a\log(\sinh(c+dx))}{d}+\frac{b\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b*Log[Cosh[c + d*x]])/d + (a*Log[Sinh[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3706

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/tan[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Dist[A, Int[1/Tan[e + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x],
x] /; FreeQ[{e, f, A, C}, x] && NeQ[A, C]
```

Rubi steps

$$\begin{aligned}\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx&=a\int \coth(c+dx)\,dx+b\int \tanh(c+dx)\,dx\\&=\frac{b\log(\cosh(c+dx))}{d}+\frac{a\log(\sinh(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 1.32

$$\frac{b\log(\cosh(c+dx))}{d}+\frac{a(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (b*Log[Cosh[c + d*x]])/d + (a*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d
```

Maple [A]
time = 1.72, size = 24, normalized size = 0.96

method	result	size
derivativedivides	$\frac{a\ln(\sinh(dx+c))+b\ln(\cosh(dx+c))}{d}$	24
default	$\frac{a\ln(\sinh(dx+c))+b\ln(\cosh(dx+c))}{d}$	24
risch	$-ax-bx-\frac{2bc}{d}-\frac{2ac}{d}+\frac{\ln(1+e^{2dx+2c})b}{d}+\frac{a\ln(e^{2dx+2c}-1)}{d}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*ln(sinh(d*x+c))+b*ln(cosh(d*x+c)))
```

Maxima [A]
time = 0.29, size = 35, normalized size = 1.40

$$\frac{b\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{a\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*log(e^(d*x + c) + e^(-d*x - c))/d + a*log(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(25) = 50$.
time = 0.47, size = 70, normalized size = 2.80

$$-\frac{(a+b)dx-b\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)-a\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x - b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - a*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^{2}\left(c+dx\right)\right)\coth\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x), x)
```

Giac [A]
time = 0.44, size = 46, normalized size = 1.84

$$-\frac{(dx+c)(a+b)-b\log\left(e^{(2\,dx+2\,c)}+1\right)-a\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*(a + b) - b*log(e^(2*d*x + 2*c) + 1) - a*log(abs(e^(2*d*x + 2*c
) - 1)))/d
```

Mupad [B]
time = 1.31, size = 228, normalized size = 9.12

$$\frac{a\ln\left(8\,a\,b-4\,a^2-4\,b^2+4\,a^2\,e^{4\,c}\,e^{4\,d\,x}+4\,b^2\,e^{4\,c}\,e^{4\,d\,x}-8\,a\,b\,e^{4\,c}\,e^{4\,d\,x}\right)}{2\,d}-b\,x-\frac{\mathrm{atan}\left(\frac{a\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2-2\,a\,b+b^2}}-\frac{b\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2-2\,a\,b+b^2}}\right)\,\sqrt{a^2-2\,a\,b+b^2}}{\sqrt{-d^2}}-a\,x+\frac{b\ln\left(8\,a\,b-4\,a^2-4\,b^2+4\,a^2\,e^{4\,c}\,e^{4\,d\,x}+4\,b^2\,e^{4\,c}\,e^{4\,d\,x}-8\,a\,b\,e^{4\,c}\,e^{4\,d\,x}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (a*log(8*a*b - 4*a^2 - 4*b^2 + 4*a^2*exp(4*c)*exp(4*d*x) + 4*b^2*exp(4*c)*e
xp(4*d*x) - 8*a*b*exp(4*c)*exp(4*d*x)))/(2*d) - b*x - (atan((a*exp(2*c)*exp
(2*d*x)*(-d^2)^(1/2))/(d*(a^2 - 2*a*b + b^2)^(1/2)) - (b*exp(2*c)*exp(2*d*x
)*(-d^2)^(1/2))/(d*(a^2 - 2*a*b + b^2)^(1/2)))*(a^2 - 2*a*b + b^2)^(1/2))/(
-d^2)^(1/2) - a*x + (b*log(8*a*b - 4*a^2 - 4*b^2 + 4*a^2*exp(4*c)*exp(4*d*x
) + 4*b^2*exp(4*c)*exp(4*d*x) - 8*a*b*exp(4*c)*exp(4*d*x)))/(2*d)
```

3.140 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=18

$$(a+b)x-\frac{a\coth(c+dx)}{d}$$

```
[Out] (a+b)*x-a*coth(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3710, 8}

$$x(a+b)-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - (a*Coth[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx&=-\frac{a\coth(c+dx)}{d}-\int(-a-b)\,dx\\&=(a+b)x-\frac{a\coth(c+dx)}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 32, normalized size = 1.78

$$bx-\frac{a\coth(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] b*x - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d
```

Maple [A]
time = 1.45, size = 28, normalized size = 1.56

method	result	size
risch	$ax + bx - \frac{2a}{d(e^{2dx+2c}-1)}$	27
derivativedivides	$\frac{a(dx+c-\coth(dx+c))+b(dx+c)}{d}$	28
default	$\frac{a(dx+c-\coth(dx+c))+b(dx+c)}{d}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(d*x+c-coth(d*x+c))+b*(d*x+c))
```

Maxima [A]
time = 0.27, size = 31, normalized size = 1.72

$$a\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2\,dx-2\,c)} - 1)}\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.
time = 0.34, size = 38, normalized size = 2.11

$$-\frac{a\cosh(dx+c) - ((a+b)dx+a)\sinh(dx+c)}{d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*cosh(d*x + c) - ((a + b)*d*x + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.

time = 6.25, size = 100, normalized size = 5.56

$$a\left(\begin{cases} x\coth^2(c) & \text{for } d=0 \\ -\frac{\log(-e^{-dx})\coth^2(dx+\log(-e^{-dx}))}{d} & \text{for } c=\log(-e^{-dx}) \\ -\frac{\log(e^{-dx})\coth^2(dx+\log(e^{-dx}))}{d} & \text{for } c=\log(e^{-dx}) \\ x-\frac{1}{d\tanh(c+dx)} & \text{otherwise} \end{cases}\right)+b\left(\begin{cases} x & \text{for } |x|<1 \\ G_{2,2}^{1,1}\left(\begin{matrix} 1 & 2 \\ 1 & 0 \end{matrix}\middle| x\right)+G_{2,2}^{0,2}\left(\begin{matrix} 2,1 & \\ & 1,0 \end{matrix}\middle| x\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] a*Piecewise((x*coth(c)**2, Eq(d, 0)), (-log(-exp(-d*x))*coth(d*x + log(-exp
(-d*x)))**2/d, Eq(c, log(-exp(-d*x)))), (-log(exp(-d*x))*coth(d*x + log(exp
(-d*x)))**2/d, Eq(c, log(exp(-d*x)))), (x - 1/(d*tanh(c + d*x)), True)) + b
*Piecewise((x, Abs(x) < 1), (meijerg(((1,), (2,)), ((1,), (0,)), x) + meije
rg(((2, 1), ()), ((), (1, 0)), x), True))
```

Giac [A]

time = 0.44, size = 30, normalized size = 1.67

$$\frac{(dx+c)(a+b)-\frac{2a}{e^{(2dx+2c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] ((d*x + c)*(a + b) - 2*a/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B]

time = 1.27, size = 25, normalized size = 1.39

$$x\,(a+b)-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x*(a + b) - (2*a)/(d*(exp(2*c + 2*d*x) - 1))
```

3.141 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$-\frac{a\coth^2(c+dx)}{2d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

```
[Out] -1/2*a*coth(d*x+c)^2/d+(a+b)*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3710, 12, 3556}

$$\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{a\coth^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*(a*Coth[c + d*x]^2)/d + ((a + b)*Log[Sinh[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\coth^2(c+dx)}{2d} + \int (a+b)\coth(c+dx)\,dx$$
$$= -\frac{a\coth^2(c+dx)}{2d} + (a+b)\int \coth(c+dx)\,dx$$
$$= -\frac{a\coth^2(c+dx)}{2d} + \frac{(a+b)\log(\sinh(c+dx))}{d}$$

Mathematica [A]
time = 0.09, size = 39, normalized size = 1.26

$$\frac{-a\coth^2(c+dx) + 2(a+b)(\log(\cosh(c+dx)) + \log(\tanh(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
[Out] (-(a*Coth[c + d*x]^2) + 2*(a + b)*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])
)/(2*d)
```

Maple [A]
time = 1.80, size = 35, normalized size = 1.13

method	result	size
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)+b\ln(\sinh(dx+c))}{d}$	35
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)+b\ln(\sinh(dx+c))}{d}$	35
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}-\frac{2a\,e^{2dx+2c}}{d\left(e^{2dx+2c}-1\right)^2}+\frac{a\ln\left(e^{2dx+2c}-1\right)}{d}+\frac{\ln\left(e^{2dx+2c}-1\right)b}{d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)+b*ln(sinh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(29) = 58.
time = 0.28, size = 106, normalized size = 3.42

$$a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)+\frac{b\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*
x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b*log(e^(d*x +
c) - e^(-d*x - c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(29) = 58$.
time = 0.35, size = 407, normalized size = 13.13

$$-\frac{(a+b)dx\cosh(dx+c)^4+4(a+b)dx\cosh(dx+c)\sinh(dx+c)^3+(a+b)dx\sinh(dx+c)^4+(a+b)dx-2\left((a+b)dx-a\right)\cosh(dx+c)^2+2\left(3(a+b)dx\cosh(dx+c)^2-(a+b)dx+a\right)\sinh(dx+c)^2-\left((a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4-2(a+b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2-a-b\right)\sinh(dx+c)^2+4\left((a+b)\cosh(dx+c)^3-(a+b)\cosh(dx+c)\right)\sinh(dx+c)+a+b\right)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)+4\left((a+b)dx\cosh(dx+c)^3-\left((a+b)dx-a\right)\cosh(dx+c)\right)\sinh(dx+c)}{d\cosh(dx+c)^4+4d\cosh(dx+c)\sinh(dx+c)^3+d\sinh(dx+c)^4-2d\cosh(dx+c)^2+2\left(3d\cosh(dx+c)^2-d\right)\sinh(dx+c)^2+4\left(d\cosh(dx+c)^3-d\cosh(dx+c)\right)\sinh(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^4 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + b)*d*x*sinh(d*x + c)^4 + (a + b)*d*x - 2*((a + b)*d*x - a)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*d*x*cosh(d*x + c)^2 - (a + b)*d*x + a)*sinh(d*x + c)
^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (
a + b)*sinh(d*x + c)^4 - 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
+ c)^2 - a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (a + b)*cosh
(d*x + c))*sinh(d*x + c) + a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh
(d*x + c))) + 4*((a + b)*d*x*cosh(d*x + c)^3 - ((a + b)*d*x - a)*cosh(d*x +
 c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3
+ d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sin
h(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)\coth^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**3, x)
```

Giac [A]
time = 0.45, size = 58, normalized size = 1.87

$$-\frac{(dx+c)(a+b)-(a+b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\,ae^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*(a + b) - (a + b)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*a*e^(2*d*x
+ 2*c)/(e^(2*d*x + 2*c) - 1)^2)/d
```

Mupad [B]
time = 1.26, size = 76, normalized size = 2.45

$$\frac{\ln\left(e^{2c}\,e^{2dx}-1\right)\,(a+b)}{d}-\frac{2a}{d\,\left(e^{2c+2dx}-1\right)}-\frac{2a}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}-x\,(a+b)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(a + b))/d - (2*a)/(d*(exp(2*c + 2*d*x) - 1))
 - (2*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - x*(a + b)
```

3.142 $\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx$

Optimal. Leaf size=36

$$(a+b)x - \frac{(a+b)\coth(c+dx)}{d} - \frac{a\coth^3(c+dx)}{3d}$$

```
[Out] (a+b)*x-(a+b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3710, 12, 3554, 8}

$$-\frac{(a+b)\coth(c+dx)}{d} + x(a+b) - \frac{a\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (a + b)*x - ((a + b)*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= -\frac{a\coth^3(c+dx)}{3d}+\int (a+b)\coth^2(c+dx)\,dx\\ &= -\frac{a\coth^3(c+dx)}{3d}+(a+b)\int \coth^2(c+dx)\,dx\\ &= -\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}+(a+b)\int 1\,dx\\ &= (a+b)x-\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 61, normalized size = 1.69

$$-\frac{a\coth^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(c+dx)\right)}{3d}-\frac{b\coth(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/3*(a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]^2])/
d - (b*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d
```

Maple [A]
time = 1.72, size = 46, normalized size = 1.28

method	result	size
derivativedivides	$\frac{a\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+b(dx+c-\coth(dx+c))}{d}$	46
default	$\frac{a\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+b(dx+c-\coth(dx+c))}{d}$	46
risch	$ax+bx-\frac{2\left(6a\,e^{4dx+4c}+3b\,e^{4dx+4c}-6a\,e^{2dx+2c}-6b\,e^{2dx+2c}+4a+3b\right)}{3d\left(e^{2dx+2c}-1\right)^3}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+b*(d*x+c-coth(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(34) = 68.

time = 0.28, size = 105, normalized size = 2.92

$$\frac{1}{3}\,a\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{\left(-2\,dx-2\,c\right)}-3\,e^{\left(-4\,dx-4\,c\right)}-2\right)}{d(3\,e^{\left(-2\,dx-2\,c\right)}-3\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}-1)}\right)+b\left(x+\frac{c}{d}+\frac{2}{d(e^{\left(-2\,dx-2\,c\right)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(3*
e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b*(x + c/
d + 2/(d*(e^(-2*d*x - 2*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(34) = 68.
time = 0.38, size = 156, normalized size = 4.33

$$-\frac{(4\,a+3\,b)\cosh\left(dx+c\right)^{3}+3\,(4\,a+3\,b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}-\left(3\,(a+b)dx+4\,a+3\,b\right)\sinh\left(dx+c\right)^{3}-3\,b\cosh\left(dx+c\right)+3\,\left(3\,(a+b)dx-\left(3\,(a+b)dx+4\,a+3\,b\right)\cosh\left(dx+c\right)^{2}+4\,a+3\,b\right)\sinh\left(dx+c\right)}{3\,\left(d\sinh\left(dx+c\right)^{3}+3\,\left(d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*((4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x +
c)^2 - (3*(a + b)*d*x + 4*a + 3*b)*sinh(d*x + c)^3 - 3*b*cosh(d*x + c) + 3*
(3*(a + b)*d*x - (3*(a + b)*d*x + 4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b)*s
inh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^{2}\left(c+dx\right)\right)\coth^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(34) = 68.
time = 0.45, size = 86, normalized size = 2.39

$$\frac{3\,(dx+c)(a+b)-\frac{2\,\left(6\,ae^{\left(4\,dx+4\,c\right)}+3\,be^{\left(4\,dx+4\,c\right)}-6\,ae^{\left(2\,dx+2\,c\right)}-6\,be^{\left(2\,dx+2\,c\right)}+4\,a+3\,b\right)}{\left(e^{\left(2\,dx+2\,c\right)}-1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*(a + b) - 2*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 6
*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + 4*a + 3*b)/(e^(2*d*x + 2*c) - 1)
^3)/d
```

Mupad [B]
time = 1.17, size = 162, normalized size = 4.50

$$\frac{\frac{2\,b}{3\,d}-\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a+b\right)}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,\left(2\,a+b\right)}{3\,d}-\frac{4\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{3\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a+b\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}+x\,\left(a+b\right)-\frac{2\,\left(2\,a+b\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] ((2*b)/(3*d) - (2*exp(2*c + 2*d*x)*(2*a + b))/(3*d))/(exp(4*c + 4*d*x) - 2*
exp(2*c + 2*d*x) + 1) - ((2*(2*a + b))/(3*d) - (4*b*exp(2*c + 2*d*x))/(3*d)
 + (2*exp(4*c + 4*d*x)*(2*a + b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) - 1) + x*(a + b) - (2*(2*a + b))/(3*d*(exp(2*c +
2*d*x) - 1))
```

3.143 $\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal. Leaf size=49

$$-\frac{(a+b)\coth^2(c+dx)}{2d}-\frac{a\coth^4(c+dx)}{4d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

```
[Out] -1/2*(a+b)*coth(d*x+c)^2/d-1/4*a*coth(d*x+c)^4/d+(a+b)*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3710, 12, 3554, 3556}

$$-\frac{(a+b)\coth^2(c+dx)}{2d}+\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{a\coth^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((a + b)*Coth[c + d*x]^2)/d - (a*Coth[c + d*x]^4)/(4*d) + ((a + b)*Log
[Sinh[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\coth^4(c+dx)}{4d} + \int (a+b)\coth^3(c+dx)\,dx$$
$$= -\frac{a\coth^4(c+dx)}{4d} + (a+b)\int \coth^3(c+dx)\,dx$$
$$= -\frac{(a+b)\coth^2(c+dx)}{2d} - \frac{a\coth^4(c+dx)}{4d} + (a+b)\int \coth(c+$$
$$= -\frac{(a+b)\coth^2(c+dx)}{2d} - \frac{a\coth^4(c+dx)}{4d} + \frac{(a+b)\log(\sinh(c}{d}$$

Mathematica [A]
time = 0.24, size = 51, normalized size = 1.04

$$-\frac{2(a+b)\coth^2(c+dx)+a\coth^4(c+dx)-4(a+b)(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/4*(2*(a + b)*Coth[c + d*x]^2 + a*Coth[c + d*x]^4 - 4*(a + b)*(Log[Cosh[c
 + d*x]] + Log[Tanh[c + d*x]]))/d
```

Maple [A]
time = 1.71, size = 56, normalized size = 1.14

method	result
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+b\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)}{d}$
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+b\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)}{d}$
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}-\frac{2\,e^{2dx+2c}\left(2a\,e^{4dx+4c}+b\,e^{4dx+4c}-2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+2a+b\right)}{d\left(e^{2dx+2c}-1\right)^4}+\frac{a\ln\left(e^{2dx+2c}-1\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+b*(ln(sinh(d*x
+c))-1/2*coth(d*x+c)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(45) = 90$.
time = 0.29, size = 206, normalized size = 4.20

$$a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}\right)+b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6
*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + b*(x + c
/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)
/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(45) = 90$.
time = 0.37, size = 1216, normalized size = 24.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^7
 + (a + b)*d*x*sinh(d*x + c)^8 - 2*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^
6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 - 2*(a + b)*d*x + 2*a + b)*sinh(d*x +
 c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 - 3*(2*(a + b)*d*x - 2*a - b)*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^4
 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x - 15*(2*(a + b)*d*x -
2*a - b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*d*x*co
sh(d*x + c)^5 - 5*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*(a + b)*d*
x - 2*a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a + b)*d*x - 2*(2*(a + b)*
d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^6 - 15*(2*
(a + b)*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*(a + b)*d*x + 6*(3*(a + b)*d*x -
 2*a - 2*b)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x
+ c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sinh(d*x + c)^8
- 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*
x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*x + c)^4 - 30*(a
 + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x
+ c)^5 - 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x + c
)^3 - 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x + c)^6 - 15*(a + b)
*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 8*(
(a + b)*cosh(d*x + c)^7 - 3*(a + b)*cosh(d*x + c)^5 + 3*(a + b)*cosh(d*x +
```

```
c)^3 - (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*sinh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*d*x*cosh(d*x + c)^7 - 3*(2*(a
 + b)*d*x - 2*a - b)*cosh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d
*x + c)^3 - (2*(a + b)*d*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh
(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*c
osh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(
d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(
35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d
*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9
*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d
*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)\coth^5\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(45) = 90.
time = 0.47, size = 93, normalized size = 1.90

$$-\frac{(dx+c)(a+b) - (a+b)\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right) + \frac{2\left((2\,a+b)e^{(6\,dx+6\,c)} - 2\,(a+b)e^{(4\,dx+4\,c)} + (2\,a+b)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*(a + b) - (a + b)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*((2*a + b)*
e^(6*d*x + 6*c) - 2*(a + b)*e^(4*d*x + 4*c) + (2*a + b)*e^(2*d*x + 2*c))/(e
^(2*d*x + 2*c) - 1)^4)/d
```

Mupad [B]
time = 1.25, size = 177, normalized size = 3.61

$$\frac{\ln\left(e^{2\,c}\,e^{2\,d\,x}-1\right)\,\left(a+b\right)}{d}-\frac{2\,\left(2\,a+b\right)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-x\,\left(a+b\right)-\frac{2\,\left(4\,a+b\right)}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{8\,a}{d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}-\frac{4\,a}{d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(a + b))/d - (2*(2*a + b))/(d*(exp(2*c + 2*d*
x) - 1)) - x*(a + b) - (2*(4*a + b))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d
*x) + 1)) - (8*a)/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6
*d*x) - 1)) - (4*a)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c
 + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.144 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=83

$$(a+b)^2x-\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] (a+b)^2*x-(a+b)^2*tanh(d*x+c)/d-1/3*(a+b)^2*tanh(d*x+c)^3/d-1/5*b*(2*a+b)*t
anh(d*x+c)^5/d-1/7*b^2*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 472, 212}

$$-\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{(a+b)^2\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - ((a + b)^2*Tanh[c + d*x])/d - ((a + b)^2*Tanh[c + d*x]^3)/(3*
d) - (b*(2*a + b)*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-(a+b)^2-(a+b)^2x^2-b(2a+b)x^4-b^2x^6+\frac{a^2+2}{1-}\right.\right.}{d}$$

$$= -\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{b(2a+b)}{}$$

$$= (a+b)^2x-\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(83) = 166.

time = 0.06, size = 190, normalized size = 2.29

$$\frac{a^2\tanh^{-1}(\tanh(c+dx))}{d}+\frac{2ab\tanh^{-1}(\tanh(c+dx))}{d}+\frac{b^2\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a^2\tanh(c+dx)}{d}-\frac{2ab\tanh(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d}-\frac{2ab\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{2ab\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*Ar
cTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d -
 (b^2*Tanh[c + d*x])/d - (a^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d*x]
^3)/(3*d) - (b^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d*x]^5)/(5*d) - (
b^2*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(77) = 154.

time = 0.38, size = 158, normalized size = 1.90

method	result
derivativedivides	$\frac{-\frac{2ab\left(\tanh^3(dx+c)\right)}{3}-\frac{2ab\left(\tanh^5(dx+c)\right)}{5}-2ab\tanh(dx+c)-a^2\tanh(dx+c)+\frac{\left(a^2+2ab+b^2\right)\ln(1+\tanh(dx+c))}{2}-b^2\tanh(dx+c)}{d}$
default	$\frac{-\frac{2ab\left(\tanh^3(dx+c)\right)}{3}-\frac{2ab\left(\tanh^5(dx+c)\right)}{5}-2ab\tanh(dx+c)-a^2\tanh(dx+c)+\frac{\left(a^2+2ab+b^2\right)\ln(1+\tanh(dx+c))}{2}-b^2\tanh(dx+c)}{d}$
risch	$a^2x+2abx+b^2x+\frac{4a^2e^{12dx+12c}+12ab\,e^{12dx+12c}+8b^2e^{12dx+12c}+20a^2e^{10dx+10c}+48ab\,e^{10dx+10c}+24b^2e^{10dx+1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/3*a*b*tanh(d*x+c)^3-2/5*a*b*tanh(d*x+c)^5-2*a*b*tanh(d*x+c)-a^2*tan
h(d*x+c)+1/2*(a^2+2*a*b+b^2)*ln(1+tanh(d*x+c))-b^2*tanh(d*x+c)-1/7*b^2*tanh
```

```
(d*x+c)^7-1/3*a^2*tanh(d*x+c)^3-1/3*b^2*tanh(d*x+c)^3-1/5*b^2*tanh(d*x+c)^5
-1/2*(a^2+2*a*b+b^2)*ln(tanh(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(77) = 154$.
time = 0.30, size = 369, normalized size = 4.45

$$\frac{1}{105}b^2\left(105x+\frac{105c}{d}-\frac{8\left(203e^{(-2dx-2c)}+609e^{(-4dx-4c)}+770e^{(-6dx-6c)}+770e^{(-8dx-8c)}+315e^{(-10dx-10c)}+105e^{(-12dx-12c)}+44\right)}{d(7e^{(-2dx-2c)}+21e^{(-4dx-4c)}+35e^{(-6dx-6c)}+35e^{(-8dx-8c)}+21e^{(-10dx-10c)}+7e^{(-12dx-12c)}+e^{(-14dx-14c)}+1)}\right)+\frac{2}{15}ab\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+\frac{1}{3}a^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c)
 + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 1
05*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-1
2*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/15*a*b*(15*x + 15*c/d - 2*(70
*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8*d
*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 1/3*a^2*(3*x + 3
*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(77) = 154$.
time = 0.36, size = 796, normalized size = 9.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x
 + c)^7 + 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cos
h(d*x + c)*sinh(d*x + c)^6 - 2*(70*a^2 + 161*a*b + 88*b^2)*sinh(d*x + c)^7
+ 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x +
c)^5 - 14*(3*(70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^2 + 40*a^2 + 71*a*b
+ 28*b^2)*sinh(d*x + c)^5 + 35*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 32
2*a*b + 176*b^2)*cosh(d*x + c)^3 + (105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 +
 322*a*b + 176*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*(a^2 + 2*a*b +
 b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 - 14*(5*(70*a^2 +
161*a*b + 88*b^2)*cosh(d*x + c)^4 + 10*(40*a^2 + 71*a*b + 28*b^2)*cosh(d*x
+ c)^2 + 60*a^2 + 123*a*b + 84*b^2)*sinh(d*x + c)^3 + 7*(3*(105*(a^2 + 2*a*
b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^5 + 10*(105*(a^2
+ 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 + 9*(105*
(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c))*sinh(
```

```
d*x + c)^2 + 35*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)
*cosh(d*x + c) - 14*((70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^6 + 5*(40*a^
2 + 71*a*b + 28*b^2)*cosh(d*x + c)^4 + 9*(20*a^2 + 41*a*b + 28*b^2)*cosh(d*
x + c)^2 + 30*a^2 + 75*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*
x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*co
sh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^
5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d
*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(70) = 140$.
time = 0.24, size = 165, normalized size = 1.99

$$\begin{cases} a^2x - \frac{a^2\tanh^3(c+dx)}{3d} - \frac{a^2\tanh(c+dx)}{d} + 2abx - \frac{2ab\tanh^5(c+dx)}{5d} - \frac{2ab\tanh^3(c+dx)}{3d} - \frac{2ab\tanh(c+dx)}{d} + b^2x - \frac{b^2\tanh^7(c+dx)}{7d} - \frac{b^2\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)^2\tanh^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*tanh(c + d*x)**3/(3*d) - a**2*tanh(c + d*x)/d + 2*
a*b*x - 2*a*b*tanh(c + d*x)**5/(5*d) - 2*a*b*tanh(c + d*x)**3/(3*d) - 2*a*b
*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**7/(7*d) - b**2*tanh(c + d*x
)**5/(5*d) - b**2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)),
 (x*(a + b*tanh(c)**2)**2*tanh(c)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(77) = 154$.
time = 0.49, size = 300, normalized size = 3.61

$$\frac{105\left(a^2 + 2ab + b^2\right)(dx + c) + \frac{4\left(105a^2e^{(12dx+12c)} + 315abe^{(12dx+12c)} + 210b^2e^{(12dx+12c)} + 525a^2e^{(10dx+10c)} + 1260abe^{(10dx+10c)} + 630b^2e^{(10dx+10c)} + 1120a^2e^{(8dx+8c)} + 2555abe^{(8dx+8c)} + 1540b^2e^{(8dx+8c)} + 1330a^2e^{(6dx+6c)} + 3080abe^{(6dx+6c)} + 1540b^2e^{(6dx+6c)} + 945a^2e^{(4dx+4c)} + 2121abe^{(4dx+4c)} + 1218b^2e^{(4dx+4c)} + 385a^2e^{(2dx+2c)} + 812abe^{(2dx+2c)} + 406b^2e^{(2dx+2c)} + 70a^2 + 161ab + 88b^2\right)}{\left(e^{(2dx+2c)} + 1\right)^7}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*(a^2 + 2*a*b + b^2)*(d*x + c) + 4*(105*a^2*e^(12*d*x + 12*c) + 3
15*a*b*e^(12*d*x + 12*c) + 210*b^2*e^(12*d*x + 12*c) + 525*a^2*e^(10*d*x +
10*c) + 1260*a*b*e^(10*d*x + 10*c) + 630*b^2*e^(10*d*x + 10*c) + 1120*a^2*e
^(8*d*x + 8*c) + 2555*a*b*e^(8*d*x + 8*c) + 1540*b^2*e^(8*d*x + 8*c) + 1330
*a^2*e^(6*d*x + 6*c) + 3080*a*b*e^(6*d*x + 6*c) + 1540*b^2*e^(6*d*x + 6*c)
+ 945*a^2*e^(4*d*x + 4*c) + 2121*a*b*e^(4*d*x + 4*c) + 1218*b^2*e^(4*d*x +
4*c) + 385*a^2*e^(2*d*x + 2*c) + 812*a*b*e^(2*d*x + 2*c) + 406*b^2*e^(2*d*x
 + 2*c) + 70*a^2 + 161*a*b + 88*b^2)/(e^(2*d*x + 2*c) + 1)^7)/d
```

Mupad [B]
time = 0.18, size = 91, normalized size = 1.10

$$x\left(a^2 + 2\,a\,b + b^2\right) - \frac{\tanh(c + d\,x)\,\left(a + b\right)^2}{d} - \frac{{\tanh(c + d\,x)}^5\,\left(b^2 + 2\,a\,b\right)}{5\,d} - \frac{b^2\,{\tanh(c + d\,x)}^7}{7\,d} - \frac{{\tanh(c + d\,x)}^3\,\left(a^2 + 2\,a\,b + b^2\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)*(a + b)^2)/d - (tanh(c + d*x)^5*(2*a
*b + b^2))/(5*d) - (b^2*tanh(c + d*x)^7)/(7*d) - (tanh(c + d*x)^3*(2*a*b +
a^2 + b^2))/(3*d)
```

3.145 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=76

$$\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{(a+b)^2\tanh^2(c+dx)}{2d}-\frac{b(2a+b)\tanh^4(c+dx)}{4d}-\frac{b^2\tanh^6(c+dx)}{6d}$$

```
[Out] (a+b)^2*ln(cosh(d*x+c))/d-1/2*(a+b)^2*tanh(d*x+c)^2/d-1/4*b*(2*a+b)*tanh(d*
x+c)^4/d-1/6*b^2*tanh(d*x+c)^6/d
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 78}

$$-\frac{b(2a+b)\tanh^4(c+dx)}{4d}-\frac{(a+b)^2\tanh^2(c+dx)}{2d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{b^2\tanh^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d - ((a + b)^2*Tanh[c + d*x]^2)/(2*d) - (b*(
2*a + b)*Tanh[c + d*x]^4)/(4*d) - (b^2*Tanh[c + d*x]^6)/(6*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{x^3\left(a+bx^2\right)^2}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int\frac{x(a+bx)^2}{1-x}\,dx,x,\tanh^2(c+dx)\right)}{2d}$$
$$=\frac{\text{Subst}\left(\int\left(-(a+b)^2-\frac{(a+b)^2}{-1+x}-b(2a+b)x-b^2x^2\right)\,dx,x,\tan\right.}{2d}$$
$$=\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{(a+b)^2\tanh^2(c+dx)}{2d}-\frac{b(2a}{}$$

Mathematica [A]
time = 0.32, size = 66, normalized size = 0.87

$$-\frac{-12(a+b)^2\log(\cosh(c+dx))+6(a+b)^2\tanh^2(c+dx)+3b(2a+b)\tanh^4(c+dx)+2b^2\tanh^6(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] -1/12*(-12*(a + b)^2*Log[Cosh[c + d*x]] + 6*(a + b)^2*Tanh[c + d*x]^2 + 3*b
*(2*a + b)*Tanh[c + d*x]^4 + 2*b^2*Tanh[c + d*x]^6)/d
```

Maple [A]
time = 0.38, size = 130, normalized size = 1.71

method	result
derivativedivides	$\frac{-\frac{ab\left(\tanh^4(dx+c)\right)}{2}-ab\left(\tanh^2(dx+c)\right)-\frac{b^2\left(\tanh^6(dx+c)\right)}{6}-\frac{a^2\left(\tanh^2(dx+c)\right)}{2}-\frac{b^2\left(\tanh^2(dx+c)\right)}{2}+\frac{\left(-a^2-2ab-b^2\right)\ln(1+\tanh}{2}}{d}$
default	$\frac{-\frac{ab\left(\tanh^4(dx+c)\right)}{2}-ab\left(\tanh^2(dx+c)\right)-\frac{b^2\left(\tanh^6(dx+c)\right)}{6}-\frac{a^2\left(\tanh^2(dx+c)\right)}{2}-\frac{b^2\left(\tanh^2(dx+c)\right)}{2}+\frac{\left(-a^2-2ab-b^2\right)\ln(1+\tanh}{2}}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2b^2c}{d}+\frac{2\,e^{2dx+2c}\left(3a^2e^{8dx+8c}+12ab\,e^{8dx+8c}+9b^2e^{8dx+8c}+12a^2e^{6dx}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/2*a*b*tanh(d*x+c)^4-a*b*tanh(d*x+c)^2-1/6*b^2*tanh(d*x+c)^6-1/2*a^2
*tanh(d*x+c)^2-1/2*b^2*tanh(d*x+c)^2+1/2*(-a^2-2*a*b-b^2)*ln(1+tanh(d*x+c))
-1/4*b^2*tanh(d*x+c)^4-1/2*(a^2+2*a*b+b^2)*ln(tanh(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(70) = 140$.
time = 0.50, size = 333, normalized size = 4.38

$$\frac{1}{3}b^2\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}+18e^{(-4dx-4c)}+34e^{(-6dx-6c)}+18e^{(-8dx-8c)}+9e^{(-10dx-10c)}\right)}{d(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(-12dx-12c)}+1)}\right)+2ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)+a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c
) + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(
-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c)
 + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*
x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + a^2*(x + c/d + lo
g(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(
-4*d*x - 4*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3441 vs. $2(70) = 140$.
time = 0.42, size = 3441, normalized size = 45.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^12 + 36*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^12 + 6*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^10
+ 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 3*(a^2 + 2*a*b + b^2)*d*x
 - a^2 - 4*a*b - 3*b^2)*sinh(d*x + c)^10 + 60*(11*(a^2 + 2*a*b + b^2)*d*x*c
osh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^9 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 1
2*b^2)*cosh(d*x + c)^8 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 1
5*(a^2 + 2*a*b + b^2)*d*x + 90*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3
*b^2)*cosh(d*x + c)^2 - 8*a^2 - 24*a*b - 12*b^2)*sinh(d*x + c)^8 + 24*(99*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 30*(3*(a^2 + 2*a*b + b^2)*d*x - a^
2 - 4*a*b - 3*b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 -
24*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(15*(a^2 + 2*a*b + b^2)
*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^6 + 4*(693*(a^2 + 2*a*b + b^2
)*d*x*cosh(d*x + c)^6 + 315*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^
2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x + 21*(15*(a^2 + 2*a*b + b^2
)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^2 - 9*a^2 - 24*a*b - 17*b^2)
```

```
*sinh(d*x + c)^6 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 63*(3*(
a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^5 + 7*(15*(a^2
+ 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^3 + (15*(a^2 +
2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c))*sinh(d*x + c)^5
+ 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^4
+ 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 420*(3*(a^2 + 2*a*b + b^
2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^6 + 70*(15*(a^2 + 2*a*b + b^2)*
d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x
 + 20*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^
2 - 8*a^2 - 24*a*b - 12*b^2)*sinh(d*x + c)^4 + 4*(165*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)^9 + 180*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*
cosh(d*x + c)^7 + 42*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)
*cosh(d*x + c)^5 + 20*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2
)*cosh(d*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2
)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*x + 6*(3*(a^2 +
2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^2 + 6*(33*(a^2 + 2*a*
b + b^2)*d*x*cosh(d*x + c)^10 + 45*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b
 - 3*b^2)*cosh(d*x + c)^8 + 14*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b
 - 12*b^2)*cosh(d*x + c)^6 + 10*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*
b - 17*b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x + 3*(15*(a^2 + 2*a*
b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^2 - a^2 - 4*a*b - 3*b
^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 12*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^2 + 2*a*b + b^2)*sinh(d*x +
 c)^12 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 6*(11*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 + 20*(11*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^9 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 15*(33*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^
2)*sinh(d*x + c)^8 + 24*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 30*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d
*x + c)^7 + 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^6 + 315*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 105*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 10*a*b + 5*b^2)*sinh(d*x + c)^6 + 2
4*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 63*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^5 + 35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(a^2 + 2*a*b + b^2)*co
sh(d*x + c))*sinh(d*x + c)^5 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*
(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^6 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 20*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^9 + 36*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 42*(a^2 + 2*
a*b + b^2)*cosh(d*x + c)^5 + 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + 6*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 45*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^8 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 50*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a
```

```
^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 12*((a^2 + 2*a*b +
b^2)*cosh(d*x + c)^11 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 10*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^7 + 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 5*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh
(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(65) = 130$.
time = 0.22, size = 170, normalized size = 2.24

$$\begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} - \frac{a^2\tanh^2(c+dx)}{2d} + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} - \frac{ab\tanh^4(c+dx)}{2d} - \frac{ab\tanh^2(c+dx)}{d} + b^2x - \frac{b^2\log(\tanh(c+dx)+1)}{d} - \frac{b^2\tanh^6(c+dx)}{6d} - \frac{b^2\tanh^4(c+dx)}{4d} - \frac{b^2\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)^2\tanh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/(
2*d) + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c + d*x)**4/(2*d
) - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d*x) + 1)/d - b**2*
tanh(c + d*x)**6/(6*d) - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*x)**
2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**2*tanh(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(70) = 140$.
time = 0.50, size = 191, normalized size = 2.51

$$-\frac{3\left(a^2+2ab+b^2\right)(dx+c) - 3\left(a^2+2ab+b^2\right)\log\left(e^{(2dx+2c)}+1\right) - \frac{2\left(3\left(a^2+4ab+3b^2\right)e^{(10dx+10c)}+6\left(2a^2+6ab+3b^2\right)e^{(8dx+8c)}+2\left(9a^2+24ab+17b^2\right)e^{(6dx+6c)}+6\left(2a^2+6ab+3b^2\right)e^{(4dx+4c)}+3\left(a^2+4ab+3b^2\right)e^{(2dx+2c)}\right)}{\left(e^{(2dx+2c)}+1\right)^6}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 3*(a^2 + 2*a*b + b^2)*log(e^(2*d*x
+ 2*c) + 1) - 2*(3*(a^2 + 4*a*b + 3*b^2)*e^(10*d*x + 10*c) + 6*(2*a^2 + 6*a
*b + 3*b^2)*e^(8*d*x + 8*c) + 2*(9*a^2 + 24*a*b + 17*b^2)*e^(6*d*x + 6*c) +
 6*(2*a^2 + 6*a*b + 3*b^2)*e^(4*d*x + 4*c) + 3*(a^2 + 4*a*b + 3*b^2)*e^(2*d
*x + 2*c))/(e^(2*d*x + 2*c) + 1)^6)/d
```

Mupad [B]
time = 1.31, size = 100, normalized size = 1.32

$$x\left(a^2+2ab+b^2\right) - \frac{\tanh(c+dx)^4\left(b^2+2ab\right)}{4d} - \frac{\ln\left(\tanh(c+dx)+1\right)\left(a^2+2ab+b^2\right)}{d} - \frac{b^2\tanh(c+dx)^6}{6d} - \frac{\tanh(c+dx)^2\left(a^2+2ab+b^2\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)^4*(2*a*b + b^2))/(4*d) - (log(tanh(c
 + d*x) + 1)*(2*a*b + a^2 + b^2))/d - (b^2*tanh(c + d*x)^6)/(6*d) - (tanh(c
 + d*x)^2*(2*a*b + a^2 + b^2))/(2*d)
```

3.146 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=63

$$(a+b)^2x-\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)^2*x-(a+b)^2*tanh(d*x+c)/d-1/3*b*(2*a+b)*tanh(d*x+c)^3/d-1/5*b^2*tanh(
d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3751, 472, 212}

$$-\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{(a+b)^2\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - ((a + b)^2*Tanh[c + d*x])/d - (b*(2*a + b)*Tanh[c + d*x]^3)/(
3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-(a+b)^2 - b(2a+b)x^2 - b^2x^4 + \frac{a^2+2ab+b^2}{1-x^2}\right)dx, x,$$
$$= -\frac{(a+b)^2\tanh(c+dx)}{d} - \frac{b(2a+b)\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(}{5}$$
$$= (a+b)^2x - \frac{(a+b)^2\tanh(c+dx)}{d} - \frac{b(2a+b)\tanh^3(c+dx)}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(63) = 126.
time = 0.04, size = 137, normalized size = 2.17

$$\frac{a^2\tanh^{-1}(\tanh(c+dx))}{d}+\frac{2ab\tanh^{-1}(\tanh(c+dx))}{d}+\frac{b^2\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a^2\tanh(c+dx)}{d}-\frac{2ab\tanh(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{2ab\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*Ar
cTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d -
 (b^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]
^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(59) = 118.
time = 0.36, size = 120, normalized size = 1.90

method	result
derivativedivides	$\frac{-\frac{2ab\left(\tanh^3(dx+c)\right)}{3}-a^2\tanh(dx+c)-2ab\tanh(dx+c)+\frac{\left(a^2+2ab+b^2\right)\ln(1+\tanh(dx+c))}{2}-\frac{b^2\left(\tanh^5(dx+c)\right)}{5}-\frac{b^2\left(\tanh^3(dx+c)\right)}{3}}{d}$
default	$\frac{-\frac{2ab\left(\tanh^3(dx+c)\right)}{3}-a^2\tanh(dx+c)-2ab\tanh(dx+c)+\frac{\left(a^2+2ab+b^2\right)\ln(1+\tanh(dx+c))}{2}-\frac{b^2\left(\tanh^5(dx+c)\right)}{5}-\frac{b^2\left(\tanh^3(dx+c)\right)}{3}}{d}$
risch	$a^2x+2abx+b^2x+\frac{2a^2e^{8dx+8c}+8ab\,e^{8dx+8c}+6b^2e^{8dx+8c}+8a^2e^{6dx+6c}+24ab\,e^{6dx+6c}+12b^2e^{6dx+6c}+12a^2e^{4dx+}}{d(1+e^{2c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/3*a*b*tanh(d*x+c)^3-a^2*tanh(d*x+c)-2*a*b*tanh(d*x+c)+1/2*(a^2+2*a*
b+b^2)*ln(1+tanh(d*x+c))-1/5*b^2*tanh(d*x+c)^5-1/3*b^2*tanh(d*x+c)^3-b^2*ta
nh(d*x+c)-1/2*(a^2+2*a*b+b^2)*ln(tanh(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(59) = 118$.

time = 0.29, size = 231, normalized size = 3.67

$$\frac{1}{15}\,b^{2}\left(15\,x+\frac{15\,c}{d}-\frac{2\,\left(70\,e^{\left(-2\,dx-2\,c\right)}+140\,e^{\left(-4\,dx-4\,c\right)}+90\,e^{\left(-6\,dx-6\,c\right)}+45\,e^{\left(-8\,dx-8\,c\right)}+23\right)}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}+10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}+5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}+1\right)}\right)+\frac{2}{3}\,ab\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{\left(-2\,dx-2\,c\right)}+3\,e^{\left(-4\,dx-4\,c\right)}+2\right)}{d\left(3\,e^{\left(-2\,dx-2\,c\right)}+3\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}+1\right)}\right)+a^{2}\left(x+\frac{c}{d}-\frac{2}{d\left(e^{\left(-2\,dx-2\,c\right)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) +
 1))) + a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(59) = 118$.

time = 0.39, size = 483, normalized size = 7.67

$$\frac{\left(15\left(a^{2}+2\,ab+b^{2}\right)dx+15\,a^{2}+40\,ab+23\,b^{2}\right)\cosh\left(dx+c\right)^{5}+5\left(15\left(a^{2}+2\,ab+b^{2}\right)dx+15\,a^{2}+40\,ab+23\,b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}-\left(15\,a^{2}+40\,ab+23\,b^{2}\right)\sinh\left(dx+c\right)^{5}+5\left(15\left(a^{2}+2\,ab+b^{2}\right)dx+15\,a^{2}+40\,ab+23\,b^{2}\right)\cosh\left(dx+c\right)^{3}-5\left(2\left(15\,a^{2}+40\,ab+23\,b^{2}\right)\cosh\left(dx+c\right)^{2}+9\,a^{2}+16\,ab+5\,b^{2}\right)\sinh\left(dx+c\right)^{3}+5\left(2\left(15\left(a^{2}+2\,ab+b^{2}\right)dx+15\,a^{2}+40\,ab+23\,b^{2}\right)\cosh\left(dx+c\right)^{3}+3\left(15\left(a^{2}+2\,ab+b^{2}\right)dx+15\,a^{2}+40\,ab+23\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+10\left(15\left(a^{2}+2\,ab+b^{2}\right)dx+15\,a^{2}+40\,ab+23\,b^{2}\right)\cosh\left(dx+c\right)-5\left(\left(15\,a^{2}+40\,ab+23\,b^{2}\right)\cosh\left(dx+c\right)^{4}+3\left(9\,a^{2}+16\,ab+5\,b^{2}\right)\cosh\left(dx+c\right)^{2}+6\,a^{2}+8\,ab+10\,b^{2}\right)\sinh\left(dx+c\right)}{15\left(d\cosh\left(dx+c\right)^{5}+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+5\,d\cosh\left(dx+c\right)^{3}+5\left(2\,d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+10\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)
^5 + 5*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c
)*sinh(d*x + c)^4 - (15*a^2 + 40*a*b + 23*b^2)*sinh(d*x + c)^5 + 5*(15*(a^2
 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^3 - 5*(2*(15*
a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^2 + 9*a^2 + 16*a*b + 5*b^2)*sinh(d*x +
 c)^3 + 5*(2*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d
*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(
d*x + c))*sinh(d*x + c)^2 + 10*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*
b + 23*b^2)*cosh(d*x + c) - 5*((15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^4 +
 3*(9*a^2 + 16*a*b + 5*b^2)*cosh(d*x + c)^2 + 6*a^2 + 8*a*b + 10*b^2)*sinh(
d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh
(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 +
 10*d*cosh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

time = 0.17, size = 117, normalized size = 1.86

$$\begin{cases} a^{2}x - \frac{a^{2}\tanh{\left(c+dx \right)}}{d} + 2abx - \frac{2ab\tanh^{3}{\left(c+dx \right)}}{3d} - \frac{2ab\tanh{\left(c+dx \right)}}{d} + b^{2}x - \frac{b^{2}\tanh^{5}{\left(c+dx \right)}}{5d} - \frac{b^{2}\tanh^{3}{\left(c+dx \right)}}{3d} - \frac{b^{2}\tanh{\left(c+dx \right)}}{d} & \text{for}\: d \neq 0 \\ x\left(a + b\tanh^{2}{\left(c \right)}\right)^{2}\tanh^{2}{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*tanh(c + d*x)/d + 2*a*b*x - 2*a*b*tanh(c + d*x)**3
/(3*d) - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b**
2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(
c)**2)**2*tanh(c)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(59) = 118.
time = 0.48, size = 218, normalized size = 3.46

$$\frac{15\,(a^2+2\,ab+b^2)(dx+c)+\frac{2\,(15\,a^2e^{(8\,dx+8\,c)}+60\,abe^{(8\,dx+8\,c)}+45\,b^2e^{(8\,dx+8\,c)}+60\,a^2e^{(6\,dx+6\,c)}+180\,abe^{(6\,dx+6\,c)}+90\,b^2e^{(6\,dx+6\,c)}+90\,a^2e^{(4\,dx+4\,c)}+220\,abe^{(4\,dx+4\,c)}+140\,b^2e^{(4\,dx+4\,c)}+60\,a^2e^{(2\,dx+2\,c)}+140\,abe^{(2\,dx+2\,c)}+70\,b^2e^{(2\,dx+2\,c)}+15\,a^2+40\,ab+23\,b^2)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*(a^2 + 2*a*b + b^2)*(d*x + c) + 2*(15*a^2*e^(8*d*x + 8*c) + 60*a*b
*e^(8*d*x + 8*c) + 45*b^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 180*a*
b*e^(6*d*x + 6*c) + 90*b^2*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x + 4*c) + 220*a
*b*e^(4*d*x + 4*c) + 140*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x + 2*c) + 140
*a*b*e^(2*d*x + 2*c) + 70*b^2*e^(2*d*x + 2*c) + 15*a^2 + 40*a*b + 23*b^2)/(
e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.30, size = 67, normalized size = 1.06

$$x\,\left(a^2+2\,a\,b+b^2\right)-\frac{\mathrm{tanh}\left(c+d\,x\right)\,{\left(a+b\right)}^2}{d}-\frac{{\mathrm{tanh}\left(c+d\,x\right)}^3\,\left(b^2+2\,a\,b\right)}{3\,d}-\frac{b^2\,{\mathrm{tanh}\left(c+d\,x\right)}^5}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)*(a + b)^2)/d - (tanh(c + d*x)^3*(2*a
*b + b^2))/(3*d) - (b^2*tanh(c + d*x)^5)/(5*d)
```

3.147 $\int \tanh(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=57

$$\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{b(a+b)\tanh^2(c+dx)}{2d}-\frac{\left(a+b\tanh^2(c+dx)\right)^2}{4d}$$

```
[Out] (a+b)^2*ln(cosh(d*x+c))/d-1/2*b*(a+b)*tanh(d*x+c)^2/d-1/4*(a+b*tanh(d*x+c)^
2)^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 455, 45}

$$-\frac{b(a+b)\tanh^2(c+dx)}{2d}-\frac{\left(a+b\tanh^2(c+dx)\right)^2}{4d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d - (b*(a + b)*Tanh[c + d*x]^2)/(2*d) - (a +
 b*Tanh[c + d*x]^2)^2/(4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(a+b)+\frac{(a+b)^2}{1-x}-b(a+bx)\right)\,dx, x, \tanh^2(c+dx\right.}{2d}$$
$$= \frac{(a+b)^2\log(\cosh(c+dx))}{d} - \frac{b(a+b)\tanh^2(c+dx)}{2d} - \frac{(a+b\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.23, size = 50, normalized size = 0.88

$$-\frac{-4(a+b)^2\log(\cosh(c+dx))+2b(2a+b)\tanh^2(c+dx)+b^2\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/4*(-4*(a + b)^2*Log[Cosh[c + d*x]] + 2*b*(2*a + b)*Tanh[c + d*x]^2 + b^2
*Tanh[c + d*x]^4)/d
```

Maple [A]
time = 0.37, size = 92, normalized size = 1.61

method	result
derivativedivides	$\frac{-\frac{b^2\left(\tanh^4(dx+c)\right)}{4}-ab\left(\tanh^2(dx+c)\right)-\frac{b^2\left(\tanh^2(dx+c)\right)}{2}-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(-a^2-2ab-b^2\right)\ln(1+\tanh(dx+c))}{2}}{d}$
default	$\frac{-\frac{b^2\left(\tanh^4(dx+c)\right)}{4}-ab\left(\tanh^2(dx+c)\right)-\frac{b^2\left(\tanh^2(dx+c)\right)}{2}-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(-a^2-2ab-b^2\right)\ln(1+\tanh(dx+c))}{2}}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2b^2c}{d}+\frac{4b\,e^{2dx+2c}\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}+b\,e^{2dx+2c}+a+b\right)}{d\left(1+e^{2dx+2c}\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*b^2*tanh(d*x+c)^4-a*b*tanh(d*x+c)^2-1/2*b^2*tanh(d*x+c)^2-1/2*(a^
2+2*a*b+b^2)*ln(tanh(d*x+c)-1)+1/2*(-a^2-2*a*b-b^2)*ln(1+tanh(d*x+c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(53) = 106$.
time = 0.49, size = 186, normalized size = 3.26

$$b^2\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d} + \frac{4\left(e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)} + 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} + e^{(-8\,dx-8\,c)} + 1)}\right) + 2\,ab\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{a^2\log\left(\cosh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d
*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x -
4*c) + 1))) + a^2*log(cosh(d*x + c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1638 vs. $2(53) = 106$.
time = 0.36, size = 1638, normalized size = 28.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 + 4*((a^
2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2
)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*sinh(d*x + c)^
6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b + b^2)*d
*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b + b^2)*d
*x - 4*a*b - 2*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x + 30*((a^2 + 2*a*b + b^2)*d*x - a*b -
b^2)*cosh(d*x + c)^2 - 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d
*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c))*sinh
(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x + 4*((a^2 + 2*a*b + b^2)*d*x - a*b -
b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((
a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^4 + (a^2 + 2*a*b + b^2)*d
*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c)^2 - a*b -
b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
8 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c
```

```
)^4 + 30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(
d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^6 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 9*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*
((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
5 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x +
c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8
*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b + b^2)*d*x - a*
b - b^2)*cosh(d*x + c)^5 + (3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh
(d*x + c)^3 + ((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x
 + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x
+ c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6
+ 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*
x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x +
 c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(
d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^
7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
 c) + d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(48) = 96$.

time = 0.13, size = 122, normalized size = 2.14

$$\begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} - \frac{ab\tanh^2(c+dx)}{d} + b^2x - \frac{b^2\log(\tanh(c+dx)+1)}{d} - \frac{b^2\tanh^4(c+dx)}{4d} - \frac{b^2\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)^2\tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d + 2*a*b*x - 2*a*b*log(tan
h(c + d*x) + 1)/d - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d*x
) + 1)/d - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*x)**2/(2*d), Ne(d,
 0)), (x*(a + b*tanh(c)**2)**2*tanh(c), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(53) = 106$.

time = 0.45, size = 116, normalized size = 2.04

$$-\frac{(a^2 + 2\,ab + b^2)(dx + c) - (a^2 + 2\,ab + b^2)\log\left(e^{(2\,dx+2\,c)} + 1\right) - \frac{4\left((ab+b^2)e^{(6\,dx+6\,c)} + (2\,ab+b^2)e^{(4\,dx+4\,c)} + (ab+b^2)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -((a^2 + 2*a*b + b^2)*(d*x + c) - (a^2 + 2*a*b + b^2)*log(e^(2*d*x + 2*c) +
 1) - 4*((a*b + b^2)*e^(6*d*x + 6*c) + (2*a*b + b^2)*e^(4*d*x + 4*c) + (a*b
 + b^2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1)^4)/d
```

Mupad [B]
time = 1.21, size = 76, normalized size = 1.33

$$x\left(a^2+2\,a\,b+b^2\right)-\frac{\tanh(c+d\,x)^2\,\left(b^2+2\,a\,b\right)}{2\,d}-\frac{\ln\left(\tanh(c+d\,x)+1\right)\,\left(a^2+2\,a\,b+b^2\right)}{d}-\frac{b^2\tanh(c+d\,x)^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)^2*(2*a*b + b^2))/(2*d) - (log(tanh(c
 + d*x) + 1)*(2*a*b + a^2 + b^2))/d - (b^2*tanh(c + d*x)^4)/(4*d)
```

3.148 $\int (a+b\tanh^2(c+dx))^2\,dx$

Optimal. Leaf size=43

$$(a+b)^2x-\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] (a+b)^2*x-b*(2*a+b)*tanh(d*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 398, 212}

$$-\frac{b(2a+b)\tanh(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - (b*(2*a + b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int (a+b\tanh^2(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(2a+b)-b^2x^2+\frac{(a+b)^2}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{b(2a+b)\tanh(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x,\right.}{d}$$
$$= (a+b)^2x - \frac{b(2a+b)\tanh(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.52, size = 65, normalized size = 1.51

$$\frac{\tanh(c+dx)\left(\frac{3(a+b)^2\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b\left(6a+b\left(3+\tanh^2(c+dx)\right)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (Tanh[c + d*x]*((3*(a + b)^2*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c +
d*x]^2] - b*(6*a + b*(3 + Tanh[c + d*x]^2))))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(41) = 82.
time = 0.30, size = 84, normalized size = 1.95

method	result
derivativedivides	$\frac{-\frac{b^2\left(\tanh^3(dx+c)\right)}{3}-2ab\tanh(dx+c)-b^2\tanh(dx+c)-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^2+2ab+b^2\right)\ln(1+\tanh(dx+c))}{2}}{d}$
default	$\frac{-\frac{b^2\left(\tanh^3(dx+c)\right)}{3}-2ab\tanh(dx+c)-b^2\tanh(dx+c)-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^2+2ab+b^2\right)\ln(1+\tanh(dx+c))}{2}}{d}$
risch	$a^2x+2abx+b^2x+\frac{4b\left(3a\,e^{4dx+4c}+3b\,e^{4dx+4c}+6a\,e^{2dx+2c}+3b\,e^{2dx+2c}+3a+2b\right)}{3d\left(1+e^{2dx+2c}\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*b^2*tanh(d*x+c)^3-2*a*b*tanh(d*x+c)-b^2*tanh(d*x+c)-1/2*(a^2+2*a*
b+b^2)*ln(tanh(d*x+c)-1)+1/2*(a^2+2*a*b+b^2)*ln(1+tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(41) = 82$.

time = 0.27, size = 114, normalized size = 2.65

$$\frac{1}{3}b^2\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right) + 2ab\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a*b*(
x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(41) = 82$.

time = 0.36, size = 201, normalized size = 4.67

$$\frac{(3(a^2+2ab+b^2)dx+6ab+4b^2)\cosh(dx+c)^3 + 3(3(a^2+2ab+b^2)dx+6ab+4b^2)\cosh(dx+c)\sinh(dx+c)^2 - 2(3ab+2b^2)\sinh(dx+c)^3 + 3(3(a^2+2ab+b^2)dx+6ab+4b^2)\cosh(dx+c) - 6((3ab+2b^2)\cosh(dx+c)^2+ab)\sinh(dx+c)}{3(d\cosh(dx+c)^3 + 3d\cosh(dx+c)\sinh(dx+c)^2 + 3d\cosh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*(a^
2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 - 2*(3*
a*b + 2*b^2)*sinh(d*x + c)^3 + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2
)*cosh(d*x + c) - 6*((3*a*b + 2*b^2)*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/
(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [A]

time = 0.11, size = 68, normalized size = 1.58

$$\begin{cases} a^2x + 2abx - \frac{2ab\tanh(c+dx)}{d} + b^2x - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c
+ d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**2
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.

time = 0.42, size = 103, normalized size = 2.40

$$\frac{3(a^2 + 2ab + b^2)(dx + c) + \frac{4\left(3abe^{(4dx+4c)} + 3b^2e^{(4dx+4c)} + 6abe^{(2dx+2c)} + 3b^2e^{(2dx+2c)} + 3ab + 2b^2\right)}{\left(e^{(2dx+2c)} + 1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) + 4*(3*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(
4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 3*a*b + 2*b^
2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B]
time = 1.24, size = 47, normalized size = 1.09

$$x\left(a^2+2\,a\,b+b^2\right)-\frac{b^2\tanh(c+d\,x)^3}{3\,d}-\frac{b\tanh(c+d\,x)\,\left(2\,a+b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (b^2*tanh(c + d*x)^3)/(3*d) - (b*tanh(c + d*x)*(2*a
 + b))/d
```

3.149 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a^2\log(\tanh(c+dx))}{d}-\frac{b^2\tanh^2(c+dx)}{2d}$$

```
[Out] (a+b)^2*ln(cosh(d*x+c))/d+a^2*ln(tanh(d*x+c))/d-1/2*b^2*tanh(d*x+c)^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 457, 84}

$$\frac{a^2\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{b^2\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d + (a^2*Log[Tanh[c + d*x]])/d - (b^2*Tanh[c
 + d*x]^2)/(2*d)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x(1-x^2)} \, dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x} \, dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2 - \frac{(a+b)^2}{-1+x} + \frac{a^2}{x}\right) \, dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{(a+b)^2 \log(\cosh(c+dx))}{d} + \frac{a^2 \log(\tanh(c+dx))}{d} - \frac{b^2 \tanh^2(}{2}$$

Mathematica [A]
time = 0.10, size = 48, normalized size = 0.98

$$\frac{2((a+b)^2 \log(\cosh(c+dx)) + a^2 \log(\tanh(c+dx))) - b^2 \tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (2*((a + b)^2*Log[Cosh[c + d*x]] + a^2*Log[Tanh[c + d*x]]) - b^2*Tanh[c + d
*x]^2)/(2*d)
```

Maple [A]
time = 1.86, size = 50, normalized size = 1.02

method	result
derivativedivides	$\frac{a^2 \ln(\sinh(dx+c)) + 2ab \ln(\cosh(dx+c)) + b^2 \left(\ln(\cosh(dx+c)) - \frac{\left(\tanh^2(dx+c)\right)}{2}\right)}{d}$
default	$\frac{a^2 \ln(\sinh(dx+c)) + 2ab \ln(\cosh(dx+c)) + b^2 \left(\ln(\cosh(dx+c)) - \frac{\left(\tanh^2(dx+c)\right)}{2}\right)}{d}$
risch	$-a^2 x - 2abx - b^2 x - \frac{2a^2 c}{d} - \frac{4abc}{d} - \frac{2b^2 c}{d} + \frac{2b^2 e^{2dx+2c}}{d\left(1+e^{2dx+2c}\right)^2} + \frac{a^2 \ln\left(e^{2dx+2c}-1\right)}{d} + \frac{2\ln\left(1+e^{2dx+2c}\right)a}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*ln(sinh(d*x+c))+2*a*b*ln(cosh(d*x+c))+b^2*(ln(cosh(d*x+c))-1/2*tan
h(d*x+c)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(47) = 94$.
time = 0.51, size = 104, normalized size = 2.12

$$b^2\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx-2\,c)} + 1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{2\,ab\log\left(e^{(dx+c)} + e^{(-dx-c)}\right)}{d} + \frac{a^2\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 2*a*b*log(e^(d*x + c) + e^(-d*x - c
))/d + a^2*log(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(47) = 94$.
time = 0.35, size = 668, normalized size = 13.63

$$-\frac{(a^2+2ab+b^2)dx\cosh(dx+c)^4 + 4(a^2+2ab+b^2)dx\cosh(dx+c)\sinh(dx+c)^3 + (a^2+2ab+b^2)dx\sinh(dx+c)^4 + (a^2+2ab+b^2)dx + 2((a^2+2ab+b^2)dx - b^2)\cosh(dx+c)^2 + 2(3(a^2+2ab+b^2)dx\cosh(dx+c)^2 + (a^2+2ab+b^2)dx - b^2)\sinh(dx+c)^2 - ((2ab+b^2)\cosh(dx+c)^4 + 4(2ab+b^2)\cosh(dx+c)\sinh(dx+c)^3 + (2ab+b^2)\sinh(dx+c)^4 + 2(2ab+b^2)\cosh(dx+c)^2 + 2(3(2ab+b^2)\cosh(dx+c)^2 + 2ab+b^2)\sinh(dx+c)^2 + 2ab + b^2 + 4((2ab+b^2)\cosh(dx+c)^3 + (2ab+b^2)\cosh(dx+c))\sinh(dx+c))\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right) - (a^2\cosh(dx+c)^4 + 4a^2\cosh(dx+c)\sinh(dx+c)^3 + a^2\sinh(dx+c)^4 + 2a^2\cosh(dx+c)^2 + 2(3a^2\cosh(dx+c)^2 + a^2)\sinh(dx+c)^2 + a^2 + 4(a^2\cosh(dx+c)^3 + a^2\cosh(dx+c))\sinh(dx+c))\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right) + 4((a^2+2ab+b^2)dx\cosh(dx+c)^3 + ((a^2+2ab+b^2)dx - b^2)\cosh(dx+c))\sinh(dx+c)}{d\cosh(dx+c)^4 + 4d\cosh(dx+c)\sinh(dx+c)^3 + d\sinh(dx+c)^4 + 2d\cosh(dx+c)^2 + 2(3d\cosh(dx+c)^2 + d)\sinh(dx+c)^2 + 4(d\cosh(dx+c)^3 + d\cosh(dx+c))\sinh(dx+c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2 +
 2*a*b + b^2)*d*x + 2*((a^2 + 2*a*b + b^2)*d*x - b^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - b^2)*
sinh(d*x + c)^2 - ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a*b + b^2)*cos
h(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x +
 c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (2*a*b + b^2)*cosh
(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh
(d*x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2)*sinh(
d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c
))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^3 + ((a^2 + 2*a*b + b^2)*d*x - b^2)*cosh(d*x + c))*
sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*s
inh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^2\left(c + dx\right)\right)^2 \coth\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(47) = 94.
time = 0.46, size = 141, normalized size = 2.88

$$\frac{a^2\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)+(2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-\frac{2\,ab\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+4\,ab-2\,b^2}{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + (2*a*b + b^2)*log(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (2*a*b*(e^(2*d*x + 2*c) + e^(-2*d*x
 - 2*c)) + b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*a*b - 2*b^2)/(e^(2*
d*x + 2*c) + e^(-2*d*x - 2*c) + 2))/d
```

Mupad [B]
time = 1.32, size = 210, normalized size = 4.29

$$\frac{2\,b^2}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-x\,{\left(a+b\right)}^2-\frac{2\,b^2}{d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}+\frac{\ln\left({\mathrm{e}}^{4\,c+4\,d\,x}-1\right)\,\left(d\,\left(b^2+2\,a\,b\right)+a^2\,d\right)}{2\,d^2}+\frac{\mathrm{atan}\left(\frac{{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\left(b^2\,\sqrt{-d^2}-a^2\,\sqrt{-d^2}+2\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4-4\,a^3\,b+2\,a^2\,b^2+4\,a\,b^3+b^4}}\right)\,\sqrt{a^4-4\,a^3\,b+2\,a^2\,b^2+4\,a\,b^3+b^4}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) - x*(a + b)^2 - (2*b^2)/(d*(2*exp(2*c +
2*d*x) + exp(4*c + 4*d*x) + 1)) + (log(exp(4*c + 4*d*x) - 1)*(d*(2*a*b + b^
2) + a^2*d))/(2*d^2) + (atan((exp(2*c)*exp(2*d*x)*(b^2*(-d^2)^(1/2) - a^2*(
-d^2)^(1/2) + 2*a*b*(-d^2)^(1/2)))/(d*(4*a*b^3 - 4*a^3*b + a^4 + b^4 + 2*a^
2*b^2)^(1/2)))*(4*a*b^3 - 4*a^3*b + a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-d^2)^(1
/2)
```

3.150 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=36

$$(a+b)^2x-\frac{a^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] (a+b)^2*x-a^2*coth(d*x+c)/d-b^2*tanh(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3751, 472, 213}

$$-\frac{a^2\coth(c+dx)}{d}+x(a+b)^2-\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - (a^2*Coth[c + d*x])/d - (b^2*Tanh[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^2(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2+\frac{a^2}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{-1+x}\right.}{d}$$
$$= (a+b)^2x-\frac{a^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 64, normalized size = 1.78

$$2abx+\frac{b^2\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a^2\coth(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (b^2*ArcTanh[Tanh[c + d*x]])/d - (a^2*Coth[c + d*x]*Hypergeometri
c2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d - (b^2*Tanh[c + d*x])/d
```

Maple [A]
time = 1.48, size = 49, normalized size = 1.36

method	result	size
derivativedivides	$\frac{a^2(dx+c-\coth(dx+c))+2ab(dx+c)+b^2(dx+c-\tanh(dx+c))}{d}$	49
default	$\frac{a^2(dx+c-\coth(dx+c))+2ab(dx+c)+b^2(dx+c-\tanh(dx+c))}{d}$	49
risch	$a^2x+2abx+b^2x-\frac{2\left(a^2e^{2dx+2c}-b^2e^{2dx+2c}+a^2+b^2\right)}{d\left(e^{2dx+2c}-1\right)\left(1+e^{2dx+2c}\right)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c))+2*a*b*(d*x+c)+b^2*(d*x+c-tanh(d*x+c)))
```

Maxima [A]
time = 0.28, size = 64, normalized size = 1.78

$$b^2\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)+a^2\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right)+2\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*(x + c/d + 2/(d*(e^(-2*d
*x - 2*c) - 1))) + 2*a*b*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(36) = 72.
time = 0.37, size = 97, normalized size = 2.69

$$-\frac{(a^2+b^2)\cosh(dx+c)^2-2((a^2+2ab+b^2)dx+a^2+b^2)\cosh(dx+c)\sinh(dx+c)+(a^2+b^2)\sinh(dx+c)^2+a^2-b^2}{2d\cosh(dx+c)\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + b^2)*cosh(d*x + c)^2 - 2*((a^2 + 2*a*b + b^2)*d*x + a^2 + b^2)
*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2)/(d*
cosh(d*x + c)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^2(c+dx)\right)^2\coth^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**2, x)
```

Giac [A]
time = 0.49, size = 71, normalized size = 1.97

$$\frac{(a^2+2ab+b^2)(dx+c)-\frac{2\left(a^2e^{(2dx+2c)}-b^2e^{(2dx+2c)}+a^2+b^2\right)}{e^{(4dx+4c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] ((a^2 + 2*a*b + b^2)*(d*x + c) - 2*(a^2*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2*
c) + a^2 + b^2)/(e^(4*d*x + 4*c) - 1))/d
```

Mupad [B]
time = 1.24, size = 59, normalized size = 1.64

$$x\,(a+b)^2-\frac{\frac{2\,(a^2+b^2)}{d}+\frac{2\,e^{2\,c+2\,d\,x}\,(a^2-b^2)}{d}}{e^{4\,c+4\,d\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(a + b)^2 - ((2*(a^2 + b^2))/d + (2*exp(2*c + 2*d*x)*(a^2 - b^2))/d)/(exp
(4*c + 4*d*x) - 1)
```

3.151 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=52

$$-\frac{a^2\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a(a+2b)\log(\tanh(c+dx))}{d}$$

```
[Out] -1/2*a^2*coth(d*x+c)^2/d+(a+b)^2*ln(cosh(d*x+c))/d+a*(a+2*b)*ln(tanh(d*x+c)
)/d
```

Rubi [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{a^2\coth^2(c+dx)}{2d}+\frac{a(a+2b)\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(a^2*Coth[c + d*x]^2)/d + ((a + b)^2*Log[Cosh[c + d*x]])/d + (a*(a + 2
*b)*Log[Tanh[c + d*x]])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^3(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{a^2}{x^2}+\frac{a(a+2b)}{x}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= -\frac{a^2\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a(a+2b)}{}$$

Mathematica [A]
time = 0.12, size = 50, normalized size = 0.96

$$\frac{-a^2\coth^2(c+dx)+2(a+b)^2\log(\cosh(c+dx))+2a(a+2b)\log(\tanh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-(a^2*Coth[c + d*x]^2) + 2*(a + b)^2*Log[Cosh[c + d*x]] + 2*a*(a + 2*b)*Lo
g[Tanh[c + d*x]])/(2*d)
```

Maple [A]
time = 1.83, size = 50, normalized size = 0.96

method	result
derivativedivides	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)+2ab\ln(\sinh(dx+c))+b^2\ln(\cosh(dx+c))}{d}$
default	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)+2ab\ln(\sinh(dx+c))+b^2\ln(\cosh(dx+c))}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2b^2c}{d}-\frac{4abc}{d}-\frac{2a^2c}{d}-\frac{2a^2e^{2dx+2c}}{d\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(1+e^{2dx+2c}\right)b^2}{d}+\frac{2a\ln\left(e^{2dx+2c}-1\right)b}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)+2*a*b*ln(sinh(d*x+c))+b^2*ln(c
osh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(50) = 100$.

time = 0.29, size = 134, normalized size = 2.58

$$a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)+\frac{b^2\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{2\,ab\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(d*
x + c) + e^(-d*x - c))/d + 2*a*b*log(e^(d*x + c) - e^(-d*x - c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(50) = 100$.

time = 0.37, size = 677, normalized size = 13.02

$$-\frac{(a^2+2ab+b^2)dx\cosh(dx+c)^4+4(a^2+2ab+b^2)dx\cosh(dx+c)\sinh(dx+c)^3+(a^2+2ab+b^2)dx\sinh(dx+c)^4+(a^2+2ab+b^2)dx-2((a^2+2ab+b^2)dx-a^2)\cosh(dx+c)^2+2(3(a^2+2ab+b^2)dx\cosh(dx+c)^2-(a^2+2ab+b^2)dx+a^2)\sinh(dx+c)^2-(b^2\cosh(dx+c)^4+4b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4-2b^2\cosh(dx+c)^2+2(3b^2\cosh(dx+c)^2-b^2)\sinh(dx+c)^2+b^2+4(b^2\cosh(dx+c)^3-b^2\cosh(dx+c))\sinh(dx+c))\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)-((a^2+2ab)\cosh(dx+c)^4+4(a^2+2ab)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2ab)\sinh(dx+c)^4-2(a^2+2ab)\cosh(dx+c)^2+2(3(a^2+2ab)\cosh(dx+c)^2-a^2-2ab)\sinh(dx+c)^2+a^2+2ab+4((a^2+2ab)\cosh(dx+c)^3-(a^2+2ab)\cosh(dx+c))\sinh(dx+c))\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)+4((a^2+2ab+b^2)dx\cosh(dx+c)^3-((a^2+2ab+b^2)dx-a^2)\cosh(dx+c))\sinh(dx+c)}{d\cosh(dx+c)^4+4d\cosh(dx+c)\sinh(dx+c)^3+d\sinh(dx+c)^4-2d\cosh(dx+c)^2+2(3d\cosh(dx+c)^2-d)\sinh(dx+c)^2+4(d\cosh(dx+c)^3-d\cosh(dx+c))\sinh(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2 +
 2*a*b + b^2)*d*x - 2*((a^2 + 2*a*b + b^2)*d*x - a^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a^2)*
sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2
- b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*
sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2
 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^2 + 2*a*b)*sinh(d*x + c)^4 - 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^
2 + 2*a*b)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4
*((a^2 + 2*a*b)*cosh(d*x + c)^3 - (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c
))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x - a^2)*cosh(d*x + c))*
sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*s
inh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x
 + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^2\left(c+dx\right)\right)^2\coth^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(50) = 100.
time = 0.49, size = 141, normalized size = 2.71

$$\frac{b^2\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+(a^2+2\,ab)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{a^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,ab\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a^2-4\,ab}{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + (a^2 + 2*a*b)*log(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^2*(e^(2*d*x + 2*c) + e^(-2*d*x -
 2*c)) + 2*a*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a^2 - 4*a*b)/(e^(2*
d*x + 2*c) + e^(-2*d*x - 2*c) - 2))/d
```

Mupad [B]
time = 1.41, size = 211, normalized size = 4.06

$$\frac{\ln\left(e^{4\,c+4\,d\,x}-1\right)\,\left(d\,\left(a^2+2\,b\,a\right)+b^2\,d\right)}{2\,d^2}-\frac{2\,a^2}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^2}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-x\,{\left(a+b\right)}^2-\frac{\mathrm{atan}\left(\frac{e^{2\,c}\,e^{2\,d\,x}\,\left(a^2\,\sqrt{-d^2}-b^2\,\sqrt{-d^2}+2\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4+4\,a^3\,b+2\,a^2\,b^2-4\,a\,b^3+b^4}}\right)\,\sqrt{a^4+4\,a^3\,b+2\,a^2\,b^2-4\,a\,b^3+b^4}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (log(exp(4*c + 4*d*x) - 1)*(d*(2*a*b + a^2) + b^2*d))/(2*d^2) - (2*a^2)/(d*
(exp(2*c + 2*d*x) - 1)) - (2*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
 + 1)) - x*(a + b)^2 - (atan((exp(2*c)*exp(2*d*x)*(a^2*(-d^2)^(1/2) - b^2*(
-d^2)^(1/2) + 2*a*b*(-d^2)^(1/2)))/(d*(4*a^3*b - 4*a*b^3 + a^4 + b^4 + 2*a^
2*b^2)^(1/2)))*(4*a^3*b - 4*a*b^3 + a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-d^2)^(1
/2)
```

3.152 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=43

$$(a+b)^2x-\frac{a(a+2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}$$

```
[Out] (a+b)^2*x-a*(a+2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 213}

$$-\frac{a^2\coth^3(c+dx)}{3d}-\frac{a(a+2b)\coth(c+dx)}{d}+x(a+b)^2$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - (a*(a + 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^4(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^4}+\frac{a(a+2b)}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a(a+2b)\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} - \frac{(a+b)^2\text{Subst}}{}$$
$$= (a+b)^2x - \frac{a(a+2b)\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.45, size = 65, normalized size = 1.51

$$-\frac{\coth(c+dx)\left(a\left(3a+6b+a\coth^2(c+dx)\right)-3(a+b)^2\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)\sqrt{\tanh^2(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/3*(Coth[c + d*x]*(a*(3*a + 6*b + a*Coth[c + d*x]^2) - 3*(a + b)^2*ArcTan
h[Sqrt[Tanh[c + d*x]^2]]*Sqrt[Tanh[c + d*x]^2]))/d
```

Maple [A]
time = 1.61, size = 59, normalized size = 1.37

method	result	size
derivativedivides	$\frac{a^2\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+2ab(dx+c-\coth(dx+c))+(dx+c)b^2}{d}$	59
default	$\frac{a^2\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+2ab(dx+c-\coth(dx+c))+(dx+c)b^2}{d}$	59
risch	$a^2x+2abx+b^2x-\frac{4a\left(3a\,e^{4dx+4c}+3b\,e^{4dx+4c}-3a\,e^{2dx+2c}-6b\,e^{2dx+2c}+2a+3b\right)}{3d\left(e^{2dx+2c}-1\right)^3}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+2*a*b*(d*x+c-coth(d*x+c))+(d
*x+c)*b^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(41) = 82$.
time = 0.29, size = 114, normalized size = 2.65

$$\frac{1}{3}\,a^{2}\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+2\,ab\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+b^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 2*a*b*(
x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b^2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(41) = 82$.
time = 0.39, size = 197, normalized size = 4.58

$$-\frac{2\,(2\,a^{2}+3\,ab)\cosh\left(dx+c\right)^{3}+6\,(2\,a^{2}+3\,ab)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}-(3\,(a^{2}+2\,ab+b^{2})dx+4\,a^{2}+6\,ab)\sinh\left(dx+c\right)^{3}-6\,ab\cosh\left(dx+c\right)+3\,(3\,(a^{2}+2\,ab+b^{2})dx-(3\,(a^{2}+2\,ab+b^{2})dx+4\,a^{2}+6\,ab)\cosh\left(dx+c\right)^{2}+4\,a^{2}+6\,ab)\sinh\left(dx+c\right)}{3\,\left(d\sinh\left(dx+c\right)^{3}+3\,\left(d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(2*a^2 + 3*a*b)*cosh(d*x + c)^3 + 6*(2*a^2 + 3*a*b)*cosh(d*x + c)*s
inh(d*x + c)^2 - (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*sinh(d*x + c)^
3 - 6*a*b*cosh(d*x + c) + 3*(3*(a^2 + 2*a*b + b^2)*d*x - (3*(a^2 + 2*a*b +
b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^2 + 4*a^2 + 6*a*b)*sinh(d*x + c))/(
d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^{2}\left(c+dx\right)\right)^{2}\coth^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.
time = 0.50, size = 103, normalized size = 2.40

$$\frac{3\,(a^{2}+2\,ab+b^{2})(dx+c)-\frac{4\,\left(3\,a^{2}e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}-3\,a^{2}e^{(2\,dx+2\,c)}-6\,abe^{(2\,dx+2\,c)}+2\,a^{2}+3\,ab\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 4*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(
4*d*x + 4*c) - 3*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) + 2*a^2 + 3*a*
b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B]
time = 0.17, size = 175, normalized size = 4.07

$$x\,{\left(a+b\right)}^2-\frac{\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2+b\,a\right)}{3\,d}-\frac{4\,a\,b}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{4\,\left(a^2+b\,a\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2+b\,a\right)}{3\,d}-\frac{8\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{4\,\left(a^2+b\,a\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] x*(a + b)^2 - ((4*exp(2*c + 2*d*x)*(a*b + a^2))/(3*d) - (4*a*b)/(3*d))/(exp
(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((4*(a*b + a^2))/(3*d) + (4*exp(4
*c + 4*d*x)*(a*b + a^2))/(3*d) - (8*a*b*exp(2*c + 2*d*x))/(3*d))/(3*exp(2*c
 + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (4*(a*b + a^2))/(3
*d*(exp(2*c + 2*d*x) - 1))
```

3.153 $\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=72

$$-\frac{a(a+2b)\coth^2(c+dx)}{2d}-\frac{a^2\coth^4(c+dx)}{4d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{(a+b)^2\log(\tanh(c+dx))}{d}$$

```
[Out] -1/2*a*(a+2*b)*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d+(a+b)^2*ln(cosh(d*x+
c))/d+(a+b)^2*ln(tanh(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{a^2\coth^4(c+dx)}{4d}-\frac{a(a+2b)\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(a*(a + 2*b)*Coth[c + d*x]^2)/d - (a^2*Coth[c + d*x]^4)/(4*d) + ((a +
b)^2*Log[Cosh[c + d*x]])/d + ((a + b)^2*Log[Tanh[c + d*x]])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^5(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{a^2}{x^3}+\frac{a(a+2b)}{x^2}+\frac{(a+b)^2}{x}\right)\,dx, x, \tanh^2(c+a\right.}{2d}$$
$$= -\frac{a(a+2b)\coth^2(c+dx)}{2d}-\frac{a^2\coth^4(c+dx)}{4d}+\frac{(a+b)^2\log(}{}$$

Mathematica [A]
time = 0.31, size = 58, normalized size = 0.81

$$-\frac{2a(a+2b)\coth^2(c+dx)+a^2\coth^4(c+dx)-4(a+b)^2(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]
[Out] -1/4*(2*a*(a + 2*b)*Coth[c + d*x]^2 + a^2*Coth[c + d*x]^4 - 4*(a + b)^2*(Lo
g[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d
```

Maple [A]
time = 1.72, size = 71, normalized size = 0.99

method	result
derivativedivides	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+2ab\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)+b^2\ln(\sinh(dx+c))}{d}$
default	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+2ab\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)+b^2\ln(\sinh(dx+c))}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2b^2c}{d}-\frac{4a\,e^{2dx+2c}\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}-a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+\right.}{d\left(e^{2dx+2c}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+2*a*b*(ln(si
nh(d*x+c))-1/2*coth(d*x+c)^2)+b^2*ln(sinh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(68) = 136$.
time = 0.28, size = 236, normalized size = 3.28

$$a^{2}\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}\right)+2\,ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)+\frac{b^{2}\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*a*b*
(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
- 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(d*x +
c) - e^(-d*x - c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1649 vs. $2(68) = 136$.
time = 0.37, size = 1649, normalized size = 22.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 - 4*((a^
2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2
)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a^2 + a*b)*sinh(d*x + c)^
6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - 3*((a^2 + 2*a*b + b^2)*d
*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b + b^2)*d
*x - 2*a^2 - 4*a*b)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 30*((a^2 + 2*a*b + b^2)*d*x - a^2 -
a*b)*cosh(d*x + c)^2 - 2*a^2 - 4*a*b)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^5 - 10*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d
*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c))*sinh
(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*d*x - a^2 -
a*b)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 - 15*((
a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^4 - (a^2 + 2*a*b + b^2)*d
*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 +
a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
8 - 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(
```

```
a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^4 - 30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(
d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 10*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3
- 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^6 - 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 9*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*
((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
5 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cosh(d*x +
c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8
*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 - 3*((a^2 + 2*a*b + b^2)*d*x - a^
2 - a*b)*cosh(d*x + c)^5 + (3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh
(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x
 + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x
+ c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6
+ 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*
x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x +
 c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*
sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(
d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^
7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x +
 c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^{2}\left(c + dx\right)\right)^{2}\coth^{5}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**5, x)
```

Giac [**A**]
time = 0.51, size = 118, normalized size = 1.64

$$-\frac{(a^2+2\,ab+b^2)(dx+c)-(a^2+2\,ab+b^2)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{4\,\left((a^2+ab)e^{(6\,dx+6\,c)}-(a^2+2\,ab)e^{(4\,dx+4\,c)}+(a^2+ab)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -((a^2 + 2*a*b + b^2)*(d*x + c) - (a^2 + 2*a*b + b^2)*log(abs(e^(2*d*x + 2*
c) - 1)) + 4*((a^2 + a*b)*e^(6*d*x + 6*c) - (a^2 + 2*a*b)*e^(4*d*x + 4*c) +
 (a^2 + a*b)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^4)/d
```

Mupad [B]
time = 1.27, size = 197, normalized size = 2.74

$$\frac{\ln\left(e^{2c}\,e^{2dx}-1\right)\,\left(a^2+2ab+b^2\right)}{d}-x\,(a+b)^2-\frac{4\,(2\,a^2+b\,a)}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}-\frac{8\,a^2}{d\,\left(3\,e^{2c+2dx}-3\,e^{4c+4dx}+e^{6c+6dx}-1\right)}-\frac{4\,a^2}{d\,\left(6\,e^{4c+4dx}-4\,e^{2c+2dx}-4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}-\frac{4\,(a^2+b\,a)}{d\,\left(e^{2c+2dx}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(2*a*b + a^2 + b^2))/d - x*(a + b)^2 - (4*(a*
b + 2*a^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^2)/(d*(3
*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*a^2)/(
d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c +
 8*d*x) + 1)) - (4*(a*b + a^2))/(d*(exp(2*c + 2*d*x) - 1))
```

3.154 $\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=63

$$(a+b)^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{a(a+2b)\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c+dx)}{5d}$$

```
[Out] (a+b)^2*x-(a+b)^2*coth(d*x+c)/d-1/3*a*(a+2*b)*coth(d*x+c)^3/d-1/5*a^2*coth(
d*x+c)^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3751, 472, 213}

$$-\frac{a^2\coth^5(c+dx)}{5d}-\frac{a(a+2b)\coth^3(c+dx)}{3d}-\frac{(a+b)^2\coth(c+dx)}{d}+x(a+b)^2$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2*x - ((a + b)^2*Coth[c + d*x])/d - (a*(a + 2*b)*Coth[c + d*x]^3)/(
3*d) - (a^2*Coth[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^6(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^6} + \frac{a(a+2b)}{x^4} + \frac{(a+b)^2}{x^2} - \frac{(a+b)^2}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)^2\coth(c+dx)}{d} - \frac{a(a+2b)\coth^3(c+dx)}{3d} - \frac{a^2\coth^5(}{5a}$$

$$= (a+b)^2x - \frac{(a+b)^2\coth(c+dx)}{d} - \frac{a(a+2b)\coth^3(c+dx)}{3d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 98, normalized size = 1.56

$$-\frac{a^2\coth^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\tanh^2(c+dx)\right)}{5d} - \frac{2ab\coth^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(c+dx)\right)}{3d} - \frac{b^2\coth(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/5*(a^2*Coth[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d*x]^2]
)/d - (2*a*b*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]
^2])/(3*d) - (b^2*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*
x]^2])/d
```

Maple [A]
time = 1.84, size = 87, normalized size = 1.38

method	result
derivativedivides	$\frac{a^2\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}-\frac{\left(\coth^5(dx+c)\right)}{5}\right)+2ab\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+b^2(dx+c-\coth(dx}{d}$
default	$\frac{a^2\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}-\frac{\left(\coth^5(dx+c)\right)}{5}\right)+2ab\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+b^2(dx+c-\coth(dx}{d}$
risch	$a^2x+2abx+b^2x-\frac{2(45a^2e^{8dx+8c}+60ab\,e^{8dx+8c}+15b^2e^{8dx+8c}-90a^2e^{6dx+6c}-180ab\,e^{6dx+6c}-60b^2e^{6dx+6c}+14}{15}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3-1/5*coth(d*x+c)^5)+2*a*b*(d*x
+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+b^2*(d*x+c-coth(d*x+c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(59) = 118$.
time = 0.28, size = 231, normalized size = 3.67

$$\frac{1}{15}a^2\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}-140e^{(-4dx-4c)}+90e^{(-6dx-6c)}-45e^{(-8dx-8c)}-23\right)}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}\right)+\frac{2}{3}ab\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+b^2\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2dx-2c)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*a^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x -
 4*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) -
 1))) + b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(59) = 118$.
time = 0.38, size = 473, normalized size = 7.51

$$-\frac{\left(23a^2+40ab+15b^2\right)\cosh\left(dx+c\right)^5+5\left(23a^2+40ab+15b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-\left(15\left(a^2+2ab+b^2\right)dx+23a^2+40ab+15b^2\right)\sinh\left(dx+c\right)^5-5\left(5a^2+16ab+9b^2\right)\cosh\left(dx+c\right)^3+5\left(15\left(a^2+2ab+b^2\right)dx-2\left(15\left(a^2+2ab+b^2\right)dx+23a^2+40ab+15b^2\right)\cosh\left(dx+c\right)^2+23a^2+40ab+15b^2\right)\sinh\left(dx+c\right)^3+5\left(2\left(23a^2+40ab+15b^2\right)\cosh\left(dx+c\right)^3-3\left(5a^2+16ab+9b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2+10\left(5a^2+4ab+3b^2\right)\cosh\left(dx+c\right)-5\left(\left(15\left(a^2+2ab+b^2\right)dx+23a^2+40ab+15b^2\right)\cosh\left(dx+c\right)^4+30\left(a^2+2ab+b^2\right)dx-3\left(15\left(a^2+2ab+b^2\right)dx+23a^2+40ab+15b^2\right)\cosh\left(dx+c\right)^2+46a^2+80ab+30b^2\right)\sinh\left(dx+c\right)}{15\left(d\sinh\left(dx+c\right)^5+5\left(2d\cosh\left(dx+c\right)^2-d\right)\sinh\left(dx+c\right)^3+5\left(d\cosh\left(dx+c\right)^4-3d\cosh\left(dx+c\right)^2+2d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 40*a*b + 15
*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2
+ 40*a*b + 15*b^2)*sinh(d*x + c)^5 - 5*(5*a^2 + 16*a*b + 9*b^2)*cosh(d*x +
c)^3 + 5*(15*(a^2 + 2*a*b + b^2)*d*x - 2*(15*(a^2 + 2*a*b + b^2)*d*x + 23*a
^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 23*a^2 + 40*a*b + 15*b^2)*sinh(d*x
+ c)^3 + 5*(2*(23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*
b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 + 4*a*b + 3*b^2)*cosh
(d*x + c) - 5*((15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2 + 40*a*b + 15*b^2)*cosh
(d*x + c)^4 + 30*(a^2 + 2*a*b + b^2)*d*x - 3*(15*(a^2 + 2*a*b + b^2)*d*x +
23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 46*a^2 + 80*a*b + 30*b^2)*sinh(
d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3
+ 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(59) = 118.
time = 0.52, size = 218, normalized size = 3.46

$$\frac{15\,(a^2+2\,ab+b^2)(dx+c)-\frac{2\,(45\,a^2e^{(8\,dx+8\,c)}+60\,abe^{(8\,dx+8\,c)}+15\,b^2e^{(8\,dx+8\,c)}-90\,a^2e^{(6\,dx+6\,c)}-180\,abe^{(6\,dx+6\,c)}-60\,b^2e^{(6\,dx+6\,c)}+140\,a^2e^{(4\,dx+4\,c)}+220\,abe^{(4\,dx+4\,c)}+90\,b^2e^{(4\,dx+4\,c)}-70\,a^2e^{(2\,dx+2\,c)}-140\,abe^{(2\,dx+2\,c)}-60\,b^2e^{(2\,dx+2\,c)}+23\,a^2+40\,ab+15\,b^2)}{(e^{(2\,dx+2\,c)}-1)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*(a^2 + 2*a*b + b^2)*(d*x + c) - 2*(45*a^2*e^(8*d*x + 8*c) + 60*a*b
*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x + 8*c) - 90*a^2*e^(6*d*x + 6*c) - 180*a*
b*e^(6*d*x + 6*c) - 60*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4*c) + 220*
a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x + 2*c) - 140
*a*b*e^(2*d*x + 2*c) - 60*b^2*e^(2*d*x + 2*c) + 23*a^2 + 40*a*b + 15*b^2)/(
e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [B]
time = 0.20, size = 529, normalized size = 8.40

$$\frac{\frac{2\,(b^2+2\,a\,b)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1}+\frac{\frac{2\,(b^2+2\,a\,b)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(b^2+2\,a\,b)}{5\,d}-\frac{2\,e^{6\,c+6\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(5\,a^2+4\,a\,b+3\,b^2)}{5\,d}}{6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}+x\,(a+b)^2-\frac{\frac{2\,(3\,a^2+4\,a\,b+b^2)}{5\,d}-\frac{8\,e^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{5\,d}-\frac{8\,e^{6\,c+6\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{2\,e^{8\,c+8\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(5\,a^2+4\,a\,b+3\,b^2)}{5\,d}}{5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1}-\frac{\frac{2\,(5\,a^2+4\,a\,b+3\,b^2)}{15\,d}-\frac{4\,e^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1}-\frac{2\,(3\,a^2+4\,a\,b+b^2)}{5\,d\,(e^{2\,c+2\,d\,x}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] ((2*(2*a*b + b^2))/(5*d) - (2*exp(2*c + 2*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d)
)/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) + ((2*(2*a*b + b^2))/(5*d) +
(6*exp(4*c + 4*d*x)*(2*a*b + b^2))/(5*d) - (2*exp(6*c + 6*d*x)*(4*a*b + 3*a
^2 + b^2))/(5*d) - (2*exp(2*c + 2*d*x)*(4*a*b + 5*a^2 + 3*b^2))/(5*d))/(6*e
xp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x
) + 1) + x*(a + b)^2 - ((2*(4*a*b + 3*a^2 + b^2))/(5*d) - (8*exp(2*c + 2*d*
x)*(2*a*b + b^2))/(5*d) - (8*exp(6*c + 6*d*x)*(2*a*b + b^2))/(5*d) + (2*exp
(8*c + 8*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d) + (4*exp(4*c + 4*d*x)*(4*a*b + 5
*a^2 + 3*b^2))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*
c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(4*a*b + 5*
a^2 + 3*b^2))/(15*d) - (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(5*d) + (2*exp(4*
c + 4*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) - 1) - (2*(4*a*b + 3*a^2 + b^2))/(5*d*(exp(2*c +
2*d*x) - 1))
```

3.155 $\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=92

$$-\frac{(a+b)^2\coth^2(c+dx)}{2d}-\frac{a(a+2b)\coth^4(c+dx)}{4d}-\frac{a^2\coth^6(c+dx)}{6d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{(a+b)^2}{}$$

```
[Out] -1/2*(a+b)^2*coth(d*x+c)^2/d-1/4*a*(a+2*b)*coth(d*x+c)^4/d-1/6*a^2*coth(d*x
+c)^6/d+(a+b)^2*ln(cosh(d*x+c))/d+(a+b)^2*ln(tanh(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{a^2\coth^6(c+dx)}{6d}-\frac{a(a+2b)\coth^4(c+dx)}{4d}-\frac{(a+b)^2\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\tanh(c+dx))}{d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a + b)^2*Coth[c + d*x]^2)/d - (a*(a + 2*b)*Coth[c + d*x]^4)/(4*d) -
(a^2*Coth[c + d*x]^6)/(6*d) + ((a + b)^2*Log[Cosh[c + d*x]])/d + ((a + b)^2
*Log[Tanh[c + d*x]])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^7(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^4}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{a^2}{x^4}+\frac{a(a+2b)}{x^3}+\frac{(a+b)^2}{x^2}+\frac{(a+b)^2}{x}\right)dx, x, \tanh\right.}{2d}$$

$$= -\frac{(a+b)^2\coth^2(c+dx)}{2d}-\frac{a(a+2b)\coth^4(c+dx)}{4d}-\frac{a^2\coth^6}{6}$$

Mathematica [A]
time = 0.35, size = 74, normalized size = 0.80

$$-\frac{6(a+b)^2\coth^2(c+dx)+3a(a+2b)\coth^4(c+dx)+2a^2\coth^6(c+dx)-12(a+b)^2(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/12*(6*(a + b)^2*Coth[c + d*x]^2 + 3*a*(a + 2*b)*Coth[c + d*x]^4 + 2*a^2*
Coth[c + d*x]^6 - 12*(a + b)^2*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d
```

Maple [A]
time = 1.81, size = 102, normalized size = 1.11

method	result
derivativedivides	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}-\frac{\left(\coth^6(dx+c)\right)}{6}\right)+2ab\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+\right.}{4}\right.}{d}$
default	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}-\frac{\left(\coth^6(dx+c)\right)}{6}\right)+2ab\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+\right.}{4}\right.}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2b^2c}{d}-\frac{2\,e^{2dx+2c}\left(9a^2e^{8dx+8c}+12ab\,e^{8dx+8c}+3b^2e^{8dx+8c}-18a^2e^{6dx+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4-1/6*coth(d*x+
c)^6)+2*a*b*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+b^2*(ln(s
inh(d*x+c))-1/2*coth(d*x+c)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(86) = 172$.
time = 0.28, size = 390, normalized size = 4.24

$$\frac{1}{3}a^2\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}-18e^{(-4dx-4c)}+34e^{(-6dx-6c)}-18e^{(-8dx-8c)}+9e^{(-10dx-10c)}\right)}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1)}\right)+2ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1)}\right)+b^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 2*a*b*(x + c/d + log(e^(-d*x - c) + 1)/d +
 log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d
*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c)
 - e^(-8*d*x - 8*c) - 1))) + b^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e
^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x
- 4*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3454 vs. $2(86) = 172$.
time = 0.39, size = 3454, normalized size = 37.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^12 + 36*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^12 - 6*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^10
+ 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x
 + 3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^10 + 60*(11*(a^2 + 2*a*b + b^2)*d*x*c
osh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x
 + c))*sinh(d*x + c)^9 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b -
8*b^2)*cosh(d*x + c)^8 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 1
5*(a^2 + 2*a*b + b^2)*d*x - 90*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b -
 b^2)*cosh(d*x + c)^2 - 12*a^2 - 24*a*b - 8*b^2)*sinh(d*x + c)^8 + 24*(99*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 30*(3*(a^2 + 2*a*b + b^2)*d*x - 3*
a^2 - 4*a*b - b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 -
 24*a*b - 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*(a^2 + 2*a*b + b^2)
*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^6 + 4*(693*(a^2 + 2*a*b + b^2
)*d*x*cosh(d*x + c)^6 - 315*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^
2)*cosh(d*x + c)^4 - 15*(a^2 + 2*a*b + b^2)*d*x + 21*(15*(a^2 + 2*a*b + b^2
```

)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^2 + 17*a^2 + 24*a*b + 9*b^2)
*sinh(d*x + c)^6 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 - 63*(3*(
a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^5 + 7*(15*(a^2
+ 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^3 - (15*(a^2 +
2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^5
+ 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^4
+ 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 - 420*(3*(a^2 + 2*a*b + b^
2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^6 + 70*(15*(a^2 + 2*a*b + b^2)*
d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x
 - 20*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^
2 - 12*a^2 - 24*a*b - 8*b^2)*sinh(d*x + c)^4 + 4*(165*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)^9 - 180*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*
cosh(d*x + c)^7 + 42*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)
*cosh(d*x + c)^5 - 20*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2
)*cosh(d*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2
)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*x - 6*(3*(a^2 +
2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^2 + 6*(33*(a^2 + 2*a*
b + b^2)*d*x*cosh(d*x + c)^10 - 45*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a
*b - b^2)*cosh(d*x + c)^8 + 14*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*
b - 8*b^2)*cosh(d*x + c)^6 - 10*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a
*b - 9*b^2)*cosh(d*x + c)^4 - 3*(a^2 + 2*a*b + b^2)*d*x + 3*(15*(a^2 + 2*a*
b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 4*a*b + b
^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 12*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^2 + 2*a*b + b^2)*sinh(d*x +
 c)^12 - 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 6*(11*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^10 + 20*(11*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^9 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 15*(33*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 - 18*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^
2)*sinh(d*x + c)^8 + 24*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 30*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d
x + c)^7 - 20(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^6 - 315*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 105*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^2 - 5*a^2 - 10*a*b - 5*b^2)*sinh(d*x + c)^6 + 2
4*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 63*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^5 + 35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 5*(a^2 + 2*a*b + b^2)*co
sh(d*x + c))*sinh(d*x + c)^5 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*
(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 84*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^6 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 20*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^9 - 36*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 42*(a^2 + 2*
a*b + b^2)*cosh(d*x + c)^5 - 20*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + 6*(11*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 - 45*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^8 + 70*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 50*(a^2

```
+ 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a
^2 - 2*a*b - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 12*((a^2 + 2*a*b +
b^2)*cosh(d*x + c)^11 - 5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 10*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^7 - 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 5*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh
(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(86) = 172.
time = 0.56, size = 192, normalized size = 2.09

$$-\frac{3\,(a^2+2\,ab+b^2)(dx+c)-3\,(a^2+2\,ab+b^2)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\,(3\,(3\,a^2+4\,ab+b^2)e^{(10\,dx+10\,c)}-6\,(3\,a^2+6\,ab+2\,b^2)e^{(8\,dx+8\,c)}+2\,(17\,a^2+24\,ab+9\,b^2)e^{(6\,dx+6\,c)}-6\,(3\,a^2+6\,ab+2\,b^2)e^{(4\,dx+4\,c)}+3\,(3\,a^2+4\,ab+b^2)e^{(2\,dx+2\,c)})}{\left(e^{(2\,dx+2\,c)}-1\right)^6}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 3*(a^2 + 2*a*b + b^2)*log(abs(e^(2*
d*x + 2*c) - 1)) + 2*(3*(3*a^2 + 4*a*b + b^2)*e^(10*d*x + 10*c) - 6*(3*a^2
+ 6*a*b + 2*b^2)*e^(8*d*x + 8*c) + 2*(17*a^2 + 24*a*b + 9*b^2)*e^(6*d*x + 6
*c) - 6*(3*a^2 + 6*a*b + 2*b^2)*e^(4*d*x + 4*c) + 3*(3*a^2 + 4*a*b + b^2)*e
^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^6)/d
```

Mupad [**B**]
time = 0.27, size = 362, normalized size = 3.93

$$\frac{\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}-1\right)\,\left(a^2+2\,a\,b+b^2\right)}{d}-\frac{2\,\left(3\,a^2+4\,a\,b+b^2\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{32\,a^2}{3\,d\,\left(15\,{\mathrm{e}}^{4\,c+4\,d\,x}-6\,{\mathrm{e}}^{2\,c+2\,d\,x}-20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}-6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+12\,d\,x}+1\right)}-x\,{\left(a+b\right)}^2-\frac{2\,\left(9\,a^2+8\,a\,b+b^2\right)}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,\left(13\,a^2+6\,b\,a\right)}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{4\,\left(11\,a^2+2\,b\,a\right)}{d\,\left(6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}-\frac{32\,a^2}{d\,\left(5\,{\mathrm{e}}^{2\,c+2\,d\,x}-10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}-5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(2*a*b + a^2 + b^2))/d - (2*(4*a*b + 3*a^2 +
b^2))/(d*(exp(2*c + 2*d*x) - 1)) - (32*a^2)/(3*d*(15*exp(4*c + 4*d*x) - 6*e
xp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c +
10*d*x) + exp(12*c + 12*d*x) + 1)) - x*(a + b)^2 - (2*(8*a*b + 9*a^2 + b^2)
)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*(6*a*b + 13*a^2))/(3
*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(
2*a*b + 11*a^2))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c +
6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*a^2)/(d*(5*exp(2*c + 2*d*x) - 10*exp(
4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x
) - 1))
```

3.156 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=114

$$(a+b)^3x-\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{(a+b)^3\tanh^3(c+dx)}{3d}-\frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d}-\frac{b^2(3a+b)\tanh^7}{7d}$$

```
[Out] (a+b)^3*x-(a+b)^3*tanh(d*x+c)/d-1/3*(a+b)^3*tanh(d*x+c)^3/d-1/5*b*(3*a^2+3*
a*b+b^2)*tanh(d*x+c)^5/d-1/7*b^2*(3*a+b)*tanh(d*x+c)^7/d-1/9*b^3*tanh(d*x+c
)^9/d
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 472, 212}

$$-\frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d}-\frac{b^2(3a+b)\tanh^7(c+dx)}{7d}-\frac{(a+b)^3\tanh^3(c+dx)}{3d}-\frac{(a+b)^3\tanh(c+dx)}{d}+x(a+b)^3-\frac{b^3\tanh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - ((a + b)^3*Tanh[c + d*x])/d - ((a + b)^3*Tanh[c + d*x]^3)/(3*
d) - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + b)*Tanh[
c + d*x]^7)/(7*d) - (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-(a+b)^3-(a+b)^3x^2-b\left(3a^2+3ab+b^2\right)x^4-b\right.\right.}{d}$$

$$= -\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{(a+b)^3\tanh^3(c+dx)}{3d}-\frac{b(3a^2+}{}$$

$$= (a+b)^3x-\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{(a+b)^3\tanh^3(c+dx)}{3d}$$

Mathematica [A]
time = 1.06, size = 123, normalized size = 1.08

$$\frac{\tanh(c+dx)\left(-315(a+b)^3-105(a+b)^3\tanh^2(c+dx)-63b\left(3a^2+3ab+b^2\right)\tanh^4(c+dx)-45b^2(3a+b)\tanh^6(c+dx)-35b^3\tanh^8(c+dx)+\frac{315(a+b)^3\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}\right)}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Tanh[c + d*x]*(-315*(a + b)^3 - 105*(a + b)^3*Tanh[c + d*x]^2 - 63*b*(3*a^
2 + 3*a*b + b^2)*Tanh[c + d*x]^4 - 45*b^2*(3*a + b)*Tanh[c + d*x]^6 - 35*b^
3*Tanh[c + d*x]^8 + (315*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tan
h[c + d*x]^2]))/(315*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(106) = 212.
time = 0.40, size = 247, normalized size = 2.17

method	result
derivativedivides	$\frac{\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln(1+\tanh(dx+c))}{2}-3a^2b\tanh(dx+c)-3a\,b^2\tanh(dx+c)-a^2b\left(\tanh^3(dx+c)\right)-a\,b^2\left(\tanh^3(dx+c)\right)-}{}$
default	$\frac{\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln(1+\tanh(dx+c))}{2}-3a^2b\tanh(dx+c)-3a\,b^2\tanh(dx+c)-a^2b\left(\tanh^3(dx+c)\right)-a\,b^2\left(\tanh^3(dx+c)\right)-}{}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x+\frac{\frac{352a\,b^2}{35}+120a\,b^2e^{14dx+14c}+308a^2b\,e^{12dx+12c}+344a\,b^2e^{12dx+12c}+540a^2b\,e^{10}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*(a^3+3*a^2*b+3*a*b^2+b^3)*ln(1+tanh(d*x+c))-3*a^2*b*tanh(d*x+c)-3*
a*b^2*tanh(d*x+c)-a^2*b*tanh(d*x+c)^3-a*b^2*tanh(d*x+c)^3-3/5*a*b^2*tanh(d*
x+c)^5-3/5*a^2*b*tanh(d*x+c)^5-3/7*a*b^2*tanh(d*x+c)^7-a^3*tanh(d*x+c)-b^3*
tanh(d*x+c)-1/9*b^3*tanh(d*x+c)^9-1/2*(a^3+3*a^2*b+3*a*b^2+b^3)*ln(tanh(d*x
+c)-1)-1/3*a^3*tanh(d*x+c)^3-1/3*b^3*tanh(d*x+c)^3-1/5*b^3*tanh(d*x+c)^5-1/
7*b^3*tanh(d*x+c)^7)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(106) = 212$.
time = 0.29, size = 583, normalized size = 5.11

$$\frac{1}{315}\,b^{3}\left(315\,x+\frac{315\,c}{d}-\frac{2\,\left(3492\,e^{\left(-2\,dx-2\,c\right)}+13968\,e^{\left(-4\,dx-4\,c\right)}+26292\,e^{\left(-6\,dx-6\,c\right)}+39438\,e^{\left(-8\,dx-8\,c\right)}+31500\,e^{\left(-10\,dx-10\,c\right)}+21000\,e^{\left(-12\,dx-12\,c\right)}+6300\,e^{\left(-14\,dx-14\,c\right)}+1575\,e^{\left(-16\,dx-16\,c\right)}+563\right)}{d(9\,e^{\left(-2\,dx-2\,c\right)}+36\,e^{\left(-4\,dx-4\,c\right)}+84\,e^{\left(-6\,dx-6\,c\right)}+126\,e^{\left(-8\,dx-8\,c\right)}+126\,e^{\left(-10\,dx-10\,c\right)}+84\,e^{\left(-12\,dx-12\,c\right)}+36\,e^{\left(-14\,dx-14\,c\right)}+9\,e^{\left(-16\,dx-16\,c\right)}+e^{\left(-18\,dx-18\,c\right)}+1)}\right)+\frac{1}{35}\,ab^{2}\left(105\,x+\frac{105\,c}{d}-\frac{8\,\left(203\,e^{\left(-2\,dx-2\,c\right)}+609\,e^{\left(-4\,dx-4\,c\right)}+770\,e^{\left(-6\,dx-6\,c\right)}+770\,e^{\left(-8\,dx-8\,c\right)}+315\,e^{\left(-10\,dx-10\,c\right)}+105\,e^{\left(-12\,dx-12\,c\right)}+44\right)}{d(7\,e^{\left(-2\,dx-2\,c\right)}+21\,e^{\left(-4\,dx-4\,c\right)}+35\,e^{\left(-6\,dx-6\,c\right)}+35\,e^{\left(-8\,dx-8\,c\right)}+21\,e^{\left(-10\,dx-10\,c\right)}+7\,e^{\left(-12\,dx-12\,c\right)}+e^{\left(-14\,dx-14\,c\right)}+1)}\right)+\frac{1}{5}\,a^{2}b\left(15\,x+\frac{15\,c}{d}-\frac{2\,\left(70\,e^{\left(-2\,dx-2\,c\right)}+140\,e^{\left(-4\,dx-4\,c\right)}+90\,e^{\left(-6\,dx-6\,c\right)}+45\,e^{\left(-8\,dx-8\,c\right)}+23\right)}{d(5\,e^{\left(-2\,dx-2\,c\right)}+10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}+5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}+1)}\right)+\frac{1}{3}\,a^{3}\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{\left(-2\,dx-2\,c\right)}+3\,e^{\left(-4\,dx-4\,c\right)}+2\right)}{d(3\,e^{\left(-2\,dx-2\,c\right)}+3\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/315*b^3*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x - 4
*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x -
10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16*d*
x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*
x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x -
12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) +
 1))) + 1/35*a*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d
*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x -
10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
- 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
+ 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a^2*b*(15*x + 15*c
/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) +
45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10
*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 1/3*a^
3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-
2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1563 vs. $2(106) = 212$.
time = 0.35, size = 1563, normalized size = 13.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/315*((420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 + 9*(420*a^3 + 1449*a^2*b + 1584*a*b^2
+ 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x
 + c)^8 - (420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*sinh(d*x + c)^9 + 9
```

```
*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^7 - 9*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^
3 + 4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^7 + 21*(4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 3*(420*a^3 + 1449*a^2*b +
1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^6 + 36*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 9*(14*(420*a^3 + 14
49*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^4 + 700*a^3 + 2016*a^2*b + 2
136*a*b^2 + 852*b^3 + 21*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563
*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 35*(420*a
^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*x)*cosh(d*x + c)^3 + 20*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 +
315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^4 + 8
4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 - 3*(28*(420*a^3 + 1449*a^2*b + 1584*a*b^2 +
 563*b^3)*cosh(d*x + c)^6 + 105*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3)
*cosh(d*x + c)^4 + 2660*a^3 + 8232*a^2*b + 8232*a*b^2 + 1764*b^3 + 120*(175
*a^3 + 504*a^2*b + 534*a*b^2 + 213*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
9*(4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 21*(420*a^3 + 1449*a^2*b + 1584*a*b^2 +
 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 40*(4
20*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x)*cosh(d*x + c)^3 + 28*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^
3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2
 + 126*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 9*((420*a^3 + 1449*a^2*b + 1584*a*b^2 +
 563*b^3)*cosh(d*x + c)^8 + 7*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3)*c
osh(d*x + c)^6 + 20*(175*a^3 + 504*a^2*b + 534*a*b^2 + 213*b^3)*cosh(d*x +
c)^4 + 420*a^3 + 1386*a^2*b + 1176*a*b^2 + 882*b^3 + 28*(95*a^3 + 294*a^2*b
 + 294*a*b^2 + 63*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^9 +
 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + 9*d*cosh(d*x + c)^7 + 21*(4*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^6 + 36*d*cosh(d*x + c)^5 + 9*(14
*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 20*d*cosh(d*x + c))*sinh(d*x +
c)^4 + 84*d*cosh(d*x + c)^3 + 9*(4*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5
 + 40*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh(d*x + c)^2 + 126*d*cosh(
d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(99) = 198$.
time = 0.37, size = 260, normalized size = 2.28

$$\begin{cases} a^3x - \frac{a^3\tanh^3(c+dx)}{3d} - \frac{a^3\tanh(c+dx)}{d} + 3a^2bx - \frac{3a^2b\tanh^5(c+dx)}{5d} - \frac{a^2b\tanh^3(c+dx)}{d} - \frac{3a^2b\tanh(c+dx)}{d} + 3ab^2x - \frac{3ab^2\tanh^7(c+dx)}{7d} - \frac{3ab^2\tanh^5(c+dx)}{5d} - \frac{ab^2\tanh^3(c+dx)}{d} - \frac{3ab^2\tanh(c+dx)}{d} + b^3x - \frac{b^3\tanh^9(c+dx)}{9d} - \frac{b^3\tanh^7(c+dx)}{7d} - \frac{b^3\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^3(c+dx)}{3d} - \frac{b^3\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)^3\tanh^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*tanh(c + d*x)**3/(3*d) - a**3*tanh(c + d*x)/d + 3*
a**2*b*x - 3*a**2*b*tanh(c + d*x)**5/(5*d) - a**2*b*tanh(c + d*x)**3/d - 3*
a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**7/(7*d) - 3*a
*b**2*tanh(c + d*x)**5/(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c
+ d*x)/d + b**3*x - b**3*tanh(c + d*x)**9/(9*d) - b**3*tanh(c + d*x)**7/(7*
d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(
c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(106) = 212$.
time = 0.60, size = 534, normalized size = 4.68

$$\frac{315\,(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c)+\frac{2\,(630\,a^3e^{(16\,dx+16\,c)}+2835\,a^2be^{(16\,dx+16\,c)}+3780\,ab^2e^{(16\,dx+16\,c)}+1575\,b^3e^{(16\,dx+16\,c)}+4410\,a^3e^{(14\,dx+14\,c)}+17010\,a^2be^{(14\,dx+14\,c)}+18900\,ab^2e^{(14\,dx+14\,c)}+6300\,b^3e^{(14\,dx+14\,c)}+13650\,a^3e^{(12\,dx+12\,c)}+48510\,a^2be^{(12\,dx+12\,c)}+54180\,ab^2e^{(12\,dx+12\,c)}+21000\,b^3e^{(12\,dx+12\,c)}+24570\,a^3e^{(10\,dx+10\,c)}+85050\,a^2be^{(10\,dx+10\,c)}+94500\,ab^2e^{(10\,dx+10\,c)}+31500\,b^3e^{(10\,dx+10\,c)}+28350\,a^3e^{(8\,dx+8\,c)}+97524\,a^2be^{(8\,dx+8\,c)}+105084\,ab^2e^{(8\,dx+8\,c)}+39438\,b^3e^{(8\,dx+8\,c)}+21630\,a^3e^{(6\,dx+6\,c)}+73206\,a^2be^{(6\,dx+6\,c)}+78876\,ab^2e^{(6\,dx+6\,c)}+26292\,b^3e^{(6\,dx+6\,c)}+10710\,a^3e^{(4\,dx+4\,c)}+35154\,a^2be^{(4\,dx+4\,c)}+38124\,ab^2e^{(4\,dx+4\,c)}+13968\,b^3e^{(4\,dx+4\,c)}+3150\,a^3e^{(2\,dx+2\,c)}+10206\,a^2be^{(2\,dx+2\,c)}+10476\,ab^2e^{(2\,dx+2\,c)}+3492\,b^3e^{(2\,dx+2\,c)}+420\,a^3+1449\,a^2b+1584\,ab^2+563\,b^3)}{(e^{(2\,dx+2\,c)}+1)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/315*(315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(630*a^3*e^(16*d*x
 + 16*c) + 2835*a^2*b*e^(16*d*x + 16*c) + 3780*a*b^2*e^(16*d*x + 16*c) + 15
75*b^3*e^(16*d*x + 16*c) + 4410*a^3*e^(14*d*x + 14*c) + 17010*a^2*b*e^(14*d
*x + 14*c) + 18900*a*b^2*e^(14*d*x + 14*c) + 6300*b^3*e^(14*d*x + 14*c) + 1
3650*a^3*e^(12*d*x + 12*c) + 48510*a^2*b*e^(12*d*x + 12*c) + 54180*a*b^2*e^
(12*d*x + 12*c) + 21000*b^3*e^(12*d*x + 12*c) + 24570*a^3*e^(10*d*x + 10*c)
 + 85050*a^2*b*e^(10*d*x + 10*c) + 94500*a*b^2*e^(10*d*x + 10*c) + 31500*b^
3*e^(10*d*x + 10*c) + 28350*a^3*e^(8*d*x + 8*c) + 97524*a^2*b*e^(8*d*x + 8*
c) + 105084*a*b^2*e^(8*d*x + 8*c) + 39438*b^3*e^(8*d*x + 8*c) + 21630*a^3*e
^(6*d*x + 6*c) + 73206*a^2*b*e^(6*d*x + 6*c) + 78876*a*b^2*e^(6*d*x + 6*c)
+ 26292*b^3*e^(6*d*x + 6*c) + 10710*a^3*e^(4*d*x + 4*c) + 35154*a^2*b*e^(4*
d*x + 4*c) + 38124*a*b^2*e^(4*d*x + 4*c) + 13968*b^3*e^(4*d*x + 4*c) + 3150
*a^3*e^(2*d*x + 2*c) + 10206*a^2*b*e^(2*d*x + 2*c) + 10476*a*b^2*e^(2*d*x +
 2*c) + 3492*b^3*e^(2*d*x + 2*c) + 420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*
b^3)/(e^(2*d*x + 2*c) + 1)^9)/d
```

Mupad [B]
time = 0.26, size = 138, normalized size = 1.21

$$x\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)-\frac{\tanh(c+d\,x)\,(a+b)^3}{d}-\frac{\tanh(c+d\,x)^5\,(3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}-\frac{\tanh(c+d\,x)^7\,(b^3+3\,a\,b^2)}{7\,d}-\frac{b^3\tanh(c+d\,x)^9}{9\,d}-\frac{\tanh(c+d\,x)^3\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)*(a + b)^3)/d - (tanh(c +
 d*x)^5*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) - (tanh(c + d*x)^7*(3*a*b^2 + b^3)
)/(7*d) - (b^3*tanh(c + d*x)^9)/(9*d) - (tanh(c + d*x)^3*(3*a*b^2 + 3*a^2*b
 + a^3 + b^3))/(3*d)
```

3.157 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=107

$$\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{(a+b)^3\tanh^2(c+dx)}{2d}-\frac{b(3a^2+3ab+b^2)\tanh^4(c+dx)}{4d}-\frac{b^2(3a+b)\tanh^6(c+$$

```
[Out] (a+b)^3*ln(cosh(d*x+c))/d-1/2*(a+b)^3*tanh(d*x+c)^2/d-1/4*b*(3*a^2+3*a*b+b^
2)*tanh(d*x+c)^4/d-1/6*b^2*(3*a+b)*tanh(d*x+c)^6/d-1/8*b^3*tanh(d*x+c)^8/d
```

Rubi [A]
time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 78}

$$-\frac{b(3a^2+3ab+b^2)\tanh^4(c+dx)}{4d}-\frac{b^2(3a+b)\tanh^6(c+dx)}{6d}-\frac{(a+b)^3\tanh^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b^3\tanh^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d - ((a + b)^3*Tanh[c + d*x]^2)/(2*d) - (b*(
3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^4)/(4*d) - (b^2*(3*a + b)*Tanh[c + d*x]^
6)/(6*d) - (b^3*Tanh[c + d*x]^8)/(8*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a+bx)^3}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-(a+b)^3 - \frac{(a+b)^3}{-1+x} - b(3a^2+3ab+b^2)\,x - b^2(3a+\right.\right.}{2d}$$
$$= \frac{(a+b)^3\log(\cosh(c+dx))}{d} - \frac{(a+b)^3\tanh^2(c+dx)}{2d} - \frac{b(3a^2+}{}$$

Mathematica [A]
time = 0.22, size = 98, normalized size = 0.92

$$\frac{2(a+b)^3\log(\cosh(c+dx)) - (a+b)^3\tanh^2(c+dx) - \frac{1}{2}b(3a^2+3ab+b^2)\tanh^4(c+dx) - \frac{1}{3}b^2(3a+b)\tanh^6(c+dx) - \frac{1}{4}b^3\tanh^8(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (2*(a + b)^3*Log[Cosh[c + d*x]] - (a + b)^3*Tanh[c + d*x]^2 - (b*(3*a^2 + 3
*a*b + b^2)*Tanh[c + d*x]^4)/2 - (b^2*(3*a + b)*Tanh[c + d*x]^6)/3 - (b^3*T
anh[c + d*x]^8)/4)/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(99) = 198.
time = 0.40, size = 205, normalized size = 1.92

method	result
derivativedivides	$\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\ln(1+\tanh(dx+c))}{2} - \frac{3a^2b\left(\tanh^2(dx+c)\right)}{2} - \frac{3a\,b^2\left(\tanh^2(dx+c)\right)}{2} - \frac{3a^2b\left(\tanh^4(dx+c)\right)}{4} - \frac{3a\,b^2\left(\tanh^4(dx+c)\right.}{4}$
default	$\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\ln(1+\tanh(dx+c))}{2} - \frac{3a^2b\left(\tanh^2(dx+c)\right)}{2} - \frac{3a\,b^2\left(\tanh^2(dx+c)\right)}{2} - \frac{3a^2b\left(\tanh^4(dx+c)\right)}{4} - \frac{3a\,b^2\left(\tanh^4(dx+c)\right.}{4}$
risch	$-a^3x - 3a^2bx - 3a\,b^2x - b^3x - \frac{2a^3c}{d} - \frac{6a^2bc}{d} - \frac{6a\,b^2c}{d} - \frac{2b^3c}{d} + \frac{2\,\mathrm{e}^{2dx+2c}\left(27a\,b^2+18a^2b\,\mathrm{e}^{12dx+12c}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*(-a^3-3*a^2*b-3*a*b^2-b^3)*ln(1+tanh(d*x+c))-3/2*a^2*b*tanh(d*x+c)
^2-3/2*a*b^2*tanh(d*x+c)^2-3/4*a^2*b*tanh(d*x+c)^4-3/4*a*b^2*tanh(d*x+c)^4-
1/2*a*b^2*tanh(d*x+c)^6-1/8*b^3*tanh(d*x+c)^8-1/2*(a^3+3*a^2*b+3*a*b^2+b^3)
*ln(tanh(d*x+c)-1)-1/2*a^3*tanh(d*x+c)^2-1/2*b^3*tanh(d*x+c)^2-1/4*b^3*tanh
(d*x+c)^4-1/6*b^3*tanh(d*x+c)^6)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(99) = 198$.
time = 0.53, size = 540, normalized size = 5.05

$$ab^2\left(3\,x+\frac{3\,c}{d}+\frac{3\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,\left(9\,e^{(-2\,dx-2\,c)}+18\,e^{(-4\,dx-4\,c)}+34\,e^{(-6\,dx-6\,c)}+18\,e^{(-8\,dx-8\,c)}+9\,e^{(-10\,dx-10\,c)}\right)}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)+\frac{1}{3}\,b^3\left(3\,x+\frac{3\,c}{d}+\frac{3\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{8\,\left(3\,e^{(-2\,dx-2\,c)}+9\,e^{(-4\,dx-4\,c)}+25\,e^{(-6\,dx-6\,c)}+26\,e^{(-8\,dx-8\,c)}+25\,e^{(-10\,dx-10\,c)}+9\,e^{(-12\,dx-12\,c)}+3\,e^{(-14\,dx-14\,c)}\right)}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56\,e^{(-10\,dx-10\,c)}+28\,e^{(-12\,dx-12\,c)}+8\,e^{(-14\,dx-14\,c)}+e^{(-16\,dx-16\,c)}+1)}\right)+3\,a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\,\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)+a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a*b^2*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c)
+ 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(-1
0*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x
- 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) +
1))) + 1/3*b^3*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 8*(3*e^(-2*d*
x - 2*c) + 9*e^(-4*d*x - 4*c) + 25*e^(-6*d*x - 6*c) + 26*e^(-8*d*x - 8*c) +
 25*e^(-10*d*x - 10*c) + 9*e^(-12*d*x - 12*c) + 3*e^(-14*d*x - 14*c))/(d*(8
*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*
x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 1
4*c) + e^(-16*d*x - 16*c) + 1))) + 3*a^2*b*(x + c/d + log(e^(-2*d*x - 2*c)
+ 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e
^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c
) + 1))) + a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/
(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7502 vs. $2(99) = 198$.
time = 0.43, size = 7502, normalized size = 70.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^16 + 48*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^15 + 3*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^16 - 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^
3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^14 + 6*(60*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 - a^3 - 6*a^2*b - 9*a*b^2 - 4*
b^3 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^14 + 84*(20*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 - (a^3 + 6*a^2*b + 9*a*b^2
```

+ 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x +
c)^13 - 12*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^12 + 6*(910*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x
*cosh(d*x + c)^4 - 6*a^3 - 30*a^2*b - 36*a*b^2 - 12*b^3 + 14*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x - 91*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 24*(546*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 91*(a^3 + 6*a^2*b + 9*a*b^
2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 6*(3*a
^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*c
osh(d*x + c))*sinh(d*x + c)^11 - 2*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^
3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 2*(12012*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 3003*(a^3 + 6*a^2*b + 9*
a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 45
*a^3 - 198*a^2*b - 237*a*b^2 - 100*b^3 + 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x - 396*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(8580*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 - 3003*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3
- 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 660*(3*a^3 + 15*
a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x
+ c)^3 - 5*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 2*(60*a^3 + 252*a^2*b
+ 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
 c)^8 + 2*(19305*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 - 9009
*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*
cosh(d*x + c)^6 - 2970*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 60*a^3 - 252*a^2*b - 312*a*b^2
- 104*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 45*(45*a^3 + 198*a^2*
b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 + 16*(2145*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh
(d*x + c)^9 - 1287*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 594*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^
3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 15*(45*a^3 + 1
98*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*co
sh(d*x + c)^3 - (60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(45*a^3 + 198*
a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c)^6 + 2*(12012*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^10
- 9009*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^8 - 5544*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 210*(45*a^3 + 198*a^2*b +
 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
)^4 - 45*a^3 - 198*a^2*b - 237*a*b^2 - 100*b^3 + 84*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x - 28*(60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(3276*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^11 - 3003*(a^3 + 6*a^2*b + 9*

```
a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 - 23
76*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^7 - 126*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 28*(60*a^3 + 252*a^2
*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*
x + c)^3 - 3*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 12*(3*a^3 + 15*a^2*b
 + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^
4 + 2*(2730*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 - 3003*(a^
3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh
(d*x + c)^10 - 2970*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 - 210*(45*a^3 + 198*a^2*b + 237*a*b^
2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 70*
(60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*x)*cosh(d*x + c)^4 - 18*a^3 - 90*a^2*b -...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(94) = 188$.
time = 0.33, size = 279, normalized size = 2.61

$$\begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} - \frac{a^3\tanh^2(c+dx)}{2d} + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\tanh^4(c+dx)}{4d} - \frac{3a^2b\tanh^2(c+dx)}{2d} + 3ab^2x - \frac{3ab^2\log(\tanh(c+dx)+1)}{d} - \frac{ab^2\tanh^6(c+dx)}{2d} - \frac{3ab^2\tanh^4(c+dx)}{4d} - \frac{3ab^2\tanh^2(c+dx)}{2d} + b^3x - \frac{b^3\log(\tanh(c+dx)+1)}{d} - \frac{b^3\tanh^8(c+dx)}{8d} - \frac{b^3\tanh^6(c+dx)}{6d} - \frac{b^3\tanh^4(c+dx)}{4d} - \frac{b^3\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)^3\tanh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)**2/(
2*d) + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c + d
*x)**4/(4*d) - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x - 3*a*b**2*log(
tanh(c + d*x) + 1)/d - a*b**2*tanh(c + d*x)**6/(2*d) - 3*a*b**2*tanh(c + d*
x)**4/(4*d) - 3*a*b**2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh(c +
d*x) + 1)/d - b**3*tanh(c + d*x)**8/(8*d) - b**3*tanh(c + d*x)**6/(6*d) - b
**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a
+ b*tanh(c)**2)**3*tanh(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(99) = 198$.
time = 0.56, size = 309, normalized size = 2.89

$$-\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c) - 3\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(e^{(2dx+2c)}+1\right) - \frac{2\left(3\left(a^3+6a^2b+9ab^2+4b^3\right)e^{(14dx+14c)} + 18\left(a^3+5a^2b+6ab^2+2b^3\right)e^{(12dx+12c)} + \left(45a^3+198a^2b+237ab^2+100b^3\right)e^{(10dx+10c)} + 4\left(15a^3+63a^2b+78ab^2+26b^3\right)e^{(8dx+8c)} + \left(45a^3+198a^2b+237ab^2+100b^3\right)e^{(6dx+6c)} + 18\left(a^3+5a^2b+6ab^2+2b^3\right)e^{(4dx+4c)} + 3\left(a^3+6a^2b+9ab^2+4b^3\right)e^{(2dx+2c)}\right)}{\left(e^{(2dx+2c)}+1\right)^8}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*log(e^(2*d*x + 2*c) + 1) - 2*(3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3
)*e^(14*d*x + 14*c) + 18*(a^3 + 5*a^2*b + 6*a*b^2 + 2*b^3)*e^(12*d*x + 12*c
```

```
) + (45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3)*e^(10*d*x + 10*c) + 4*(15*a^
3 + 63*a^2*b + 78*a*b^2 + 26*b^3)*e^(8*d*x + 8*c) + (45*a^3 + 198*a^2*b + 2
37*a*b^2 + 100*b^3)*e^(6*d*x + 6*c) + 18*(a^3 + 5*a^2*b + 6*a*b^2 + 2*b^3)*
e^(4*d*x + 4*c) + 3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*e^(2*d*x + 2*c))/(e^(
2*d*x + 2*c) + 1)^8)/d
```

Mupad [B]
time = 1.24, size = 155, normalized size = 1.45

$$x\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)-\frac{{\mathrm{tanh}\left(c+d\,x\right)}^4\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{4\,d}-\frac{\ln\left(\mathrm{tanh}\left(c+d\,x\right)+1\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d}-\frac{{\mathrm{tanh}\left(c+d\,x\right)}^6\,\left(b^3+3\,a\,b^2\right)}{6\,d}-\frac{b^3\,{\mathrm{tanh}\left(c+d\,x\right)}^8}{8\,d}-\frac{{\mathrm{tanh}\left(c+d\,x\right)}^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^4*(3*a*b^2 + 3*a^2*b + b
^3))/(4*d) - (log(tanh(c + d*x) + 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d - (
tanh(c + d*x)^6*(3*a*b^2 + b^3))/(6*d) - (b^3*tanh(c + d*x)^8)/(8*d) - (tan
h(c + d*x)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(2*d)
```

3.158 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=94

$$(a+b)^3x-\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+b)\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^7(c+$$

```
[Out] (a+b)^3*x-(a+b)^3*tanh(d*x+c)/d-1/3*b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)^3/d-1/5
*b^2*(3*a+b)*tanh(d*x+c)^5/d-1/7*b^3*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 472, 212}

$$-\frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^3\tanh(c+dx)}{d}+x(a+b)^3-\frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - ((a + b)^3*Tanh[c + d*x])/d - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c
 + d*x]^3)/(3*d) - (b^2*(3*a + b)*Tanh[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*
x]^7)/(7*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-(a+b)^3 - b(3a^2+3ab+b^2)\,x^2 - b^2(3a+b)x^4 - \right.\right.}{d}$$

$$= -\frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d} -$$

$$= (a+b)^3x - \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b(3a^2+3ab+b^2)\tanh^3(c}{3d}$$

Mathematica [A]
time = 1.19, size = 108, normalized size = 1.15

$$\frac{\tanh(c+dx)\left(-105(a+b)^3 - 35b(3a^2+3ab+b^2)\tanh^2(c+dx) - 21b^2(3a+b)\tanh^4(c+dx) - 15b^3\tanh^6(c+dx) + \frac{105(a+b)^3\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Tanh[c + d*x]*(-105*(a + b)^3 - 35*b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^2
 - 21*b^2*(3*a + b)*Tanh[c + d*x]^4 - 15*b^3*Tanh[c + d*x]^6 + (105*(a + b)
^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c + d*x]^2]))/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(88) = 176.
time = 0.40, size = 193, normalized size = 2.05

method	result
derivativedivides	$\frac{-3a\,b^2\tanh(dx+c)-3a^2b\tanh(dx+c)-a^2b\left(\tanh^3(dx+c)\right)-a\,b^2\left(\tanh^3(dx+c)\right)-\frac{3a\,b^2\left(\tanh^5(dx+c)\right)}{5}+\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right.}{2}}{}$
default	$\frac{-3a\,b^2\tanh(dx+c)-3a^2b\tanh(dx+c)-a^2b\left(\tanh^3(dx+c)\right)-a\,b^2\left(\tanh^3(dx+c)\right)-\frac{3a\,b^2\left(\tanh^5(dx+c)\right)}{5}+\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right.}{2}}{}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x+\frac{\frac{46a\,b^2}{5}+12a^2b\,\mathrm{e}^{12dx+12c}+18a\,b^2\mathrm{e}^{12dx+12c}+60a^2b\,\mathrm{e}^{10dx+10c}+72a\,b^2\mathrm{e}^{10dx+10c}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-3*a*b^2*tanh(d*x+c)-3*a^2*b*tanh(d*x+c)-a^2*b*tanh(d*x+c)^3-a*b^2*tan
h(d*x+c)^3-3/5*a*b^2*tanh(d*x+c)^5+1/2*(a^3+3*a^2*b+3*a*b^2+b^3)*ln(1+tanh(
d*x+c))-b^3*tanh(d*x+c)-a^3*tanh(d*x+c)-1/7*b^3*tanh(d*x+c)^7-1/2*(a^3+3*a^
2*b+3*a*b^2+b^3)*ln(tanh(d*x+c)-1)-1/3*b^3*tanh(d*x+c)^3-1/5*b^3*tanh(d*x+c
)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(88) = 176$.
time = 0.29, size = 400, normalized size = 4.26

$$\frac{1}{105}\,b^3\left(105\,x+\frac{105\,c}{d}-\frac{8\,\left(203\,e^{(-2\,dx-2\,c)}+609\,e^{(-4\,dx-4\,c)}+770\,e^{(-6\,dx-6\,c)}+770\,e^{(-8\,dx-8\,c)}+315\,e^{(-10\,dx-10\,c)}+105\,e^{(-12\,dx-12\,c)}+44\right)}{d(7\,e^{(-2\,dx-2\,c)}+21\,e^{(-4\,dx-4\,c)}+35\,e^{(-6\,dx-6\,c)}+35\,e^{(-8\,dx-8\,c)}+21\,e^{(-10\,dx-10\,c)}+7\,e^{(-12\,dx-12\,c)}+e^{(-14\,dx-14\,c)}+1)}\right)+\frac{1}{5}\,ab^2\left(15\,x+\frac{15\,c}{d}-\frac{2\,\left(70\,e^{(-2\,dx-2\,c)}+140\,e^{(-4\,dx-4\,c)}+90\,e^{(-6\,dx-6\,c)}+45\,e^{(-8\,dx-8\,c)}+23\right)}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)+a^2b\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)+a^3\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/105*b^3*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c)
 + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 1
05*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-1
2*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a*b^2*(15*x + 15*c/d - 2*(7
0*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8*
d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^2*b*(3*x + 3*
c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c
) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^3*(x + c/d - 2/(d*(e^(
-2*d*x - 2*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(88) = 176$.
time = 0.40, size = 1036, normalized size = 11.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 7*(105*a^3 + 420*a^2*b + 483*a*b^2 + 17
6*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c
)^6 - (105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*sinh(d*x + c)^7 + 7*(105*
a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^5 - 7*(75*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3 + 3*(105
*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 +
35*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 + (105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3
 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^4
```

```
+ 21*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 7*(5*(105*a^3 + 420*a^2*b + 483*a*b^2 + 1
76*b^3)*cosh(d*x + c)^4 + 135*a^3 + 360*a^2*b + 369*a*b^2 + 168*b^3 + 10*(7
5*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
7*(3*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 10*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176
*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 9*(105*a^
3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3 + 420*a^2*b + 483*a*b^2 +
176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 7*((105*
a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^6 + 5*(75*a^3 + 240*a^
2*b + 213*a*b^2 + 56*b^3)*cosh(d*x + c)^4 + 75*a^3 + 180*a^2*b + 225*a*b^2
+ 9*(45*a^3 + 120*a^2*b + 123*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x +
c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh
(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x
+ c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(82) = 164$.
time = 0.25, size = 192, normalized size = 2.04

$$\begin{cases} a^3x - \frac{a^3\tanh(c+dx)}{d} + 3a^2bx - \frac{a^2b\tanh^3(c+dx)}{d} - \frac{3a^2b\tanh(c+dx)}{d} + 3ab^2x - \frac{3ab^2\tanh^5(c+dx)}{5d} - \frac{ab^2\tanh^3(c+dx)}{d} - \frac{3ab^2\tanh(c+dx)}{d} + b^3x - \frac{b^3\tanh^7(c+dx)}{7d} - \frac{b^3\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^3(c+dx)}{3d} - \frac{b^3\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)^3\tanh^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*tanh(c + d*x)/d + 3*a**2*b*x - a**2*b*tanh(c + d*x
)**3/d - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**5/
(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**
3*tanh(c + d*x)**7/(7*d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)
**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(
c)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(88) = 176$.
time = 0.55, size = 418, normalized size = 4.45

$$\frac{105\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)+\frac{2\left(105a^3e^{(12dx+12c)}+630a^2be^{(12dx+12c)}+945ab^2e^{(12dx+12c)}+420b^3e^{(12dx+12c)}+630a^3e^{(10dx+10c)}+3150a^2be^{(10dx+10c)}+3780ab^2e^{(10dx+10c)}+1260b^3e^{(10dx+10c)}+1575a^3e^{(8dx+8c)}+6720a^2be^{(8dx+8c)}+7665ab^2e^{(8dx+8c)}+3080b^3e^{(8dx+8c)}+2100a^3e^{(6dx+6c)}+7980a^2be^{(6dx+6c)}+9240ab^2e^{(6dx+6c)}+3080b^3e^{(6dx+6c)}+1575a^3e^{(4dx+4c)}+5670a^2be^{(4dx+4c)}+6363ab^2e^{(4dx+4c)}+2436b^3e^{(4dx+4c)}+630a^3e^{(2dx+2c)}+2310a^2be^{(2dx+2c)}+2436ab^2e^{(2dx+2c)}+812b^3e^{(2dx+2c)}+105a^3+420a^2b+483ab^2+176b^3\right)}{\left(e^{(2dx+2c)}+1\right)^7}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/105*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(105*a^3*e^(12*d*x
 + 12*c) + 630*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) + 420*
```

```
b^3*e^(12*d*x + 12*c) + 630*a^3*e^(10*d*x + 10*c) + 3150*a^2*b*e^(10*d*x +
10*c) + 3780*a*b^2*e^(10*d*x + 10*c) + 1260*b^3*e^(10*d*x + 10*c) + 1575*a^
3*e^(8*d*x + 8*c) + 6720*a^2*b*e^(8*d*x + 8*c) + 7665*a*b^2*e^(8*d*x + 8*c)
 + 3080*b^3*e^(8*d*x + 8*c) + 2100*a^3*e^(6*d*x + 6*c) + 7980*a^2*b*e^(6*d*
x + 6*c) + 9240*a*b^2*e^(6*d*x + 6*c) + 3080*b^3*e^(6*d*x + 6*c) + 1575*a^3
*e^(4*d*x + 4*c) + 5670*a^2*b*e^(4*d*x + 4*c) + 6363*a*b^2*e^(4*d*x + 4*c)
+ 2436*b^3*e^(4*d*x + 4*c) + 630*a^3*e^(2*d*x + 2*c) + 2310*a^2*b*e^(2*d*x
+ 2*c) + 2436*a*b^2*e^(2*d*x + 2*c) + 812*b^3*e^(2*d*x + 2*c) + 105*a^3 + 4
20*a^2*b + 483*a*b^2 + 176*b^3)/(e^(2*d*x + 2*c) + 1)^7)/d
```

Mupad [B]
time = 1.22, size = 106, normalized size = 1.13

$$x\left(a^3+3a^2b+3ab^2+b^3\right)-\frac{\tanh(c+dx)\,(a+b)^3}{d}-\frac{\tanh(c+dx)^3\,(3a^2b+3ab^2+b^3)}{3d}-\frac{\tanh(c+dx)^5\,(b^3+3ab^2)}{5d}-\frac{b^3\tanh(c+dx)^7}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)*(a + b)^3)/d - (tanh(c +
 d*x)^3*(3*a*b^2 + 3*a^2*b + b^3))/(3*d) - (tanh(c + d*x)^5*(3*a*b^2 + b^3)
)/(5*d) - (b^3*tanh(c + d*x)^7)/(7*d)
```

3.159 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b(a+b)^2\tanh^2(c+dx)}{2d}-\frac{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}{4d}-\frac{\left(a+b\tanh^2(c+dx)\right)^3}{6d}$$

```
[Out] (a+b)^3*ln(cosh(d*x+c))/d-1/2*b*(a+b)^2*tanh(d*x+c)^2/d-1/4*(a+b)*(a+b*tanh
(d*x+c)^2)^2/d-1/6*(a+b*tanh(d*x+c)^2)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 455, 45}

$$-\frac{b(a+b)^2\tanh^2(c+dx)}{2d}-\frac{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}{4d}-\frac{\left(a+b\tanh^2(c+dx)\right)^3}{6d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d - (b*(a + b)^2*Tanh[c + d*x]^2)/(2*d) - ((
a + b)*(a + b*Tanh[c + d*x]^2)^2)/(4*d) - (a + b*Tanh[c + d*x]^2)^3/(6*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(a+b)^2 + \frac{(a+b)^3}{1-x} - b(a+b)(a+bx) - b(a+bx)\right.\right.}{2d}$$
$$= \frac{(a+b)^3\log(\cosh(c+dx))}{d} - \frac{b(a+b)^2\tanh^2(c+dx)}{2d} - \frac{(a+}{}$$

Mathematica [A]
time = 0.17, size = 76, normalized size = 0.92

$$-\frac{-2(a+b)^3\log(\cosh(c+dx)) + b(a+b)^2\tanh^2(c+dx) + \frac{1}{2}(a+b)\left(a+b\tanh^2(c+dx)\right)^2 + \frac{1}{3}\left(a+b\tanh^2(c+dx)\right)^3}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(-2*(a + b)^3*Log[Cosh[c + d*x]] + b*(a + b)^2*Tanh[c + d*x]^2 + ((a +
 b)*(a + b*Tanh[c + d*x]^2)^2)/2 + (a + b*Tanh[c + d*x]^2)^3/3)/d
```

Maple [A]
time = 0.39, size = 151, normalized size = 1.82

method	result
derivativedivides	$\frac{\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\ln(1+\tanh(dx+c))}{2} - \frac{3a\,b^2\left(\tanh^2(dx+c)\right)}{2} - \frac{3a^2b\left(\tanh^2(dx+c)\right)}{2} - \frac{3a\,b^2\left(\tanh^4(dx+c)\right)}{4} - \frac{b^3\left(\tanh^6(dx+c)\right)}{6}}{d}$
default	$\frac{\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\ln(1+\tanh(dx+c))}{2} - \frac{3a\,b^2\left(\tanh^2(dx+c)\right)}{2} - \frac{3a^2b\left(\tanh^2(dx+c)\right)}{2} - \frac{3a\,b^2\left(\tanh^4(dx+c)\right)}{4} - \frac{b^3\left(\tanh^6(dx+c)\right)}{6}}{d}$
risch	$-a^3x - 3a^2bx - 3a\,b^2x - b^3x - \frac{2a^3c}{d} - \frac{6a^2bc}{d} - \frac{6a\,b^2c}{d} - \frac{2b^3c}{d} + \frac{2b\,e^{2dx+2c}\left(9a^2e^{8dx+8c}+18ab\,e^{8dx-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*(-a^3-3*a^2*b-3*a*b^2-b^3)*ln(1+tanh(d*x+c))-3/2*a*b^2*tanh(d*x+c)
^2-3/2*a^2*b*tanh(d*x+c)^2-3/4*a*b^2*tanh(d*x+c)^4-1/6*b^3*tanh(d*x+c)^6-1/
2*(a^3+3*a^2*b+3*a*b^2+b^3)*ln(tanh(d*x+c)-1)-1/2*b^3*tanh(d*x+c)^2-1/4*b^3
*tanh(d*x+c)^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(77) = 154$.
time = 0.51, size = 351, normalized size = 4.23

$$\frac{1}{3}b^3\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}+18e^{(-4dx-4c)}+34e^{(-6dx-6c)}+18e^{(-8dx-8c)}+9e^{(-10dx-10c)}\right)}{d(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(-12dx-12c)}+1)}\right)+3ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)+3a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)+\frac{a^3\log\left(\cosh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c
) + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(
-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) + 3*a*b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*
c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a^2*b*(x + c/
d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c)
 + e^(-4*d*x - 4*c) + 1))) + a^3*log(cosh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4298 vs. $2(77) = 154$.
time = 0.46, size = 4298, normalized size = 51.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 + 36*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^12 - 18*(a^2*b + 2*a*b^2 + b^3 - (a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 18*(11*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 - a^2*b - 2*a*b^2 - b^3 + (a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^10 + 60*(11*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x*cosh(d*x + c)^3 - 3*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 9*(8*a^2*b + 12*a*b^2 + 4
*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 + 9*(165*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 8*a^2*b - 12*a*b^2 - 4*b^
3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 90*(a^2*b + 2*a*b^2 + b^3 - (a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 30*(a^2*b + 2*a*b^2
+ b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (8*a^2*b + 1
2*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sin
h(d*x + c)^7 - 4*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^6 + 4*(693*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
```

cosh(d*x + c)^6 - 945*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d*x)*cosh(d*x + c)^4 - 27*a^2*b - 36*a*b^2 - 17*b^3 + 15*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x - 63*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 - 189*(a^2*b + 2*a*b^2 + b^3 - (a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 21*(8*a^2*b + 12*a*b^2 +
 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (27*a^2*b
 + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 9*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 3*(495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x*cosh(d*x + c)^8 - 1260*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x)*cosh(d*x + c)^6 - 210*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 24*a^2*b - 36*a*b^2 - 12*b^3
 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 20*(27*a^2*b + 36*a*b^2 + 17*b^
3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 4*(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^9 - 540*(a^2*b
 + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 1
26*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c)^5 - 20*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 - 9*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x - 18*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 6*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x*cosh(d*x + c)^10 - 135*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x)*cosh(d*x + c)^8 - 42*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 10*(27*a^2*b + 36*a*b^2 + 17*b
^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 3*a^2*b - 6*
a*b^2 - 3*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 9*(8*a^2*b + 12*a*b
^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d
x + c)^2 - 3((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 12*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*sinh(d*x + c)^12 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d
x + c)^10 + 6(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 11*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*
sinh(d*x + c)^9 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 15*(
33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 18*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^8 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 30*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^7 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 4*(231*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 315*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^3 + 15*a^2*b + 15*a*b^2 +
 5*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^6 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 63*(a^3 + 3*a

```
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*(33*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 84*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^6 + 70*(a^3 + 3*a^2*b + 3*a*...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(71) = 142$.

time = 0.21, size = 211, normalized size = 2.54

$$\begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\tanh^2(c+dx)}{2d} + 3ab^2x - \frac{3ab^2\log(\tanh(c+dx)+1)}{d} - \frac{3ab^2\tanh^4(c+dx)}{4d} - \frac{3ab^2\tanh^2(c+dx)}{2d} + b^3x - \frac{b^3\log(\tanh(c+dx)+1)}{d} - \frac{b^3\tanh^6(c+dx)}{6d} - \frac{b^3\tanh^4(c+dx)}{4d} - \frac{b^3\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)^3\tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d + 3*a**2*b*x - 3*a**2*b*l
og(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x - 3*
a*b**2*log(tanh(c + d*x) + 1)/d - 3*a*b**2*tanh(c + d*x)**4/(4*d) - 3*a*b**
2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh(c + d*x) + 1)/d - b**3*ta
nh(c + d*x)**6/(6*d) - b**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2/
(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(77) = 154$.

time = 0.53, size = 216, normalized size = 2.60

$$-\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c) - 3\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(e^{(2dx+2c)}+1\right) - \frac{2\left(9\left(a^2b+2ab^2+b^3\right)e^{(10dx+10c)}+18\left(2a^2b+3ab^2+b^3\right)e^{(8dx+8c)}+2\left(27a^2b+36ab^2+17b^3\right)e^{(6dx+6c)}+18\left(2a^2b+3ab^2+b^3\right)e^{(4dx+4c)}+9\left(a^2b+2ab^2+b^3\right)e^{(2dx+2c)}\right)}{\left(e^{(2dx+2c)}+1\right)^6}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*log(e^(2*d*x + 2*c) + 1) - 2*(9*(a^2*b + 2*a*b^2 + b^3)*e^(10*d*
x + 10*c) + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^(8*d*x + 8*c) + 2*(27*a^2*b + 36
*a*b^2 + 17*b^3)*e^(6*d*x + 6*c) + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^(4*d*x +
4*c) + 9*(a^2*b + 2*a*b^2 + b^3)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1)^6)/
d
```

Mupad [B]

time = 1.25, size = 123, normalized size = 1.48

$$x\left(a^3+3a^2b+3ab^2+b^3\right) - \frac{\tanh(c+dx)^2\left(3a^2b+3ab^2+b^3\right)}{2d} - \frac{\ln\left(\tanh(c+dx)+1\right)\left(a^3+3a^2b+3ab^2+b^3\right)}{d} - \frac{\tanh(c+dx)^4\left(b^3+3ab^2\right)}{4d} - \frac{b^3\tanh(c+dx)^6}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^2*(3*a*b^2 + 3*a^2*b + b
^3))/(2*d) - (log(tanh(c + d*x) + 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d - (
tanh(c + d*x)^4*(3*a*b^2 + b^3))/(4*d) - (b^3*tanh(c + d*x)^6)/(6*d)
```

3.160 $\int \left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=74

$$(a+b)^3x-\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)^3*x-b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(3*a+b)*tanh(d*x+c)^3/d
-1/5*b^3*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 398, 212}

$$-\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}+x(a+b)^3-\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(3*a + b)*Ta
nh[c + d*x]^3)/(3*d) - (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(3a^2+3ab+b^2)-b^2(3a+b)x^2-b^3x^4+\frac{(a+b)^3}{1-x^2}\right)\,dx, x, \tanh(c$$
$$= -\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3\tanh^5(c+}{5d}$$
$$= (a+b)^3x-\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d}-\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3}{}$$

Mathematica [A]
time = 0.42, size = 95, normalized size = 1.28

$$\frac{\tanh(c+dx)\left(\frac{15(a+b)^3\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}-b\left(45a^2+15ab\left(3+\tanh^2(c+dx)\right)+b^2\left(15+5\tanh^2(c+dx)+3\tanh^4(c+dx)\right)\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Tanh[c + d*x]*((15*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c +
 d*x]^2] - b*(45*a^2 + 15*a*b*(3 + Tanh[c + d*x]^2) + b^2*(15 + 5*Tanh[c +
d*x]^2 + 3*Tanh[c + d*x]^4))))/(15*d)
```

Maple [A]
time = 0.31, size = 141, normalized size = 1.91

method	result
derivativedivides	$\frac{-3a\,b^2\tanh(dx+c)-3a^2b\tanh(dx+c)-a\,b^2\left(\tanh^3(dx+c)\right)+\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln(1+\tanh(dx+c))}{2}-\frac{b^3\left(\tanh^3(dx+c)\right)}{3}-\frac{b^3}{}}{d}$
default	$\frac{-3a\,b^2\tanh(dx+c)-3a^2b\tanh(dx+c)-a\,b^2\left(\tanh^3(dx+c)\right)+\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln(1+\tanh(dx+c))}{2}-\frac{b^3\left(\tanh^3(dx+c)\right)}{3}-\frac{b^3}{}}{d}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x+\frac{2b\left(45a^2\mathrm{e}^{8dx+8c}+90ab\,\mathrm{e}^{8dx+8c}+45b^2\mathrm{e}^{8dx+8c}+180a^2\mathrm{e}^{6dx+6c}+270ab\,\mathrm{e}^{6dx+6c}+9\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-3*a*b^2*tanh(d*x+c)-3*a^2*b*tanh(d*x+c)-a*b^2*tanh(d*x+c)^3+1/2*(a^3+
3*a^2*b+3*a*b^2+b^3)*ln(1+tanh(d*x+c))-1/3*b^3*tanh(d*x+c)^3-1/5*b^3*tanh(d
*x+c)^5-b^3*tanh(d*x+c)-1/2*(a^3+3*a^2*b+3*a*b^2+b^3)*ln(tanh(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(70) = 140$.
time = 0.28, size = 239, normalized size = 3.23

$$\frac{1}{15}\,b^{3}\left(15\,x+\frac{15\,c}{d}-\frac{2\,\left(70\,e^{\left(-2\,dx-2\,c\right)}+140\,e^{\left(-4\,dx-4\,c\right)}+90\,e^{\left(-6\,dx-6\,c\right)}+45\,e^{\left(-8\,dx-8\,c\right)}+23\right)}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}+10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}+5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}+1\right)}\right)+ab^{2}\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{\left(-2\,dx-2\,c\right)}+3\,e^{\left(-4\,dx-4\,c\right)}+2\right)}{d\left(3\,e^{\left(-2\,dx-2\,c\right)}+3\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}+1\right)}\right)+3\,a^{2}b\left(x+\frac{c}{d}-\frac{2}{d\left(e^{\left(-2\,dx-2\,c\right)}+1\right)}\right)+a^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4
*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1
))) + 3*a^2*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(70) = 140$.
time = 0.34, size = 567, normalized size = 7.66

$$\frac{\left(45\,a^{2}b+60\,ab^{2}+23\,b^{3}+15\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})dx\right)\cosh\left(dx+c\right)^{5}+5\left(45\,a^{2}b+60\,ab^{2}+23\,b^{3}+15\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})dx\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}-\left(45\,a^{2}b+60\,ab^{2}+23\,b^{3}\right)\sinh\left(dx+c\right)^{5}+5\left(45\,a^{2}b+60\,ab^{2}+23\,b^{3}+15\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})dx\right)\cosh\left(dx+c\right)^{3}-5\left(27\,a^{2}b+24\,ab^{2}+5\,b^{3}+2\left(45\,a^{2}b+60\,ab^{2}+23\,b^{3}\right)\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{3}+5\left(2\left(45\,a^{2}b+60\,ab^{2}+23\,b^{3}+15\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})dx\right)\cosh\left(dx+c\right)^{3}+3\left(45\,a^{2}b+60\,ab^{2}+23\,b^{3}+15\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})dx\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+10\left(45\,a^{2}b+60\,ab^{2}+23\,b^{3}+15\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})dx\right)\cosh\left(dx+c\right)-5\left(\left(45\,a^{2}b+60\,ab^{2}+23\,b^{3}\right)\cosh\left(dx+c\right)^{4}+18\,a^{2}b+12\,ab^{2}+10\,b^{3}+3\left(27\,a^{2}b+24\,ab^{2}+5\,b^{3}\right)\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)}{15\left(d\cosh\left(dx+c\right)^{5}+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+5\,d\cosh\left(dx+c\right)^{3}+5\left(2\,d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+10\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*((45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x)*cosh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 - (45*a^2*b + 60*a*b^2 +
23*b^3)*sinh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 5*(27*a^2*b + 24*a*b^2 + 5*b^3 +
 2*(45*a^2*b + 60*a*b^2 + 23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(
45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh
(d*x + c)^3 + 3*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(45*a^2*b + 60*a*b^2 + 23*
b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 5*((45*a^2*b
+ 60*a*b^2 + 23*b^3)*cosh(d*x + c)^4 + 18*a^2*b + 12*a*b^2 + 10*b^3 + 3*(27
*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)
^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [A]
time = 0.17, size = 126, normalized size = 1.70

$$\begin{cases} a^{3}x + 3a^{2}bx - \frac{3a^{2}b\tanh{\left(c+dx \right)}}{d} + 3ab^{2}x - \frac{ab^{2}\tanh^{3}{\left(c+dx \right)}}{d} - \frac{3ab^{2}\tanh{\left(c+dx \right)}}{d} + b^{3}x - \frac{b^{3}\tanh^{5}{\left(c+dx \right)}}{5d} - \frac{b^{3}\tanh^{3}{\left(c+dx \right)}}{3d} - \frac{b^{3}\tanh{\left(c+dx \right)}}{d} & \text{for}\: d \neq 0 \\ x\left(a+b\tanh^{2}{\left(c \right)}\right)^{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - a*
b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**3*tanh(c +
 d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d,
0)), (x*(a + b*tanh(c)**2)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(70) = 140$.
time = 0.43, size = 241, normalized size = 3.26

$$\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c)+\frac{2\,(45\,a^2be^{(8\,dx+8\,c)}+90\,ab^2e^{(8\,dx+8\,c)}+45\,b^3e^{(8\,dx+8\,c)}+180\,a^2be^{(6\,dx+6\,c)}+270\,ab^2e^{(6\,dx+6\,c)}+90\,b^3e^{(6\,dx+6\,c)}+270\,a^2be^{(4\,dx+4\,c)}+330\,ab^2e^{(4\,dx+4\,c)}+140\,b^3e^{(4\,dx+4\,c)}+180\,a^2be^{(2\,dx+2\,c)}+210\,ab^2e^{(2\,dx+2\,c)}+70\,b^3e^{(2\,dx+2\,c)}+45\,a^2b+60\,ab^2+23\,b^3)}{(e^{(2\,dx+2\,c)}+1)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(45*a^2*b*e^(8*d*x +
 8*c) + 90*a*b^2*e^(8*d*x + 8*c) + 45*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*
d*x + 6*c) + 270*a*b^2*e^(6*d*x + 6*c) + 90*b^3*e^(6*d*x + 6*c) + 270*a^2*b
*e^(4*d*x + 4*c) + 330*a*b^2*e^(4*d*x + 4*c) + 140*b^3*e^(4*d*x + 4*c) + 18
0*a^2*b*e^(2*d*x + 2*c) + 210*a*b^2*e^(2*d*x + 2*c) + 70*b^3*e^(2*d*x + 2*c
) + 45*a^2*b + 60*a*b^2 + 23*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.28, size = 86, normalized size = 1.16

$$x\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)-\frac{{\mathrm{tanh}\left(c+d\,x\right)}^3\,\left(b^3+3\,a\,b^2\right)}{3\,d}-\frac{b^3\,{\mathrm{tanh}\left(c+d\,x\right)}^5}{5\,d}-\frac{b\,\mathrm{tanh}\left(c+d\,x\right)\,\left(3\,a^2+3\,a\,b+b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^3*(3*a*b^2 + b^3))/(3*d)
 - (b^3*tanh(c + d*x)^5)/(5*d) - (b*tanh(c + d*x)*(3*a*b + 3*a^2 + b^2))/d
```

3.161 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=72

$$\frac{(a+b)^3\log(\cosh(c+dx))}{d}+\frac{a^3\log(\tanh(c+dx))}{d}-\frac{b^2(3a+b)\tanh^2(c+dx)}{2d}-\frac{b^3\tanh^4(c+dx)}{4d}$$

```
[Out] (a+b)^3*ln(cosh(d*x+c))/d+a^3*ln(tanh(d*x+c))/d-1/2*b^2*(3*a+b)*tanh(d*x+c)
^2/d-1/4*b^3*tanh(d*x+c)^4/d
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 457, 84}

$$\frac{a^3\log(\tanh(c+dx))}{d}-\frac{b^2(3a+b)\tanh^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b^3\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d + (a^3*Log[Tanh[c + d*x]])/d - (b^2*(3*a +
 b)*Tanh[c + d*x]^2)/(2*d) - (b^3*Tanh[c + d*x]^4)/(4*d)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2(3a+b) - \frac{(a+b)^3}{-1+x} + \frac{a^3}{x} - b^3x\right) dx, x, \tanh^2(c+dx\right.}{2d}$$
$$= \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{a^3\log(\tanh(c+dx))}{d} - \frac{b^2(3a+b)}{}$$

Mathematica [A]
time = 0.38, size = 67, normalized size = 0.93

$$\frac{2(a+b)^3\log(\cosh(c+dx)) + 2a^3\log(\tanh(c+dx)) - b^2(3a+b)\tanh^2(c+dx) - \frac{1}{2}b^3\tanh^4(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (2*(a + b)^3*Log[Cosh[c + d*x]] + 2*a^3*Log[Tanh[c + d*x]] - b^2*(3*a + b)*
Tanh[c + d*x]^2 - (b^3*Tanh[c + d*x]^4)/2)/(2*d)
```

Maple [A]
time = 1.99, size = 86, normalized size = 1.19

method	result
derivativedivides	$\frac{a^3\ln(\sinh(dx+c))+3a^2b\ln(\cosh(dx+c))+3a\,b^2\left(\ln(\cosh(dx+c))-\frac{\left(\tanh^2(dx+c)\right)}{2}\right)+b^3\left(\ln(\cosh(dx+c))-\frac{\left(\tanh^2(dx+c)\right)}{2}\right.}{d}$
default	$\frac{a^3\ln(\sinh(dx+c))+3a^2b\ln(\cosh(dx+c))+3a\,b^2\left(\ln(\cosh(dx+c))-\frac{\left(\tanh^2(dx+c)\right)}{2}\right)+b^3\left(\ln(\cosh(dx+c))-\frac{\left(\tanh^2(dx+c)\right)}{2}\right.}{d}$
risch	$-a^3x-3a^2bx-3a\,b^2x-b^3x-\frac{6a^2bc}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}-\frac{2a^3c}{d}+\frac{2b^2e^{2dx+2c}\left(3a\,e^{4dx+4c}+2b\,e^{4dx+4c}+\right.}{d\left(1+e^{2dx}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*ln(sinh(d*x+c))+3*a^2*b*ln(cosh(d*x+c))+3*a*b^2*(ln(cosh(d*x+c))-1
/2*tanh(d*x+c)^2)+b^3*(ln(cosh(d*x+c))-1/2*tanh(d*x+c)^2-1/4*tanh(d*x+c)^4)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(68) = 136$.
time = 0.48, size = 214, normalized size = 2.97

$$b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)+3\,ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{3\,a^2b\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{a^3\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] b^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a*b^2*(x + c/d + log(e^(-2
*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x
- 4*c) + 1))) + 3*a^2*b*log(e^(d*x + c) + e^(-d*x - c))/d + a^3*log(sinh(d*
x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2381 vs. $2(68) = 136$.
time = 0.39, size = 2381, normalized size = 33.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*sinh(d*x + c)^8 - 2*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^2 - 3*a*b^2 - 2*b^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*
sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3
 - 3*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
+ c)^4 - 6*a*b^2 - 2*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 15*(3*a*
b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 5*(
3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 -
(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(3*a*b^2 + 2*b^3 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 15*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 3*a*b^2 - 2*b^3 + 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x - 6*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^2*b + 3*a*b^2 + b^3)
```

```
*cosh(d*x + c)^8 + 8*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^
7 + (3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^6 + 4*(3*a^2*b + 3*a*b^2 + b^3 + 7*(3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 30*(3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^5 + 10*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d
*x + c)^6 + 15*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*a^2*b + 3*a*b^
2 + b^3 + 9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
((3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(3*a^2*b + 3*a*b^2 + b^3)*co
sh(d*x + c)^5 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (3*a^2*b + 3*
a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x +
c) - sinh(d*x + c))) - (a^3*cosh(d*x + c)^8 + 8*a^3*cosh(d*x + c)*sinh(d*x
+ c)^7 + a^3*sinh(d*x + c)^8 + 4*a^3*cosh(d*x + c)^6 + 6*a^3*cosh(d*x + c)^
4 + 4*(7*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 8*(7*a^3*cosh(d*x + c
)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 4*a^3*cosh(d*x + c)^2 + 2*(35*
a^3*cosh(d*x + c)^4 + 30*a^3*cosh(d*x + c)^2 + 3*a^3)*sinh(d*x + c)^4 + 8*(
7*a^3*cosh(d*x + c)^5 + 10*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(
d*x + c)^3 + a^3 + 4*(7*a^3*cosh(d*x + c)^6 + 15*a^3*cosh(d*x + c)^4 + 9*a^
3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 8*(a^3*cosh(d*x + c)^7 + 3*a^3*c
osh(d*x + c)^5 + 3*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 - 3*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 2*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (3*a*b^2 + 2*b^3 - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^
8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x +
c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3
 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(
d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x
+ c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*co
sh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5
+ 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\coth\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(68) = 136$.
time = 0.51, size = 267, normalized size = 3.71

$$\frac{2\,a^3\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)+2\left(3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-\frac{9\,a^2b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+9\,ab^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b^3\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+36\,a^2b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+12\,ab^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)-4\,b^3\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+36\,a^2b-12\,ab^2-4\,b^3}{\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + 2*(3*a^2*b + 3*a*b
^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (9*a^2*b*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c))^2 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))
^2 + 3*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 36*a^2*b*(e^(2*d*x + 2*
c) + e^(-2*d*x - 2*c)) + 12*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*
b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 36*a^2*b - 12*a*b^2 - 4*b^3)/(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)^2)/d
```

Mupad [B]
time = 0.40, size = 380, normalized size = 5.28

$$\frac{\ln\left(e^{4c+4dx}-1\right)\left(a^3\,d+d\left(3\,a^2\,b+3\,a\,b^2+b^3\right)\right)}{2\,d^2}-x\left(a+b\right)^3+\frac{2\left(2\,b^3+3\,a\,b^2\right)}{d\left(e^{2c+2dx}+1\right)}+\frac{8\,b^3}{d\left(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1\right)}+\frac{\mathrm{atan}\left(\frac{e^{2c}\,e^{2dx}\left(b^3\sqrt{-d^2}-a^3\sqrt{-d^2}+3\,a\,b^2\sqrt{-d^2}+3\,a^2\,b\sqrt{-d^2}\right)}{d\sqrt{a^6-6\,a^5\,b+3\,a^4\,b^2+16\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5+b^6}}\right)\sqrt{a^6-6\,a^5\,b+3\,a^4\,b^2+16\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5+b^6}}{\sqrt{-d^2}}-\frac{2\left(4\,b^3+3\,a\,b^2\right)}{d\left(2\,e^{2c+2dx}+e^{4c+4dx}+1\right)}-\frac{4\,b^3}{d\left(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (log(exp(4*c + 4*d*x) - 1)*(a^3*d + d*(3*a*b^2 + 3*a^2*b + b^3)))/(2*d^2) -
 x*(a + b)^3 + (2*(3*a*b^2 + 2*b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (8*b^3)/(
d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (atan
((exp(2*c)*exp(2*d*x)*(b^3*(-d^2)^(1/2) - a^3*(-d^2)^(1/2) + 3*a*b^2*(-d^2)
^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a*b^5 - 6*a^5*b + a^6 + b^6 + 15*a^2*
b^4 + 16*a^3*b^3 + 3*a^4*b^2)^(1/2)))*(6*a*b^5 - 6*a^5*b + a^6 + b^6 + 15*a
^2*b^4 + 16*a^3*b^3 + 3*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (2*(3*a*b^2 + 4*b^3)
)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (4*b^3)/(d*(4*exp(2*c +
 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.162 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=59

$$(a+b)^3x-\frac{a^3\coth(c+dx)}{d}-\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] (a+b)^3*x-a^3*coth(d*x+c)/d-b^2*(3*a+b)*tanh(d*x+c)/d-1/3*b^3*tanh(d*x+c)^3
/d
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 472, 213}

$$-\frac{a^3\coth(c+dx)}{d}-\frac{b^2(3a+b)\tanh(c+dx)}{d}+x(a+b)^3-\frac{b^3\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (a^3*Coth[c + d*x])/d - (b^2*(3*a + b)*Tanh[c + d*x])/d - (b^
3*Tanh[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^2(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-b^2(3a+b)+\frac{a^3}{x^2}-b^3x^2-\frac{(a+b)^3}{-1+x^2}\right)\,dx, x, \tanh(c+$$

$$= -\frac{a^3\coth(c+dx)}{d}-\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+}{3d}$$

$$= (a+b)^3x-\frac{a^3\coth(c+dx)}{d}-\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\,\text{t}}{}$$

Mathematica [A]
time = 1.63, size = 81, normalized size = 1.37

$$\frac{\tanh(c+dx)\left(-3a^3\coth^2(c+dx)+3(a+b)^3\tanh^{-1}\left(\sqrt{\coth^2(c+dx)}\right)\sqrt{\coth^2(c+dx)}-b^2\left(9a+3b+b\tanh^2(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (Tanh[c + d*x]*(-3*a^3*Coth[c + d*x]^2 + 3*(a + b)^3*ArcTanh[Sqrt[Coth[c +
d*x]^2]]*Sqrt[Coth[c + d*x]^2] - b^2*(9*a + 3*b + b*Tanh[c + d*x]^2)))/(3*d
)
```

Maple [A]
time = 1.63, size = 80, normalized size = 1.36

method	result
derivativedivides	$\frac{a^3(dx+c-\coth(dx+c))+3a^2b(dx+c)+3a\,b^2(dx+c-\tanh(dx+c))+b^3\left(dx+c-\tanh(dx+c)-\frac{\left(\tanh^3(dx+c)\right)}{3}\right)}{d}$
default	$\frac{a^3(dx+c-\coth(dx+c))+3a^2b(dx+c)+3a\,b^2(dx+c-\tanh(dx+c))+b^3\left(dx+c-\tanh(dx+c)-\frac{\left(\tanh^3(dx+c)\right)}{3}\right)}{d}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x-\frac{2\left(3a^3\mathrm{e}^{6dx+6c}-9a\,b^2\mathrm{e}^{6dx+6c}-6b^3\mathrm{e}^{6dx+6c}+9a^3\mathrm{e}^{4dx+4c}-9a\,b^2\mathrm{e}^{4dx+4c}+9a^3\mathrm{e}^{2}\right.}{3d\left(1+\mathrm{e}^{2dx+2c}\right)^3\left(\mathrm{e}^{2dx+2c}-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c))+3*a^2*b*(d*x+c)+3*a*b^2*(d*x+c-tanh(d*x+c))+b^
3*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(57) = 114$.
time = 0.28, size = 147, normalized size = 2.49

$$\frac{1}{3}\,b^3\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)+3\,ab^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)+a^3\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+3\,a^2bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 3*a*b^2
*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^3*(x + c/d + 2/(d*(e^(-2*d*x
- 2*c) - 1))) + 3*a^2*b*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(57) = 114$.
time = 0.35, size = 341, normalized size = 5.78

$$-\frac{(3\,a^3+9\,ab^2+4\,b^3)\cosh\left(dx+c\right)^4-4\,(3\,a^3+9\,ab^2+4\,b^3+3\,(a^3+3\,a^2b+3\,ab^2+b^3)dx)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(3\,a^3+9\,ab^2+4\,b^3)\sinh\left(dx+c\right)^4+9\,a^3-9\,ab^2+4\,(3\,a^3-b^3)\cosh\left(dx+c\right)^2+2\,(6\,a^3-2\,b^3+3\,(3\,a^3+9\,ab^2+4\,b^3)\cosh\left(dx+c\right)^2)\sinh\left(dx+c\right)^2-4\,((3\,a^3+9\,ab^2+4\,b^3+3\,(a^3+3\,a^2b+3\,ab^2+b^3)dx)\cosh\left(dx+c\right)^3+(3\,a^3+9\,ab^2+4\,b^3+3\,(a^3+3\,a^2b+3\,ab^2+b^3)dx)\cosh\left(dx+c\right))\sinh\left(dx+c\right)}{12\,(d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(d\cosh\left(dx+c\right)^3+d\cosh\left(dx+c\right))\sinh\left(dx+c\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*((3*a^3 + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 4*(3*a^3 + 9*a*b^2 + 4*b
^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (3*a^3 + 9*a*b^2 + 4*b^3)*sinh(d*x + c)^4 + 9*a^3 - 9*a*b^2 + 4*(3*a^3 - b
^3)*cosh(d*x + c)^2 + 2*(6*a^3 - 2*b^3 + 3*(3*a^3 + 9*a*b^2 + 4*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 - 4*((3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh^{2}\left(c+dx\right)\right)^{3}\coth^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(57) = 114$.
time = 0.54, size = 135, normalized size = 2.29

$$\frac{3\,(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c)-\frac{6\,a^3}{e^{(2\,dx+2\,c)}-1}+\frac{2\,\left(9\,ab^2e^{(4\,dx+4\,c)}+6\,b^3e^{(4\,dx+4\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+6\,b^3e^{(2\,dx+2\,c)}+9\,ab^2+4\,b^3\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 6*a^3/(e^(2*d*x + 2*c) -
 1) + 2*(9*a*b^2*e^(4*d*x + 4*c) + 6*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*
x + 2*c) + 6*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 4*b^3)/(e^(2*d*x + 2*c) + 1)^3
)/d
```

Mupad [B]
time = 1.31, size = 218, normalized size = 3.69

$$x\,{\left(a+b\right)}^3+\frac{\frac{2\,a\,b^2}{d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}+\frac{\frac{2\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}+\frac{4\,a\,b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{2\,a^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}+\frac{2\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(a + b)^3 + ((2*a*b^2)/d + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 2*b^3))/(3*d))/
(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(3*a*b^2 + 2*b^3))/(3*d)
+ (2*exp(4*c + 4*d*x)*(3*a*b^2 + 2*b^3))/(3*d) + (4*a*b^2*exp(2*c + 2*d*x))
/d)/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*a
^3)/(d*(exp(2*c + 2*d*x) - 1)) + (2*(3*a*b^2 + 2*b^3))/(3*d*(exp(2*c + 2*d*
x) + 1))
```

3.163 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=72

$$-\frac{a^3\coth^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}+\frac{a^2(a+3b)\log(\tanh(c+dx))}{d}-\frac{b^3\tanh^2(c+dx)}{2d}$$

```
[Out] -1/2*a^3*coth(d*x+c)^2/d+(a+b)^3*ln(cosh(d*x+c))/d+a^2*(a+3*b)*ln(tanh(d*x+
c))/d-1/2*b^3*tanh(d*x+c)^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{a^3\coth^2(c+dx)}{2d}+\frac{a^2(a+3b)\log(\tanh(c+dx))}{d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-\frac{b^3\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(a^3*Coth[c + d*x]^2)/d + ((a + b)^3*Log[Cosh[c + d*x]])/d + (a^2*(a +
 3*b)*Log[Tanh[c + d*x]])/d - (b^3*Tanh[c + d*x]^2)/(2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^3(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^3 - \frac{(a+b)^3}{-1+x} + \frac{a^3}{x^2} + \frac{a^2(a+3b)}{x}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= -\frac{a^3\coth^2(c+dx)}{2d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{a^2(a+3b)}{}$$

Mathematica [A]
time = 0.33, size = 63, normalized size = 0.88

$$-\frac{a^3\coth^2(c+dx) - 2(a+b)^3\log(\cosh(c+dx)) - 2a^2(a+3b)\log(\tanh(c+dx)) + b^3\tanh^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(a^3*Coth[c + d*x]^2 - 2*(a + b)^3*Log[Cosh[c + d*x]] - 2*a^2*(a + 3*b
)*Log[Tanh[c + d*x]] + b^3*Tanh[c + d*x]^2)/d
```

Maple [A]
time = 1.97, size = 76, normalized size = 1.06

method	result
derivativedivides	$\frac{a^3\left(\ln(\sinh(dx+c)) - \frac{\left(\coth^2(dx+c)\right)}{2}\right) + 3a^2b\ln(\sinh(dx+c)) + 3a\,b^2\ln(\cosh(dx+c)) + b^3\left(\ln(\cosh(dx+c)) - \frac{\left(\tanh^2(dx+c)\right)}{2}\right.}{d}$
default	$\frac{a^3\left(\ln(\sinh(dx+c)) - \frac{\left(\coth^2(dx+c)\right)}{2}\right) + 3a^2b\ln(\sinh(dx+c)) + 3a\,b^2\ln(\cosh(dx+c)) + b^3\left(\ln(\cosh(dx+c)) - \frac{\left(\tanh^2(dx+c)\right)}{2}\right.}{d}$
risch	$-a^3x - 3a^2bx - 3a\,b^2x - b^3x - \frac{2a^3c}{d} - \frac{6a^2bc}{d} - \frac{6a\,b^2c}{d} - \frac{2b^3c}{d} - \frac{2\,e^{2dx+2c}\left(a^3e^{4dx+4c} - b^3e^{4dx+4c} +\right.}{d\left(1+e^{2dx+2c}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)+3*a^2*b*ln(sinh(d*x+c))+3*a*b^
2*ln(cosh(d*x+c))+b^3*(ln(cosh(d*x+c))-1/2*tanh(d*x+c)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(68) = 136$.
time = 0.49, size = 203, normalized size = 2.82

$$a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)+b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{3\,ab^2\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{3\,a^2b\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^3*(x + c/d
+ log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) +
 e^(-4*d*x - 4*c) + 1))) + 3*a*b^2*log(e^(d*x + c) + e^(-d*x - c))/d + 3*a^
2*b*log(e^(d*x + c) - e^(-d*x - c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1686 vs. $2(68) = 136$.
time = 0.38, size = 1686, normalized size = 23.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*sinh(d*x + c)^8 + 2*(a^3 - b^3)*cosh(d*x + c)^6 + 2*(14*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^3 - b^3)*sinh(d*x + c)^6 +
 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + 3*(a^3 - b^3)*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*c
osh(d*x + c)^4 + 2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 15*(
a^3 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*cosh(d*x + c)^5 + 5*(a^3 - b^3)*cosh(d*x + c)^3 + (2*a^3 + 2*b^
3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + (
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(a^3 - b^3)*cosh(d*x + c)^2 + 2*(14*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(a^3 - b^3)*cosh(d
*x + c)^4 + a^3 - b^3 + 6*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a*b^2 + b^3)*cosh(d*x + c)^8 +
56*(3*a*b^2 + b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*a*b^2 + b^3)*cos
h(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c
)^7 + (3*a*b^2 + b^3)*sinh(d*x + c)^8 - 2*(3*a*b^2 + b^3)*cosh(d*x + c)^4 +
 2*(35*(3*a*b^2 + b^3)*cosh(d*x + c)^4 - 3*a*b^2 - b^3)*sinh(d*x + c)^4 + 8
*(7*(3*a*b^2 + b^3)*cosh(d*x + c)^5 - (3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d
*x + c)^3 + 3*a*b^2 + b^3 + 4*(7*(3*a*b^2 + b^3)*cosh(d*x + c)^6 - 3*(3*a*b
```

```
^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a*b^2 + b^3)*cosh(d*x +
c)^7 - (3*a*b^2 + b^3)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*cosh(d*x + c)/
(cosh(d*x + c) - sinh(d*x + c))) - ((a^3 + 3*a^2*b)*cosh(d*x + c)^8 + 56*(a
^3 + 3*a^2*b)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(a^3 + 3*a^2*b)*cosh(d*x
 + c)^2*sinh(d*x + c)^6 + 8*(a^3 + 3*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 +
 (a^3 + 3*a^2*b)*sinh(d*x + c)^8 - 2*(a^3 + 3*a^2*b)*cosh(d*x + c)^4 + 2*(3
5*(a^3 + 3*a^2*b)*cosh(d*x + c)^4 - a^3 - 3*a^2*b)*sinh(d*x + c)^4 + 8*(7*(
a^3 + 3*a^2*b)*cosh(d*x + c)^5 - (a^3 + 3*a^2*b)*cosh(d*x + c))*sinh(d*x +
c)^3 + a^3 + 3*a^2*b + 4*(7*(a^3 + 3*a^2*b)*cosh(d*x + c)^6 - 3*(a^3 + 3*a^
2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 + 3*a^2*b)*cosh(d*x + c)^7
- (a^3 + 3*a^2*b)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh
(d*x + c) - sinh(d*x + c))) + 4*(2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh
(d*x + c)^7 + 3*(a^3 - b^3)*cosh(d*x + c)^5 + 2*(2*a^3 + 2*b^3 - (a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (a^3 - b^3)*cosh(d*x + c))*sin
h(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*
d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*s
inh(d*x + c)^8 - 2*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - d)*sinh(d*
x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7
*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*cosh(d*x +
 c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh^2\left(c + dx\right)\right)^3 \coth^3\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(68) = 136.
time = 0.58, size = 274, normalized size = 3.81

$$\frac{2\left(3ab^2+b^3\right)\log\left(e^{(2dx+2c)}+e^{(-2dx-2c)}+2\right)+2\left(a^3+3a^2b\right)\log\left(e^{(2dx+2c)}+e^{(-2dx-2c)}-2\right)-\frac{a^3\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2+3a^2b\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2+3ab^2\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2+b^3\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2+8a^3\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)-8b^3\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)+12a^3-12a^2b-12ab^2+12b^3}{\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2-4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + 2*(a^3
 + 3*a^2*b)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^3*(e^(2*d*x +
2*c) + e^(-2*d*x - 2*c))^2 + 3*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2
 + 3*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + b^3*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c))^2 + 8*a^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 8*b^3*(e
```

```
^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 12*a^3 - 12*a^2*b - 12*a*b^2 + 12*b^3)
/((e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 - 4))/d
```

Mupad [B]
time = 2.48, size = 327, normalized size = 4.54

$$\frac{\ln\left(e^{4c+4dx}-1\right)\left(d\,a^3+3\,d\,a^2\,b+3\,d\,a\,b^2+d\,b^3\right)}{2\,d^2}-\frac{\frac{4\,(a^3+b^3)}{d}+\frac{2\,e^{2c+2dx}\,(a^3-b^3)}{d}}{e^{4c+4dx}-1}-\frac{\frac{4\,(a^3+b^3)}{d}+\frac{4\,e^{2c+2dx}\,(a^3-b^3)}{d}}{e^{8c+8dx}-2\,e^{4c+4dx}+1}-\frac{\operatorname{atan}\left(\frac{e^{2c}\,e^{2dx}\,\left(a^3\,\sqrt{-d^2}-b^3\,\sqrt{-d^2}-3\,a\,b^2\,\sqrt{-d^2}+3\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+6\,a^5\,b+3\,a^4\,b^2-20\,a^3\,b^3+3\,a^2\,b^4+6\,a\,b^5+b^6}}\right)\sqrt{a^6+6\,a^5\,b+3\,a^4\,b^2-20\,a^3\,b^3+3\,a^2\,b^4+6\,a\,b^5+b^6}}{\sqrt{-d^2}}-x\,(a+b)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (log(exp(4*c + 4*d*x) - 1)*(a^3*d + b^3*d + 3*a*b^2*d + 3*a^2*b*d))/(2*d^2)
 - ((4*(a^3 + b^3))/d + (2*exp(2*c + 2*d*x)*(a^3 - b^3))/d)/(exp(4*c + 4*d*
x) - 1) - ((4*(a^3 + b^3))/d + (4*exp(2*c + 2*d*x)*(a^3 - b^3))/d)/(exp(8*c
 + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (atan((exp(2*c)*exp(2*d*x)*(a^3*(-d^2
)^(1/2) - b^3*(-d^2)^(1/2) - 3*a*b^2*(-d^2)^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/
(d*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 3*a^2*b^4 - 20*a^3*b^3 + 3*a^4*b^2)^(1/
2)))*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 3*a^2*b^4 - 20*a^3*b^3 + 3*a^4*b^2)^(
1/2))/(-d^2)^(1/2) - x*(a + b)^3
```

3.164 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=59

$$(a+b)^3x-\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{b^3\tanh(c+dx)}{d}$$

```
[Out] (a+b)^3*x-a^2*(a+3*b)*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d-b^3*tanh(d*x+c)
/d
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 213}

$$-\frac{a^3\coth^3(c+dx)}{3d}-\frac{a^2(a+3b)\coth(c+dx)}{d}+x(a+b)^3-\frac{b^3\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d)
 - (b^3*Tanh[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^4(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-b^3+\frac{a^3}{x^4}+\frac{a^2(a+3b)}{x^2}-\frac{(a+b)^3}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a^2(a+3b)\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{b^3\tanh(c+dx}{d}$$

$$= (a+b)^3x - \frac{a^2(a+3b)\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{b^3 \text{t}}{}$$

Mathematica [A]
time = 0.88, size = 82, normalized size = 1.39

$$\frac{\left(-3b^3-3a^2(a+3b)\coth^2(c+dx)-a^3\coth^4(c+dx)+3(a+b)^3\tanh^{-1}\left(\sqrt{\coth^2(c+dx)}\right)\sqrt{\coth^2(c+dx)}\right)\tanh(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((-3*b^3 - 3*a^2*(a + 3*b)*Coth[c + d*x]^2 - a^3*Coth[c + d*x]^4 + 3*(a + b
)^3*ArcTanh[Sqrt[Coth[c + d*x]^2]]*Sqrt[Coth[c + d*x]^2])*Tanh[c + d*x])/(3
*d)
```

Maple [A]
time = 1.60, size = 80, normalized size = 1.36

method	result
derivativedivides	$\frac{a^3\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+3a^2b(dx+c-\coth(dx+c))+3a\,b^2(dx+c)+b^3(dx+c-\tanh(dx+c))}{d}$
default	$\frac{a^3\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+3a^2b(dx+c-\coth(dx+c))+3a\,b^2(dx+c)+b^3(dx+c-\tanh(dx+c))}{d}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x-\frac{2(6a^3e^{6dx+6c}+9a^2b\,e^{6dx+6c}-3b^3e^{6dx+6c}-9a^2b\,e^{4dx+4c}+9b^3e^{4dx+4c}-2a^3e^{2dx}}{3d(e^{2dx+2c}-1)^3(1+e^{2dx+2c})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+3*a^2*b*(d*x+c-coth(d*x+c))+
3*a*b^2*(d*x+c)+b^3*(d*x+c-tanh(d*x+c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(57) = 114$.

time = 0.31, size = 147, normalized size = 2.49

$$\frac{1}{3}a^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+b^3\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right)+3a^2b\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2dx-2c)}-1\right)}\right)+3ab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^3*(x
+ c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 3*a^2*b*(x + c/d + 2/(d*(e^(-2*d*x
- 2*c) - 1))) + 3*a*b^2*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(57) = 114$.

time = 0.35, size = 341, normalized size = 5.78

$$-\frac{(4a^3+9a^2b+3b^3)\cosh(dx+c)^4-4(4a^3+9a^2b+3b^3+3(a^3+3a^2b+3ab^2+b^3)dx)\cosh(dx+c)\sinh(dx+c)^3+(4a^3+9a^2b+3b^3)\sinh(dx+c)^4-9a^2b+9b^3+4(a^3-3b^3)\cosh(dx+c)^2+2(2a^3-6b^3+3(4a^3+9a^2b+3b^3)\cosh(dx+c)^2)\sinh(dx+c)^2-4((4a^3+9a^2b+3b^3+3(a^3+3a^2b+3ab^2+b^3)dx)\cosh(dx+c)^3-(4a^3+9a^2b+3b^3+3(a^3+3a^2b+3ab^2+b^3)dx)\cosh(dx+c))\sinh(dx+c)}{12(d\cosh(dx+c)\sinh(dx+c)^3+(d\cosh(dx+c)^3-d\cosh(dx+c))\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*((4*a^3 + 9*a^2*b + 3*b^3)*cosh(d*x + c)^4 - 4*(4*a^3 + 9*a^2*b + 3*b
^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (4*a^3 + 9*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 9*a^2*b + 9*b^3 + 4*(a^3 - 3*b
^3)*cosh(d*x + c)^2 + 2*(2*a^3 - 6*b^3 + 3*(4*a^3 + 9*a^2*b + 3*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 - 4*((4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(57) = 114$.

time = 0.58, size = 135, normalized size = 2.29

$$\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)+\frac{6b^3}{e^{(2dx+2c)}+1}-\frac{2\left(6a^3e^{(4dx+4c)}+9a^2be^{(4dx+4c)}-6a^3e^{(2dx+2c)}-18a^2be^{(2dx+2c)}+4a^3+9a^2b\right)}{\left(e^{(2dx+2c)}-1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 6*b^3/(e^(2*d*x + 2*c) +
 1) - 2*(6*a^3*e^(4*d*x + 4*c) + 9*a^2*b*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x +
 2*c) - 18*a^2*b*e^(2*d*x + 2*c) + 4*a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3
)/d
```

Mupad [B]
time = 1.29, size = 219, normalized size = 3.71

$$x\,{\left(a+b\right)}^3+\frac{\frac{2\,a^2\,b}{d}-\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a^3+3\,b\,a^2\right)}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,\left(2\,a^3+3\,b\,a^2\right)}{3\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a^3+3\,b\,a^2\right)}{3\,d}-\frac{4\,a^2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}+\frac{2\,b^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\left(2\,a^3+3\,b\,a^2\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(a + b)^3 + ((2*a^2*b)/d - (2*exp(2*c + 2*d*x)*(3*a^2*b + 2*a^3))/(3*d))/
(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*(3*a^2*b + 2*a^3))/(3*d)
+ (2*exp(4*c + 4*d*x)*(3*a^2*b + 2*a^3))/(3*d) - (4*a^2*b*exp(2*c + 2*d*x))
/d)/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (2*b
^3)/(d*(exp(2*c + 2*d*x) + 1)) - (2*(3*a^2*b + 2*a^3))/(3*d*(exp(2*c + 2*d*
x) - 1))
```

3.165 $\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$-\frac{a^2(a+3b)\coth^2(c+dx)}{2d}-\frac{a^3\coth^4(c+dx)}{4d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}+\frac{a(a^2+3ab+3b^2)\log(\tanh(c+$$

```
[Out] -1/2*a^2*(a+3*b)*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d+(a+b)^3*ln(cosh(d*
x+c))/d+a*(a^2+3*a*b+3*b^2)*ln(tanh(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{a^3\coth^4(c+dx)}{4d}+\frac{a(a^2+3ab+3b^2)\log(\tanh(c+dx))}{d}-\frac{a^2(a+3b)\coth^2(c+dx)}{2d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(a^2*(a + 3*b)*Coth[c + d*x]^2)/d - (a^3*Coth[c + d*x]^4)/(4*d) + ((a
+ b)^3*Log[Cosh[c + d*x]])/d + (a*(a^2 + 3*a*b + 3*b^2)*Log[Tanh[c + d*x]])
/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^3}{x^5(1-x^2)}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^3}{(1-x)x^3}\,dx,x,\tanh^2(c+dx)\right)}{2d}$$
$$=\frac{\text{Subst}\left(\int\left(-\frac{(a+b)^3}{-1+x}+\frac{a^3}{x^3}+\frac{a^2(a+3b)}{x^2}+\frac{a(a^2+3ab+3b^2)}{x}\right)\,dx,x,\tanh^2\right.}{2d}$$
$$=-\frac{a^2(a+3b)\coth^2(c+dx)}{2d}-\frac{a^3\coth^4(c+dx)}{4d}+\frac{(a+b)^3\log(c}{d}$$

Mathematica [A]
time = 0.39, size = 67, normalized size = 0.81

$$\frac{-a^2(a+3b)\coth^2(c+dx)-\frac{1}{2}a^3\coth^4(c+dx)+2(a+b)^3\log(\sinh(c+dx))-2b^3\log(\tanh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (-(a^2*(a + 3*b)*Coth[c + d*x]^2) - (a^3*Coth[c + d*x]^4)/2 + 2*(a + b)^3*L
og[Sinh[c + d*x]] - 2*b^3*Log[Tanh[c + d*x]])/(2*d)
```

Maple [A]
time = 1.73, size = 86, normalized size = 1.04

method	result
derivativedivides	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+3a^2b\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)+3a\,b^2\ln(\sinh(dx+c))+b^3}{d}$
default	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+3a^2b\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)+3a\,b^2\ln(\sinh(dx+c))+b^3}{d}$
risch	$-a^3x-3a^2bx-3a\,b^2x-b^3x-\frac{2b^3c}{d}-\frac{2a^3c}{d}-\frac{6a^2bc}{d}-\frac{6a\,b^2c}{d}-\frac{2a^2\mathrm{e}^{2dx+2c}\left(2a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}-\right.}{d\left(\mathrm{e}^{2dx+2c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+3*a^2*b*(ln(
sinh(d*x+c))-1/2*coth(d*x+c)^2)+3*a*b^2*ln(sinh(d*x+c))+b^3*ln(cosh(d*x+c))
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(79) = 158$.
time = 0.28, size = 264, normalized size = 3.18

$$a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}\right)+3\,a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)+\frac{b^3\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{3\,ab^2\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 3*a^2*
b*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*
x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^3*log(e^(d*x
+ c) + e^(-d*x - c))/d + 3*a*b^2*log(e^(d*x + c) - e^(-d*x - c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2393 vs. $2(79) = 158$.
time = 0.40, size = 2393, normalized size = 28.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x*sinh(d*x + c)^8 + 2*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^2 + 2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*
sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3
 + 3*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
+ c)^4 - 2*a^3 - 6*a^2*b + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 15*(2*a^
3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 5*(
2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 -
(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(2*a^3 + 3*a^2*b -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*a^3 + 3*a^2*b - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x - 6*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - (b^3*cosh(d*x + c)^8 + 8*b
```

```
^3*cosh(d*x + c)*sinh(d*x + c)^7 + b^3*sinh(d*x + c)^8 - 4*b^3*cosh(d*x + c
)^6 + 6*b^3*cosh(d*x + c)^4 + 4*(7*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)
^6 + 8*(7*b^3*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c))*sinh(d*x + c)^5 - 4*b^
3*cosh(d*x + c)^2 + 2*(35*b^3*cosh(d*x + c)^4 - 30*b^3*cosh(d*x + c)^2 + 3*
b^3)*sinh(d*x + c)^4 + 8*(7*b^3*cosh(d*x + c)^5 - 10*b^3*cosh(d*x + c)^3 +
3*b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + 4*(7*b^3*cosh(d*x + c)^6 - 15*
b^3*cosh(d*x + c)^4 + 9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^2 + 8*(b^3
*cosh(d*x + c)^7 - 3*b^3*cosh(d*x + c)^5 + 3*b^3*cosh(d*x + c)^3 - b^3*cosh
(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) - ((a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b + 3*a*b
^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2)*sinh(d*x + c)
^8 - 4*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^6 - 4*(a^3 + 3*a^2*b + 3*a*b
^2 - 7*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a
^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^5 + 6*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 + 2
*(35*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 + 3*a^3 + 9*a^2*b + 9*a*b^2
- 30*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3
 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 - 10*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d
*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + a^
3 + 3*a^2*b + 3*a*b^2 - 4*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^2 + 4*(7*
(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^6 - 15*(a^3 + 3*a^2*b + 3*a*b^2)*co
sh(d*x + c)^4 - a^3 - 3*a^2*b - 3*a*b^2 + 9*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^7
- 3*(a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 3*(a^3 + 3*a^2*b + 3*a*b^2)
*cosh(d*x + c)^3 - (a^3 + 3*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 + 3*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (2*a^3 + 3*a^2*b - 2*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^
8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x +
c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3
 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(
d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x
+ c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*co
sh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*
x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5
+ 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(79) = 158.
time = 0.65, size = 267, normalized size = 3.22

$$\frac{2\,b^{3}\log\left(e^{\left(2\,dx+2\,c\right)}+e^{\left(-2\,dx-2\,c\right)}+2\right)+2\,\left(a^{3}+3\,a^{2}b+3\,ab^{2}\right)\log\left(e^{\left(2\,dx+2\,c\right)}+e^{\left(-2\,dx-2\,c\right)}-2\right)-\frac{3\,a^{3}\left(e^{\left(2\,dx+2\,c\right)}+e^{\left(-2\,dx-2\,c\right)}\right)^{2}+9\,a^{2}b\left(e^{\left(2\,dx+2\,c\right)}+e^{\left(-2\,dx-2\,c\right)}\right)^{2}+9\,ab^{2}\left(e^{\left(2\,dx+2\,c\right)}+e^{\left(-2\,dx-2\,c\right)}\right)^{2}+4\,a^{3}\left(e^{\left(2\,dx+2\,c\right)}+e^{\left(-2\,dx-2\,c\right)}\right)-12\,a^{2}b\left(e^{\left(2\,dx+2\,c\right)}+e^{\left(-2\,dx-2\,c\right)}\right)-36\,ab^{2}\left(e^{\left(2\,dx+2\,c\right)}+e^{\left(-2\,dx-2\,c\right)}\right)-4\,a^{3}-12\,a^{2}b+36\,ab^{2}}{\left(e^{\left(2\,dx+2\,c\right)}+e^{\left(-2\,dx-2\,c\right)}-2\right)^{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + 2*(a^3 + 3*a^2*b +
 3*a*b^2)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (3*a^3*(e^(2*d*x +
2*c) + e^(-2*d*x - 2*c))^2 + 9*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2
 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 4*a^3*(e^(2*d*x + 2*c)
+ e^(-2*d*x - 2*c)) - 12*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 36*a*
b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*a^3 - 12*a^2*b + 36*a*b^2)/(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)^2)/d
```

Mupad [B]
time = 0.40, size = 381, normalized size = 4.59

$$\frac{\ln\left(e^{4c+4dx}-1\right)\,\left(b^{3}d+d\,\left(a^{3}+3\,a^{2}b+3\,a\,b^{2}\right)\right)}{2\,d^{2}}-x\,\left(a+b\right)^{3}-\frac{2\,\left(2\,a^{3}+3\,b\,a^{2}\right)}{d\,\left(e^{2c+2dx}-1\right)}-\frac{8\,a^{3}}{d\,\left(3\,e^{2c+2dx}-3\,e^{4c+4dx}+e^{6c+6dx}-1\right)}-\frac{\mathrm{atan}\left(\frac{e^{2c}\,e^{2dx}\,\left(a^{3}\,\sqrt{-d^{2}}-b^{3}\,\sqrt{-d^{2}}+3\,a\,b^{2}\,\sqrt{-d^{2}}+3\,a^{2}b\,\sqrt{-d^{2}}\right)}{d\,\sqrt{a^{6}+6\,a^{5}b+15\,a^{4}b^{2}+16\,a^{3}b^{3}+3\,a^{2}b^{4}-6\,a\,b^{5}+b^{6}}}\right)\,\sqrt{a^{6}+6\,a^{5}b+15\,a^{4}b^{2}+16\,a^{3}b^{3}+3\,a^{2}b^{4}-6\,a\,b^{5}+b^{6}}}{\sqrt{-d^{2}}}-\frac{2\,\left(4\,a^{3}+3\,b\,a^{2}\right)}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}-\frac{4\,a^{3}}{d\,\left(6\,e^{4c+4dx}-4\,e^{2c+2dx}-4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (log(exp(4*c + 4*d*x) - 1)*(b^3*d + d*(3*a*b^2 + 3*a^2*b + a^3)))/(2*d^2) -
 x*(a + b)^3 - (2*(3*a^2*b + 2*a^3))/(d*(exp(2*c + 2*d*x) - 1)) - (8*a^3)/(
d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (atan
((exp(2*c)*exp(2*d*x)*(a^3*(-d^2)^(1/2) - b^3*(-d^2)^(1/2) + 3*a*b^2*(-d^2)
^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a^5*b - 6*a*b^5 + a^6 + b^6 + 3*a^2*b
^4 + 16*a^3*b^3 + 15*a^4*b^2)^(1/2)))*(6*a^5*b - 6*a*b^5 + a^6 + b^6 + 3*a^
2*b^4 + 16*a^3*b^3 + 15*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (2*(3*a^2*b + 4*a^3)
)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (4*a^3)/(d*(6*exp(4*c +
 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.166 $\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=74

$$(a+b)^3x-\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}-\frac{a^2(a+3b)\coth^3(c+dx)}{3d}-\frac{a^3\coth^5(c+dx)}{5d}$$

```
[Out] (a+b)^3*x-a*(a^2+3*a*b+3*b^2)*coth(d*x+c)/d-1/3*a^2*(a+3*b)*coth(d*x+c)^3/d
-1/5*a^3*coth(d*x+c)^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3751, 472, 213}

$$-\frac{a^3\coth^5(c+dx)}{5d}-\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}-\frac{a^2(a+3b)\coth^3(c+dx)}{3d}+x(a+b)^3$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a + b)^3*x - (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d - (a^2*(a + 3*b)*Co
th[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^6(1-x^2)}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^6}+\frac{a^2(a+3b)}{x^4}+\frac{a(a^2+3ab+3b^2)}{x^2}-\frac{(a+b)^3}{-1+x^2}\right)dx, x, \tanh(\right.}{d}$$

$$= -\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}-\frac{a^2(a+3b)\coth^3(c+dx)}{3d}$$

$$= (a+b)^3x-\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}-\frac{a^2(a+3b)\coth}{3d}$$

Mathematica [A]
time = 1.24, size = 100, normalized size = 1.35

$$-\frac{a\coth(c+dx)\left(15(a^2+3ab+3b^2)+5a(a+3b)\coth^2(c+dx)+3a^2\coth^4(c+dx)\right)}{15d}+\frac{(a+b)^3\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)\tanh(c+dx)}{d\sqrt{\tanh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/15*(a*Coth[c + d*x]*(15*(a^2 + 3*a*b + 3*b^2) + 5*a*(a + 3*b)*Coth[c + d
*x]^2 + 3*a^2*Coth[c + d*x]^4))/d + ((a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2
]]*Tanh[c + d*x])/(d*Sqrt[Tanh[c + d*x]^2])
```

Maple [A]
time = 1.80, size = 100, normalized size = 1.35

method	result
derivativedivides	$\frac{a^3\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}-\frac{\left(\coth^5(dx+c)\right)}{5}\right)+3a^2b\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+3a\,b^2(dx+c-\text{co}}{d}$
default	$\frac{a^3\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}-\frac{\left(\coth^5(dx+c)\right)}{5}\right)+3a^2b\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+3a\,b^2(dx+c-\text{co}}{d}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x-\frac{2a\left(45a^2e^{8dx+8c}+90ab\,e^{8dx+8c}+45b^2e^{8dx+8c}-90a^2e^{6dx+6c}-270ab\,e^{6dx+6c}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3-1/5*coth(d*x+c)^5)+3*a^2*b*(d
*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+3*a*b^2*(d*x+c-coth(d*x+c))+(d*x+c)*b^3
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(70) = 140$.

time = 0.30, size = 239, normalized size = 3.23

$$\frac{1}{15}a^3\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}-140e^{(-4dx-4c)}+90e^{(-6dx-6c)}-45e^{(-8dx-8c)}-23\right)}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}\right)+a^2b\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+3ab^2\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2dx-2c)}-1\right)}\right)+b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*a^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + a^2*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4
*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1
))) + 3*a*b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(70) = 140$.

time = 0.36, size = 557, normalized size = 7.53

$$-\frac{(23a^3+60a^2b+45ab^2)\cosh(dx+c)^5+5(23a^3+60a^2b+45ab^2)\cosh(dx+c)\sinh(dx+c)^4-(23a^3+60a^2b+45ab^2+15(a^3+3a^2b+3ab^2+b^3)dx)\sinh(dx+c)^5-5(5a^3+24a^2b+27ab^2)\cosh(dx+c)^3+5(23a^3+60a^2b+45ab^2+15(a^3+3a^2b+3ab^2+b^3)dx-2(23a^3+60a^2b+45ab^2+15(a^3+3a^2b+3ab^2+b^3)dx)\cosh(dx+c)^2)\sinh(dx+c)^3+5(2(23a^3+60a^2b+45ab^2)\cosh(dx+c)^3-3(5a^3+24a^2b+27ab^2)\cosh(dx+c))\sinh(dx+c)^2+10(5a^3+6a^2b+9ab^2)\cosh(dx+c)-5((23a^3+60a^2b+45ab^2+15(a^3+3a^2b+3ab^2+b^3)dx)\cosh(dx+c)^4+46a^3+120a^2b+90ab^2+30(a^3+3a^2b+3ab^2+b^3)dx-3(23a^3+60a^2b+45ab^2+15(a^3+3a^2b+3ab^2+b^3)dx)\cosh(dx+c)^2)\sinh(dx+c)}{15\left(d\sinh(dx+c)^5+5(2d\cosh(dx+c)^2-d)\sinh(dx+c)^3+5(d\cosh(dx+c)^4-3d\cosh(dx+c)^2+2d)\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^5 + 5*(23*a^3 + 60*a^2*
b + 45*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (23*a^3 + 60*a^2*b + 45*a*b^2
 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^5 - 5*(5*a^3 + 24*
a^2*b + 27*a*b^2)*cosh(d*x + c)^3 + 5*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*
(23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^3 - 3*(5*a^3 + 24*a^2*b + 27*a
*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 6*a^2*b + 9*a*b^2)*cosh(
d*x + c) - 5*((23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x)*cosh(d*x + c)^4 + 46*a^3 + 120*a^2*b + 90*a*b^2 + 30*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x - 3*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^
5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*
d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(70) = 140.
time = 0.63, size = 241, normalized size = 3.26

$$\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c)-\frac{2\,\left(45\,a^3e^{(8\,dx+8\,c)}+90\,a^2be^{(8\,dx+8\,c)}+45\,ab^2e^{(8\,dx+8\,c)}-90\,a^3e^{(6\,dx+6\,c)}-270\,a^2be^{(6\,dx+6\,c)}-180\,ab^2e^{(6\,dx+6\,c)}+140\,a^3e^{(4\,dx+4\,c)}+330\,a^2be^{(4\,dx+4\,c)}+270\,ab^2e^{(4\,dx+4\,c)}-70\,a^3e^{(2\,dx+2\,c)}-210\,a^2be^{(2\,dx+2\,c)}-180\,ab^2e^{(2\,dx+2\,c)}+23\,a^3+60\,a^2b+45\,ab^2\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 2*(45*a^3*e^(8*d*x + 8
*c) + 90*a^2*b*e^(8*d*x + 8*c) + 45*a*b^2*e^(8*d*x + 8*c) - 90*a^3*e^(6*d*x
 + 6*c) - 270*a^2*b*e^(6*d*x + 6*c) - 180*a*b^2*e^(6*d*x + 6*c) + 140*a^3*e
^(4*d*x + 4*c) + 330*a^2*b*e^(4*d*x + 4*c) + 270*a*b^2*e^(4*d*x + 4*c) - 70
*a^3*e^(2*d*x + 2*c) - 210*a^2*b*e^(2*d*x + 2*c) - 180*a*b^2*e^(2*d*x + 2*c
) + 23*a^3 + 60*a^2*b + 45*a*b^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [B]
time = 1.30, size = 568, normalized size = 7.68

$$x\,(a+b)^3-\frac{\frac{6\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}+\frac{6\,\mathrm{e}^{8\,c+8\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}-\frac{24\,\mathrm{e}^{2\,c+2\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}-\frac{24\,\mathrm{e}^{6\,c+6\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}+\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(5\,a^3+6\,a^2\,b+9\,a\,b^2)}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1}-\frac{\frac{2\,(5\,a^3+6\,a^2\,b+9\,a\,b^2)}{15\,d}+\frac{6\,\mathrm{e}^{4\,c+4\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}-\frac{12\,\mathrm{e}^{2\,c+2\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}+\frac{\frac{6\,(a^2\,b+a\,b^2)}{5\,d}-\frac{6\,\mathrm{e}^{6\,c+6\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}+\frac{18\,\mathrm{e}^{4\,c+4\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}-\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(5\,a^3+6\,a^2\,b+9\,a\,b^2)}{5\,d}}{6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1}+\frac{\frac{6\,(a^2\,b+a\,b^2)}{5\,d}-\frac{6\,\mathrm{e}^{2\,c+2\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1}-\frac{6\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d\,(\mathrm{e}^{2\,c+2\,d\,x}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] x*(a + b)^3 - ((6*(a*b^2 + 2*a^2*b + a^3))/(5*d) + (6*exp(8*c + 8*d*x)*(a*b
^2 + 2*a^2*b + a^3))/(5*d) - (24*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d) -
(24*exp(6*c + 6*d*x)*(a*b^2 + a^2*b))/(5*d) + (4*exp(4*c + 4*d*x)*(9*a*b^2
+ 6*a^2*b + 5*a^3))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(9*a*b
^2 + 6*a^2*b + 5*a^3))/(15*d) + (6*exp(4*c + 4*d*x)*(a*b^2 + 2*a^2*b + a^3)
)/(5*d) - (12*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d))/(3*exp(2*c + 2*d*x)
- 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + ((6*(a*b^2 + a^2*b))/(5*d) -
 (6*exp(6*c + 6*d*x)*(a*b^2 + 2*a^2*b + a^3))/(5*d) + (18*exp(4*c + 4*d*x)*
(a*b^2 + a^2*b))/(5*d) - (2*exp(2*c + 2*d*x)*(9*a*b^2 + 6*a^2*b + 5*a^3))/(
5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1) + ((6*(a*b^2 + a^2*b))/(5*d) - (6*exp(2*c + 2*d*x)*(a*b^2
+ 2*a^2*b + a^3))/(5*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (6*(
a*b^2 + 2*a^2*b + a^3))/(5*d*(exp(2*c + 2*d*x) - 1))
```

3.167 $\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=103

$$-\frac{a(a^2+3ab+3b^2)\coth^2(c+dx)}{2d}-\frac{a^2(a+3b)\coth^4(c+dx)}{4d}-\frac{a^3\coth^6(c+dx)}{6d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}-$$

```
[Out] -1/2*a*(a^2+3*a*b+3*b^2)*coth(d*x+c)^2/d-1/4*a^2*(a+3*b)*coth(d*x+c)^4/d-1/
6*a^3*coth(d*x+c)^6/d+(a+b)^3*ln(cosh(d*x+c))/d+(a+b)^3*ln(tanh(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3751, 457, 90}

$$-\frac{a^3\coth^6(c+dx)}{6d}-\frac{a(a^2+3ab+3b^2)\coth^2(c+dx)}{2d}-\frac{a^2(a+3b)\coth^4(c+dx)}{4d}+\frac{(a+b)^3\log(\tanh(c+dx))}{d}+\frac{(a+b)^3\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x]^2)/d - (a^2*(a + 3*b)*Coth[c +
d*x]^4)/(4*d) - (a^3*Coth[c + d*x]^6)/(6*d) + ((a + b)^3*Log[Cosh[c + d*x]]
)/d + ((a + b)^3*Log[Tanh[c + d*x]])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^7(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x^4}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^3}{-1+x}+\frac{a^3}{x^4}+\frac{a^2(a+3b)}{x^3}+\frac{a(a^2+3ab+3b^2)}{x^2}+\frac{(a+b)^3}{x}\right)\,d\right.}{2d}$$

$$= -\frac{a(a^2+3ab+3b^2)\coth^2(c+dx)}{2d} - \frac{a^2(a+3b)\coth^4(c+dx}{4d}$$

Mathematica [A]
time = 0.18, size = 76, normalized size = 0.74

$$-\frac{a(a+b)^2\coth^2(c+dx)+\frac{1}{2}(a+b)\left(b+a\coth^2(c+dx)\right)^2+\frac{1}{3}\left(b+a\coth^2(c+dx)\right)^3-2(a+b)^3\log(\sinh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(a*(a + b)^2*Coth[c + d*x]^2 + ((a + b)*(b + a*Coth[c + d*x]^2)^2)/2 +
 (b + a*Coth[c + d*x]^2)^3/3 - 2*(a + b)^3*Log[Sinh[c + d*x]])/d
```

Maple [A]
time = 1.78, size = 117, normalized size = 1.14

method	result
derivativedivides	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}-\frac{\left(\coth^6(dx+c)\right)}{6}\right)+3a^2b\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(\right.}{4}\right.}{d}$
default	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}-\frac{\left(\coth^6(dx+c)\right)}{6}\right)+3a^2b\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(\right.}{4}\right.}{d}$
risch	$-a^3x-3a^2bx-3a\,b^2x-b^3x-\frac{2a^3c}{d}-\frac{6a^2bc}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}-\frac{2a\,e^{2dx+2c}\left(9a^2e^{8dx+8c}+18ab\,e^{8dx}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4-1/6*coth(d*x+
c)^6)+3*a^2*b*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+3*a*b^2
*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)+b^3*ln(sinh(d*x+c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(97) = 194$.
time = 0.30, size = 420, normalized size = 4.08

$$\frac{1}{3}a^3\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}-18e^{(-4dx-4c)}+34e^{(-6dx-6c)}-18e^{(-8dx-8c)}+9e^{(-10dx-10c)}\right)}{d\left(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1\right)}\right)+3a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1\right)}\right)+3ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1\right)}\right)+\frac{b^3\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 3*a^2*b*(x + c/d + log(e^(-d*x - c) + 1)/d
 + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*
c) - e^(-8*d*x - 8*c) - 1))) + 3*a*b^2*(x + c/d + log(e^(-d*x - c) + 1)/d +
 log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-
4*d*x - 4*c) - 1))) + b^3*log(e^(d*x + c) - e^(-d*x - c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4305 vs. $2(97) = 194$.
time = 0.39, size = 4305, normalized size = 41.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 + 36*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^12 + 18*(a^3 + 2*a^2*b + a*b^2 - (a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 18*(11*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^10 + 60*(11*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 9*(4*a^3 + 12*a^2*b + 8*a
*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 + 9*(165*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 4*a^3 - 12*a^2*b - 8*a*b^
2 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 90*(a^3 + 2*a^2*b + a*b^2 - (a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 30*(a^3 + 2*a^2*b +
a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 12*
a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sin
h(d*x + c)^7 + 4*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^
```

2 + b^3)*d*x)*cosh(d*x + c)^6 + 4*(693*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^6 + 945*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d*x)*cosh(d*x + c)^4 + 17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x - 63*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 + 189*(a^3 + 2*a^2*b + a*b^2 - (a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 21*(4*a^3 + 12*a^2*b + 8
*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (17*a^3 +
 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 9*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 3*(495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x*cosh(d*x + c)^8 + 1260*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d*x)*cosh(d*x + c)^6 - 210*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 12*a^3 - 36*a^2*b - 24*a*b^2
 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 20*(17*a^3 + 36*a^2*b + 27*a*b^
2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 4*(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^9 + 540*(a^3 +
 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 1
26*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c)^5 + 20*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x)*cosh(d*x + c)^3 - 9*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x + 18*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 6*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x*cosh(d*x + c)^10 + 135*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*d*x)*cosh(d*x + c)^8 - 42*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 10*(17*a^3 + 36*a^2*b + 27*a*b
^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 3*a^3 + 6*a^
2*b + 3*a*b^2 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 9*(4*a^3 + 12*a^2*b
 + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d
x + c)^2 - 3((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^12 + 12*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*sinh(d*x + c)^12 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d
x + c)^10 - 6(a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 11*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*
sinh(d*x + c)^9 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 15*(
33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 - 18*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^8 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 - 30*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^7 - 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 4*(231*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 315*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 5*a^3 - 15*a^2*b - 15*a*b^2 -
 5*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)

```
^6 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 - 63*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*(33*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 - 84*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^6 + 70*(a^3 + 3*a^2*b + 3*a*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(97) = 194.
time = 0.70, size = 217, normalized size = 2.11

$$-\frac{3\,(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c)-3\,(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\,\left(9\,(a^3+2\,a^2b+ab^2)e^{(10\,dx+10\,c)}-18\,(a^3+3\,a^2b+2\,ab^2)e^{(8\,dx+8\,c)}+2\,(17\,a^3+36\,a^2b+27\,ab^2)e^{(6\,dx+6\,c)}-18\,(a^3+3\,a^2b+2\,ab^2)e^{(4\,dx+4\,c)}+9\,(a^3+2\,a^2b+ab^2)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^6}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*(9*(a^3 + 2*a^2*b + a*b^2)*e^(
10*d*x + 10*c) - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^(8*d*x + 8*c) + 2*(17*a^3 +
 36*a^2*b + 27*a*b^2)*e^(6*d*x + 6*c) - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^(4*d
*x + 4*c) + 9*(a^3 + 2*a^2*b + a*b^2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1
)^6)/d
```

Mupad [B]
time = 0.32, size = 380, normalized size = 3.69

$$\frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d}-\frac{4\,\left(11\,a^3+3\,b\,a^2\right)}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-\frac{32\,a^3}{3\,d\,\left(15\,\mathrm{e}^{4\,c+4\,d\,x}-6\,\mathrm{e}^{2\,c+2\,d\,x}-20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}-6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}-\frac{6\,\left(3\,a^3+4\,a^2\,b+a\,b^2\right)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{6\,\left(a^3+2\,a^2\,b+a\,b^2\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{8\,\left(13\,a^3+9\,b\,a^2\right)}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{32\,a^3}{d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1\right)}-x\,{\left(a+b\right)}^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d - (4*(3*a^
2*b + 11*a^3))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*
d*x) + exp(8*c + 8*d*x) + 1)) - (32*a^3)/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(
2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*
d*x) + exp(12*c + 12*d*x) + 1)) - (6*(a*b^2 + 4*a^2*b + 3*a^3))/(d*(exp(4*c
```

```
 + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (6*(a*b^2 + 2*a^2*b + a^3))/(d*(exp(
2*c + 2*d*x) - 1)) - (8*(9*a^2*b + 13*a^3))/(3*d*(3*exp(2*c + 2*d*x) - 3*ex
p(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (32*a^3)/(d*(5*exp(2*c + 2*d*x) -
 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c
+ 10*d*x) - 1)) - x*(a + b)^3
```

3.168 $\int \left(a+b\tanh^2(c+dx)\right)^4 \, dx$

Optimal. Leaf size=110

$$(a+b)^4x-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^3(4a+b)\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)^4*x-b*(2*a+b)*(2*a^2+2*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(6*a^2+4*a*b+b^
2)*tanh(d*x+c)^3/d-1/5*b^3*(4*a+b)*tanh(d*x+c)^5/d-1/7*b^4*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 398, 212}

$$-\frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^3(4a+b)\tanh^5(c+dx)}{5d}+x(a+b)^4-\frac{b^4\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^4,x]
```

```
[Out] (a + b)^4*x - (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(6
*a^2 + 4*a*b + b^2)*Tanh[c + d*x]^3)/(3*d) - (b^3*(4*a + b)*Tanh[c + d*x]^5
)/(5*d) - (b^4*Tanh[c + d*x]^7)/(7*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tanh^2(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^4}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-b(2a+b)\left(2a^2+2ab+b^2\right)-b^2\left(6a^2+4ab+b^2\right)x^2-b^3(4a+b\right.\right.}{d}$$

$$= -\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+d}{3d}$$

$$= (a+b)^4x-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2\left(6a^2+4ab+b^2\right)\text{ta}}{3d}$$

Mathematica [A]
time = 1.24, size = 128, normalized size = 1.16

$$\frac{\tanh(c+dx)\left(\frac{105(a+b)^4\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}-b\left(105\left(4a^3+6a^2b+4ab^2+b^3\right)+35b\left(6a^2+4ab+b^2\right)\tanh^2(c+dx)+21b^2(4a+b)\tanh^4(c+dx)+15b^3\tanh^6(c+dx)\right)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^4,x]
```

```
[Out] (Tanh[c + d*x]*((105*(a + b)^4*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
+ d*x]^2] - b*(105*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3) + 35*b*(6*a^2 + 4*a*b
+ b^2)*Tanh[c + d*x]^2 + 21*b^2*(4*a + b)*Tanh[c + d*x]^4 + 15*b^3*Tanh[c +
 d*x]^6)))/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(104) = 208$.
time = 0.31, size = 214, normalized size = 1.95

method	result
derivativedivides	$\frac{\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\ln(1+\tanh(dx+c))}{2}-4a\,b^3\tanh(dx+c)-4a^3b\tanh(dx+c)-6a^2b^2\tanh(dx+c)-\frac{4a\,b^3\left(\tanh^3(dx+\right.}{3}$
default	$\frac{\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\ln(1+\tanh(dx+c))}{2}-4a\,b^3\tanh(dx+c)-4a^3b\tanh(dx+c)-6a^2b^2\tanh(dx+c)-\frac{4a\,b^3\left(\tanh^3(dx+\right.}{3}$
risch	$x\,a^4+4a^3bx+6a^2b^2x+4a\,b^3x+b^4x+\frac{8b\left(161a\,b^2+315a^2b\,e^{12dx+12c}+315a\,b^2e^{12dx+12c}+1575a^2b\,e^{10d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*ln(1+tanh(d*x+c))-4*a*b^3*tanh
(d*x+c)-4*a^3*b*tanh(d*x+c)-6*a^2*b^2*tanh(d*x+c)-4/3*a*b^3*tanh(d*x+c)^3-2
```

```
*a^2*b^2*tanh(d*x+c)^3-4/5*a*b^3*tanh(d*x+c)^5-1/2*(a^4+4*a^3*b+6*a^2*b^2+4
*a*b^3+b^4)*ln(tanh(d*x+c)-1)-b^4*tanh(d*x+c)-1/7*b^4*tanh(d*x+c)^7-1/3*b^4
*tanh(d*x+c)^3-1/5*b^4*tanh(d*x+c)^5)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(104) = 208$.
time = 0.30, size = 410, normalized size = 3.73

$$\frac{1}{105}\,b^4\left(105\,x+\frac{105\,c}{d}-\frac{8\,\left(203\,e^{(-2\,dx-2\,c)}+609\,e^{(-4\,dx-4\,c)}+770\,e^{(-6\,dx-6\,c)}+770\,e^{(-8\,dx-8\,c)}+315\,e^{(-10\,dx-10\,c)}+105\,e^{(-12\,dx-12\,c)}+44\right)}{d(7\,e^{(-2\,dx-2\,c)}+21\,e^{(-4\,dx-4\,c)}+35\,e^{(-6\,dx-6\,c)}+35\,e^{(-8\,dx-8\,c)}+21\,e^{(-10\,dx-10\,c)}+7\,e^{(-12\,dx-12\,c)}+e^{(-14\,dx-14\,c)}+1)}\right)+\frac{4}{15}\,ab^3\left(15\,x+\frac{15\,c}{d}-\frac{2\,\left(70\,e^{(-2\,dx-2\,c)}+140\,e^{(-4\,dx-4\,c)}+90\,e^{(-6\,dx-6\,c)}+45\,e^{(-8\,dx-8\,c)}+23\right)}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)+2\,a^2b^2\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)+4\,a^3b\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)+a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/105*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c)
 + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 1
05*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-1
2*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 4/15*a*b^3*(15*x + 15*c/d - 2*(
70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8
*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^2*b^2*(3*x
 + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x
- 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a^3*b*(x + c/d -
2/(d*(e^(-2*d*x - 2*c) + 1))) + a^4*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. $2(104) = 208$.
time = 0.37, size = 1176, normalized size = 10.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*((420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^7 + 7*(420*a^3*b + 840*a^2*
b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
*d*x)*cosh(d*x + c)*sinh(d*x + c)^6 - 4*(105*a^3*b + 210*a^2*b^2 + 161*a*b^
3 + 44*b^4)*sinh(d*x + c)^7 + 7*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*
b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^5
- 28*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 + 14*b^4 + 3*(105*a^3*b + 210*a^2*b
^2 + 161*a*b^3 + 44*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((420*a^3*b
+ 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4)*d*x)*cosh(d*x + c)^3 + (420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 17
6*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c))
*sinh(d*x + c)^4 + 21*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*
```

```
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 - 28*(5*(1
05*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)*cosh(d*x + c)^4 + 135*a^3*b +
180*a^2*b^2 + 123*a*b^3 + 42*b^4 + 10*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 +
14*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(3*(420*a^3*b + 840*a^2*b^2 +
644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*
cosh(d*x + c)^5 + 10*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 + 9*(420*a^
3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(420*a^3*b + 840*a^
2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d*x)*cosh(d*x + c) - 28*((105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)*
cosh(d*x + c)^6 + 5*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 + 14*b^4)*cosh(d*x +
 c)^4 + 75*a^3*b + 90*a^2*b^2 + 75*a*b^3 + 9*(45*a^3*b + 60*a^2*b^2 + 41*a*
b^3 + 14*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh
(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d
*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x +
c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cos
h(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(99) = 198$.
time = 0.28, size = 209, normalized size = 1.90

$$\begin{cases} a^4x + 4a^3bx - \frac{4a^3b\tanh(c+dx)}{d} + 6a^2b^2x - \frac{2a^2b^2\tanh^3(c+dx)}{d} - \frac{6a^2b^2\tanh(c+dx)}{d} + 4ab^3x - \frac{4ab^3\tanh^5(c+dx)}{5d} - \frac{4ab^3\tanh^3(c+dx)}{3d} - \frac{4ab^3\tanh(c+dx)}{d} + b^4x - \frac{b^4\tanh^7(c+dx)}{7d} - \frac{b^4\tanh^5(c+dx)}{5d} - \frac{b^4\tanh^3(c+dx)}{3d} - \frac{b^4\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**2)**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*x - 4*a**3*b*tanh(c + d*x)/d + 6*a**2*b**2*x -
 2*a**2*b**2*tanh(c + d*x)**3/d - 6*a**2*b**2*tanh(c + d*x)/d + 4*a*b**3*x
- 4*a*b**3*tanh(c + d*x)**5/(5*d) - 4*a*b**3*tanh(c + d*x)**3/(3*d) - 4*a*b
**3*tanh(c + d*x)/d + b**4*x - b**4*tanh(c + d*x)**7/(7*d) - b**4*tanh(c +
d*x)**5/(5*d) - b**4*tanh(c + d*x)**3/(3*d) - b**4*tanh(c + d*x)/d, Ne(d, 0
)), (x*(a + b*tanh(c)**2)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(104) = 208$.
time = 0.46, size = 447, normalized size = 4.06

$$\frac{105\left(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\right)(dx + c) + \frac{8\left(105a^3be^{(12dx+12c)} + 315a^2b^2e^{(12dx+12c)} + 315ab^3e^{(12dx+12c)} + 105b^4e^{(12dx+12c)} + 630a^3be^{(10dx+10c)} + 1575a^2b^2e^{(10dx+10c)} + 1260ab^3e^{(10dx+10c)} + 315b^4e^{(10dx+10c)} + 1575a^3be^{(8dx+8c)} + 3360a^2b^2e^{(8dx+8c)} + 2555ab^3e^{(8dx+8c)} + 770b^4e^{(8dx+8c)} + 2100a^3be^{(6dx+6c)} + 3990a^2b^2e^{(6dx+6c)} + 3080ab^3e^{(6dx+6c)} + 770b^4e^{(6dx+6c)} + 1575a^3be^{(4dx+4c)} + 2835a^2b^2e^{(4dx+4c)} + 2121ab^3e^{(4dx+4c)} + 609b^4e^{(4dx+4c)} + 630a^3be^{(2dx+2c)} + 1155a^2b^2e^{(2dx+2c)} + 812ab^3e^{(2dx+2c)} + 203b^4e^{(2dx+2c)} + 105a^3b + 210a^2b^2 + 161ab^3 + 44b^4\right)}{\left(e^{(2dx+2c)} + 1\right)^7}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(d*x + c) + 8*(105*a
^3*b*e^(12*d*x + 12*c) + 315*a^2*b^2*e^(12*d*x + 12*c) + 315*a*b^3*e^(12*d*
```

```
x + 12*c) + 105*b^4*e^(12*d*x + 12*c) + 630*a^3*b*e^(10*d*x + 10*c) + 1575*
a^2*b^2*e^(10*d*x + 10*c) + 1260*a*b^3*e^(10*d*x + 10*c) + 315*b^4*e^(10*d*
x + 10*c) + 1575*a^3*b*e^(8*d*x + 8*c) + 3360*a^2*b^2*e^(8*d*x + 8*c) + 255
5*a*b^3*e^(8*d*x + 8*c) + 770*b^4*e^(8*d*x + 8*c) + 2100*a^3*b*e^(6*d*x + 6
*c) + 3990*a^2*b^2*e^(6*d*x + 6*c) + 3080*a*b^3*e^(6*d*x + 6*c) + 770*b^4*e
^(6*d*x + 6*c) + 1575*a^3*b*e^(4*d*x + 4*c) + 2835*a^2*b^2*e^(4*d*x + 4*c)
+ 2121*a*b^3*e^(4*d*x + 4*c) + 609*b^4*e^(4*d*x + 4*c) + 630*a^3*b*e^(2*d*x
 + 2*c) + 1155*a^2*b^2*e^(2*d*x + 2*c) + 812*a*b^3*e^(2*d*x + 2*c) + 203*b^
4*e^(2*d*x + 2*c) + 105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)/(e^(2*d*x
 + 2*c) + 1)^7)/d
```

Mupad [B]
time = 0.20, size = 133, normalized size = 1.21

$$x\,\left(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right)-\frac{\mathrm{tanh}\left(c+d\,x\right)^3\,\left(6\,a^2\,b^2+4\,a\,b^3+b^4\right)}{3\,d}-\frac{\mathrm{tanh}\left(c+d\,x\right)^5\,\left(b^4+4\,a\,b^3\right)}{5\,d}-\frac{b^4\,\mathrm{tanh}\left(c+d\,x\right)^7}{7\,d}-\frac{b\,\mathrm{tanh}\left(c+d\,x\right)\,\left(4\,a^3+6\,a^2\,b+4\,a\,b^2+b^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^4,x)
```

```
[Out] x*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2) - (tanh(c + d*x)^3*(4*a*b^3 +
 b^4 + 6*a^2*b^2))/(3*d) - (tanh(c + d*x)^5*(4*a*b^3 + b^4))/(5*d) - (b^4*t
anh(c + d*x)^7)/(7*d) - (b*tanh(c + d*x)*(4*a*b^2 + 6*a^2*b + 4*a^3 + b^3))
/d
```

3.169 $\int \left(a+b\tanh^2(c+dx)\right)^5\,dx$

Optimal. Leaf size=160

$$(a+b)^5x-\frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d}-\frac{b^2(10a^3+10a^2b+5ab^2+b^3)\tanh^3(c+dx)}{3d}-$$

```
[Out] (a+b)^5*x-b*(5*a^4+10*a^3*b+10*a^2*b^2+5*a*b^3+b^4)*tanh(d*x+c)/d-1/3*b^2*(
10*a^3+10*a^2*b+5*a*b^2+b^3)*tanh(d*x+c)^3/d-1/5*b^3*(10*a^2+5*a*b+b^2)*tan
h(d*x+c)^5/d-1/7*b^4*(5*a+b)*tanh(d*x+c)^7/d-1/9*b^5*tanh(d*x+c)^9/d
```

Rubi [A]
time = 0.07, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 398, 212}

$$-\frac{b^3(10a^2+5ab+b^2)\tanh^5(c+dx)}{5d}-\frac{b^2(10a^3+10a^2b+5ab^2+b^3)\tanh^3(c+dx)}{3d}-\frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d}-\frac{b^4(5a+b)\tanh^7(c+dx)}{7d}+x(a+b)^5-\frac{b^5\tanh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^5,x]
```

```
[Out] (a + b)^5*x - (b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4)*Tanh[c + d
*x])/d - (b^2*(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[c + d*x]^3)/(3*d) -
(b^3*(10*a^2 + 5*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^4*(5*a + b)*Tanh[c
+ d*x]^7)/(7*d) - (b^5*Tanh[c + d*x]^9)/(9*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tanh^2(c+dx)\right)^5\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^5}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)-b^2(10a^3+10a^2b+5ab^2+\right.\right.}{}$$
$$= -\frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d} - \frac{b^2(10a^3+10a^2b+5ab}{3}$$
$$= (a+b)^5x - \frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d} - \frac{b^2(10a^3+1}{}$$

Mathematica [A]
time = 1.58, size = 170, normalized size = 1.06

$$\frac{\tanh(c+dx)\left(\frac{315(a+b)^5\tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b(315(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)+105b(10a^3+10a^2b+5ab^2+b^3)\tanh^2(c+dx)+63b^2(10a^2+5ab+b^2)\tanh^4(c+dx)+45b^3(5a+b)\tanh^6(c+dx)+35b^4\tanh^8(c+dx))\right)}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^5,x]
```

```
[Out] (Tanh[c + d*x]*((315*(a + b)^5*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
+ d*x]^2] - b*(315*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4) + 105*b*
(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[c + d*x]^2 + 63*b^2*(10*a^2 + 5*a*
b + b^2)*Tanh[c + d*x]^4 + 45*b^3*(5*a + b)*Tanh[c + d*x]^6 + 35*b^4*Tanh[c
 + d*x]^8)))/(315*d)
```

Maple [A]
time = 0.34, size = 303, normalized size = 1.89

method	result
derivativedivides	$\frac{-5a\,b^4\tanh(dx+c)-10a^2b^3\tanh(dx+c)-10a^3b^2\tanh(dx+c)-5a^4b\tanh(dx+c)-\frac{5a\,b^4\left(\tanh^3(dx+c)\right)}{3}-\frac{10a^2b^3\left(\tanh^3(dx+c\right.}{3}}{}$
default	$\frac{-5a\,b^4\tanh(dx+c)-10a^2b^3\tanh(dx+c)-10a^3b^2\tanh(dx+c)-5a^4b\tanh(dx+c)-\frac{5a\,b^4\left(\tanh^3(dx+c)\right)}{3}-\frac{10a^2b^3\left(\tanh^3(dx+c\right.}{3}}{}$
risch	$a^5x+5a^4xb+10a^3b^2x+10a^2b^3x+5a\,b^4x+b^5x+\frac{2b\left(88200a^4e^{10dx+10c}+39438b^4e^{8dx+8c}+44100a^4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c)^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-5*a*b^4*tanh(d*x+c)-10*a^2*b^3*tanh(d*x+c)-10*a^3*b^2*tanh(d*x+c)-5*a
^4*b*tanh(d*x+c)-5/3*a*b^4*tanh(d*x+c)^3-10/3*a^2*b^3*tanh(d*x+c)^3-10/3*a^
3*b^2*tanh(d*x+c)^3-2*a^2*b^3*tanh(d*x+c)^5-a*b^4*tanh(d*x+c)^5-5/7*a*b^4*t
anh(d*x+c)^7-1/9*b^5*tanh(d*x+c)^9-1/2*(a^5+5*a^4*b+10*a^3*b^2+10*a^2*b^3+5
*a*b^4+b^5)*ln(tanh(d*x+c)-1)-1/3*b^5*tanh(d*x+c)^3-1/7*b^5*tanh(d*x+c)^7-1
/5*b^5*tanh(d*x+c)^5-b^5*tanh(d*x+c)+1/2*(a^5+5*a^4*b+10*a^3*b^2+10*a^2*b^3
+5*a*b^4+b^5)*ln(1+tanh(d*x+c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(152) = 304$.
time = 0.31, size = 624, normalized size = 3.90

$$\frac{1}{315}b^5\left(315x+\frac{315c}{d}-\frac{2\left(3492e^{(-2dx-2c)}+13968e^{(-4dx-4c)}+26292e^{(-6dx-6c)}+39438e^{(-8dx-8c)}+31500e^{(-10dx-10c)}+21000e^{(-12dx-12c)}+6300e^{(-14dx-14c)}+1575e^{(-16dx-16c)}+563\right)}{d(9e^{(-2dx-2c)}+36e^{(-4dx-4c)}+84e^{(-6dx-6c)}+126e^{(-8dx-8c)}+126e^{(-10dx-10c)}+84e^{(-12dx-12c)}+36e^{(-14dx-14c)}+9e^{(-16dx-16c)}+e^{(-18dx-18c)}+1)}\right)+\frac{1}{21}ab^4\left(105x+\frac{105c}{d}-\frac{8\left(203e^{(-2dx-2c)}+609e^{(-4dx-4c)}+770e^{(-6dx-6c)}+770e^{(-8dx-8c)}+315e^{(-10dx-10c)}+105e^{(-12dx-12c)}+44\right)}{d(7e^{(-2dx-2c)}+21e^{(-4dx-4c)}+35e^{(-6dx-6c)}+35e^{(-8dx-8c)}+21e^{(-10dx-10c)}+7e^{(-12dx-12c)}+e^{(-14dx-14c)}+1)}\right)+\frac{2}{3}a^2b^3\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+\frac{10}{3}a^3b^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+5a^4b\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="maxima")
```

```
[Out] 1/315*b^5*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x - 4
*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x -
10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16*d*
x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*
x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x -
12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) +
 1))) + 1/21*a*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d
*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x -
10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
- 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
+ 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/3*a^2*b^3*(15*x + 15
*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c)
+ 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 10/3
*a^3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 5*a^4*
b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^5*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. $2(152) = 304$.
time = 0.40, size = 2133, normalized size = 13.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="fricas")
```

```
[Out] 1/315*((1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 3
15*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x
```

+ c)^9 + 9*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5
 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(
d*x + c)*sinh(d*x + c)^8 - (1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640
*a*b^4 + 563*b^5)*sinh(d*x + c)^9 + 9*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2
*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
+ 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^7 - 9*(1225*a^4*b + 2800*a^3*b^2 + 2730
*a^2*b^3 + 1240*a*b^4 + 213*b^5 + 4*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b
^3 + 2640*a*b^4 + 563*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(1575*a
^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^3 + 3*(15
75*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 +
5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c))*sinh
(d*x + c)^6 + 36*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 5
63*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)
*cosh(d*x + c)^5 - 9*(3500*a^4*b + 7000*a^3*b^2 + 6720*a^2*b^3 + 3560*a*b^4
 + 852*b^5 + 14*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 56
3*b^5)*cosh(d*x + c)^4 + 21*(1225*a^4*b + 2800*a^3*b^2 + 2730*a^2*b^3 + 124
0*a*b^4 + 213*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(1575*a^4*b + 4
200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^5 + 35*(1575*a^4*
b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b
 + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^3 + 20*(1575
*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*
a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c))*sinh(d
x + c)^4 + 84(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563
b^5 + 315(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*c
osh(d*x + c)^3 - 3*(28*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b
^4 + 563*b^5)*cosh(d*x + c)^6 + 14700*a^4*b + 26600*a^3*b^2 + 27440*a^2*b^3
 + 13720*a*b^4 + 1764*b^5 + 105*(1225*a^4*b + 2800*a^3*b^2 + 2730*a^2*b^3 +
 1240*a*b^4 + 213*b^5)*cosh(d*x + c)^4 + 120*(875*a^4*b + 1750*a^3*b^2 + 16
80*a^2*b^3 + 890*a*b^4 + 213*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(4*(
1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5
+ 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^7 +
 21*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*
(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c
)^5 + 40*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 +
 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*
x + c)^3 + 28*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*
b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^2 + 126*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^
3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5
*a*b^4 + b^5)*d*x)*cosh(d*x + c) - 9*((1575*a^4*b + 4200*a^3*b^2 + 4830*a^2
*b^3 + 2640*a*b^4 + 563*b^5)*cosh(d*x + c)^8 + 7*(1225*a^4*b + 2800*a^3*b^2
 + 2730*a^2*b^3 + 1240*a*b^4 + 213*b^5)*cosh(d*x + c)^6 + 2450*a^4*b + 4200
*a^3*b^2 + 4620*a^2*b^3 + 1960*a*b^4 + 882*b^5 + 20*(875*a^4*b + 1750*a^3*b

```
^2 + 1680*a^2*b^3 + 890*a*b^4 + 213*b^5)*cosh(d*x + c)^4 + 28*(525*a^4*b +
950*a^3*b^2 + 980*a^2*b^3 + 490*a*b^4 + 63*b^5)*cosh(d*x + c)^2)*sinh(d*x +
 c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + 9*d*cosh(d*x
+ c)^7 + 21*(4*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^6 + 36*
d*cosh(d*x + c)^5 + 9*(14*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 20*d*c
osh(d*x + c))*sinh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 9*(4*d*cosh(d*x + c)
^7 + 21*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh
(d*x + c)^2 + 126*d*cosh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(148) = 296$.
time = 0.38, size = 308, normalized size = 1.92

$$\begin{cases} a^5x + 5a^4bx - \frac{5a^4b\tanh(c+dx)}{d} + 10a^3b^2x - \frac{10a^3b^2\tanh^3(c+dx)}{3d} - \frac{10a^3b^2\tanh(c+dx)}{d} + 10a^2b^3x - \frac{2a^2b^3\tanh^5(c+dx)}{d} - \frac{10a^2b^3\tanh^3(c+dx)}{3d} - \frac{10a^2b^3\tanh(c+dx)}{d} + 5ab^4x - \frac{5ab^4\tanh^7(c+dx)}{7d} - \frac{ab^4\tanh^5(c+dx)}{d} - \frac{5ab^4\tanh^3(c+dx)}{3d} - \frac{5ab^4\tanh(c+dx)}{d} + b^5x - \frac{b^5\tanh^9(c+dx)}{9d} - \frac{b^5\tanh^7(c+dx)}{7d} - \frac{b^5\tanh^5(c+dx)}{5d} - \frac{b^5\tanh^3(c+dx)}{3d} - \frac{b^5\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x(a + b\tanh^2(c))^5 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**2)**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*x - 5*a**4*b*tanh(c + d*x)/d + 10*a**3*b**2*x
- 10*a**3*b**2*tanh(c + d*x)**3/(3*d) - 10*a**3*b**2*tanh(c + d*x)/d + 10*a
**2*b**3*x - 2*a**2*b**3*tanh(c + d*x)**5/d - 10*a**2*b**3*tanh(c + d*x)**3
/(3*d) - 10*a**2*b**3*tanh(c + d*x)/d + 5*a*b**4*x - 5*a*b**4*tanh(c + d*x)
**7/(7*d) - a*b**4*tanh(c + d*x)**5/d - 5*a*b**4*tanh(c + d*x)**3/(3*d) - 5
*a*b**4*tanh(c + d*x)/d + b**5*x - b**5*tanh(c + d*x)**9/(9*d) - b**5*tanh(
c + d*x)**7/(7*d) - b**5*tanh(c + d*x)**5/(5*d) - b**5*tanh(c + d*x)**3/(3*
d) - b**5*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(152) = 304$.
time = 0.47, size = 721, normalized size = 4.51

$$\frac{315\,(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)(dx + c) + \frac{2\,(1575a^4be^{(16dx+16c)} + 6300a^3b^2e^{(16dx+16c)} + 9450a^2b^3e^{(16dx+16c)} + 6300ab^4e^{(16dx+16c)} + 1575b^5e^{(16dx+16c)} + 12600a^4be^{(14dx+14c)} + 44100a^3b^2e^{(14dx+14c)} + 56700a^2b^3e^{(14dx+14c)} + 31500ab^4e^{(14dx+14c)} + 6300b^5e^{(14dx+14c)} + \text{[illegible]}}{\text{[illegible]}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="giac")
```

```
[Out] 1/315*(315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(d*x +
 c) + 2*(1575*a^4*b*e^(16*d*x + 16*c) + 6300*a^3*b^2*e^(16*d*x + 16*c) + 94
50*a^2*b^3*e^(16*d*x + 16*c) + 6300*a*b^4*e^(16*d*x + 16*c) + 1575*b^5*e^(1
6*d*x + 16*c) + 12600*a^4*b*e^(14*d*x + 14*c) + 44100*a^3*b^2*e^(14*d*x + 1
4*c) + 56700*a^2*b^3*e^(14*d*x + 14*c) + 31500*a*b^4*e^(14*d*x + 14*c) + 63
00*b^5*e^(14*d*x + 14*c) + 44100*a^4*b*e^(12*d*x + 12*c) + 136500*a^3*b^2*e
^(12*d*x + 12*c) + 161700*a^2*b^3*e^(12*d*x + 12*c) + 90300*a*b^4*e^(12*d*x
 + 12*c) + 21000*b^5*e^(12*d*x + 12*c) + 88200*a^4*b*e^(10*d*x + 10*c) + 24
5700*a^3*b^2*e^(10*d*x + 10*c) + 283500*a^2*b^3*e^(10*d*x + 10*c) + 157500*
a*b^4*e^(10*d*x + 10*c) + 31500*b^5*e^(10*d*x + 10*c) + 110250*a^4*b*e^(8*d
```

```
*x + 8*c) + 283500*a^3*b^2*e^(8*d*x + 8*c) + 325080*a^2*b^3*e^(8*d*x + 8*c)
 + 175140*a*b^4*e^(8*d*x + 8*c) + 39438*b^5*e^(8*d*x + 8*c) + 88200*a^4*b*e
^(6*d*x + 6*c) + 216300*a^3*b^2*e^(6*d*x + 6*c) + 244020*a^2*b^3*e^(6*d*x +
 6*c) + 131460*a*b^4*e^(6*d*x + 6*c) + 26292*b^5*e^(6*d*x + 6*c) + 44100*a^
4*b*e^(4*d*x + 4*c) + 107100*a^3*b^2*e^(4*d*x + 4*c) + 117180*a^2*b^3*e^(4*
d*x + 4*c) + 63540*a*b^4*e^(4*d*x + 4*c) + 13968*b^5*e^(4*d*x + 4*c) + 1260
0*a^4*b*e^(2*d*x + 2*c) + 31500*a^3*b^2*e^(2*d*x + 2*c) + 34020*a^2*b^3*e^(
2*d*x + 2*c) + 17460*a*b^4*e^(2*d*x + 2*c) + 3492*b^5*e^(2*d*x + 2*c) + 157
5*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5)/(e^(2*d*x + 2
*c) + 1)^9)/d
```

Mupad [B]
time = 1.32, size = 188, normalized size = 1.18

$$x\,\left(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right)-\frac{\mathrm{tanh}\left(c+d\,x\right)^3\,\left(10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right)}{3\,d}-\frac{\mathrm{tanh}\left(c+d\,x\right)^5\,\left(10\,a^2\,b^3+5\,a\,b^4+b^5\right)}{5\,d}-\frac{\mathrm{tanh}\left(c+d\,x\right)^7\,\left(b^5+5\,a\,b^4\right)}{7\,d}-\frac{b^5\,\mathrm{tanh}\left(c+d\,x\right)^9}{9\,d}-\frac{b\,\mathrm{tanh}\left(c+d\,x\right)\,\left(5\,a^4+10\,a^3\,b+10\,a^2\,b^2+5\,a\,b^3+b^4\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^5,x)
```

```
[Out] x*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2) - (tanh(c + d*x
)^3*(5*a*b^4 + b^5 + 10*a^2*b^3 + 10*a^3*b^2))/(3*d) - (tanh(c + d*x)^5*(5*
a*b^4 + b^5 + 10*a^2*b^3))/(5*d) - (tanh(c + d*x)^7*(5*a*b^4 + b^5))/(7*d)
- (b^5*tanh(c + d*x)^9)/(9*d) - (b*tanh(c + d*x)*(5*a*b^3 + 10*a^3*b + 5*a^
4 + b^4 + 10*a^2*b^2))/d
```

3.170 $\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)d}-\frac{\tanh^2(c+dx)}{2bd}$$

```
[Out] ln(cosh(d*x+c))/(a+b)/d+1/2*a^2*ln(a+b*tanh(d*x+c)^2)/b^2/(a+b)/d-1/2*tanh(
d*x+c)^2/b/d
```

Rubi [A]
time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3751, 457, 84}

$$\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2d(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}-\frac{\tanh^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + (a^2*Log[a + b*Tanh[c + d*x]^2])/(2*b^2*(a
 + b)*d) - Tanh[c + d*x]^2/(2*b*d)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b}-\frac{1}{(a+b)(-1+x)}+\frac{a^2}{b(a+b)(a+bx)}\right)dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)d}-\frac{\tanh^2(c+dx)}{2bd}$$

Mathematica [A]
time = 0.13, size = 60, normalized size = 0.91

$$-\frac{-\frac{2\log(\cosh(c+dx))}{a+b}-\frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{b^2(a+b)}+\frac{\tanh^2(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*((-2*Log[Cosh[c + d*x]])/(a + b) - (a^2*Log[a + b*Tanh[c + d*x]^2])/(b
^2*(a + b)) + Tanh[c + d*x]^2/b)/d
```

Maple [A]
time = 0.54, size = 85, normalized size = 1.29

method	result
derivativedivides	$\frac{-\frac{\tanh^2(dx+c)}{2b}-\frac{\ln(1+\tanh(dx+c))}{2b+2a}-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}+\frac{a^2\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{2(a+b)b^2}}{d}$
default	$\frac{-\frac{\tanh^2(dx+c)}{2b}-\frac{\ln(1+\tanh(dx+c))}{2b+2a}-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}+\frac{a^2\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{2(a+b)b^2}}{d}$
risch	$\frac{x}{a+b}+\frac{2ax}{b^2}+\frac{2ac}{b^2d}-\frac{2x}{b}-\frac{2c}{bd}-\frac{2a^2x}{b^2(a+b)}-\frac{2a^2c}{b^2d(a+b)}+\frac{2\,e^{2dx+2c}}{bd\left(1+e^{2dx+2c}\right)^2}-\frac{\ln\left(1+e^{2dx+2c}\right)a}{b^2d}+\frac{\ln\left(1+e^{2dx+2\right.}}{bd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*tanh(d*x+c)^2/b-1/(2*b+2*a)*ln(1+tanh(d*x+c))-1/(2*b+2*a)*ln(tanh
(d*x+c)-1)+1/2*a^2/(a+b)/b^2*ln(a+b*tanh(d*x+c)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(62) = 124$.
time = 0.50, size = 133, normalized size = 2.02

$$\frac{a^2 \log\left(2\,(a-b)e^{(-2\,dx-2\,c)} + (a+b)e^{(-4\,dx-4\,c)} + a + b\right)}{2\,(ab^2+b^3)d} + \frac{dx+c}{(a+b)d} + \frac{2\,e^{(-2\,dx-2\,c)}}{(2\,be^{(-2\,dx-2\,c)} + be^{(-4\,dx-4\,c)} + b)d} - \frac{(a-b)\log\left(e^{(-2\,dx-2\,c)} + 1\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a*b^2 + b^3)*d) + (d*x + c)/((a + b)*d) + 2*e^(-2*d*x - 2*c)/((2*b*e^(-2*
d*x - 2*c) + b*e^(-4*d*x - 4*c) + b)*d) - (a - b)*log(e^(-2*d*x - 2*c) + 1)
/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(62) = 124$.
time = 0.41, size = 742, normalized size = 11.24

$$-\frac{2\,b^2dx\cosh(dx+c)^4 + 8\,b^2dx\cosh(dx+c)\sinh(dx+c)^3 + 2\,b^2dx\sinh(dx+c)^4 + 2\,b^2dx + 4\,(b^2dx-ab-b^2)\cosh(dx+c)^2 + 4\,(3\,b^2dx\cosh(dx+c)^2 + b^2dx - ab - b^2)\sinh(dx+c)^2 - \left(a^2\cosh(dx+c)^4 + 4\,a^2\cosh(dx+c)\sinh(dx+c)^3 + a^2\sinh(dx+c)^4 + 2\,a^2\cosh(dx+c)^2 + 2\,(3\,a^2\cosh(dx+c)^2 + a^2)\sinh(dx+c)^2 + a^2 + 4\,(a^2\cosh(dx+c)^3 + a^2\cosh(dx+c))\sinh(dx+c)\right)\log\left(\frac{2\,((a+b)\cosh(dx+c)^2 + (a+b)\sinh(dx+c)^2 + a - b)}{\cosh(dx+c)^2 - 2\cosh(dx+c)\sinh(dx+c) + \sinh(dx+c)^2}\right) + 2\,\left((a^2-b^2)\cosh(dx+c)^4 + 4\,(a^2-b^2)\cosh(dx+c)\sinh(dx+c)^3 + (a^2-b^2)\sinh(dx+c)^4 + 2\,(a^2-b^2)\cosh(dx+c)^2 + 2\,(3\,(a^2-b^2)\cosh(dx+c)^2 + a^2 - b^2)\sinh(dx+c)^2 + a^2 - b^2 + 4\,((a^2-b^2)\cosh(dx+c)^3 + (a^2-b^2)\cosh(dx+c))\sinh(dx+c)\right)\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right) + 8\,(b^2dx\cosh(dx+c)^3 + (b^2dx-ab-b^2)\cosh(dx+c))\sinh(dx+c)}{2\,\left((ab^2+b^3)d\cosh(dx+c)^4 + 4\,(ab^2+b^3)d\cosh(dx+c)\sinh(dx+c)^3 + (ab^2+b^3)d\sinh(dx+c)^4 + 2\,(ab^2+b^3)d\cosh(dx+c)^2 + 2\,(3\,(ab^2+b^3)d\cosh(dx+c)^2 + (ab^2+b^3)d)\sinh(dx+c)^2 + (ab^2+b^3)d + 4\,((ab^2+b^3)d\cosh(dx+c)^3 + (ab^2+b^3)d\cosh(dx+c))\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x - a*b - b^2)*cosh(d*x +
 c)^2 + 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x - a*b - b^2)*sinh(d*x + c)^2
 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*
x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x
 + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*
log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x
 + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 2*((a^2 - b^2
)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^
2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(
d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d
*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(
cosh(d*x + c) - sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 + (b^2*d*x - a
*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/((a*b^2 + b^3)*d*cosh(d*x + c)^4 +
4*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2 + b^3)*d*sinh(d*x
+ c)^4 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)^2 + 2*(3*(a*b^2 + b^3)*d*cosh(d*x
+ c)^2 + (a*b^2 + b^3)*d)*sinh(d*x + c)^2 + (a*b^2 + b^3)*d + 4*((a*b^2 + b
^3)*d*cosh(d*x + c)^3 + (a*b^2 + b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(53) = 106$.

time = 12.23, size = 415, normalized size = 6.29

$$\begin{cases} \tilde{\infty} x \tanh^3(c) & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x - \frac{\log(\tanh(c+dx)+1)}{d} - \frac{\tanh^4(c+dx)}{4d} - \frac{\tanh^2(c+dx)}{2d}}{a} & \text{for } b = 0 \\ \frac{4dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} - \frac{4dx}{2bd\tanh^2(c+dx)-2bd} - \frac{4\log(\tanh(c+dx)+1)\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} + \frac{4\log(\tanh(c+dx)+1)}{2bd\tanh^2(c+dx)-2bd} - \frac{\tanh^4(c+dx)}{2bd\tanh^2(c+dx)-2bd} + \frac{2}{2bd\tanh^2(c+dx)-2bd} & \text{for } a = -b \\ \frac{x\tanh^5(c)}{a+b\tanh^2(c)} & \text{for } d = 0 \\ \frac{a^2\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d+2b^3d} + \frac{a^2\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d+2b^3d} - \frac{ab\tanh^2(c+dx)}{2ab^2d+2b^3d} + \frac{2b^2dx}{2ab^2d+2b^3d} - \frac{2b^2\log(\tanh(c+dx)+1)}{2ab^2d+2b^3d} - \frac{b^2\tanh^2(c+dx)}{2ab^2d+2b^3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tan
h(c + d*x) + 1)/d - tanh(c + d*x)**4/(4*d) - tanh(c + d*x)**2/(2*d))/a, Eq(
b, 0)), (4*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*d*x/(2
*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/
(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 4*log(tanh(c + d*x) + 1)/(2*b*d*tanh(c +
 d*x)**2 - 2*b*d) - tanh(c + d*x)**4/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 2/(
2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**5/(a + b*tanh(c)**
2), Eq(d, 0)), (a**2*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b**2*d + 2*b**3*
d) + a**2*log(sqrt(-a/b) + tanh(c + d*x))/(2*a*b**2*d + 2*b**3*d) - a*b*tan
h(c + d*x)**2/(2*a*b**2*d + 2*b**3*d) + 2*b**2*d*x/(2*a*b**2*d + 2*b**3*d)
- 2*b**2*log(tanh(c + d*x) + 1)/(2*a*b**2*d + 2*b**3*d) - b**2*tanh(c + d*x
)**2/(2*a*b**2*d + 2*b**3*d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(62) = 124.
time = 0.48, size = 132, normalized size = 2.00

$$\frac{\frac{a^2\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{ab^2+b^3} - \frac{2\,(dx+c)}{a+b} - \frac{2\,(a-b)\log\left(e^{(2\,dx+2\,c)}+1\right)}{b^2} + \frac{4\,e^{(2\,dx+2\,c)}}{b\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)/(a*b^2 + b^3) - 2*(d*x + c)/(a + b) - 2*(a - b
)*log(e^(2*d*x + 2*c) + 1)/b^2 + 4*e^(2*d*x + 2*c)/(b*(e^(2*d*x + 2*c) + 1)
^2))/d
```

Mupad [B]
time = 0.27, size = 72, normalized size = 1.09

$$-\frac{b^2\left(\ln\left(\tanh(c+d\,x)+1\right)-d\,x+\frac{\tanh(c+d\,x)^2}{2}\right)-\frac{a^2\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}+\frac{a\,b\tanh(c+d\,x)^2}{2}}{b^2\,d\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] -(b^2*(log(tanh(c + d*x) + 1) - d*x + tanh(c + d*x)^2/2) - (a^2*log(a + b*t
anh(c + d*x)^2))/2 + (a*b*tanh(c + d*x)^2)/2)/(b^2*d*(a + b))
```

3.171 $\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{a+b}+\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d}-\frac{\tanh(c+dx)}{bd}$$

```
[Out] x/(a+b)+a^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/(a+b)/d-tanh(d*
x+c)/b/d
```

Rubi [A]
time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3751, 490, 536, 212, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}d(a+b)}+\frac{x}{a+b}-\frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(b^(3/2)*(a +
 b)*d) - Tanh[c + d*x]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+bx^2)}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= -\frac{\tanh(c+dx)}{bd}+\frac{\text{Subst}\left(\int \frac{a+(-a+b)x^2}{(1-x^2)(a+bx^2)}\,dx,x,\tanh(c+dx)\right)}{bd}\\ &= -\frac{\tanh(c+dx)}{bd}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{(a+b)d}+\frac{a^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\right.}{b(a+b)d}\\ &= \frac{x}{a+b}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d}-\frac{\tanh(c+dx)}{bd}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 66, normalized size = 1.12

$$\frac{c+dx}{(a+b)d}+\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d}-\frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (c + d*x)/((a + b)*d) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
b^(3/2)*(a + b)*d) - Tanh[c + d*x]/(b*d)
```

Maple [A]
time = 0.74, size = 87, normalized size = 1.47

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)}{b}+\frac{a^2 \arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{b(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}+\frac{\ln(1+\tanh(dx+c))}{2b+2a}}{d}$
default	$\frac{-\frac{\tanh(dx+c)}{b}+\frac{a^2 \arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{b(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}+\frac{\ln(1+\tanh(dx+c))}{2b+2a}}{d}$
risch	$\frac{x}{a+b}+\frac{2}{bd\left(1+e^{2dx+2c}\right)}+\frac{\sqrt{-ab}\, a\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2b^2(a+b)d}-\frac{\sqrt{-ab}\, a\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2b^2(a+b)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*tanh(d*x+c)+1/b/(a+b)*a^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^
(1/2))-1/(2*b+2*a)*ln(tanh(d*x+c)-1)+1/(2*b+2*a)*ln(1+tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(51) = 102.
time = 0.64, size = 509, normalized size = 8.63

$$-\frac{(a-b)\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{8(ab+b^2)d}+\frac{(a-b)\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{8(ab+b^2)d}+\frac{(a^2-6ab+b^2)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{16(ab+b^2)\sqrt{ab}\,d}+\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}\,bd}-\frac{(a^2-6ab+b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{16(ab+b^2)\sqrt{ab}\,d}-\frac{3(a+b)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}\,bd}-\frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}\,bd}-\frac{\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{4bd}+\frac{\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{4bd}+\frac{3\log\left(e^{(2dx+2c)}+1\right)}{4bd}-\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{4bd}+\frac{5}{8\left(be^{(2dx+2c)}+b\right)d}-\frac{11}{8\left(be^{(-2dx-2c)}+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(a - b)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a +
b)/((a*b + b^2)*d) + 1/8*(a - b)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e
^(-4*d*x - 4*c) + a + b)/((a*b + b^2)*d) + 1/16*(a^2 - 6*a*b + b^2)*arctan(
1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a*b + b^2)*sqrt(a*b)*d)
+ 1/4*(a - b)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt
(a*b)*b*d) - 1/16*(a^2 - 6*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c)
+ a - b)/sqrt(a*b))/((a*b + b^2)*sqrt(a*b)*d) - 3/8*(a + b)*arctan(1/2*((a
+ b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*b*d) - 1/4*(a - b)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*b*d) - 1/4
*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/(b*d) + 1
/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(b*d)
 + 3/4*log(e^(2*d*x + 2*c) + 1)/(b*d) - 3/4*log(e^(-2*d*x - 2*c) + 1)/(b*d)
 + 5/8/((b*e^(2*d*x + 2*c) + b)*d) - 11/8/((b*e^(-2*d*x - 2*c) + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(51) = 102$.

time = 0.37, size = 777, normalized size = 13.17

$$\left[\frac{2\,bdx\cosh\left(dx+c\right)^2+4\,bdx\cosh\left(dx+c\right)\sinh\left(dx+c\right)+2\,bdx\sinh\left(dx+c\right)^2+2\,bdx+\left(a\cosh\left(dx+c\right)^2+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^2+a\right)\sqrt{-\frac{a}{b}}\log\left(\frac{(a^2+2ab+b^2)\cosh(dx+c)^4+4(a^2+2ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2ab+b^2)\sinh(dx+c)^4+2(a^2-b^2)\cosh(dx+c)^2+2\left(3(a^2+2ab+b^2)\cosh(dx+c)^2+a^2-b^2\right)\sinh(dx+c)^2+a^2-6ab+b^2+4\left((a^2+2ab+b^2)\cosh(dx+c)^3+(a^2-b^2)\cosh(dx+c)\right)\sinh(dx+c)+4\left((ab+b^2)\cosh(dx+c)^2+2(ab+b^2)\cosh(dx+c)\sinh(dx+c)+(ab+b^2)\sinh(dx+c)^2+ab-b^2\right)\sqrt{-\frac{a}{b}}}{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2(a-b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2+a-b\right)\sinh(dx+c)^2+4\left((a+b)\cosh(dx+c)^3+(a-b)\cosh(dx+c)\right)\sinh(dx+c)+a+b}\right)+4\,a+4\,b}{2\left((ab+b^2)d\cosh\left(dx+c\right)^2+2\,(ab+b^2)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(ab+b^2)d\sinh\left(dx+c\right)^2+(ab+b^2)d\right)}, \frac{bdx\cosh\left(dx+c\right)^2+2\,bdx\cosh\left(dx+c\right)\sinh\left(dx+c\right)+bdx\sinh\left(dx+c\right)^2+bdx+\left(a\cosh\left(dx+c\right)^2+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^2+a\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{\left((a+b)\cosh(dx+c)^2+2(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a-b\right)\sqrt{\frac{a}{b}}}{2a}\right)+2\,a+2\,b}{(ab+b^2)d\cosh\left(dx+c\right)^2+2\,(ab+b^2)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(ab+b^2)d\sinh\left(dx+c\right)^2+(ab+b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*d
*x*sinh(d*x + c)^2 + 2*b*d*x + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(
d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(
d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2
*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^
2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sin
h(d*x + c) + 4*((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*s
inh(d*x + c) + (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a + b
)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
 b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b)) + 4*a + 4*b)/((a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a*b
 + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*sinh(d*x + c)^2 + (a*
b + b^2)*d), (b*d*x*cosh(d*x + c)^2 + 2*b*d*x*cosh(d*x + c)*sinh(d*x + c) +
 b*d*x*sinh(d*x + c)^2 + b*d*x + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sin
h(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x
+ c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a - b)*sqrt(a/b)/a) + 2*a + 2*b)/((a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a*b +
b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*sinh(d*x + c)^2 + (a*b +
 b^2)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(48) = 96$.

time = 6.21, size = 428, normalized size = 7.25

$$\begin{cases} \tilde{\infty}x\tanh^2(c) & \text{for}\ a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x - \frac{\tanh^3(c+dx)}{3d} - \frac{\tanh(c+dx)}{d}}{a} & \text{for}\ b = 0 \\ \frac{x - \frac{\tanh(c+dx)}{d}}{b} & \text{for}\ a = 0 \\ \frac{3dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} - \frac{3dx}{2bd\tanh^2(c+dx)-2bd} - \frac{2\tanh^3(c+dx)}{2bd\tanh^2(c+dx)-2bd} + \frac{3\tanh(c+dx)}{2bd\tanh^2(c+dx)-2bd} & \text{for}\ a = -b \\ \frac{x\tanh^4(c)}{a+b\tanh^2(c)} & \text{for}\ d = 0 \\ \frac{a^2\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} - \frac{a^2\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} - \frac{2ab\sqrt{-\frac{a}{b}}\tanh(c+dx)}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} + \frac{2b^2dx\sqrt{-\frac{a}{b}}}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} - \frac{2b^2\sqrt{-\frac{a}{b}}\tanh(c+dx)}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c
+ d*x)**3/(3*d) - tanh(c + d*x)/d)/a, Eq(b, 0)), ((x - tanh(c + d*x)/d)/b,
Eq(a, 0)), (3*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 3*d*x
/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*tanh(c + d*x)**3/(2*b*d*tanh(c + d*x)
**2 - 2*b*d) + 3*tanh(c + d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b))
, (x*tanh(c)**4/(a + b*tanh(c)**2), Eq(d, 0)), (a**2*log(-sqrt(-a/b) + tanh
(c + d*x))/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)) - a**2*log(sqrt(-a
/b) + tanh(c + d*x))/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)) - 2*a*b*
sqrt(-a/b)*tanh(c + d*x)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)) + 2*
b**2*d*x*sqrt(-a/b)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)) - 2*b**2*
sqrt(-a/b)*tanh(c + d*x)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)), Tru
e))
```

Giac [A]
time = 0.47, size = 87, normalized size = 1.47

$$\frac{\frac{a^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(ab+b^2)\sqrt{ab}}+\frac{dx+c}{a+b}+\frac{2}{b\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (a^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
((a*b + b^2)*sqrt(a*b)) + (d*x + c)/(a + b) + 2/(b*(e^(2*d*x + 2*c) + 1)))/
d
```

Mupad [B]
time = 1.23, size = 56, normalized size = 0.95

$$\frac{x}{a+b}-\frac{\tanh(c+d\,x)}{b\,d}+\frac{a^2\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{b\,d\,\sqrt{a\,b}\,\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x/(a + b) - tanh(c + d*x)/(b*d) + (a^2*atan((b*tanh(c + d*x))/(a*b)^(1/2)))
/(b*d*(a*b)^(1/2)*(a + b))
```

3.172 $\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{\log(\cosh(c+dx))}{(a+b)d}-\frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2b(a+b)d}$$

```
[Out] ln(cosh(d*x+c))/(a+b)/d-1/2*a*ln(a+b*tanh(d*x+c)^2)/b/(a+b)/d
```

Rubi [A]
time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 78}

$$\frac{\log(\cosh(c+dx))}{d(a+b)}-\frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) - (a*Log[a + b*Tanh[c + d*x]^2])/(2*b*(a + b
)*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)}-\frac{a}{(a+b)(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\
&= \frac{\log(\cosh(c+dx))}{(a+b)d}-\frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2b(a+b)d}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.91

$$\frac{2b\log(\cosh(c+dx))-a\log\left(a+b\tanh^2(c+dx)\right)}{2abd+2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
[Out] (2*b*Log[Cosh[c + d*x]] - a*Log[a + b*Tanh[c + d*x]^2])/(2*a*b*d + 2*b^2*d)
```

Maple [A]
time = 0.69, size = 70, normalized size = 1.52

method	result	size
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}-\frac{a\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{2(a+b)b}-\frac{\ln(1+\tanh(dx+c))}{2b+2a}}{d}$	70
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}-\frac{a\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{2(a+b)b}-\frac{\ln(1+\tanh(dx+c))}{2b+2a}}{d}$	70
risch	$\frac{x}{a+b}-\frac{2x}{b}-\frac{2c}{bd}+\frac{2ax}{b(a+b)}+\frac{2ac}{bd(a+b)}+\frac{\ln\left(1+e^{2dx+2c}\right)}{bd}-\frac{a\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2c}}{a+b}+1\right)}{2bd(a+b)}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/(2*b+2*a)*ln(tanh(d*x+c)-1)-1/2/(a+b)*a/b*ln(a+b*tanh(d*x+c)^2)-1/(
2*b+2*a)*ln(1+tanh(d*x+c)))
```

Maxima [A]
time = 0.49, size = 82, normalized size = 1.78

$$-\frac{a\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(ab+b^2)d}+\frac{dx+c}{(a+b)d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(
(a*b + b^2)*d) + (d*x + c)/((a + b)*d) + log(e^(-2*d*x - 2*c) + 1)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(44) = 88$.
time = 0.39, size = 118, normalized size = 2.57

$$-\frac{2\,bdx+a\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)-2\,(a+b)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,(ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + a*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2
+ a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
)) - 2*(a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a*b
+ b^2)*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(36) = 72$.
time = 4.28, size = 306, normalized size = 6.65

$$\begin{cases} \tilde{\infty}x\tanh(c) & \text{for}\ a=0\wedge b=0\wedge d=0 \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}-\frac{\tanh^2(c+dx)}{2d}}{a} & \text{for}\ b=0 \\ \frac{2dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}-\frac{2dx}{2bd\tanh^2(c+dx)-2bd}-\frac{2\log(\tanh(c+dx)+1)\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}+\frac{2\log(\tanh(c+dx)+1)}{2bd\tanh^2(c+dx)-2bd}+\frac{1}{2bd\tanh^2(c+dx)-2bd} & \text{for}\ a=-b \\ \frac{x\tanh^3(c)}{a+b\tanh^2(c)} & \text{for}\ d=0 \\ -\frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd+2b^2d}-\frac{a\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd+2b^2d}+\frac{2bdx}{2abd+2b^2d}-\frac{2b\log(\tanh(c+dx)+1)}{2abd+2b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(c
 + d*x) + 1)/d - tanh(c + d*x)**2/(2*d))/a, Eq(b, 0)), (2*d*x*tanh(c + d*x)
**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*
```

```
d) - 2*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*
b*d) + 2*log(tanh(c + d*x) + 1)/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 1/(2*b*d
*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**3/(a + b*tanh(c)**2), E
q(d, 0)), (-a*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d + 2*b**2*d) - a*log
(sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d + 2*b**2*d) + 2*b*d*x/(2*a*b*d + 2*b*
*2*d) - 2*b*log(tanh(c + d*x) + 1)/(2*a*b*d + 2*b**2*d), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.
time = 0.46, size = 96, normalized size = 2.09

$$-\frac{\frac{a\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{ab+b^2}+\frac{2\,(dx+c)}{a+b}-\frac{2\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2
*b*e^(2*d*x + 2*c) + a + b)/(a*b + b^2) + 2*(d*x + c)/(a + b) - 2*log(e^(2*
d*x + 2*c) + 1)/b)/d
```

Mupad [**B**]
time = 1.24, size = 46, normalized size = 1.00

$$-\frac{\frac{a\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}+b\,\left(\ln\left(\tanh(c+d\,x)+1\right)-d\,x\right)}{b\,d\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] -((a*log(a + b*tanh(c + d*x)^2))/2 + b*(log(tanh(c + d*x) + 1) - d*x))/(b*d
*(a + b))
```

3.173 $\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{a+b}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}\,(a+b)d}$$

```
[Out] x/(a+b)-arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/(a+b)/d/b^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3751, 492, 212, 211}

$$\frac{x}{a+b}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}\,d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b]*(a +
 b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} - \frac{a\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d}\\ &= \frac{x}{a+b} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}\,(a+b)d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.02

$$\frac{-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}} + \tanh^{-1}(\tanh(c+dx))}{(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[b]) + ArcTanh[Tan
h[c + d*x]])/((a + b)*d)
```

Maple [A]
time = 0.74, size = 72, normalized size = 1.57

method	result	size
derivativedivides	$\frac{-\frac{a\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}} + \frac{\ln(1+\tanh(dx+c))}{2b+2a} - \frac{\ln(\tanh(dx+c)-1)}{2b+2a}}{d}$	72

default	$\frac{-\frac{a\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln(1+\tanh(dx+c))}{2b+2a}-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}}{d}$	72
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2b(a+b)d}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2b(a+b)d}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)*a/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))+1/(2*b+2*a)*l
n(1+tanh(d*x+c))-1/(2*b+2*a)*ln(tanh(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(38) = 76$.
time = 0.52, size = 215, normalized size = 4.67

$$-\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}(a+b)d}+\frac{\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}d}+\frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}(a+b)d}+\frac{\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{4(a+b)d}-\frac{\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{4(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(a - b)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt(
a*b)*(a + b)*d) + 1/2*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*
b))/(sqrt(a*b)*d) + 1/4*(a - b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a -
b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) + 1/4*log((a + b)*e^(4*d*x + 4*c) + 2*(
a - b)*e^(2*d*x + 2*c) + a + b)/((a + b)*d) - 1/4*log(2*(a - b)*e^(-2*d*x -
 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.
time = 0.39, size = 486, normalized size = 10.57

$$\left[\frac{2dx+\sqrt{-\frac{a}{b}}\log\left(\frac{(a^2+2ab+b^2)\cosh(dx+c)^4+4(a^2+2ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2ab+b^2)\sinh(dx+c)^4+2(a^2-b^2)\cosh(dx+c)^2+2\left(3(a^2+2ab+b^2)\cosh(dx+c)^2+a^2-b^2\right)\sinh(dx+c)^2+a^2-6ab+b^2+4\left((a^2+2ab+b^2)\cosh(dx+c)^3+(a^2-b^2)\cosh(dx+c)\right)\sinh(dx+c)-4\left((ab+b^2)\cosh(dx+c)^2+2(ab+b^2)\cosh(dx+c)\sinh(dx+c)+(ab+b^2)\sinh(dx+c)^2+ab-b^2\right)\sqrt{-\frac{a}{b}}}{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2(a-b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2+a-b\right)\sinh(dx+c)^2+4\left((a+b)\cosh(dx+c)^3+(a-b)\cosh(dx+c)\right)\sinh(dx+c)+a+b}\right)}{2(a+b)d}, \frac{dx-\sqrt{\frac{a}{b}}\arctan\left(\frac{\left((a+b)\cosh(dx+c)^2+2(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a-b\right)\sqrt{\frac{a}{b}}}{2a}\right)}{(a+b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a*b
```

```
 + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b
+ b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(d*x + c)^4 +
4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)))/((a + b)*d), (d*x - sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(
a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(
a/b)/a))/((a + b)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(37) = 74$.

time = 3.46, size = 253, normalized size = 5.50

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x - \frac{\tanh(c+dx)}{d}}{a} & \text{for } b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx) - 2bd} - \frac{dx}{2bd\tanh^2(c+dx) - 2bd} + \frac{\tanh(c+dx)}{2bd\tanh^2(c+dx) - 2bd} & \text{for } a = -b \\ \frac{x\tanh^2(c)}{a + b\tanh^2(c)} & \text{for } d = 0 \\ -\frac{a\log\left(-\sqrt{-\frac{a}{b}} + \tanh(c+dx)\right)}{2abd\sqrt{-\frac{a}{b}} + 2b^2d\sqrt{-\frac{a}{b}}} + \frac{a\log\left(\sqrt{-\frac{a}{b}} + \tanh(c+dx)\right)}{2abd\sqrt{-\frac{a}{b}} + 2b^2d\sqrt{-\frac{a}{b}}} + \frac{2bdx\sqrt{-\frac{a}{b}}}{2abd\sqrt{-\frac{a}{b}} + 2b^2d\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c + d*x)/d)/a
, Eq(b, 0)), (x/b, Eq(a, 0)), (d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2
 - 2*b*d) - d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(c + d*x)/(2*b*d*tan
h(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**2/(a + b*tanh(c)**2), Eq(d,
 0)), (-a*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d*sqrt(-a/b) + 2*b**2*d*s
qrt(-a/b)) + a*log(sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d*sqrt(-a/b) + 2*b**2
*d*sqrt(-a/b)) + 2*b*d*x*sqrt(-a/b)/(2*a*b*d*sqrt(-a/b) + 2*b**2*d*sqrt(-a/
b)), True))
```

Giac [A]

time = 0.44, size = 65, normalized size = 1.41

$$-\frac{\frac{a\arctan\left(\frac{ae^{(2\,dx+2\,c)} + be^{(2\,dx+2\,c)} + a - b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)} - \frac{dx+c}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(
sqrt(a*b)*(a + b)) - (d*x + c)/(a + b))/d
```

Mupad [B]
time = 0.11, size = 38, normalized size = 0.83

$$\frac{x}{a+b}-\frac{a\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}\,\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x/(a + b) - (a*atan((b*tanh(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a + b))
```

3.174 $\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=42

$$\frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)d}$$

```
[Out] ln(cosh(d*x+c))/(a+b)/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)/d
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3751, 455, 36, 31}

$$\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tanh^2(c+dx)\right)}{2(a+b)d} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \tanh^2(c+dx)\right)}{2(a+b)d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.83

$$\frac{2\log(\cosh(c+dx)) + \log\left(a+b\tanh^2(c+dx)\right)}{2ad+2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
[Out] (2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2])/(2*a*d + 2*b*d)
```

Maple [A]
time = 0.53, size = 66, normalized size = 1.57

method	result	size
risch	$-\frac{x}{a+b}-\frac{2c}{d(a+b)}+\frac{\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2c}}{a+b}+1\right)}{2d(a+b)}$	64
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}-\frac{\ln(1+\tanh(dx+c))}{2b+2a}+\frac{\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{2b+2a}}{d}$	66
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}-\frac{\ln(1+\tanh(dx+c))}{2b+2a}+\frac{\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{2b+2a}}{d}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(2*b+2*a)*ln(tanh(d*x+c)-1)-1/(2*b+2*a)*ln(1+tanh(d*x+c))+1/2/(a+b)
*ln(a+b*tanh(d*x+c)^2))
```

Maxima [A]
time = 0.30, size = 58, normalized size = 1.38

$$\frac{dx+c}{(a+b)d}+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a + b)*d) + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*
d*x - 4*c) + a + b)/((a + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(40) = 80.
time = 0.36, size = 82, normalized size = 1.95

$$-\frac{2\,dx-\log\left(\frac{2\,\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)}{2\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*x - log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a
- b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/
((a + b)*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(32) = 64.
time = 3.38, size = 146, normalized size = 3.48

$$\begin{cases} \frac{\tilde{\infty}x}{\tanh{(c)}} & \text{for}\: a=0\wedge b=0\wedge d=0 \\ \frac{1}{2bd\tanh^2{(c+dx)}-2bd} & \text{for}\: a=-b \\ \frac{x\tanh{(c)}}{a+b\tanh^2{(c)}} & \text{for}\: d=0 \\ \frac{x-\frac{\log{(\tanh{(c+dx)}+1)}}{d}}{a} & \text{for}\: b=0 \\ \frac{2dx}{2ad+2bd}+\frac{\log{\left(-\sqrt{-\frac{a}{b}}+\tanh{(c+dx)}\right)}}{2ad+2bd}+\frac{\log{\left(\sqrt{-\frac{a}{b}}+\tanh{(c+dx)}\right)}}{2ad+2bd}-\frac{2\log{(\tanh{(c+dx)}+1)}}{2ad+2bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(2*b*d*tanh(c
 + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)/(a + b*tanh(c)**2), Eq(d, 0)),
((x - log(tanh(c + d*x) + 1)/d)/a, Eq(b, 0)), (2*d*x/(2*a*d + 2*b*d) + log(
-sqrt(-a/b) + tanh(c + d*x))/(2*a*d + 2*b*d) + log(sqrt(-a/b) + tanh(c + d*
x))/(2*a*d + 2*b*d) - 2*log(tanh(c + d*x) + 1)/(2*a*d + 2*b*d), True))
```

Giac [A]
time = 0.44, size = 61, normalized size = 1.45

$$\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{2\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e
^(-2*d*x - 2*c)) + 2*a - 2*b))/((a + b)*d)
```

Mupad [B]
time = 1.17, size = 43, normalized size = 1.02

$$\frac{x}{a+b}-\frac{\ln\left(\tanh(c+d\,x)+1\right)-\frac{\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}}{d\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x/(a + b) - (log(tanh(c + d*x) + 1) - log(a + b*tanh(c + d*x)^2)/2)/(d*(a +
 b))
```

3.175 $\int \frac{1}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{x}{a+b}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)d}$$

```
[Out] x/(a+b)+arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/(a+b)/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3741, 3756, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a+b)}+\frac{x}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-1),x]
```

```
[Out] x/(a + b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a +
 b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3741

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} + \frac{b\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx}{a+b}$$
$$= \frac{x}{a+b} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d}$$
$$= \frac{x}{a+b} + \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)d}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 1.44

$$\frac{\frac{2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} - \log(1-\tanh(c+dx)) + \log(1+\tanh(c+dx))}{2ad+2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-1),x]
```

```
[Out] ((2*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - Log[1 - Tanh
[c + d*x]] + Log[1 + Tanh[c + d*x]])/(2*a*d + 2*b*d)
```

Maple [A]
time = 0.90, size = 71, normalized size = 1.58

method	result	size
derivativedivides	$\frac{\frac{\ln(1+\tanh(dx+c))}{2b+2a}+\frac{b\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}}{d}$	71
default	$\frac{\frac{\ln(1+\tanh(dx+c))}{2b+2a}+\frac{b\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2b+2a}}{d}$	71
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a(a+b)d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2a(a+b)d}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(2*b+2*a)*ln(1+tanh(d*x+c))+b/(a+b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)
/(a*b)^(1/2))-1/(2*b+2*a)*ln(tanh(d*x+c)-1))
```

Maxima [A]
time = 0.51, size = 57, normalized size = 1.27

$$-\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)d}+\frac{dx+c}{(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a +
 b)*d) + (d*x + c)/((a + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(37) = 74$.
time = 0.38, size = 484, normalized size = 10.76

$$\left[\frac{2\,dx+\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)\sinh(dx+c)^4+2\,(a^2-b^2)\cosh(dx+c)^2+2\left(3\,(a^2+2\,ab+b^2)\cosh(dx+c)^2+a^2-b^2\right)\sinh(dx+c)^2+a^2-6\,ab+b^2+4\left((a^2+2\,ab+b^2)\cosh(dx+c)^3+(a^2-b^2)\cosh(dx+c)\right)\sinh(dx+c)+4\left((a^2+ab)\cosh(dx+c)^2+2\,(a^2+ab)\cosh(dx+c)\sinh(dx+c)+(a^2+ab)\sinh(dx+c)^2+a^2-ab\right)\sqrt{-\frac{b}{a}}}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2\,(a-b)\cosh(dx+c)^2+2\left(3\,(a+b)\cosh(dx+c)^2+a-b\right)\sinh(dx+c)^2+4\left((a+b)\cosh(dx+c)^3+(a-b)\cosh(dx+c)\right)\sinh(dx+c)+a+b}\right)}{2\,(a+b)d},\ \frac{dx+\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cosh(dx+c)^2+2\,(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a-b\right)\sqrt{\frac{b}{a}}}{2b}\right)}{(a+b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x
 + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2
 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 +
4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)))/((a + b)*d), (d*x + sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(
a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(
b/a)/b))/((a + b)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(37) = 74$.

time = 3.41, size = 240, normalized size = 5.33

$$\begin{cases} \frac{\tilde{\infty}x}{\tanh^2(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x - \frac{1}{d\tanh(c+dx)}}{b} & \text{for } a = 0 \\ -\frac{dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} + \frac{dx}{2bd\tanh^2(c+dx)-2bd} + \frac{\tanh(c+dx)}{2bd\tanh^2(c+dx)-2bd} & \text{for } a = -b \\ \frac{x}{a+b\tanh^2(c)} & \text{for } d = 0 \\ \frac{x}{a} & \text{for } b = 0 \\ \frac{2dx\sqrt{-\frac{a}{b}}}{2ad\sqrt{-\frac{a}{b}}+2bd\sqrt{-\frac{a}{b}}} + \frac{\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}+2bd\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}+2bd\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - 1/(d*ta
nh(c + d*x)))/b, Eq(a, 0)), (-d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2
- 2*b*d) + d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(c + d*x)/(2*b*d*tanh
(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x/(a + b*tanh(c)**2), Eq(d, 0)), (x/a,
Eq(b, 0)), (2*d*x*sqrt(-a/b)/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)) + log(-s
qrt(-a/b) + tanh(c + d*x))/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)) - log(sqrt
(-a/b) + tanh(c + d*x))/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)), True))
```

Giac [A]
time = 0.41, size = 63, normalized size = 1.40

$$\frac{\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)} + \frac{dx+c}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(s
qrt(a*b)*(a + b)) + (d*x + c)/(a + b))/d
```

Mupad [B]
time = 0.09, size = 37, normalized size = 0.82

$$\frac{x}{a+b} + \frac{b\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x/(a + b) + (b*atan((b*tanh(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a + b))
```

3.176 $\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{\log(\tanh(c+dx))}{ad}-\frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2a(a+b)d}$$

```
[Out] ln(cosh(d*x+c))/(a+b)/d+ln(tanh(d*x+c))/a/d-1/2*b*ln(a+b*tanh(d*x+c)^2)/a/(
a+b)/d
```

Rubi [A]
time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 457, 84}

$$-\frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2ad(a+b)}+\frac{\log(\cosh(c+dx))}{d(a+b)}+\frac{\log(\tanh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)*d) + Log[Tanh[c + d*x]]/(a*d) - (b*Log[a + b*Ta
nh[c + d*x]^2])/(2*a*(a + b)*d)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)} + \frac{1}{ax} - \frac{b^2}{a(a+b)(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log(\tanh(c+dx))}{ad} - \frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2a(a+b)d}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 0.90

$$\frac{2a\log(\cosh(c+dx)) + 2(a+b)\log(\tanh(c+dx)) - b\log\left(a+b\tanh^2(c+dx)\right)}{2a(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (2*a*Log[Cosh[c + d*x]] + 2*(a + b)*Log[Tanh[c + d*x]] - b*Log[a + b*Tanh[c
 + d*x]^2])/(2*a*(a + b)*d)
```

Maple [A]
time = 2.90, size = 113, normalized size = 1.88

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a+b} - \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a+b} - \frac{b\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)}{2a(a+b)} + \ln(\tanh}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a+b} - \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a+b} - \frac{b\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)}{2a(a+b)} + \ln(\tanh}{d}$
risch	$\frac{x}{a+b} - \frac{2x}{a} - \frac{2c}{ad} + \frac{2bx}{a(a+b)} + \frac{2bc}{ad(a+b)} + \frac{\ln\left(e^{2dx+2c}-1\right)}{ad} - \frac{b\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2c}}{a+b}+1\right)}{2ad(a+b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)*ln(tanh(1/2*d*x+1/2*c)+1)-1/(a+b)*ln(tanh(1/2*d*x+1/2*c)-1)-1
/2*b/a/(a+b)*ln(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(
1/2*d*x+1/2*c)^2+a)+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.27, size = 101, normalized size = 1.68

$$-\frac{b\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a^2+ab)d}+\frac{dx+c}{(a+b)d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(
(a^2 + a*b)*d) + (d*x + c)/((a + b)*d) + log(e^(-d*x - c) + 1)/(a*d) + log(
e^(-d*x - c) - 1)/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(58) = 116.
time = 0.42, size = 118, normalized size = 1.97

$$-\frac{2adx+b\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)-2(a+b)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2(a^2+ab)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x + b*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2
+ a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
)) - 2*(a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2
+ a*b)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Giac [A]
time = 0.44, size = 97, normalized size = 1.62

$$-\frac{\frac{b\log\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}{a^2+ab}+\frac{2(dx+c)}{a+b}-\frac{2\log\left(\left|e^{(2dx+2c)}-1\right|\right)}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2
*b*e^(2*d*x + 2*c) + a + b)/(a^2 + a*b) + 2*(d*x + c)/(a + b) - 2*log(abs(e
^(2*d*x + 2*c) - 1))/a)/d
```

Mupad [B]
time = 1.47, size = 194, normalized size = 3.23

$$\frac{\ln\left(12\,a\,b^2+4\,a^2\,b+9\,b^3-9\,b^3\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}-12\,a\,b^2\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}-4\,a^2\,b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\right)}{a\,d}-\frac{b\,\ln\left(5\,a\,b+2\,a^2+3\,b^2+4\,a^2\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+2\,a^2\,{\mathrm{e}}^{4\,c}\,{\mathrm{e}}^{4\,d\,x}-6\,b^2\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+3\,b^2\,{\mathrm{e}}^{4\,c}\,{\mathrm{e}}^{4\,d\,x}+2\,a\,b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+5\,a\,b\,{\mathrm{e}}^{4\,c}\,{\mathrm{e}}^{4\,d\,x}\right)}{2\,d\,a^2+2\,b\,d\,a}-\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] log(12*a*b^2 + 4*a^2*b + 9*b^3 - 9*b^3*exp(2*c)*exp(2*d*x) - 12*a*b^2*exp(2
*c)*exp(2*d*x) - 4*a^2*b*exp(2*c)*exp(2*d*x))/(a*d) - (b*log(5*a*b + 2*a^2
+ 3*b^2 + 4*a^2*exp(2*c)*exp(2*d*x) + 2*a^2*exp(4*c)*exp(4*d*x) - 6*b^2*exp
(2*c)*exp(2*d*x) + 3*b^2*exp(4*c)*exp(4*d*x) + 2*a*b*exp(2*c)*exp(2*d*x) +
5*a*b*exp(4*c)*exp(4*d*x)))/(2*a^2*d + 2*a*b*d) - x/(a + b)
```

3.177 $\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{x}{a+b}-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d}-\frac{\coth(c+dx)}{ad}$$

```
[Out] x/(a+b)-b^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a+b)/d-coth(d*
x+c)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3751, 491, 536, 212, 211}

$$-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d(a+b)}+\frac{x}{a+b}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) - (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a +
 b)*d) - Coth[c + d*x]/(a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)}{ad} + \frac{\text{Subst}\left(\int \frac{a-b+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{ad}$$

$$= -\frac{\coth(c+dx)}{ad} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} - \frac{b^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \right.}{a(a+b)}$$

$$= \frac{x}{a+b} - \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d} - \frac{\coth(c+dx)}{ad}$$

Mathematica [A]
time = 0.14, size = 67, normalized size = 1.12

$$\frac{c+dx}{(a+b)d} - \frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d} - \frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

[Out] (c + d*x)/((a + b)*d) - (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
a^(3/2)*(a + b)*d) - Coth[c + d*x]/(a*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(52) = 104$.
time = 2.74, size = 232, normalized size = 3.87

method	result
risch	$\frac{x}{a+b}-\frac{2}{da\left(e^{2dx+2c}-1\right)}+\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2a^{2}(a+b)d}-\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a^{2}(a+b)d}$
derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a+b}+\frac{2b^{2}\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right.}{a}$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a+b}+\frac{2b^{2}\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/(a+b)*ln(tanh(1/2*d*x+1/2*c)-1)+2*b^2/(a+
b)*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2
))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)))+
1/(a+b)*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/a/tanh(1/2*d*x+1/2*c))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(52) = 104$.
time = 0.52, size = 329, normalized size = 5.48

$$-\frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^{2}+ab)d}+\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^{2}+ab)d}+\frac{(ab-b^{2})\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^{2}+ab)\sqrt{ab}\,d}-\frac{(ab-b^{2})\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^{2}+ab)\sqrt{ab}\,d}+\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}\,ad}+\frac{\log\left(e^{(2\,dx+2\,c)}-1\right)}{2\,ad}-\frac{\log\left(e^{(-2\,dx-2\,c)}-1\right)}{2\,ad}-\frac{1}{2\,(ae^{(2\,dx+2\,c)}-a)d}+\frac{3}{2\,(ae^{(-2\,dx-2\,c)}-a)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] -1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + a*b)*d) + 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + a*b)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + a*b)*sqrt(a*b)*d) - 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + a*b)*sqrt(a*b)*d) + 1/2*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*log(e^(2*d*x + 2*c) - 1)/(a*d) - 1/2*log(e^(-2*d*x - 2*c) - 1)/(a*d) - 1/2/((a*e^(2*d*x + 2*c) - a)*d) + 3/2/((a*e^(-2*d*x - 2*c) - a)*d)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(52) = 104$.
time = 0.39, size = 784, normalized size = 13.07

$$\left[\frac{2 a d x \cosh\left(d x + c\right)^{2} + 4 a d x \cosh\left(d x + c\right) \sinh\left(d x + c\right) + 2 a d x \sinh\left(d x + c\right)^{2} - 2 a d x + \left(b \cosh\left(d x + c\right)^{2} + 2 b \cosh\left(d x + c\right) \sinh\left(d x + c\right) + b \sinh\left(d x + c\right)^{2} - b\right) \sqrt{-\frac{b}{a}} \log\left(\frac{\left(a^{2} + 2 a b + b^{2}\right) \cosh\left(d x + c\right)^{4} + 4 \left(a^{2} + 2 a b + b^{2}\right) \cosh\left(d x + c\right) \sinh\left(d x + c\right)^{3} + \left(a^{2} + 2 a b + b^{2}\right) \sinh\left(d x + c\right)^{4} + 2 \left(a^{2} - b^{2}\right) \cosh\left(d x + c\right)^{2} + 2 \left(3 \left(a^{2} + 2 a b + b^{2}\right) \cosh\left(d x + c\right)^{2} + a^{2} - b^{2}\right) \sinh\left(d x + c\right)^{2} + a^{2} - 6 a b + b^{2} + 4 \left(\left(a^{2} + 2 a b + b^{2}\right) \cosh\left(d x + c\right)^{3} + \left(a^{2} - b^{2}\right) \cosh\left(d x + c\right)\right) \sinh\left(d x + c\right) - 4 \left(\left(a^{2} + a b\right) \cosh\left(d x + c\right)^{2} + 2 \left(a^{2} + a b\right) \cosh\left(d x + c\right) \sinh\left(d x + c\right) + \left(a^{2} + a b\right) \sinh\left(d x + c\right)^{2} + a^{2} - a b\right) \sqrt{-\frac{b}{a}}}{\left(a + b\right) \cosh\left(d x + c\right)^{4} + 4 \left(a + b\right) \cosh\left(d x + c\right) \sinh\left(d x + c\right)^{3} + \left(a + b\right) \sinh\left(d x + c\right)^{4} + 2 \left(a - b\right) \cosh\left(d x + c\right)^{2} + 2 \left(3 \left(a + b\right) \cosh\left(d x + c\right)^{2} + a - b\right) \sinh\left(d x + c\right)^{2} + 4 \left(\left(a + b\right) \cosh\left(d x + c\right)^{3} + \left(a - b\right) \cosh\left(d x + c\right)\right) \sinh\left(d x + c\right) + a + b}\right) - 4 a - 4 b}{2 \left(\left(a^{2} + a b\right) d \cosh\left(d x + c\right)^{2} + 2 \left(a^{2} + a b\right) d \cosh\left(d x + c\right) \sinh\left(d x + c\right) + \left(a^{2} + a b\right) d \sinh\left(d x + c\right)^{2} - \left(a^{2} + a b\right) d\right)}, \frac{a d x \cosh\left(d x + c\right)^{2} + 2 a d x \cosh\left(d x + c\right) \sinh\left(d x + c\right) + a d x \sinh\left(d x + c\right)^{2} - a d x - \left(b \cosh\left(d x + c\right)^{2} + 2 b \cosh\left(d x + c\right) \sinh\left(d x + c\right) + b \sinh\left(d x + c\right)^{2} - b\right) \sqrt{\frac{b}{a}} \arctan\left(\frac{\left(\left(a + b\right) \cosh\left(d x + c\right)^{2} + 2 \left(a + b\right) \cosh\left(d x + c\right) \sinh\left(d x + c\right) + \left(a + b\right) \sinh\left(d x + c\right)^{2} + a - b\right) \sqrt{\frac{b}{a}}}{2 b}\right) - 2 a - 2 b}{\left(a^{2} + a b\right) d \cosh\left(d x + c\right)^{2} + 2 \left(a^{2} + a b\right) d \cosh\left(d x + c\right) \sinh\left(d x + c\right) + \left(a^{2} + a b\right) d \sinh\left(d x + c\right)^{2} - \left(a^{2} + a b\right) d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] [1/2*(2*a*d*x*cosh(d*x + c)^2 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*a*d*x*sinh(d*x + c)^2 - 2*a*d*x + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*a - 4*b)/((a^2 + a*b)*d*cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 - (a^2 + a*b)*d), (a*d*x*cosh(d*x + c)^2 + 2*a*d*x*cosh(d*x + c)*sinh(d*x + c) + a*d*x*sinh(d*x + c)^2 - a*d*x - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - 2*a - 2*b)/((a^2 + a*b)*d*cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 - (a^2 + a*b)*d)]

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Giac [A]
time = 0.46, size = 89, normalized size = 1.48

$$-\frac{\frac{b^2 \arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^2+ab)\sqrt{ab}}-\frac{dx+c}{a+b}+\frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))
/((a^2 + a*b)*sqrt(a*b)) - (d*x + c)/(a + b) + 2/(a*(e^(2*d*x + 2*c) - 1)))
/d
```

Mupad [B]
time = 1.62, size = 402, normalized size = 6.70

$$\frac{x}{a+b}-\frac{\mathrm{atan}\left(\left(e^{2c}\,e^{2dx}\left(\frac{4b^2}{a\,d\,(a+b)^3\,(a^2+b\,a)\,\sqrt{b^3}}+\frac{\left(a^3\,d\,\sqrt{b^3}-a\,b^2\,d\,\sqrt{b^3}\right)(a-b)}{b^2\,(a+b)^2\,(a^2+b\,a)\,\sqrt{a^5\,d^2+2\,a^4\,b\,d^2+a^3\,b^2\,d^2}\,\sqrt{a^3\,d^2\,(a+b)^2}}\right)+\frac{(a-b)\left(a^3\,d\,\sqrt{b^3}+a\,b^2\,d\,\sqrt{b^3}+2\,a^2\,b\,d\,\sqrt{b^3}\right)}{b^2\,(a+b)^2\,(a^2+b\,a)\,\sqrt{a^5\,d^2+2\,a^4\,b\,d^2+a^3\,b^2\,d^2}\,\sqrt{a^3\,d^2\,(a+b)^2}}\right)\left(\frac{a^3\,\sqrt{a^5\,d^2+2\,a^4\,b\,d^2+a^3\,b^2\,d^2}}{2}+\frac{a\,b^2\,\sqrt{a^5\,d^2+2\,a^4\,b\,d^2+a^3\,b^2\,d^2}}{2}+a^2\,b\,\sqrt{a^5\,d^2+2\,a^4\,b\,d^2+a^3\,b^2\,d^2}\right)\right)\sqrt{b^3}}{\sqrt{a^5\,d^2+2\,a^4\,b\,d^2+a^3\,b^2\,d^2}}-\frac{2}{a\,d\,\left(e^{2c+2dx}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x/(a + b) - (atan((exp(2*c)*exp(2*d*x)*((4*b^2)/(a*d*(a + b)^3*(a*b + a^2)*
(b^3)^(1/2)) + ((a^3*d*(b^3)^(1/2) - a*b^2*d*(b^3)^(1/2))*(a - b))/(b^2*(a
+ b)^2*(a*b + a^2)*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)*(a^3*d^2*(a
+ b)^2)^(1/2))) + ((a - b)*(a^3*d*(b^3)^(1/2) + a*b^2*d*(b^3)^(1/2) + 2*a^2
*b*d*(b^3)^(1/2)))/(b^2*(a + b)^2*(a*b + a^2)*(a^5*d^2 + 2*a^4*b*d^2 + a^3*
b^2*d^2)^(1/2)*(a^3*d^2*(a + b)^2)^(1/2)))*((a^3*(a^5*d^2 + 2*a^4*b*d^2 + a
^3*b^2*d^2)^(1/2))/2 + (a*b^2*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2))/
2 + a^2*b*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)))*(b^3)^(1/2))/(a^5*d
^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2) - 2/(a*d*(exp(2*c + 2*d*x) - 1))
```

3.178 $\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=85

$$-\frac{\coth^2(c+dx)}{2ad}+\frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{(a-b)\log(\tanh(c+dx))}{a^2d}+\frac{b^2\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)d}$$

```
[Out] -1/2*coth(d*x+c)^2/a/d+ln(cosh(d*x+c))/(a+b)/d+(a-b)*ln(tanh(d*x+c))/a^2/d+
1/2*b^2*ln(a+b*tanh(d*x+c)^2)/a^2/(a+b)/d
```

Rubi [A]
time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 84}

$$\frac{b^2\log\left(a+b\tanh^2(c+dx)\right)}{2a^2d(a+b)}+\frac{(a-b)\log(\tanh(c+dx))}{a^2d}+\frac{\log(\cosh(c+dx))}{d(a+b)}-\frac{\coth^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] -1/2*Coth[c + d*x]^2/(a*d) + Log[Cosh[c + d*x]]/((a + b)*d) + ((a - b)*Log[
Tanh[c + d*x]])/(a^2*d) + (b^2*Log[a + b*Tanh[c + d*x]^2])/(2*a^2*(a + b)*d
)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

alQ[n]))

Rubi steps

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)}+\frac{1}{ax^2}+\frac{a-b}{a^2x}+\frac{b^3}{a^2(a+b)(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= -\frac{\coth^2(c+dx)}{2ad}+\frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{(a-b)\log(\tanh(c+dx))}{a^2d}+\frac{b^2\log\left(a+\right.}{2a^2}$$

Mathematica [A]
time = 0.11, size = 60, normalized size = 0.71

$$-\frac{\frac{\coth^2(c+dx)}{a}-\frac{b^2\log\left(b+a\coth^2(c+dx)\right)}{a^2(a+b)}-\frac{2\log(\sinh(c+dx))}{a+b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
[Out] -1/2*(Coth[c + d*x]^2/a - (b^2*Log[b + a*Coth[c + d*x]^2])/(a^2*(a + b)) -
(2*Log[Sinh[c + d*x]])/(a + b))/d
```

Maple [A]
time = 3.00, size = 155, normalized size = 1.82

method	result
derivativedivides	$\frac{-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a+b}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(4a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a+b}+\frac{b^2\ln\left(a\left(\tanh^4\left(\right.\right.\right.}{}}{d}$
default	$\frac{-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a+b}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(4a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a+b}+\frac{b^2\ln\left(a\left(\tanh^4\left(\right.\right.\right.}{}}{d}$
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2c}{ad}+\frac{2bx}{a^2}+\frac{2bc}{a^2d}-\frac{2b^2x}{a^2(a+b)}-\frac{2b^2c}{a^2d(a+b)}-\frac{2\,\mathrm{e}^{2dx+2c}}{ad\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\frac{\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{ad}-\frac{b\ln\left(\mathrm{e}^{2dx+2c}\right.}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*tanh(1/2*d*x+1/2*c)^2/a-1/(a+b)*ln(tanh(1/2*d*x+1/2*c)+1)-1/8/a/t
anh(1/2*d*x+1/2*c)^2+1/4/a^2*(4*a-4*b)*ln(tanh(1/2*d*x+1/2*c))-1/(a+b)*ln(t
anh(1/2*d*x+1/2*c)-1)+1/2*b^2/a^2/(a+b)*ln(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh
(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a))
```

Maxima [A]
time = 0.28, size = 159, normalized size = 1.87

$$\frac{b^2\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+a^2b)d}+\frac{dx+c}{(a+b)d}+\frac{2\,e^{(-2\,dx-2\,c)}}{\left(2\,ae^{(-2\,dx-2\,c)}-ae^{(-4\,dx-4\,c)}-a\right)d}+\frac{(a-b)\log\left(e^{(-dx-c)}+1\right)}{a^2d}+\frac{(a-b)\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^3 + a^2*b)*d) + (d*x + c)/((a + b)*d) + 2*e^(-2*d*x - 2*c)/((2*a*e^(-2*
d*x - 2*c) - a*e^(-4*d*x - 4*c) - a)*d) + (a - b)*log(e^(-d*x - c) + 1)/(a^
2*d) + (a - b)*log(e^(-d*x - c) - 1)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(81) = 162$.
time = 0.49, size = 747, normalized size = 8.79

$$-\frac{2\,a^2dx\cosh\left(dx+c\right)^4+8\,a^2dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,a^2dx\sinh\left(dx+c\right)^4+2\,a^2dx-4\,\left(a^2dx-a^2-ab\right)\cosh\left(dx+c\right)^2+4\,\left(3\,a^2dx\cosh\left(dx+c\right)^2-a^2dx+a^2+ab\right)\sinh\left(dx+c\right)^2-\left(b^2\cosh\left(dx+c\right)^4+4\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+b^2\sinh\left(dx+c\right)^4-2\,b^2\cosh\left(dx+c\right)^2+2\,\left(3\,b^2\cosh\left(dx+c\right)^2-b^2\right)\sinh\left(dx+c\right)^2+b^2+4\,\left(b^2\cosh\left(dx+c\right)^3-b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)\log\left(\frac{2\,\left((a+b)\cosh\left(dx+c\right)^2+(a+b)\sinh\left(dx+c\right)^2+a-b\right)}{\cosh\left(dx+c\right)^2-2\,\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^2}\right)-2\,\left((a^2-b^2)\cosh\left(dx+c\right)^4+4\,(a^2-b^2)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(a^2-b^2)\sinh\left(dx+c\right)^4-2\,(a^2-b^2)\cosh\left(dx+c\right)^2+2\,\left(3\,(a^2-b^2)\cosh\left(dx+c\right)^2-a^2+b^2\right)\sinh\left(dx+c\right)^2+a^2-b^2+4\,\left((a^2-b^2)\cosh\left(dx+c\right)^3-(a^2-b^2)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)\log\left(\frac{2\,\sinh\left(dx+c\right)}{\cosh\left(dx+c\right)-\sinh\left(dx+c\right)}\right)+8\,\left(a^2dx\cosh\left(dx+c\right)^3-\left(a^2dx-a^2-ab\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{2\,\left((a^3+a^2b)d\cosh\left(dx+c\right)^4+4\,(a^3+a^2b)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(a^3+a^2b)d\sinh\left(dx+c\right)^4-2\,(a^3+a^2b)d\cosh\left(dx+c\right)^2+2\,\left(3\,(a^3+a^2b)d\cosh\left(dx+c\right)^2-(a^3+a^2b)d\right)\sinh\left(dx+c\right)^2+(a^3+a^2b)d+4\,\left((a^3+a^2b)d\cosh\left(dx+c\right)^3-(a^3+a^2b)d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^4 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*a^2*d*x*sinh(d*x + c)^4 + 2*a^2*d*x - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x +
 c)^2 + 4*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c)^2
 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*
x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x
 + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*
log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x
 + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^2 - b^2
)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^
2)*sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(
d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d
*x + c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(
cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^3 - (a^2*d*x - a
^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh(d*x + c)^4 +
4*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + a^2*b)*d*sinh(d*x
+ c)^4 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + a^2*b)*d*cosh(d*x
+ c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^2 + (a^3 + a^2*b)*d + 4*((a^3 + a^2
*b)*d*cosh(d*x + c)^3 - (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Giac [A]
time = 0.48, size = 133, normalized size = 1.56

$$\frac{\frac{b^2\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^3+a^2b}-\frac{2\,(dx+c)}{a+b}+\frac{2\,(a-b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^2}-\frac{4\,e^{(2\,dx+2\,c)}}{a\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)/(a^3 + a^2*b) - 2*(d*x + c)/(a + b) + 2*(a - b
)*log(abs(e^(2*d*x + 2*c) - 1))/a^2 - 4*e^(2*d*x + 2*c)/(a*(e^(2*d*x + 2*c)
 - 1)^2))/d
```

Mupad [B]
time = 1.54, size = 313, normalized size = 3.68

$$\frac{b^2\,\ln\left(3\,a\,b^2-2\,a^2\,b-2\,a^3+3\,b^3-4\,a^3\,e^{2\,c}\,e^{2\,d\,x}-2\,a^3\,e^{4\,c}\,e^{4\,d\,x}-6\,b^3\,e^{2\,c}\,e^{2\,d\,x}+3\,b^3\,e^{4\,c}\,e^{4\,d\,x}+6\,a\,b^2\,e^{2\,c}\,e^{2\,d\,x}+4\,a^2\,b\,e^{2\,c}\,e^{2\,d\,x}+3\,a\,b^2\,e^{4\,c}\,e^{4\,d\,x}-2\,a^2\,b\,e^{4\,c}\,e^{4\,d\,x}\right)}{2\,d\,a^3+2\,b\,d\,a^2}-\frac{x}{a+b}-\frac{2}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}+\frac{\ln\left(4\,a^4\,b+9\,b^5-12\,a^2\,b^3-9\,b^5\,e^{2\,c}\,e^{2\,d\,x}-4\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x}+12\,a^2\,b^3\,e^{2\,c}\,e^{2\,d\,x}\right)\,\left(a-b\right)}{a^2\,d}-\frac{2\,\left(a^2+b\,a\right)}{a^2\,d\,\left(e^{2\,c+2\,d\,x}-1\right)\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (b^2*log(3*a*b^2 - 2*a^2*b - 2*a^3 + 3*b^3 - 4*a^3*exp(2*c)*exp(2*d*x) - 2*
a^3*exp(4*c)*exp(4*d*x) - 6*b^3*exp(2*c)*exp(2*d*x) + 3*b^3*exp(4*c)*exp(4*
d*x) + 6*a*b^2*exp(2*c)*exp(2*d*x) + 4*a^2*b*exp(2*c)*exp(2*d*x) + 3*a*b^2*
exp(4*c)*exp(4*d*x) - 2*a^2*b*exp(4*c)*exp(4*d*x)))/(2*a^3*d + 2*a^2*b*d) -
 x/(a + b) - 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (log(4*a
^4*b + 9*b^5 - 12*a^2*b^3 - 9*b^5*exp(2*c)*exp(2*d*x) - 4*a^4*b*exp(2*c)*ex
p(2*d*x) + 12*a^2*b^3*exp(2*c)*exp(2*d*x))*(a - b))/(a^2*d) - (2*(a*b + a^2
))/(a^2*d*(exp(2*c + 2*d*x) - 1)*(a + b))
```

3.179 $\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{x}{a+b}+\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)d}-\frac{(a-b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

```
[Out] x/(a+b)+b^(5/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a+b)/d-(a-b)*c
oth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d
```

Rubi [A]
time = 0.13, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 491, 597, 536, 212, 211}

$$\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d(a+b)}-\frac{(a-b)\coth(c+dx)}{a^2d}+\frac{x}{a+b}-\frac{\coth^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] x/(a + b) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*(a +
 b)*d) - ((a - b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\coth^3(c+dx)}{3ad} + \frac{\text{Subst}\left(\int \frac{3(a-b)+3bx^2}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right)}{3ad}$$

$$= -\frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} - \frac{\text{Subst}\left(\int \frac{-3(a^2-ab+b^2)-3(a-b)bx^2}{(1-x^2)(a+bx^2)}\,dx, x, t\right.}{3a^2d}$$

$$= -\frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} +$$

$$= \frac{x}{a+b} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)d} - \frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}$$

Mathematica [A]
time = 0.49, size = 91, normalized size = 1.11

$$\frac{\frac{6\left(c+dx+\frac{b^{5/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}}\right)}{a+b}-\frac{(-2a+3b+(4a-3b)\cosh(2(c+dx)))\coth(c+dx)\operatorname{csch}^2(c+dx)}{a^2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
[Out] ((6*(c + d*x + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(5/2)))/
(a + b) - ((-2*a + 3*b + (4*a - 3*b)*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csch[
c + d*x]^2)/a^2)/(6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(72) = 144.
time = 2.74, size = 288, normalized size = 3.51

method	result
risch	$\frac{x}{a+b}-\frac{2\left(6a\,e^{4dx+4c}-3b\,e^{4dx+4c}-6a\,e^{2dx+2c}+6b\,e^{2dx+2c}+4a-3b\right)}{3a^2d\left(e^{2dx+2c}-1\right)^3}+\frac{\sqrt{-ab}\,b^2\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a^3(a+b)d}$
derivativedivides	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a+b}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{5a-4b}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-$ ... $2b^3$
default	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a+b}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{5a-4b}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-$ ... $2b^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a^2*(1/3*a*tanh(1/2*d*x+1/2*c)^3+5*a*tanh(1/2*d*x+1/2*c)-4*b*tanh
(1/2*d*x+1/2*c))-1/(a+b)*ln(tanh(1/2*d*x+1/2*c)-1)-1/24/a/tanh(1/2*d*x+1/2*
c)^3-1/8*(5*a-4*b)/a^2/tanh(1/2*d*x+1/2*c)-2*b^3/(a+b)/a*(-1/2*(-a+(b*(a+b)
)^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(
1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)))+1/(a+b)*ln(tanh(1/2*d
*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(72) = 144$.
time = 0.62, size = 1038, normalized size = 12.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(a*b - b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) +
a + b)/((a^3 + a^2*b)*d) + 1/8*(a*b - b^2)*log(2*(a - b)*e^(-2*d*x - 2*c) +
 (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + a^2*b)*d) + 1/16*(a^2*b - 6*a*b^
2 + b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^3 + a^
2*b)*sqrt(a*b)*d) - 1/16*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*
d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + a^2*b)*sqrt(a*b)*d) - 1/24*(3*(12*a
- b)*e^(4*d*x + 4*c) - 6*(9*a - b)*e^(2*d*x + 2*c) + 22*a - 3*b)/((a^2*e^(6
*d*x + 6*c) - 3*a^2*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - a^2)*d) - 1/6
*(3*(4*a - b)*e^(4*d*x + 4*c) - 6*(2*a - b)*e^(2*d*x + 2*c) + 4*a - 3*b)/((
a^2*e^(6*d*x + 6*c) - 3*a^2*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - a^2)*
d) - 1/24*(6*(9*a - b)*e^(-2*d*x - 2*c) - 3*(12*a - b)*e^(-4*d*x - 4*c) - 2
2*a + 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c) + a^2*e^(-6*d*
x - 6*c) - a^2)*d) - 1/6*(6*(2*a - b)*e^(-2*d*x - 2*c) - 3*(4*a - b)*e^(-4*
d*x - 4*c) - 4*a + 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c) +
 a^2*e^(-6*d*x - 6*c) - a^2)*d) + 1/4*(6*(a + b)*e^(-2*d*x - 2*c) - 3*b*e^(
-4*d*x - 4*c) - 2*a - 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c
) + a^2*e^(-6*d*x - 6*c) - a^2)*d) + 1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*
(a - b)*e^(2*d*x + 2*c) + a + b)/(a^2*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x -
2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(a^2*d) + 1/4*(2*a - b)*log(e^(2*d
*x + 2*c) - 1)/(a^2*d) - 1/2*b*log(e^(2*d*x + 2*c) - 1)/(a^2*d) - 1/4*(2*a
- b)*log(e^(-2*d*x - 2*c) - 1)/(a^2*d) + 1/2*b*log(e^(-2*d*x - 2*c) - 1)/(a
^2*d) - 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a
*b))/(sqrt(a*b)*a^2*d) - 3/8*(a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*
c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/4*(a*b - b^2)*arctan(1/2*((a +
 b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(72) = 144$.
time = 0.40, size = 2368, normalized size = 28.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^2*d*x*cosh(d*x + c)^6 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^5
+ 6*a^2*d*x*sinh(d*x + c)^6 - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*
x + c)^4 + 6*(15*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x - 4*a^2 - 2*a*b + 2*b^
2)*sinh(d*x + c)^4 - 6*a^2*d*x + 24*(5*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x
 + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(3*a^2*d*x + 4
*a^2 - 4*b^2)*cosh(d*x + c)^2 + 6*(15*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x -
 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*b^2)*sin
h(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5
 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 -
 b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 -
3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cos
h(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*
cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 +
2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)
^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2
+ a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*
cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*
b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*s
qrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cos
h(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b
)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 16*a^2 - 4*a*b + 12*b^2 + 12*(3*
a^2*d*x*cosh(d*x + c)^5 - 2*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x +
c)^3 + (3*a^2*d*x + 4*a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^
2*b)*d*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 +
(a^3 + a^2*b)*d*sinh(d*x + c)^6 - 3*(a^3 + a^2*b)*d*cosh(d*x + c)^4 + 3*(5*
(a^3 + a^2*b)*d*cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^4 + 3*(a^3
 + a^2*b)*d*cosh(d*x + c)^2 + 4*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^3 - 3*(a^3
 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + a^2*b)*d*cosh(d*x
+ c)^4 - 6*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + (a^3 + a^2*b)*d)*sinh(d*x + c)
^2 - (a^3 + a^2*b)*d + 6*((a^3 + a^2*b)*d*cosh(d*x + c)^5 - 2*(a^3 + a^2*b)
*d*cosh(d*x + c)^3 + (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(3*
a^2*d*x*cosh(d*x + c)^6 + 18*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 3*a^2*
d*x*sinh(d*x + c)^6 - 3*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4
 + 3*(15*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x - 4*a^2 - 2*a*b + 2*b^2)*sinh(
```

```
d*x + c)^4 - 3*a^2*d*x + 12*(5*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x + 4*a^2
 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(3*a^2*d*x + 4*a^2 - 4
*b^2)*cosh(d*x + c)^2 + 3*(15*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x - 6*(3*a^
2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*b^2)*sinh(d*x +
c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*s
inh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*si
nh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*co
sh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x +
c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x
 + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*c
osh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x +
 c)^2 + a - b)*sqrt(b/a)/b) - 8*a^2 - 2*a*b + 6*b^2 + 6*(3*a^2*d*x*cosh(d*x
 + c)^5 - 2*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^3 + (3*a^2*d*
x + 4*a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh(d*x
+ c)^6 + 6*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + a^2*b)*d*
sinh(d*x + c)^6 - 3*(a^3 + a^2*b)*d*cosh(d*x + c)^4 + 3*(5*(a^3 + a^2*b)*d*
cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^4 + 3*(a^3 + a^2*b)*d*cosh
(d*x + c)^2 + 4*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^3 - 3*(a^3 + a^2*b)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^4 - 6*(a^3
+ a^2*b)*d*cosh(d*x + c)^2 + (a^3 + a^2*b)*d)*sinh(d*x + c)^2 - (a^3 + a^2*
b)*d + 6*((a^3 + a^2*b)*d*cosh(d*x + c)^5 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)
^3 + (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(72) = 144.
time = 0.46, size = 147, normalized size = 1.79

$$\frac{\frac{3\,b^3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3+a^2b)\sqrt{ab}}+\frac{3\,(dx+c)}{a+b}-\frac{2\left(6\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+4\,a-3\,b\right)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/((a^3 + a^2*b)*sqrt(a*b)) + 3*(d*x + c)/(a + b) - 2*(6*a*e^(4*d*x + 4
*c) - 3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 4*a
 - 3*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B]
time = 1.61, size = 519, normalized size = 6.33

$$\frac{x}{a+b}+\frac{\mathrm{atan}\left(\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\left(\frac{4\,b^3}{a^2\,d\,{\left(a+b\right)}^3\,\left(a^3+b\,a^2\right)\,\sqrt{b^5}}+\frac{\left(a^4\,d\,\sqrt{b^5}-a^2\,b^2\,d\,\sqrt{b^5}\right)\,\left(a-b\right)}{b^3\,{\left(a+b\right)}^2\,\left(a^3+b\,a^2\right)\,\sqrt{a^7\,d^2+2\,a^6\,b\,d^2+a^5\,b^2\,d^2}\,\sqrt{a^5\,d^2\,{\left(a+b\right)}^2}}\right)+\frac{\left(a-b\right)\,\left(a^4\,d\,\sqrt{b^5}+2\,a^3\,b\,d\,\sqrt{b^5}+a^2\,b^2\,d\,\sqrt{b^5}\right)}{b^3\,{\left(a+b\right)}^2\,\left(a^3+b\,a^2\right)\,\sqrt{a^7\,d^2+2\,a^6\,b\,d^2+a^5\,b^2\,d^2}\,\sqrt{a^5\,d^2\,{\left(a+b\right)}^2}}\right)\,\left(\frac{a^4\,\sqrt{a^7\,d^2+2\,a^6\,b\,d^2+a^5\,b^2\,d^2}}{2}+a^3\,b\,\sqrt{a^7\,d^2+2\,a^6\,b\,d^2+a^5\,b^2\,d^2}+\frac{a^2\,b^2\,\sqrt{a^7\,d^2+2\,a^6\,b\,d^2+a^5\,b^2\,d^2}}{2}\right)\right)\,\sqrt{b^5}}{\sqrt{a^7\,d^2+2\,a^6\,b\,d^2+a^5\,b^2\,d^2}}-\frac{8}{3\,a\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{4\,\left(a^2+b\,a\right)}{a^2\,d\,\left(a+b\right)\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\left(2\,a^2+a\,b-b^2\right)}{a^2\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] x/(a + b) + (atan((exp(2*c)*exp(2*d*x)*((4*b^3)/(a^2*d*(a + b)^3*(a^2*b + a
^3)*(b^5)^(1/2)) + ((a^4*d*(b^5)^(1/2) - a^2*b^2*d*(b^5)^(1/2))*(a - b))/(b
^3*(a + b)^2*(a^2*b + a^3)*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2)*(a^5
*d^2*(a + b)^2)^(1/2))) + ((a - b)*(a^4*d*(b^5)^(1/2) + 2*a^3*b*d*(b^5)^(1/
2) + a^2*b^2*d*(b^5)^(1/2)))/(b^3*(a + b)^2*(a^2*b + a^3)*(a^7*d^2 + 2*a^6*
b*d^2 + a^5*b^2*d^2)^(1/2)*(a^5*d^2*(a + b)^2)^(1/2)))*((a^4*(a^7*d^2 + 2*a
^6*b*d^2 + a^5*b^2*d^2)^(1/2))/2 + a^3*b*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d
^2)^(1/2) + (a^2*b^2*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2))/2))*(b^5)
^(1/2))/(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2) - 8/(3*a*d*(3*exp(2*c +
 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(a*b + a^2))/(a^
2*d*(a + b)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (2*(a*b + 2*a^2
- b^2))/(a^2*d*(exp(2*c + 2*d*x) - 1)*(a + b))
```

3.180 $\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} \, dx$

Optimal. Leaf size=83

$$\frac{\log(\cosh(c+dx))}{(a+b)^2 d} - \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)^2 d} - \frac{a^2}{2b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^2/d-1/2*a*(a+2*b)*ln(a+b*tanh(d*x+c)^2)/b^2/(a+b)^2/d
-1/2*a^2/b^2/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{a^2}{2b^2 d(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2 d(a+b)^2} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) - (a*(a + 2*b)*Log[a + b*Tanh[c + d*x]^2])
/(2*b^2*(a + b)^2*d) - a^2/(2*b^2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{a^2}{b(a+b)(a+bx)^2} - \frac{a(a+2b)}{b(a+b)^2(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right.}{2d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)^2d} - \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)^2d} - \frac{}{2b^2(a+b)d\left(a-\right.}$$

Mathematica [A]
time = 0.38, size = 69, normalized size = 0.83

$$-\frac{-2\log(\cosh(c+dx)) + \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{b^2} + \frac{a^2(a+b)}{b^2\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
[Out] -1/2*(-2*Log[Cosh[c + d*x]] + (a*(a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/b^2
+ (a^2*(a + b))/(b^2*(a + b*Tanh[c + d*x]^2)))/((a + b)^2*d)
```

Maple [A]
time = 0.73, size = 91, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{(a+b)a}{b^2\left(a+b\left(\tanh^2(dx+c)\right)\right)}+\frac{(2b+a)\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{b^2}\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$\frac{-\frac{a\left(\frac{(a+b)a}{b^2\left(a+b\left(\tanh^2(dx+c)\right)\right)}+\frac{(2b+a)\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{b^2}\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x}{b^2}-\frac{2c}{b^2d}+\frac{4ax}{b(a^2+2ab+b^2)}+\frac{4ac}{bd(a^2+2ab+b^2)}+\frac{2a^2x}{b^2(a^2+2ab+b^2)}+\frac{2a^2c}{b^2d(a^2+2ab+b^2)}-\frac{}{bd(a+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a/(a+b)^2*((a+b)/b^2*a/(a+b*tanh(d*x+c)^2)+(2*b+a)/b^2*ln(a+b*tan
h(d*x+c)^2))-1/2/(a+b)^2*ln(tanh(d*x+c)-1)-1/2/(a+b)^2*ln(1+tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(79) = 158$.
time = 0.52, size = 217, normalized size = 2.61

$$-\frac{2\,a^{2}e^{(-2\,dx-2\,c)}}{(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4}+2\,(a^{3}b+a^{2}b^{2}-ab^{3}-b^{4})e^{(-2\,dx-2\,c)}+(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4})e^{(-4\,dx-4\,c)})d}-\frac{(a^{2}+2\,ab)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^{2}b^{2}+2\,ab^{3}+b^{4})d}+\frac{dx+c}{(a^{2}+2\,ab+b^{2})d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -2*a^2*e^(-2*d*x - 2*c)/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4 + 2*(a^3*b + a^
2*b^2 - a*b^3 - b^4)*e^(-2*d*x - 2*c) + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*e^(-4*d*x - 4*c))*d) - 1/2*(a^2 + 2*a*b)*log(2*(a - b)*e^(-2*d*x - 2*c) +
(a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*d) + (d*x + c)
/((a^2 + 2*a*b + b^2)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(79) = 158$.
time = 0.46, size = 1141, normalized size = 13.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*b^2 + b^3)*d*x*cosh(d*x + c)^4 + 8*(a*b^2 + b^3)*d*x*cosh(d*x +
c)*sinh(d*x + c)^3 + 2*(a*b^2 + b^3)*d*x*sinh(d*x + c)^4 + 2*(a*b^2 + b^3)*
d*x + 4*(a^2*b + (a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a*b^2 + b^3)*d*
x*cosh(d*x + c)^2 + a^2*b + (a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((a^3 + 3*
a^2*b + 2*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*sinh(d*x + c)^4 + a^3 + 3*a^2
*b + 2*a*b^2 + 2*(a^3 + a^2*b - 2*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b -
 2*a*b^2 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (a^3 + a^2*b - 2*a*b^2)*cosh(
d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x
+ c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
 + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*
```

```
a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x
 + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
+ 8*((a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + (a^2*b + (a*b^2 - b^3)*d*x)*cosh(d
*x + c))*sinh(d*x + c))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x +
 c)^4 + 4*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*sinh(d*x + c)^4 + 2*(a^3*b^2
 + a^2*b^3 - a*b^4 - b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + 3*a^2*b^3 + 3
*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*d)*sinh
(d*x + c)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d + 4*((a^3*b^2 + 3*a^2
*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x + c)^3 + (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)
*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(79) = 158.
time = 0.53, size = 194, normalized size = 2.34

$$-\frac{\frac{(a^2+2\,ab)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^2b^2+2\,ab^3+b^4}+\frac{2\,(dx+c)}{a^2+2\,ab+b^2}+\frac{4\,a^2e^{(2\,dx+2\,c)}}{\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)(a+b)^2b}-\frac{2\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a^2 + 2*a*b)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^2*b^2 + 2*a*b^3 + b^4) + 2*(d*x
+ c)/(a^2 + 2*a*b + b^2) + 4*a^2*e^(2*d*x + 2*c)/((a*e^(4*d*x + 4*c) + b*e^
(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a + b)^
2*b) - 2*log(e^(2*d*x + 2*c) + 1)/b^2)/d
```

Mupad [**B**]
time = 1.61, size = 170, normalized size = 2.05

$$-\frac{a^2}{2\,\left(d\,a^2\,b^2+d\,a\,b^3\,\mathrm{tanh}\left(c+d\,x\right)^2+d\,a\,b^3+d\,b^4\,\mathrm{tanh}\left(c+d\,x\right)^2\right)}-\frac{\ln\left(\mathrm{tanh}(c+d\,x)^2-1\right)}{2\,\left(d\,a^2+2\,d\,a\,b+d\,b^2\right)}-\frac{a^2\,\ln\left(b\,\mathrm{tanh}(c+d\,x)^2+a\right)}{2\,\left(d\,a^2\,b^2+2\,d\,a\,b^3+d\,b^4\right)}-\frac{a\,b\,\ln\left(b\,\mathrm{tanh}(c+d\,x)^2+a\right)}{d\,a^2\,b^2+2\,d\,a\,b^3+d\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] - a^2/(2*(a^2*b^2*d + b^4*d*tanh(c + d*x)^2 + a*b^3*d + a*b^3*d*tanh(c + d*
x)^2)) - log(tanh(c + d*x)^2 - 1)/(2*(a^2*d + b^2*d + 2*a*b*d)) - (a^2*log(
a + b*tanh(c + d*x)^2))/(2*(b^4*d + a^2*b^2*d + 2*a*b^3*d)) - (a*b*log(a +
b*tanh(c + d*x)^2))/(b^4*d + a^2*b^2*d + 2*a*b^3*d)
```

3.181 $\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=89

$$\frac{x}{(a+b)^2}-\frac{\sqrt{a}\,(a+3b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}(a+b)^2d}+\frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^2-1/2*(a+3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/b^(3/2)/(
a+b)^2/d+1/2*a*tanh(d*x+c)/b/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3751, 481, 536, 212, 211}

$$-\frac{\sqrt{a}\,(a+3b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}d(a+b)^2}+\frac{a\tanh(c+dx)}{2bd(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] x/(a + b)^2 - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
2*b^(3/2)*(a + b)^2*d) + (a*Tanh[c + d*x])/(2*b*(a + b)*d*(a + b*Tanh[c + d
*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(-a-2b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+dx)\right.}{2b(a+b)d}$$

$$= \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2 d} - \frac{(a(}{}$$

$$= \frac{x}{(a+b)^2} - \frac{\sqrt{a}\,(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}(a+b)^2 d} + \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c\right.}$$

Mathematica [A]
time = 0.41, size = 90, normalized size = 1.01

$$\frac{2(c+dx) - \frac{\sqrt{a}\,(a+3b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}} + \frac{a(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))}}{2(a+b)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
b^(3/2) + (a*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d*x
)])))/(2*(a + b)^2*d)
```

Maple [A]
time = 0.81, size = 104, normalized size = 1.17

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{a\left(-\frac{(a+b)\tanh(dx+c)}{2b\left(a+b\left(\tanh^2(dx+c)\right)\right)}+\frac{(a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{a\left(-\frac{(a+b)\tanh(dx+c)}{2b\left(a+b\left(\tanh^2(dx+c)\right)\right)}+\frac{(a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{a\left(a\,e^{2dx+2c}-b\,e^{2dx+2c}+a+b\right)}{d(a+b)^2b\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}}{a+b}\right.}{4b^2(a+b)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^2*ln(tanh(d*x+c)-1)-a/(a+b)^2*(-1/2*(a+b)/b*tanh(d*x+c)/(a+
b*tanh(d*x+c)^2)+1/2*(a+3*b)/b/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)
))+1/2/(a+b)^2*ln(1+tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. $2(77) = 154$.
time = 0.75, size = 1010, normalized size = 11.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) +
 a - b)/sqrt(a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/32*(a^3 +
9*a^2*b - 9*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt
(a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/16*(a^3 - 5*a^2*b - 5*
a*b^2 + b^3 + (a^3 - 15*a^2*b + 15*a*b^2 - b^3)*e^(2*d*x + 2*c))/((a^4*b +
3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*e^(
```

```
4*d*x + 4*c) + 2*(a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*e^(2*d*x + 2*c))*d) +
1/16*(a^3 - 5*a^2*b - 5*a*b^2 + b^3 + (a^3 - 15*a^2*b + 15*a*b^2 - b^3)*e^(
-2*d*x - 2*c))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + 2*(a^4*b + a^3*b^2
 - a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b
^4)*e^(-4*d*x - 4*c))*d) - 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^(2*d*x +
2*c))/((a^3*b + 2*a^2*b^2 + a*b^3 + (a^3*b + 2*a^2*b^2 + a*b^3)*e^(4*d*x +
4*c) + 2*(a^3*b - a*b^3)*e^(2*d*x + 2*c))*d) + 1/4*(a^2 - b^2 + (a^2 - 6*a*
b + b^2)*e^(-2*d*x - 2*c))/((a^3*b + 2*a^2*b^2 + a*b^3 + 2*(a^3*b - a*b^3)*
e^(-2*d*x - 2*c) + (a^3*b + 2*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*c))*d) + 3/8*(
(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2*b + a*b^2 + 2*(a^2*b - a*b^2)*e^(-2
*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*log((a + b)*e^(4*d
*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/
4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2
+ 2*a*b + b^2)*d) - 1/8*(a + b)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b
)/sqrt(a*b))/(sqrt(a*b)*a*b*d) + 1/8*(a + b)*arctan(1/2*((a + b)*e^(-2*d*x
- 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b*d) + 3/16*(a - b)*arctan(1/2*((a
+ b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(77) = 154$.
time = 0.44, size = 1950, normalized size = 21.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b + b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b + b^2)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a*b + b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b + b^2)*d*x + 4
*(2*(a*b - b^2)*d*x - a^2 + a*b)*cosh(d*x + c)^2 + 4*(6*(a*b + b^2)*d*x*cos
h(d*x + c)^2 + 2*(a*b - b^2)*d*x - a^2 + a*b)*sinh(d*x + c)^2 + ((a^2 + 4*a
*b + 3*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a^2 + 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b - 3*b^2)*
cosh(d*x + c)^2 + 2*(3*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b
- 3*b^2)*sinh(d*x + c)^2 + a^2 + 4*a*b + 3*b^2 + 4*((a^2 + 4*a*b + 3*b^2)*c
osh(d*x + c)^3 + (a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-
a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b
^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)
*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)
^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a*b + b^2)*cosh(d*x + c
)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*sinh(d*x + c)
^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
```

```
x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*a^2 - 4*a*b +
 8*(2*(a*b + b^2)*d*x*cosh(d*x + c)^3 + (2*(a*b - b^2)*d*x - a^2 + a*b)*cos
h(d*x + c))*sinh(d*x + c))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x
+ c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*sinh(d*x + c)^4 + 2*(a^3*b + a^
2*b^2 - a*b^3 - b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4)*d*cosh(d*x + c)^2 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d)*sinh(d*x + c)
^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d + 4*((a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*d*cosh(d*x + c)^3 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d*cosh(d*x + c
))*sinh(d*x + c)), 1/2*(2*(a*b + b^2)*d*x*cosh(d*x + c)^4 + 8*(a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a*b + b^2)*d*x*sinh(d*x + c)^4 + 2*(a
*b + b^2)*d*x + 2*(2*(a*b - b^2)*d*x - a^2 + a*b)*cosh(d*x + c)^2 + 2*(6*(a
*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a*b - b^2)*d*x - a^2 + a*b)*sinh(d*x + c
)^2 - ((a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 4*a*b + 3*b^2)*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^2 + 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 2*(a^2
+ 2*a*b - 3*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)
^2 + a^2 + 2*a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 + 4*a*b + 3*b^2 + 4*((a^2 +
 4*a*b + 3*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh
(d*x + c))*sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d
*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a/b)/a) - 2*a
^2 - 2*a*b + 4*(2*(a*b + b^2)*d*x*cosh(d*x + c)^3 + (2*(a*b - b^2)*d*x - a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*d*cosh(d*x + c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*sinh(d*x + c)^4 + 2
*(a^3*b + a^2*b^2 - a*b^3 - b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b + 3*a^2*b^
2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d)*s
inh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d + 4*((a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a^3*b + a^2*b^2 - a*b^3 - b^4)*d*
cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(77) = 154.
time = 0.50, size = 195, normalized size = 2.19

$$-\frac{\frac{(a^2+3ab)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^2b+2ab^2+b^3)\sqrt{ab}}-\frac{2(dx+c)}{a^2+2ab+b^2}+\frac{2\left(a^2e^{(2dx+2c)}-abe^{(2dx+2c)}+a^2+ab\right)}{(a^2b+2ab^2+b^3)\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/2*((a^2 + 3*a*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b)) - 2*(d*x + c)/(a^2 + 2*a*b + b^2) + 2*(a^2*e^(2*d*x + 2*c) - a*b*e^(2*d*x + 2*c) + a^2 + a*b)/((a^2*b + 2*a*b^2 + b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d

Mupad [B]

time = 1.69, size = 1655, normalized size = 18.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)

[Out] log(tanh(c + d*x) + 1)/(2*a^2*d + 2*b^2*d + 4*a*b*d) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^2) - (atan((((a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) + ((a + 3*b)*(-a*b^3)^(1/2)*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) - (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b^7*d^2 + 32*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)))*1i)/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)) + ((a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) - ((a + 3*b)*(-a*b^3)^(1/2)*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) + (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b^7*d^2 + 32*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)))*1i)/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)))/((3*a*b^2 + (5*a^2*b)/2 + a^3/2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) - ((a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) + ((a + 3*b)*(-a*b^3)^(1/2)*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) - (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b^7*d^2 + 32*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d))))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)) + ((a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) - ((a +

```
 3*b)*(-a*b^3)^(1/2)*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4
*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^
3) + (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b^7*d^2 + 3
2*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*
d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))))/(4*(b^5*d
 + a^2*b^3*d + 2*a*b^4*d))))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d))))*(a + 3*b
)*(-a*b^3)^(1/2)*1i)/(2*(b^5*d + a^2*b^3*d + 2*a*b^4*d)) + (a*tanh(c + d*x)
)/(2*b*(a + b)*(a*d + b*d*tanh(c + d*x)^2))
```

3.182 $\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=72

$$\frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2d}+\frac{a}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^2/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^2/d+1/2*a/b/(a+b)
/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 78}

$$\frac{a}{2bd(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^2}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^2*
d) + a/(2*b*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
```

```
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)}-\frac{a}{(a+b)(a+bx)^2}+\frac{b}{(a+b)^2(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}\\ &= \frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2d}+\frac{a}{2b(a+b)d\left(a+b\tanh^2(c\right.}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 57, normalized size = 0.79

$$\frac{2\log(\cosh(c+dx))+\log\left(a+b\tanh^2(c+dx)\right)+\frac{a(a+b)}{b\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2] + (a*(a + b))/(b*(a + b*
Tanh[c + d*x]^2)))/(2*(a + b)^2*d)
```

Maple [A]
time = 0.71, size = 84, normalized size = 1.17

method	result
derivativedivides	$\frac{-\frac{-\frac{a(a+b)}{b\left(a+b\left(\tanh^2(dx+c)\right)\right)}-\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$\frac{-\frac{-\frac{a(a+b)}{b\left(a+b\left(\tanh^2(dx+c)\right)\right)}-\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}}{d}$
risch	$-\frac{x}{a^2+2ab+b^2}-\frac{2c}{d(a^2+2ab+b^2)}+\frac{2a\,e^{2dx+2c}}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{\ln\left(e^{4dx+4c}+\right.}{2d(a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/(a+b)^2*(-a*(a+b)/b/(a+b*tanh(d*x+c)^2)-ln(a+b*tanh(d*x+c)^2))-1/2/(a+b)^2*ln(tanh(d*x+c)-1)-1/2/(a+b)^2*ln(1+tanh(d*x+c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(68) = 136$.
time = 0.28, size = 170, normalized size = 2.36

$$\frac{2\,ae^{(-2\,dx-2\,c)}}{(a^3+3\,a^2b+3\,ab^2+b^3+2\,(a^3+a^2b-ab^2-b^3)e^{(-2\,dx-2\,c)}+(a^3+3\,a^2b+3\,ab^2+b^3)e^{(-4\,dx-4\,c)})d}+\frac{dx+c}{(a^2+2\,ab+b^2)d}+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+2\,ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 2*a*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*e^(-2*d*x - 2*c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x - 4*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(68) = 136$.
time = 0.37, size = 629, normalized size = 8.74

$$-\frac{2\,(a+b)dx\cosh\left(dx+c\right)^4+8\,(a+b)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,(a+b)dx\sinh\left(dx+c\right)^4+2\,(a+b)dx+4\,((a-b)dx-a)\cosh\left(dx+c\right)^2+4\,\left(3\,(a+b)dx\cosh\left(dx+c\right)^2+(a-b)dx-a\right)\sinh\left(dx+c\right)^2-\left((a+b)\cosh\left(dx+c\right)^4+4\,(a+b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(a+b)\sinh\left(dx+c\right)^4+2\,(a-b)\cosh\left(dx+c\right)^2+2\,\left(3\,(a+b)\cosh\left(dx+c\right)^2+a-b\right)\sinh\left(dx+c\right)^2+4\,\left((a+b)\cosh\left(dx+c\right)^3+(a-b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+a+b\right)\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)+8\,\left((a+b)dx\cosh\left(dx+c\right)^3+((a-b)dx-a)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{2\,\left((a^3+3\,a^2b+3\,ab^2+b^3)d\cosh\left(dx+c\right)^4+4\,(a^3+3\,a^2b+3\,ab^2+b^3)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(a^3+3\,a^2b+3\,ab^2+b^3)d\sinh\left(dx+c\right)^4+2\,(a^3+a^2b-ab^2-b^3)d\cosh\left(dx+c\right)^2+2\,\left(3\,(a^3+3\,a^2b+3\,ab^2+b^3)d\cosh\left(dx+c\right)^2+(a^3+a^2b-ab^2-b^3)d\right)\sinh\left(dx+c\right)^2+(a^3+3\,a^2b+3\,ab^2+b^3)d+4\,\left((a^3+3\,a^2b+3\,ab^2+b^3)d\cosh\left(dx+c\right)^3+(a^3+a^2b-ab^2-b^3)d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x - a)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x - a)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((a - b)*d*x - a)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a^3 + 3*

```
a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x +
c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(68) = 136.
time = 0.49, size = 149, normalized size = 2.07

$$\frac{\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^2+2\,ab+b^2}-\frac{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2}{\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right)(a+b)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^2 + 2*a*b + b^2) - (e^(2*d*x + 2*c) + e^
(-2*d*x - 2*c) - 2)/((a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)*(a + b)))/d
```

Mupad [**B**]
time = 0.43, size = 210, normalized size = 2.92

$$-\frac{-a^2+a\,b\left(-1+\operatorname{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh(c+d\,x)^2\,1\mathrm{i}}{2\,a-a\tanh(c+d\,x)^2+b\tanh(c+d\,x)^2}\right)\,2\mathrm{i}\right)+b^2\tanh(c+d\,x)^2\operatorname{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh(c+d\,x)^2\,1\mathrm{i}}{2\,a-a\tanh(c+d\,x)^2+b\tanh(c+d\,x)^2}\right)\,2\mathrm{i}}{2\,d\,a^3\,b+2\,d\,a^2\,b^2\tanh\left(c+d\,x\right)^2+4\,d\,a^2\,b^2+4\,d\,a\,b^3\tanh\left(c+d\,x\right)^2+2\,d\,a\,b^3+2\,d\,b^4\tanh\left(c+d\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] -(a*b*(atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c +
 d*x)^2 + b*tanh(c + d*x)^2))*2i - 1) - a^2 + b^2*tanh(c + d*x)^2*atan((a*t
anh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh
(c + d*x)^2))*2i)/(4*a^2*b^2*d + 2*b^4*d*tanh(c + d*x)^2 + 2*a*b^3*d + 2*a^
3*b*d + 2*a^2*b^2*d*tanh(c + d*x)^2 + 4*a*b^3*d*tanh(c + d*x)^2)
```

3.183 $\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=85

$$\frac{x}{(a+b)^2}-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,(a+b)^2d}-\frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

[Out] x/(a+b)^2-1/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/(a+b)^2/d/a^(1/2)/b^(1/2)-1/2*tanh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {3751, 482, 536, 212, 211}

$$-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d(a+b)^2}-\frac{\tanh(c+dx)}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] x/(a + b)^2 - ((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*Sqrt[a]*
Sqrt[b]*(a + b)^2*d) - Tanh[c + d*x]/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1+x^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+d$$

$$= -\frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d} - $$

$$= \frac{x}{(a+b)^2} - \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(a+b)^2d} - \frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx\right.}$$

Mathematica [A]
time = 0.32, size = 86, normalized size = 1.01

$$\frac{2(c+dx)+\frac{(-a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}-\frac{(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2(a+b)^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (2*(c + d*x) + ((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - ((a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*(a + b)^2*d)

Maple [A]
time = 0.79, size = 100, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\frac{\left(\frac{a}{2}+\frac{b}{2}\right)\tanh(dx+c)}{a+b\left(\tanh^2(dx+c)\right)}+\frac{(a-b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a+b)^2}}{d}$
default	$\frac{\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\frac{\left(\frac{a}{2}+\frac{b}{2}\right)\tanh(dx+c)}{a+b\left(\tanh^2(dx+c)\right)}+\frac{(a-b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}+\frac{a\,e^{2dx+2c}-b\,e^{2dx+2c}+a+b}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+}{(a+b)\sqrt{-ab}}\right.}{4\sqrt{-ab}\,(a+b)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2/(a+b)^2*ln(1+tanh(d*x+c))-1/2/(a+b)^2*ln(tanh(d*x+c)-1)-1/(a+b)^2*((1/2*a+1/2*b)*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+1/2*(a-b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(73) = 146.
time = 0.62, size = 614, normalized size = 7.22

$$-\frac{(a^2-4ab-b^2)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^3+2a^2b+ab^2)\sqrt{ab}\,d}+\frac{(a^2-4ab-b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^3+2a^2b+ab^2)\sqrt{ab}\,d}+\frac{a^2-b^2+(a^2-6ab+b^2)e^{(2dx+2c)}}{4(a^4+3a^3b+3a^2b^2+ab^3+(a^4+3a^3b+3a^2b^2+ab^3)e^{(4dx+4c)}+2(a^4+a^3b-a^2b^2-ab^3)e^{(2dx+2c)})d}-\frac{a^2-b^2+(a^2-6ab+b^2)e^{(-2dx-2c)}}{4(a^4+3a^3b+3a^2b^2+ab^3+2(a^4+a^3b-a^2b^2-ab^3)e^{(-2dx-2c)}+(a^4+3a^3b+3a^2b^2+ab^3)e^{(-4dx-4c)})d}-\frac{(a-b)e^{(-2dx-2c)}+a+b}{2(a^3+2a^2b+ab^2+2(a^3-ab^2)e^{(-2dx-2c)}+(a^3+2a^2b+ab^2)e^{(-4dx-4c)})d}+\frac{\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{4(a^2+2ab+b^2)d}-\frac{\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{4(a^2+2ab+b^2)d}+\frac{\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -1/8*(a^2 - 4*a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)*d) + 1/8*(a^2 - 4*a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)*d) + 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^(2*d*x + 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(4*d*x + 4*c) + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(2*d*x + 2*c))*d) - 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^

```
2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a
^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*c))*d) - 1/2*((a - b)*e^(-2*d*x - 2
*c) + a + b)/((a^3 + 2*a^2*b + a*b^2 + 2*(a^3 - a*b^2)*e^(-2*d*x - 2*c) + (
a^3 + 2*a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*log((a + b)*e^(4*d*x + 4*
c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*log(2
*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b
 + b^2)*d) + 1/4*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(
sqrt(a*b)*a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(73) = 146$.
time = 0.42, size = 2025, normalized size = 23.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 4*a^2*b +
 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x
)*cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 + a^2*b - a*b^
2 + 2*(a^2*b - a*b^2)*d*x)*sinh(d*x + c)^2 + ((a^2 - b^2)*cosh(d*x + c)^4 +
 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*sinh(d*x + c)^4
+ 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x + c)^2
+ a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x
+ c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log((
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d
*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x
+ c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2
- b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b
)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b)
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*
x + c))*sinh(d*x + c) + a + b)) + 8*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3
+ (a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a
^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^4*b + 3*a^3*
b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 3*a^3*b
^2 + 3*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^4*b + a^3*b^2 - a^2*b^3 -
a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*c
osh(d*x + c)^2 + (a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (
a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d + 4*((a^4*b + 3*a^3*b^2 + 3*a^2*b^
3 + a*b^4)*d*cosh(d*x + c)^3 + (a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d
```

```
*x + c))*sinh(d*x + c)), 1/2*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^
2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2*b + a*b^2)*d*x*sinh
(d*x + c)^4 + 2*a^2*b + 2*a*b^2 + 2*(a^2*b + a*b^2)*d*x + 2*(a^2*b - a*b^2
+ 2*(a^2*b - a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(6*(a^2*b + a*b^2)*d*x*cosh(d*
x + c)^2 + a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*sinh(d*x + c)^2 - ((a^2 -
 b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
- b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
- b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4
*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x
 + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x +
 c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 4*(
2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 + (a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*
d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*d*cosh(d*x + c)^4 + 4*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*sinh(d*x +
c)^4 + 2*(a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*
b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^4*b + a^3*b^2 - a
^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4
)*d + 4*((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^4*b
 + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(73) = 146.
time = 0.49, size = 177, normalized size = 2.08

$$-\frac{\frac{(a-b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2+2\,ab+b^2}-\frac{2\,(ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a+b)}{(a^2+2\,ab+b^2)\,(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a - b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sq
rt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2 + 2*a*b + b^2)
- 2*(a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*(a
```

```
*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x +
 2*c) + a + b)))/d
```

Mupad [B]
time = 0.66, size = 106, normalized size = 1.25

$$\frac{\frac{a\,x}{(a+b)^2}-\frac{\tanh(c+d\,x)}{2\,a\,d+2\,b\,d}+\frac{b\,x\,\tanh(c+d\,x)^2}{(a+b)^2}}{b\tanh\left(c+d\,x\right)^2+a}-\frac{\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(a-b\right)}{\sqrt{a\,b}\,\left(2\,d\,a^2+4\,d\,a\,b+2\,d\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] ((a*x)/(a + b)^2 - tanh(c + d*x)/(2*a*d + 2*b*d) + (b*x*tanh(c + d*x)^2)/(a
 + b)^2)/(a + b*tanh(c + d*x)^2) - (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(a
- b))/((a*b)^(1/2)*(2*a^2*d + 2*b^2*d + 4*a*b*d))
```

3.184 $\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$\frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2d}-\frac{1}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^2/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^2/d-1/2/(a+b)/d/(
a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 455, 46}

$$-\frac{1}{2d(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^2}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^2*
d) - 1/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{b}{(a+b)(a+bx)^2} + \frac{b}{(a+b)^2(a+bx)}\right)\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)^2 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2 d} - \frac{1}{2(a+b)d\left(a+b\tanh^2(c\right.}$$

Mathematica [A]
time = 0.32, size = 55, normalized size = 0.81

$$-\frac{-2\log(\cosh(c+dx)) - \log\left(a+b\tanh^2(c+dx)\right) + \frac{a+b}{a+b\tanh^2(c+dx)}}{2(a+b)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
[Out] -1/2*(-2*Log[Cosh[c + d*x]] - Log[a + b*Tanh[c + d*x]^2] + (a + b)/(a + b*T
anh[c + d*x]^2))/((a + b)^2*d)
```

Maple [A]
time = 0.72, size = 86, normalized size = 1.26

method	result
derivativedivides	$\frac{-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}+\frac{b\left(-\frac{a+b}{b\left(a+b\left(\tanh^2(dx+c)\right)\right)}+\frac{\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{b}\right)}{2(a+b)^2}}{d}$
default	$\frac{-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}+\frac{b\left(-\frac{a+b}{b\left(a+b\left(\tanh^2(dx+c)\right)\right)}+\frac{\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{b}\right)}{2(a+b)^2}}{d}$
risch	$-\frac{x}{a^2+2ab+b^2}-\frac{2c}{d(a^2+2ab+b^2)}-\frac{2b\,e^{2dx+2c}}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{\ln\left(e^{4dx+4c}+\right.}{2d(a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/(a+b)^2*ln(1+tanh(d*x+c))-1/2/(a+b)^2*ln(tanh(d*x+c)-1)+1/2*b/(a+b)^2*(-(a+b)/b/(a+b*tanh(d*x+c)^2)+1/b*ln(a+b*tanh(d*x+c)^2)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(64) = 128$.
time = 0.28, size = 170, normalized size = 2.50

$$-\frac{2\,be^{(-2\,dx-2\,c)}}{(a^3+3\,a^2b+3\,ab^2+b^3+2\,(a^3+a^2b-ab^2-b^3)e^{(-2\,dx-2\,c)}+(a^3+3\,a^2b+3\,ab^2+b^3)e^{(-4\,dx-4\,c)})d}+\frac{dx+c}{(a^2+2\,ab+b^2)d}+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+2\,ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -2*b*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*e^(-2*d*x - 2*c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x - 4*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(64) = 128$.
time = 0.37, size = 623, normalized size = 9.16

$$-\frac{2\,(a+b)dx\cosh\left(dx+c\right)^4+8\,(a+b)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,(a+b)dx\sinh\left(dx+c\right)^4+2\,(a+b)dx+4\,((a-b)dx+b)\cosh\left(dx+c\right)^2+4\,\left(3\,(a+b)dx\cosh\left(dx+c\right)^2+(a-b)dx+b\right)\sinh\left(dx+c\right)^2-\left((a+b)\cosh\left(dx+c\right)^4+4\,(a+b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(a+b)\sinh\left(dx+c\right)^4+2\,(a-b)\cosh\left(dx+c\right)^2+2\,\left(3\,(a+b)\cosh\left(dx+c\right)^2+a-b\right)\sinh\left(dx+c\right)^2+4\,\left((a+b)\cosh\left(dx+c\right)^3+(a-b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+a+b\right)\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)+8\,\left((a+b)dx\cosh\left(dx+c\right)^3+((a-b)dx+b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{2\,\left((a^3+3\,a^2b+3\,ab^2+b^3)d\cosh\left(dx+c\right)^4+4\,(a^3+3\,a^2b+3\,ab^2+b^3)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(a^3+3\,a^2b+3\,ab^2+b^3)d\sinh\left(dx+c\right)^4+2\,(a^3+a^2b-ab^2-b^3)d\cosh\left(dx+c\right)^2+2\,\left(3\,(a^3+3\,a^2b+3\,ab^2+b^3)d\cosh\left(dx+c\right)^2+(a^3+a^2b-ab^2-b^3)d\right)\sinh\left(dx+c\right)^2+(a^3+3\,a^2b+3\,ab^2+b^3)d+4\,\left((a^3+3\,a^2b+3\,ab^2+b^3)d\cosh\left(dx+c\right)^3+(a^3+a^2b-ab^2-b^3)d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x + b)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x + b)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((a - b)*d*x + b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a^3 + 3*

```
a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x +
c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(64) = 128.
time = 0.47, size = 149, normalized size = 2.19

$$\frac{\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^2+2\,ab+b^2}-\frac{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2}{\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right)(a+b)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^2 + 2*a*b + b^2) - (e^(2*d*x + 2*c) + e^
(-2*d*x - 2*c) + 2)/((a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)*(a + b)))/d
```

Mupad [**B**]
time = 1.47, size = 129, normalized size = 1.90

$$\frac{\frac{a\,x}{a^2+2\,a\,b+b^2}+\frac{b\,x\,\tanh(c+d\,x)^2}{a^2+2\,a\,b+b^2}+\frac{b\tanh(c+d\,x)^2}{2\,a\,d\,(a+b)}}{b\tanh\left(c+d\,x\right)^2+a}+\frac{\ln\left(b\tanh(c+d\,x)^2+a\right)}{2\,d\,\left(a^2+2\,a\,b+b^2\right)}-\frac{\ln\left(\tanh(c+d\,x)+1\right)}{d\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] ((a*x)/(2*a*b + a^2 + b^2) + (b*x*tanh(c + d*x)^2)/(2*a*b + a^2 + b^2) + (b
*tanh(c + d*x)^2)/(2*a*d*(a + b)))/(a + b*tanh(c + d*x)^2) + log(a + b*tanh
(c + d*x)^2)/(2*d*(2*a*b + a^2 + b^2)) - log(tanh(c + d*x) + 1)/(d*(a + b)^
2)
```

3.185 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=89

$$\frac{x}{(a+b)^2}+\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2d}+\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^2+1/2*(3*a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(3/2)/(
a+b)^2/d+1/2*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3742, 425, 536, 212, 211}

$$\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^2}+\frac{b\tanh(c+dx)}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-2),x]
```

```
[Out] x/(a + b)^2 + (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(
2*a^(3/2)*(a + b)^2*d) + (b*Tanh[c + d*x])/(2*a*(a + b)*d*(a + b*Tanh[c + d
*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{b-2(a+b)+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+d\right.}{2a(a+b)d}$$

$$= \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d} + \frac{(b}{}$$

$$= \frac{x}{(a+b)^2} + \frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2d} + \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(\right.}$$

Mathematica [A]
time = 0.39, size = 97, normalized size = 1.09

$$\frac{\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} - \log(1-\tanh(c+dx)) + \log(1+\tanh(c+dx)) + \frac{b(a+b)\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-2),x]
```

```
[Out] ((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) - Log[
1 - Tanh[c + d*x]] + Log[1 + Tanh[c + d*x]] + (b*(a + b)*Tanh[c + d*x])/(a*
(a + b*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)
```

Maple [A]
time = 1.12, size = 103, normalized size = 1.16

method	result
derivativedivides	$\frac{\frac{b\left(\frac{(a+b)\tanh(dx+c)}{2a\left(a+b\left(\tanh^2(dx+c)\right)\right)}+\frac{(3a+b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{\frac{b\left(\frac{(a+b)\tanh(dx+c)}{2a\left(a+b\left(\tanh^2(dx+c)\right)\right)}+\frac{(3a+b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{b\left(a\,e^{2dx+2c}-b\,e^{2dx+2c}+a+b\right)}{d(a+b)^2a\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-a}}{a+b}\right.}{4a(a+b)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/(a+b)^2*(1/2/a*(a+b)*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+1/2*(3*a+b)/a/(
a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^2*ln(1+tanh(d*x+c))
-1/2/(a+b)^2*ln(tanh(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(77) = 154$.
time = 0.53, size = 206, normalized size = 2.31

$$-\frac{(3\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,(a^3+2\,a^2b+ab^2)\sqrt{ab}\,d}+\frac{ab+b^2+(ab-b^2)e^{(-2\,dx-2\,c)}}{(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4\,c)})d}+\frac{dx+c}{(a^2+2\,ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)*d) + (a*b + b^2 + (a*b - b^2)*e^(-2*d*x
 - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a
*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*
c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(77) = 154$.
time = 0.42, size = 1942, normalized size = 21.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x + 4
*(2*(a^2 - a*b)*d*x - a*b + b^2)*cosh(d*x + c)^2 + 4*(6*(a^2 + a*b)*d*x*cos
h(d*x + c)^2 + 2*(a^2 - a*b)*d*x - a*b + b^2)*sinh(d*x + c)^2 + ((3*a^2 + 4
*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - 2*a*b - b^2)*
cosh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*
b - b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + b^2 + 4*((3*a^2 + 4*a*b + b^2)*c
osh(d*x + c)^3 + (3*a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-
b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b
^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)
*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)
^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c
)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)
^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 +
 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*a*b - 4*b^2 +
 8*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 - a*b)*d*x - a*b + b^2)*cos
h(d*x + c))*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x
+ c)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(a^4 + a^3*
b - a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*d*cosh(d*x + c)^2 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d)*sinh(d*x + c)
^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 4*((a^4 + 3*a^3*b + 3*a^2*b^2
+ a*b^3)*d*cosh(d*x + c)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d*cosh(d*x + c
))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 8*(a^2 + a*b)*d
*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 2*(a
^2 + a*b)*d*x + 2*(2*(a^2 - a*b)*d*x - a*b + b^2)*cosh(d*x + c)^2 + 2*(6*(a
^2 + a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 - a*b)*d*x - a*b + b^2)*sinh(d*x + c
)^2 + ((3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 4*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 2*(3*a^
2 - 2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)
^2 + 3*a^2 - 2*a*b - b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + b^2 + 4*((3*a^2
 + 4*a*b + b^2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh
```

```
(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d
*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - 2*a
*b - 2*b^2 + 4*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 - a*b)*d*x - a*
b + b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)
*d*cosh(d*x + c)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2
*(a^4 + a^3*b - a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d)*s
inh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 4*((a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*d*
cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(77) = 154.
time = 0.45, size = 195, normalized size = 2.19

$$\frac{\frac{(3ab+b^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^3+2a^2b+ab^2)\sqrt{ab}}+\frac{2(dx+c)}{a^2+2ab+b^2}-\frac{2\left(abe^{(2dx+2c)}-b^2e^{(2dx+2c)}+ab+b^2\right)}{(a^3+2a^2b+ab^2)\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((3*a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a -
b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) + 2*(d*x + c)/(a^2 + 2*a*
b + b^2) - 2*(a*b*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2*c) + a*b + b^2)/((a^3
+ 2*a^2*b + a*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

Mupad [**B**]
time = 1.54, size = 110, normalized size = 1.24

$$\frac{\frac{a\,x}{(a+b)^2}+\frac{b\,x\,\tanh(c+d\,x)^2}{(a+b)^2}+\frac{b\tanh(c+d\,x)}{2\,a\,d\,(a+b)}}{b\tanh\left(c+d\,x\right)^2+a}+\frac{\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(b^2+3\,a\,b\right)}{\sqrt{a\,b}\,\left(2\,a^3\,d+a\,b\,\left(4\,a\,d+2\,b\,d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] ((a*x)/(a + b)^2 + (b*x*tanh(c + d*x)^2)/(a + b)^2 + (b*tanh(c + d*x))/(2*a
*d*(a + b)))/(a + b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))
*(3*a*b + b^2))/((a*b)^(1/2)*(2*a^3*d + a*b*(4*a*d + 2*b*d)))
```

3.186 $\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log(\tanh(c+dx))}{a^2d}-\frac{b(2a+b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)^2d}+\frac{b}{2a(a+b)d\left(a+b\tanh^2(c+dx\right)}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^2/d+ln(tanh(d*x+c))/a^2/d-1/2*b*(2*a+b)*ln(a+b*tanh(d
*x+c)^2)/a^2/(a+b)^2/d+1/2*b/a/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 457, 84}

$$-\frac{b(2a+b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^2d(a+b)^2}+\frac{\log(\tanh(c+dx))}{a^2d}+\frac{b}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^2*d) + Log[Tanh[c + d*x]]/(a^2*d) - (b*(2*a + b
)*Log[a + b*Tanh[c + d*x]^2])/(2*a^2*(a + b)^2*d) + b/(2*a*(a + b)*d*(a + b
*Tanh[c + d*x]^2))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)(a+bx^2)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^2}\,dx,x,\tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)}+\frac{1}{a^2x}-\frac{b^2}{a(a+b)(a+bx)^2}-\frac{b^2(2a+b)}{a^2(a+b)^2(a+bx)}\right)\,dx,x,\tanh^2(\right.}{2d}$$

$$= \frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log(\tanh(c+dx))}{a^2d}-\frac{b(2a+b)\log\left(a+b\tanh^2(c+dx\right.}{2a^2(a+b)^2d}$$

Mathematica [A]
time = 1.47, size = 83, normalized size = 0.87

$$\frac{\frac{2\log(\cosh(c+dx))}{(a+b)^2}+\frac{2\log(\tanh(c+dx))+\frac{b\left(-\left((2a+b)\log\left(a+b\tanh^2(c+dx)\right)\right)+\frac{a(a+b)}{a+b\tanh^2(c+dx)}\right)}{(a+b)^2}}{a^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((2*Log[Cosh[c + d*x]])/(a + b)^2 + (2*Log[Tanh[c + d*x]] + (b*(-((2*a + b)
*Log[a + b*Tanh[c + d*x]^2]) + (a*(a + b))/(a + b*Tanh[c + d*x]^2)))/(a + b
)^2)/a^2)/(2*d)
```

Maple [A]
time = 3.20, size = 183, normalized size = 1.93

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{(a+b)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{(a+b)^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{b\left(\frac{2b(a+b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\right.\right.}\right.}{}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{(a+b)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{(a+b)^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{b\left(\frac{2b(a+b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\right.\right.}\right.}{}}{d}$

risch	$\frac{x}{a^2+2ab+b^2} - \frac{2x}{a^2} - \frac{2c}{a^2 d} + \frac{4bx}{a(a^2+2ab+b^2)} + \frac{4bc}{ad(a^2+2ab+b^2)} + \frac{2b^2 x}{a^2(a^2+2ab+b^2)} + \frac{2b^2 c}{a^2 d(a^2+2ab+b^2)} + \frac{}{ad(a+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+
1)+1/a^2*ln(tanh(1/2*d*x+1/2*c))-b/(a+b)^2/a^2*(2*b*(a+b)*tanh(1/2*d*x+1/2*
c)^2/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/
2*c)^2+a)+1/2*(2*a+b)*ln(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+
4*b*tanh(1/2*d*x+1/2*c)^2+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(91) = 182$.
time = 0.30, size = 235, normalized size = 2.47

$$\frac{2\,b^2 e^{(-2\,dx-2\,c)}}{(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4\,c)})d} - \frac{(2\,ab+b^2)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^4+2\,a^3b+a^2b^2)d} + \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{a^2d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*b^2*e^(-2*d*x - 2*c)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b
 - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
e^(-4*d*x - 4*c))*d) - 1/2*(2*a*b + b^2)*log(2*(a - b)*e^(-2*d*x - 2*c) + (
a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + (d*x + c)/
((a^2 + 2*a*b + b^2)*d) + log(e^(-d*x - c) + 1)/(a^2*d) + log(e^(-d*x - c)
- 1)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(91) = 182$.
time = 0.44, size = 1148, normalized size = 12.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 + a^2*b)*d*x*cosh(d*x + c)^4 + 8*(a^3 + a^2*b)*d*x*cosh(d*x +
c)*sinh(d*x + c)^3 + 2*(a^3 + a^2*b)*d*x*sinh(d*x + c)^4 + 2*(a^3 + a^2*b)*
d*x - 4*(a*b^2 - (a^3 - a^2*b)*d*x)*cosh(d*x + c)^2 + 4*(3*(a^3 + a^2*b)*d*
x*cosh(d*x + c)^2 - a*b^2 + (a^3 - a^2*b)*d*x)*sinh(d*x + c)^2 + ((2*a^2*b
+ 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)*sinh(d*x + c)^3 + (2*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^4 + 2*a^2*b + 3
*a*b^2 + b^3 + 2*(2*a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(2*a^2*b - a*b
```

```
^2 - b^3 + 3*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^2*b - a*b^2 - b^3)*cosh(
d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x
+ c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
 + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x
 + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
+ 8*((a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (a*b^2 - (a^3 - a^2*b)*d*x)*cosh(d
*x + c))*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x +
 c)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^5 + a
^4*b - a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d)*sinh
(d*x + c)^2 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 4*((a^5 + 3*a^4*b +
 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)
*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(91) = 182.
time = 0.50, size = 195, normalized size = 2.05

$$-\frac{\frac{(2ab+b^2)\log\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}{a^4+2a^3b+a^2b^2}+\frac{2(dx+c)}{a^2+2ab+b^2}-\frac{4b^2e^{(2dx+2c)}}{\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)(a+b)^2a}-\frac{2\log\left(\left|e^{(2dx+2c)}-1\right|\right)}{a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^4 + 2*a^3*b + a^2*b^2) + 2*(d*x
+ c)/(a^2 + 2*a*b + b^2) - 4*b^2*e^(2*d*x + 2*c)/((a*e^(4*d*x + 4*c) + b*e^
(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a + b)^
2*a) - 2*log(abs(e^(2*d*x + 2*c) - 1))/a^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

3.187 $\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal. Leaf size=119

$$\frac{x}{(a+b)^2}-\frac{b^{3/2}(5a+3b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^2d}-\frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d}+\frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^2-1/2*b^(3/2)*(5*a+3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)
/(a+b)^2/d-1/2*(2*a+3*b)*coth(d*x+c)/a^2/(a+b)/d+1/2*b*coth(d*x+c)/a/(a+b)/
d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 483, 597, 536, 212, 211}

$$-\frac{b^{3/2}(5a+3b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^2}-\frac{(2a+3b)\coth(c+dx)}{2a^2d(a+b)}+\frac{b\coth(c+dx)}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] x/(a + b)^2 - (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])
/(2*a^(5/2)*(a + b)^2*d) - ((2*a + 3*b)*Coth[c + d*x])/(2*a^2*(a + b)*d) +
(b*Coth[c + d*x])/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a-3b+3bx^2}{x^2(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+\right.}{2a(a+b)d}$$

$$= -\frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a}{}\right.}{}$$

$$= -\frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{}{1-}\right.}{}$$

$$= \frac{x}{(a+b)^2} - \frac{b^{3/2}(5a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^2d} - \frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d}$$

Mathematica [A]
time = 1.23, size = 111, normalized size = 0.93

$$-\frac{-\frac{2(c+dx)}{(a+b)^2} + \frac{b^{3/2}(5a+3b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^2} + \frac{2\coth(c+dx)}{a^2} + \frac{b^2\sinh(2(c+dx))}{a^2(a+b)(a-b+(a+b)\cosh(2(c+dx)))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((-2*(c + d*x))/(a + b)^2 + (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[
c + d*x])/Sqrt[a]])/(a^(5/2)*(a + b)^2) + (2*Coth[c + d*x])/a^2 + (b^2*Sinh
[2*(c + d*x)])/(a^2*(a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(105) = 210.
time = 2.86, size = 329, normalized size = 2.76

method	result

derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{(a+b)^2}+\frac{2b^2\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(5a+3b)a\left(-\frac{\left(-a\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{(a+b)^2}+\frac{2b^2\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(5a+3b)a\left(-\frac{\left(-a\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2a^3e^{4dx+4c}+6a^2b\,e^{4dx+4c}+5a\,b^2e^{4dx+4c}+3b^3e^{4dx+4c}+4a^3e^{2dx+2c}+4a^2b\,e^{2dx+2c}-4a\,b^2e^{2dx+2c}-6b^3e^{2d\cdots}}{d\,a^2(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)\left(e^{2dx+2c}-1\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2*tanh(1/2*d*x+1/2*c)-1/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)+2*b^2
```

```
/(a+b)^2/a^2*(((-1/2*a-1/2*b)*tanh(1/2*d*x+1/2*c)^3+(-1/2*a-1/2*b)*tanh(1/2
*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/
2*d*x+1/2*c)^2+a)+1/2*(5*a+3*b)*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^
(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/
2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*
(a+b))^(1/2)+a+2*b)*a)^(1/2))))-1/2/a^2/tanh(1/2*d*x+1/2*c)+1/(a+b)^2*ln(ta
nh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(105) = 210$.
time = 0.68, size = 976, normalized size = 8.20

$$-\frac{(2ab+b^2)\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{4(a^4+2a^3b+a^2b^2)d}+\frac{(2ab+b^2)\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{4(a^4+2a^3b+a^2b^2)d}+\frac{(3a^2b-4ab^2-3b^3)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^4+2a^3b+a^2b^2)\sqrt{ab}d}-\frac{(3a^2b-4ab^2-3b^3)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^4+2a^3b+a^2b^2)\sqrt{ab}d}+\frac{2a^3+5a^2b+6ab^2+3b^3+(2a^3+7a^2b+3b^3)e^{(4dx+4c)}+2(2a^3+2a^2b+ab^2-3b^3)e^{(2dx+2c)}}{4(a^5+3a^4b+3a^3b^2+a^2b^3-(a^5+3a^4b+3a^3b^2+a^2b^3)e^{(6dx+6c)}-(a^5-a^4b-5a^3b^2-3a^2b^3)e^{(4dx+4c)}+(a^5-a^4b-5a^3b^2-3a^2b^3)e^{(2dx+2c)})d}-\frac{2a^3+5a^2b+6ab^2+3b^3+2(2a^3+2a^2b+ab^2-3b^3)e^{(-2dx-2c)}+(2a^3+7a^2b+3b^3)e^{(-4dx-4c)}}{4(a^5+3a^4b+3a^3b^2+a^2b^3+(a^5-a^4b-5a^3b^2-3a^2b^3)e^{(-2dx-2c)}-(a^5-a^4b-5a^3b^2-3a^2b^3)e^{(-4dx-4c)}-(a^5+3a^4b+3a^3b^2+a^2b^3)e^{(-6dx-6c)})d}-\frac{2a^2+5ab+3b^2+2(2a^2-3b^2)e^{(-2dx-2c)}+(2a^2+3ab+3b^2)e^{(-4dx-4c)}}{2(a^4+2a^3b+a^2b^2+(a^4-2a^3b-3a^2b^2)e^{(-2dx-2c)}-(a^4-2a^3b-3a^2b^2)e^{(-4dx-4c)}-(a^4+2a^3b+a^2b^2)e^{(-6dx-6c)})d}+\frac{3b\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}a^2d}+\frac{\log\left(e^{(2dx+2c)}-1\right)}{2a^2d}-\frac{\log\left(e^{(-2dx-2c)}-1\right)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*b + b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
+ a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + 1/4*(2*a*b + b^2)*log(2*(a - b)*e^
(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 2*a^3*b + a^2*b^
2)*d) + 1/8*(3*a^2*b - 4*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c)
 + a - b)/sqrt(a*b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)*d) - 1/8*(3*a^2*b
 - 4*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)*d) + 1/4*(2*a^3 + 5*a^2*b + 6*a*b^2
+ 3*b^3 + (2*a^3 + 7*a^2*b + 3*b^3)*e^(4*d*x + 4*c) + 2*(2*a^3 + 2*a^2*b +
a*b^2 - 3*b^3)*e^(2*d*x + 2*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - (a^
5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(6*d*x + 6*c) - (a^5 - a^4*b - 5*a^3*b
^2 - 3*a^2*b^3)*e^(4*d*x + 4*c) + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^(
2*d*x + 2*c))*d) - 1/4*(2*a^3 + 5*a^2*b + 6*a*b^2 + 3*b^3 + 2*(2*a^3 + 2*a^
2*b + a*b^2 - 3*b^3)*e^(-2*d*x - 2*c) + (2*a^3 + 7*a^2*b + 3*b^3)*e^(-4*d*x
 - 4*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5 - a^4*b - 5*a^3*b^2 -
 3*a^2*b^3)*e^(-2*d*x - 2*c) - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^(-4*
d*x - 4*c) - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c))*d) - 1
/2*(2*a^2 + 5*a*b + 3*b^2 + 2*(2*a^2 - 3*b^2)*e^(-2*d*x - 2*c) + (2*a^2 + 3
*a*b + 3*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b
- 3*a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 - 2*a^3*b - 3*a^2*b^2)*e^(-4*d*x - 4*c
) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 3/4*b*arctan(1/2*((a +
 b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/2*log(e^(2*d
*x + 2*c) - 1)/(a^2*d) - 1/2*log(e^(-2*d*x - 2*c) - 1)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1702 vs. $2(105) = 210$.
time = 0.43, size = 3725, normalized size = 31.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*(a^3 + a^2*b)*d*x*cosh(d*x + c)^6 + 24*(a^3 + a^2*b)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^3 + a^2*b)*d*x*sinh(d*x + c)^6 - 4*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^4 + 4*(15*(a^3 + a^2*b)*d*x*cosh(d*x + c)^2 - 2*a^3 - 6*a^2*b - 5*a*b^2 - 3*b^3 + (a^3 - 3*a^2*b)*d*x)*sinh(d*x + c)^4 + 16*(5*(a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*a^3 - 24*a^2*b - 28*a*b^2 - 12*b^3 - 4*(a^3 + a^2*b)*d*x - 4*(4*a^3 + 4*a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^2 + 4*(15*(a^3 + a^2*b)*d*x*cosh(d*x + c)^4 - 4*a^3 - 4*a^2*b + 4*a*b^2 + 6*b^3 - (a^3 - 3*a^2*b)*d*x - 6*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^6 + 6*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 8*a*b^2 + 3*b^3)*sinh(d*x + c)^6 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^4 + (5*a^2*b - 12*a*b^2 - 9*b^3 + 15*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^3 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*a^2*b - 8*a*b^2 - 3*b^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2 + (15*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^4 - 5*a^2*b + 12*a*b^2 + 9*b^3 + 6*(5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 2*(5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(3*(a^3 + a^2*b)*d*x*cosh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 4*a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^6 + 6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^6 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^4 + (15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d)*sinh(d*x + c)^4 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x

```
+ c))*sinh(d*x + c)^3 + (15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*
x + c)^4 + 6*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^2 - (a^5
 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d)*sinh(d*x + c)^2 - (a^5 + 3*a^4*b + 3*a
^3*b^2 + a^2*b^3)*d + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x
 + c)^5 + 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c)^3 - (a^5
- a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a
^3 + a^2*b)*d*x*cosh(d*x + c)^6 + 12*(a^3 + a^2*b)*d*x*cosh(d*x + c)*sinh(d
*x + c)^5 + 2*(a^3 + a^2*b)*d*x*sinh(d*x + c)^6 - 2*(2*a^3 + 6*a^2*b + 5*a*
b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^4 + 2*(15*(a^3 + a^2*b)*d*
x*cosh(d*x + c)^2 - 2*a^3 - 6*a^2*b - 5*a*b^2 - 3*b^3 + (a^3 - 3*a^2*b)*d*x
)*sinh(d*x + c)^4 + 8*(5*(a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (2*a^3 + 6*a^2
*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3
- 4*a^3 - 12*a^2*b - 14*a*b^2 - 6*b^3 - 2*(a^3 + a^2*b)*d*x - 2*(4*a^3 + 4*
a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^2 + 2*(15*(a^3
 + a^2*b)*d*x*cosh(d*x + c)^4 - 4*a^3 - 4*a^2*b + 4*a*b^2 + 6*b^3 - (a^3 -
3*a^2*b)*d*x - 6*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x +
c)^6 + 6*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2
*b + 8*a*b^2 + 3*b^3)*sinh(d*x + c)^6 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d
*x + c)^4 + (5*a^2*b - 12*a*b^2 - 9*b^3 + 15*(5*a^2*b + 8*a*b^2 + 3*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x
+ c)^3 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*a^
2*b - 8*a*b^2 - 3*b^3 - (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2 + (15*
(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^4 - 5*a^2*b + 12*a*b^2 + 9*b^3 +
6*(5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a
^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 2*(5*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(105) = 210.
time = 0.53, size = 336, normalized size = 2.82

$$-\frac{\frac{(5ab^2+3b^3)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^4+2a^3b+a^2b^2)\sqrt{ab}} - \frac{2(dx+c)}{a^2+2ab+b^2} + \frac{2\left(2a^3e^{(4dx+4c)}+6a^2be^{(4dx+4c)}+5ab^2e^{(4dx+4c)}+3b^3e^{(4dx+4c)}+4a^3e^{(2dx+2c)}+4a^2be^{(2dx+2c)}-4ab^2e^{(2dx+2c)}-6b^3e^{(2dx+2c)}+2a^3+6a^2b+7ab^2+3b^3\right)}{(a^4+2a^3b+a^2b^2)\left(ae^{(6dx+6c)}+be^{(6dx+6c)}+ae^{(4dx+4c)}-3be^{(4dx+4c)}-ae^{(2dx+2c)}+3be^{(2dx+2c)}-a-b\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((5*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) +
 a - b)/sqrt(a*b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2
 + 2*a*b + b^2) + 2*(2*a^3*e^(4*d*x + 4*c) + 6*a^2*b*e^(4*d*x + 4*c) + 5*a*
b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 4*a^3*e^(2*d*x + 2*c) + 4*a^2
*b*e^(2*d*x + 2*c) - 4*a*b^2*e^(2*d*x + 2*c) - 6*b^3*e^(2*d*x + 2*c) + 2*a^
3 + 6*a^2*b + 7*a*b^2 + 3*b^3)/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(6*d*x + 6*c
) + b*e^(6*d*x + 6*c) + a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - a*e^(2*d*
x + 2*c) + 3*b*e^(2*d*x + 2*c) - a - b)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

3.188 $\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal. Leaf size=124

$$-\frac{\coth^2(c+dx)}{2a^2d}+\frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{(a-2b)\log(\tanh(c+dx))}{a^3d}+\frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3(a+b)^2d}-2$$

[Out] -1/2*coth(d*x+c)^2/a^2/d+ln(cosh(d*x+c))/(a+b)^2/d+(a-2*b)*ln(tanh(d*x+c))/a^3/d+1/2*b^2*(3*a+2*b)*ln(a+b*tanh(d*x+c)^2)/a^3/(a+b)^2/d-1/2*b^2/a^2/(a+b)/d/(a+b*tanh(d*x+c)^2)

Rubi [A]
time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$\frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3d(a+b)^2}+\frac{(a-2b)\log(\tanh(c+dx))}{a^3d}-\frac{b^2}{2a^2d(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\coth^2(c+dx)}{2a^2d}+\frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/2*Coth[c + d*x]^2/(a^2*d) + Log[Cosh[c + d*x]]/((a + b)^2*d) + ((a - 2*b)*Log[Tanh[c + d*x]])/(a^3*d) + (b^2*(3*a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/(2*a^3*(a + b)^2*d) - b^2/(2*a^2*(a + b)*d*(a + b*Tanh[c + d*x]^2))

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)^2}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{1}{a^2x^2} + \frac{a-2b}{a^3x} + \frac{b^3}{a^2(a+b)(a+bx)^2} + \frac{b^3(3a+2b)}{a^3(a+b)^2(a+bx)}\right)dx, x\right.}{2d}$$
$$= -\frac{\coth^2(c+dx)}{2a^2d} + \frac{\log(\cosh(c+dx))}{(a+b)^2d} + \frac{(a-2b)\log(\tanh(c+dx))}{a^3d} + \frac{b^2(3a}{}$$

Mathematica [A]
time = 0.61, size = 93, normalized size = 0.75

$$\frac{-\frac{\coth^2(c+dx)}{a^2} + \frac{b^3}{a^3(a+b)\left(b+a\coth^2(c+dx)\right)} + \frac{b^2(3a+2b)\log\left(b+a\coth^2(c+dx)\right)}{a^3(a+b)^2} + \frac{2\log(\sinh(c+dx))}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] (-(Coth[c + d*x]^2/a^2) + b^3/(a^3*(a + b)*(b + a*Coth[c + d*x]^2)) + (b^2*
(3*a + 2*b)*Log[b + a*Coth[c + d*x]^2])/(a^3*(a + b)^2) + (2*Log[Sinh[c + d
*x]])/(a + b)^2)/(2*d)
```

Maple [A]
time = 3.16, size = 226, normalized size = 1.82

method	result
derivativedivides	$$\frac{-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2} + \frac{b^2\left(\frac{2b(a+b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a} + \frac{(3a+2b)\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\right.\right.}{2}\right.}{(a+b)^2a^3}}{}$$
default	$$\frac{-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2} + \frac{b^2\left(\frac{2b(a+b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a} + \frac{(3a+2b)\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\right.\right.}{2}\right.}{(a+b)^2a^3}}{}$$

risch	$\frac{x}{a^2+2ab+b^2}+\frac{4bx}{a^3}+\frac{4bc}{a^3d}-\frac{2x}{a^2}-\frac{2c}{a^2d}-\frac{6b^2x}{a^2(a^2+2ab+b^2)}-\frac{6b^2c}{a^2d(a^2+2ab+b^2)}-\frac{4b^3x}{a^3(a^2+2ab+b^2)}-\frac{4}{a^3d(a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*tanh(1/2*d*x+1/2*c)^2/a^2+b^2/(a+b)^2/a^3*(2*b*(a+b)*tanh(1/2*d*x
+1/2*c)^2/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d
*x+1/2*c)^2+a)+1/2*(3*a+2*b)*ln(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/
2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a))-1/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/
(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/8/a^2/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-8
*b+4*a)*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(118) = 236$.
time = 0.28, size = 402, normalized size = 3.24

$$\frac{(3\,ab^2+2\,b^3)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^5+2\,a^4b+a^3b^2)d}+\frac{dx+c}{(a^2+2\,ab+b^2)d}-\frac{2\left((a^3+3\,a^2b+3\,ab^2+2\,b^3)e^{(-2\,dx-2\,c)}+2\,(a^3+a^2b-ab^2-2\,b^3)e^{(-4\,dx-4\,c)}+(a^3+3\,a^2b+3\,ab^2+2\,b^3)e^{(-6\,dx-6\,c)}\right)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3-4\,(a^4b+2\,a^3b^2+a^2b^3)e^{(-2\,dx-2\,c)}-2\,(a^5-a^4b-5\,a^3b^2-3\,a^2b^3)e^{(-4\,dx-4\,c)}-4\,(a^4b+2\,a^3b^2+a^2b^3)e^{(-6\,dx-6\,c)}+(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)e^{(-8\,dx-8\,c)})d}+\frac{(a-2\,b)\log\left(e^{(-dx-c)}+1\right)}{a^3d}+\frac{(a-2\,b)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*b^2 + 2*b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
4*c) + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + (d*x + c)/((a^2 + 2*a*b + b^2
)*d) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*e^(-2*d*x - 2*c) + 2*(a^3 + a^2
*b - a*b^2 - 2*b^3)*e^(-4*d*x - 4*c) + (a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*e^
(-6*d*x - 6*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - 4*(a^4*b + 2*a^3*b^
2 + a^2*b^3)*e^(-2*d*x - 2*c) - 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^(
-4*d*x - 4*c) - 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c) + (a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-8*d*x - 8*c))*d) + (a - 2*b)*log(e^(-d*x
- c) + 1)/(a^3*d) + (a - 2*b)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3468 vs. $2(118) = 236$.
time = 0.52, size = 3468, normalized size = 27.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4 + a^3*b)*d*x*cosh(d*x + c)^8 + 16*(a^4 + a^3*b)*d*x*cosh(d*x +
 c)*sinh(d*x + c)^7 + 2*(a^4 + a^3*b)*d*x*sinh(d*x + c)^8 - 4*(2*a^3*b*d*x
- a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^6 - 4*(2*a^3*b*d*x - 1
```

4*(a^4 + a^3*b)*d*x*cosh(d*x + c)^2 - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*
sinh(d*x + c)^6 + 8*(14*(a^4 + a^3*b)*d*x*cosh(d*x + c)^3 - 3*(2*a^3*b*d*x
- a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(
2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*cosh(d*x + c)^
4 + 4*(35*(a^4 + a^3*b)*d*x*cosh(d*x + c)^4 + 2*a^4 + 2*a^3*b - 2*a^2*b^2 -
4*a*b^3 - (a^4 - 3*a^3*b)*d*x - 15*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^
2 - 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^4 + a^3*b)*d*x*cos
h(d*x + c)^5 - 5*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d
*x + c)^3 + (2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*c
osh(d*x + c))*sinh(d*x + c)^3 + 2*(a^4 + a^3*b)*d*x - 4*(2*a^3*b*d*x - a^4
- 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2 + 4*(14*(a^4 + a^3*b)*d*x*
cosh(d*x + c)^6 - 2*a^3*b*d*x - 15*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2
- 2*a*b^3)*cosh(d*x + c)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 6*(2*a^
4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 - ((3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^8 + 8*(3*a^2*
b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + 5*a*b^3
+ 2*b^4)*sinh(d*x + c)^8 - 4*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^6 - 4*(3*a*b^
3 + 2*b^4 - 7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 8*(7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^3 - 3*(3*a*b^3 + 2*b^4
)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2*b^2 - 7*a*b^3 - 6*b^4)*cosh(d*x
+ c)^4 + 2*(35*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^4 - 3*a^2*b^2 +
7*a*b^3 + 6*b^4 - 30*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 +
3*a^2*b^2 + 5*a*b^3 + 2*b^4 + 8*(7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x +
c)^5 - 10*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^3 - (3*a^2*b^2 - 7*a*b^3 - 6*b^4
)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^2 + 4*
(7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^6 - 15*(3*a*b^3 + 2*b^4)*cos
h(d*x + c)^4 - 3*a*b^3 - 2*b^4 - 3*(3*a^2*b^2 - 7*a*b^3 - 6*b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)^7 -
3*(3*a*b^3 + 2*b^4)*cosh(d*x + c)^5 - (3*a^2*b^2 - 7*a*b^3 - 6*b^4)*cosh(d
*x + c)^3 - (3*a*b^3 + 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*
cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^4 + a^3*b - 3*a^2*b^2
- 5*a*b^3 - 2*b^4)*cosh(d*x + c)^8 + 8*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 -
2*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3
- 2*b^4)*sinh(d*x + c)^8 - 4*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^6 - 4*
(a^3*b - 3*a*b^3 - 2*b^4 - 7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 -
2*b^4)*cosh(d*x + c)^3 - 3*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c))*sinh(d*
x + c)^5 - 2*(a^4 - 3*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 6*b^4)*cosh(d*x + c)^4
+ 2*(35*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^4 - a^4 +
3*a^3*b + 3*a^2*b^2 - 7*a*b^3 - 6*b^4 - 30*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4 + 8
(7(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^5 - 10*(a^3*b
- 3*a*b^3 - 2*b^4)*cosh(d*x + c)^3 - (a^4 - 3*a^3*b - 3*a^2*b^2 + 7*a*b^3
+ 6*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(

```
d*x + c)^2 + 4*(7*(a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)
^6 - 15*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^4 - a^3*b + 3*a*b^3 + 2*b^4
 - 3*(a^4 - 3*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 6*b^4)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 8*((a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 - 2*b^4)*cosh(d*x + c)^7 -
 3*(a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)^5 - (a^4 - 3*a^3*b - 3*a^2*b^2 +
 7*a*b^3 + 6*b^4)*cosh(d*x + c)^3 - (a^3*b - 3*a*b^3 - 2*b^4)*cosh(d*x + c)
)*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(
2*(a^4 + a^3*b)*d*x*cosh(d*x + c)^7 - 3*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^
2*b^2 - 2*a*b^3)*cosh(d*x + c)^5 + 2*(2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3
 - (a^4 - 3*a^3*b)*d*x)*cosh(d*x + c)^3 - (2*a^3*b*d*x - a^4 - 3*a^3*b - 3*
a^2*b^2 - 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^
2 + a^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*
cosh(d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sin
h(d*x + c)^8 - 4*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^
6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 - (a^5*b + 2*a^4*b^2 +
 a^3*b^3)*d)*sinh(d*x + c)^6 - 2*(a^6 - a^5*b - 5*a^4*b^2 - 3*a^3*b^3)*d*co
sh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^
3 - 3*(a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
5*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(118) = 236.
time = 0.52, size = 323, normalized size = 2.60

$$\frac{\frac{(3ab^2+2b^3)\log\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}{a^5+2a^4b+a^3b^2}-\frac{2(dx+c)}{a^2+2ab+b^2}+\frac{2(a-2b)\log\left(\left|e^{(2dx+2c)}-1\right|\right)}{a^3}-\frac{4\left(\frac{(a^4+3a^3b+3a^2b^2+2ab^3)e^{(6dx+6c)}}{a+b}+\frac{2(a^4+a^3b-a^2b^2-2ab^3)e^{(4dx+4c)}}{a+b}+\frac{(a^4+3a^3b+3a^2b^2+2ab^3)e^{(2dx+2c)}}{a+b}\right)}{\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)(a+b)a^3\left(e^{(2dx+2c)}-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((3*a*b^2 + 2*b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2
*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^5 + 2*a^4*b + a^3*b^2) - 2*(d
*x + c)/(a^2 + 2*a*b + b^2) + 2*(a - 2*b)*log(abs(e^(2*d*x + 2*c) - 1))/a^3
 - 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^(6*d*x + 6*c)/(a + b) + 2*(a^
4 + a^3*b - a^2*b^2 - 2*a*b^3)*e^(4*d*x + 4*c)/(a + b) + (a^4 + 3*a^3*b + 3
```

```
*a^2*b^2 + 2*a*b^3)*e^(2*d*x + 2*c)/(a + b))/((a*e^(4*d*x + 4*c) + b*e^(4*d
*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a + b)*a^3*
(e^(2*d*x + 2*c) - 1)^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

3.189 $\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^2} dx$

Optimal. Leaf size=159

$$\frac{x}{(a+b)^2}+\frac{b^{5/2}(7a+5b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^2d}-\frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3(a+b)d}-\frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d}$$

```
[Out] x/(a+b)^2+1/2*b^(5/2)*(7*a+5*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)
/(a+b)^2/d-1/2*(2*a^2-2*a*b-5*b^2)*coth(d*x+c)/a^3/(a+b)/d-1/6*(2*a+5*b)*co
th(d*x+c)^3/a^2/(a+b)/d+1/2*b*coth(d*x+c)^3/a/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.19, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 483, 597, 536, 212, 211}

$$\frac{b^{5/2}(7a+5b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^2}-\frac{(2a+5b)\coth^3(c+dx)}{6a^2d(a+b)}-\frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3d(a+b)}+\frac{b\coth^3(c+dx)}{2ad(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] x/(a + b)^2 + (b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])
/(2*a^(7/2)*(a + b)^2*d) - ((2*a^2 - 2*a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(
a + b)*d) - ((2*a + 5*b)*Coth[c + d*x]^3)/(6*a^2*(a + b)*d) + (b*Coth[c + d
*x]^3)/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a-5b+5bx^2}{x^4(1-x^2)(a+bx^2)}\,dx, x, \tanh(c+\right.}{2a(a+b)d}$$

$$= -\frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{3}{}\right.}{}$$

$$= -\frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\,\text{co}}{2a(a+b)d\,(}$$

$$= -\frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\,\text{co}}{2a(a+b)d\,(}$$

$$= \frac{x}{(a+b)^2} + \frac{b^{5/2}(7a+5b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^2d} - \frac{(2a^2-2ab-5b^2)\coth(}{2a^3(a+b)d}$$

Mathematica [A]
time = 1.12, size = 139, normalized size = 0.87

$$\frac{\frac{6(c+dx)}{(a+b)^2} + \frac{3b^{5/2}(7a+5b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^2} + \frac{4(-2a+3b)\coth(c+dx)}{a^3} - \frac{2\coth(c+dx)\text{csch}^2(c+dx)}{a^2} + \frac{3b^3\sinh(2(c+dx))}{a^3(a+b)(a-b+(a+b)\cosh(2(c+dx)))}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

```
[Out] ((6*(c + d*x))/(a + b)^2 + (3*b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^2) + (4*(-2*a + 3*b)*Coth[c + d*x])/a^3 -
(2*Coth[c + d*x]*Csch[c + d*x]^2)/a^2 + (3*b^3*Sinh[2*(c + d*x)])/(a^3*(a +
 b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(143) = 286$.
time = 3.00, size = 382, normalized size = 2.40

method	result

derivativedivides	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{2b^3\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\right.}{}$
default	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{2b^3\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\right.}{}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{12a^4e^{8dx+8c}+24a^3b\,e^{8dx+8c}-21a\,b^3e^{8dx+8c}-15b^4e^{8dx+8c}+12a^4e^{6dx+6c}-12a^3b\,e^{6dx+6c}-12a^2b^2e^{6dx+6}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/8/a^3*(1/3*a*tanh(1/2*d*x+1/2*c)^3+5*a*tanh(1/2*d*x+1/2*c)-8*b*tanh
```

```
(1/2*d*x+1/2*c))-2*b^3/(a+b)^2/a^3*(((-1/2*a-1/2*b)*tanh(1/2*d*x+1/2*c)^3+(
-1/2*a-1/2*b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*
x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(7*a+5*b)*a*(-1/2*(-a+(b*(a+b))
^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1
/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))-1/24/a^2/tanh(1/2*d*x
+1/2*c)^3-1/8*(5*a-8*b)/a^3/tanh(1/2*d*x+1/2*c)-1/(a+b)^2*ln(tanh(1/2*d*x+1
/2*c)-1)+1/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2345 vs. $2(143) = 286$.
time = 0.87, size = 2345, normalized size = 14.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*b - a*b^2 - b^3)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x
 + 2*c) + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + 1/4*(a^2*b - a*b^2 - b^3)*
log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^5 +
2*a^4*b + a^3*b^2)*d) + 1/32*(3*a^3*b - 29*a^2*b^2 - 11*a*b^3 + 5*b^4)*arct
an(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + a^3*b
^2)*sqrt(a*b)*d) - 1/32*(3*a^3*b - 29*a^2*b^2 - 11*a*b^3 + 5*b^4)*arctan(1/
2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + a^3*b^2)*
sqrt(a*b)*d) + 1/48*(44*a^4 + 117*a^3*b + 111*a^2*b^2 + 23*a*b^3 - 15*b^4 +
 3*(24*a^4 + 69*a^3*b + 45*a^2*b^2 + 27*a*b^3 - 5*b^4)*e^(8*d*x + 8*c) + 6*
(6*a^4 - 31*a^3*b - 50*a^2*b^2 - 51*a*b^3 + 10*b^4)*e^(6*d*x + 6*c) - 2*(50
*a^4 - 78*a^3*b - 225*a^2*b^2 - 196*a*b^3 + 45*b^4)*e^(4*d*x + 4*c) - 2*(10
*a^4 + 115*a^3*b + 182*a^2*b^2 + 95*a*b^3 - 30*b^4)*e^(2*d*x + 2*c))/((a^6
+ 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*e^(
10*d*x + 10*c) + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*e^(8*d*x + 8*c) +
 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(6*d*x + 6*c) - 2*(a^6 - 3*a^5
*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(4*d*x + 4*c) - (a^6 + 7*a^5*b + 11*a^4*b^2 +
 5*a^3*b^3)*e^(2*d*x + 2*c))*d) - 1/48*(44*a^4 + 117*a^3*b + 111*a^2*b^2 +
23*a*b^3 - 15*b^4 - 2*(10*a^4 + 115*a^3*b + 182*a^2*b^2 + 95*a*b^3 - 30*b^4
)*e^(-2*d*x - 2*c) - 2*(50*a^4 - 78*a^3*b - 225*a^2*b^2 - 196*a*b^3 + 45*b^
4)*e^(-4*d*x - 4*c) + 6*(6*a^4 - 31*a^3*b - 50*a^2*b^2 - 51*a*b^3 + 10*b^4)
*e^(-6*d*x - 6*c) + 3*(24*a^4 + 69*a^3*b + 45*a^2*b^2 + 27*a*b^3 - 5*b^4)*e
^(-8*d*x - 8*c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 + 7*a^5*b + 1
1*a^4*b^2 + 5*a^3*b^3)*e^(-2*d*x - 2*c) - 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*
a^3*b^3)*e^(-4*d*x - 4*c) + 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(-6
*d*x - 6*c) + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*e^(-8*d*x - 8*c) - (
a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*e^(-10*d*x - 10*c))*d) + 1/12*(8*a^3 +
```

```
 7*a^2*b - 16*a*b^2 - 15*b^3 + 3*(8*a^3 + 11*a^2*b + 6*a*b^2 - 5*b^3)*e^(8*
d*x + 8*c) + 6*(4*a^3 - 7*a^2*b - 13*a*b^2 + 10*b^3)*e^(6*d*x + 6*c) - 2*(8
*a^3 - 44*a^2*b - 43*a*b^2 + 45*b^3)*e^(4*d*x + 4*c) - 2*(4*a^3 + 27*a^2*b
+ 5*a*b^2 - 30*b^3)*e^(2*d*x + 2*c))/((a^5 + 2*a^4*b + a^3*b^2 - (a^5 + 2*a
^4*b + a^3*b^2)*e^(10*d*x + 10*c) + (a^5 + 6*a^4*b + 5*a^3*b^2)*e^(8*d*x +
8*c) + 2*(a^5 - 4*a^4*b - 5*a^3*b^2)*e^(6*d*x + 6*c) - 2*(a^5 - 4*a^4*b - 5
*a^3*b^2)*e^(4*d*x + 4*c) - (a^5 + 6*a^4*b + 5*a^3*b^2)*e^(2*d*x + 2*c))*d)
 - 1/12*(8*a^3 + 7*a^2*b - 16*a*b^2 - 15*b^3 - 2*(4*a^3 + 27*a^2*b + 5*a*b^
2 - 30*b^3)*e^(-2*d*x - 2*c) - 2*(8*a^3 - 44*a^2*b - 43*a*b^2 + 45*b^3)*e^(
-4*d*x - 4*c) + 6*(4*a^3 - 7*a^2*b - 13*a*b^2 + 10*b^3)*e^(-6*d*x - 6*c) +
3*(8*a^3 + 11*a^2*b + 6*a*b^2 - 5*b^3)*e^(-8*d*x - 8*c))/((a^5 + 2*a^4*b +
a^3*b^2 - (a^5 + 6*a^4*b + 5*a^3*b^2)*e^(-2*d*x - 2*c) - 2*(a^5 - 4*a^4*b -
 5*a^3*b^2)*e^(-4*d*x - 4*c) + 2*(a^5 - 4*a^4*b - 5*a^3*b^2)*e^(-6*d*x - 6*
c) + (a^5 + 6*a^4*b + 5*a^3*b^2)*e^(-8*d*x - 8*c) - (a^5 + 2*a^4*b + a^3*b^
2)*e^(-10*d*x - 10*c))*d) + 1/8*(4*a^2 + 19*a*b + 15*b^2 - 2*(2*a^2 + 13*a*
b + 30*b^2)*e^(-2*d*x - 2*c) - 2*(10*a^2 - 2*a*b - 45*b^2)*e^(-4*d*x - 4*c)
 - 6*(2*a^2 + a*b + 10*b^2)*e^(-6*d*x - 6*c) + 3*(3*a*b + 5*b^2)*e^(-8*d*x
- 8*c))/((a^4 + a^3*b - (a^4 + 5*a^3*b)*e^(-2*d*x - 2*c) - 2*(a^4 - 5*a^3*b
)*e^(-4*d*x - 4*c) + 2*(a^4 - 5*a^3*b)*e^(-6*d*x - 6*c) + (a^4 + 5*a^3*b)*e
^(-8*d*x - 8*c) - (a^4 + a^3*b)*e^(-10*d*x - 10*c))*d) + 1/2*b*log((a + b)*
e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/(a^3*d) - 1/2*b*log(2*
(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(a^3*d) + 1/2*
(a - b)*log(e^(2*d*x + 2*c) - 1)/(a^3*d) - b*log(e^(2*d*x + 2*c) - 1)/(a^3*
d) - 1/2*(a - b)*log(e^(-2*d*x - 2*c) - 1)/(a^3*d) + b*log(e^(-2*d*x - 2*c)
 - 1)/(a^3*d) - 1/8*(3*a*b - 5*b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a
 - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d) - 3/16*(3*a*b + 5*b^2)*arctan(1/2*((a +
b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d) + 1/8*(3*a*b - 5*
b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^
3*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4080 vs. $2(143) = 286$.
time = 0.45, size = 8482, normalized size = 53.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^4 + a^3*b)*d*x*cosh(d*x + c)^10 + 120*(a^4 + a^3*b)*d*x*cosh(d
*x + c)*sinh(d*x + c)^9 + 12*(a^4 + a^3*b)*d*x*sinh(d*x + c)^10 - 12*(4*a^4
 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^8 + 12*(4
5*(a^4 + a^3*b)*d*x*cosh(d*x + c)^2 - 4*a^4 - 8*a^3*b + 7*a*b^3 + 5*b^4 - (
a^4 + 5*a^3*b)*d*x)*sinh(d*x + c)^8 + 96*(15*(a^4 + a^3*b)*d*x*cosh(d*x + c
)^3 - (4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x +
```

```
c))*sinh(d*x + c)^7 - 24*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 +
(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^6 + 24*(105*(a^4 + a^3*b)*d*x*cosh(d*x +
 c)^4 - 2*a^4 + 2*a^3*b + 2*a^2*b^2 - 9*a*b^3 - 10*b^4 - (a^4 - 5*a^3*b)*d*
x - 14*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x +
 c)^2)*sinh(d*x + c)^6 + 48*(63*(a^4 + a^3*b)*d*x*cosh(d*x + c)^5 - 14*(4*a
^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^3 - 3*(
2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(
d*x + c))*sinh(d*x + c)^5 + 8*(2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 4
5*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^4 + 8*(315*(a^4 + a^3*b)*d*x*c
osh(d*x + c)^6 - 105*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d
*x)*cosh(d*x + c)^4 + 2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 45*b^4 + 3
*(a^4 - 5*a^3*b)*d*x - 45*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 +
 (a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 32*a^4 - 48*a^3*b
+ 48*a^2*b^2 + 124*a*b^3 + 60*b^4 + 32*(45*(a^4 + a^3*b)*d*x*cosh(d*x + c)^
7 - 21*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x +
 c)^5 - 15*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b
)*d*x)*cosh(d*x + c)^3 + (2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 45*b^4
 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 12*(a^4 + a^3*b)
*d*x - 4*(4*a^4 - 20*a^3*b - 4*a^2*b^2 + 74*a*b^3 + 60*b^4 - 3*(a^4 + 5*a^3
*b)*d*x)*cosh(d*x + c)^2 + 4*(135*(a^4 + a^3*b)*d*x*cosh(d*x + c)^8 - 84*(4
*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^6 - 9
0*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*co
sh(d*x + c)^4 - 4*a^4 + 20*a^3*b + 4*a^2*b^2 - 74*a*b^3 - 60*b^4 + 3*(a^4 +
 5*a^3*b)*d*x + 12*(2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 45*b^4 + 3*(
a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((7*a^2*b^2 + 12*a
*b^3 + 5*b^4)*cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x
 + c)*sinh(d*x + c)^9 + (7*a^2*b^2 + 12*a*b^3 + 5*b^4)*sinh(d*x + c)^10 - (
7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^8 - (7*a^2*b^2 + 40*a*b^3 + 25
*b^4 - 45*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
 8*(15*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^3 - (7*a^2*b^2 + 40*a*b
^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(7*a^2*b^2 - 30*a*b^3 - 25*
b^4)*cosh(d*x + c)^6 + 2*(105*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^
4 - 7*a^2*b^2 + 30*a*b^3 + 25*b^4 - 14*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*
x + c)^5 - 14*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^3 - 3*(7*a^2*b^
2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(7*a^2*b^2 - 30*a
*b^3 - 25*b^4)*cosh(d*x + c)^4 + 2*(105*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh
(d*x + c)^6 - 35*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 7*a^2*b^
2 - 30*a*b^3 - 25*b^4 - 15*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 - 7*a^2*b^2 - 12*a*b^3 - 5*b^4 + 8*(15*(7*a^2*b^2 + 12*a*b
^3 + 5*b^4)*cosh(d*x + c)^7 - 7*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x +
c)^5 - 5*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^3 + (7*a^2*b^2 - 30*
a*b^3 - 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (7*a^2*b^2 + 40*a*b^3 + 25
*b^4)*cosh(d*x + c)^2 + (45*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^8
- 28*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^6 - 30*(7*a^2*b^2 - 30*a
```

*b^3 - 25*b^4)*cosh(d*x + c)^4 + 7*a^2*b^2 + 40*a*b^3 + 25*b^4 + 12*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^9 - 4*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^7 - 6*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^5 + 4*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^3 + (7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(15*(a^4 + a^3*b)*d*x*...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.56, size = 281, normalized size = 1.77

$$\frac{\frac{3\,(7\,ab^3+5\,b^4)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{ab}}+\frac{6\,(dx+c)}{a^2+2\,ab+b^2}-\frac{6\,(ab^3e^{(2\,dx+2\,c)}-b^4e^{(2\,dx+2\,c)}+ab^3+b^4)}{(a^5+2\,a^4b+a^3b^2)(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b)}-\frac{8\,(3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-3\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+2\,a-3\,b)}{a^3\left(e^{(2\,dx+2\,c)}-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(7*a*b^3 + 5*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b)) + 6*(d*x + c)/(a^2 + 2*a*b + b^2) - 6*(a*b^3*e^(2*d*x + 2*c) - b^4*e^(2*d*x + 2*c) + a*b^3 + b^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)) - 8*(3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 2*a - 3*b)/(a^3*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c+d\,x)^4}{{\left(b\tanh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
[Out] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)
```

3.190 $\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{x}{(a+b)^3}-\frac{\sqrt{a}\,(3a^2+10ab+15b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}(a+b)^3d}+\frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}+\frac{a(3a\cdots}{8b^2(a+b)^2\cdots}$$

```
[Out] x/(a+b)^3-1/8*(3*a^2+10*a*b+15*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(
1/2)/b^(5/2)/(a+b)^3/d+1/4*a*tanh(d*x+c)^3/b/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+
1/8*a*(3*a+7*b)*tanh(d*x+c)/b^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 481, 592, 536, 212, 211}

$$-\frac{\sqrt{a}\,(3a^2+10ab+15b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}d(a+b)^3}+\frac{a(3a+7b)\tanh(c+dx)}{8b^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{a\tanh^3(c+dx)}{4bd(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{x}{(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(8*b^(5/2)*(a + b)^3*d) + (a*Tanh[c + d*x]^3)/(4*b*(a + b)*d
*(a + b*Tanh[c + d*x]^2)^2) + (a*(3*a + 7*b)*Tanh[c + d*x])/(8*b^2*(a + b)^
2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2(3a+(-3a-4b)x^2)}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+\right.}{4b(a+b)d}$$

$$= \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{Su}}{}$$

$$= \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Su}}{}$$

$$= \frac{x}{(a+b)^3} - \frac{\sqrt{a}\,(3a^2+10ab+15b^2)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}(a+b)^3d} + \frac{a\tan}{4b(a+b)d\,(a}$$

Mathematica [A]
time = 0.89, size = 144, normalized size = 1.00

$$\frac{8(c+dx) - \frac{\sqrt{a}\,(3a^2+10ab+15b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{5/2}} - \frac{4a^2(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{3a(a+b)(a+3b)\sinh(2(c+dx))}{b^2(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/b^(5/2) - (4*a^2*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a
+ b)*Cosh[2*(c + d*x)])^2) + (3*a*(a + b)*(a + 3*b)*Sinh[2*(c + d*x)])/(b^2
*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

Maple [A]
time = 0.89, size = 157, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3} - \frac{a\left(\frac{-\frac{(5a^2+14ab+9b^2)(\tanh^3(dx+c))}{8b} - \frac{a(3a^2+10ab+7b^2)\tanh(dx+c)}{8b^2}}{(a+b(\tanh^2(dx+c)))^2} + \frac{(3a^2+10ab+15b^2)\arctan\left(\frac{b\tanh(dx+}{\sqrt{ab}}\right.}{8b^2\sqrt{ab}}\right.}{(a+b)^3}}{d}$

default	$\frac{\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{a\left(\frac{-\frac{\left(5a^2+14ab+9b^2\right)\left(\tanh^3(dx+c)\right)}{8b}-\frac{a\left(3a^2+10ab+7b^2\right)\tanh(dx+c)}{8b^2}}{\left(a+b\left(\tanh^2(dx+c)\right)\right)^2}+\frac{\left(3a^2+10ab+15b^2\right)\arctan\left(\frac{b\tanh(dx}{\sqrt{ab}}\right.}{8b^2\sqrt{ab}}\right.}{(a+b)^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{a\left(3a^3e^{6dx+6c}+13a^2b\,e^{6dx+6c}+a\,b^2e^{6dx+6c}-9b^3e^{6dx+6c}+9a^3e^{4dx+4c}+21a^2b\,e^{4dx+4c}-9a\,b^2e^{4}\right.}{4\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2/(a+b)^3*ln(1+tanh(d*x+c))-a/(a+b)^3*((-1/8*(5*a^2+14*a*b+9*b^2)/b*tanh(d*x+c)^3-1/8*a*(3*a^2+10*a*b+7*b^2)/b^2*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(3*a^2+10*a*b+15*b^2)/b^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))-1/2/(a+b)^3*ln(tanh(d*x+c)-1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3354 vs. $2(130) = 260$.
time = 1.49, size = 3354, normalized size = 23.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/512*(3*a^5 + 25*a^4*b + 150*a^3*b^2 - 150*a^2*b^3 - 25*a*b^4 - 3*b^5)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) + 1/512*(3*a^5 + 25*a^4*b + 150*a^3*b^2 - 150*a^2*b^3 - 25*a*b^4 - 3*b^5)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) - 1/256*(3*a^6 + 30*a^5*b - 99*a^4*b^2 - 252*a^3*b^3 - 99*a^2*b^4 + 30*a*b^5 + 3*b^6 + (3*a^6 + 28*a^5*b - 465*a^4*b^2 + 465*a^2*b^4 - 28*a*b^5 - 3*b^6)*e^(6*d*x + 6*c) + (9*a^6 + 66*a^5*b - 905*a^4*b^2 + 1148*a^3*b^3 - 905*a^2*b^4 + 66*a*b^5 + 9*b^6)*e^(4*d*x + 4*c) + (9*a^6 + 68*a^5*b - 659*a^4*b^2 + 659*a^2*b^4 - 68*a*b^5 - 9*b^6)*e^(2*d*x + 2*c))/((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7 + (a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*e^(8*d*x + 8*c) + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^(6*d*x + 6*c) + 2*(3*a^7*b^2 + 7*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 + 7*a^3*b^6 + 3*a^2*b^7)*e^(4*d*x + 4*c) + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^(2*d*x + 2*c))*d) + 1/256*(3*a^6 + 30*a^5*b - 99*a^4*b^2 - 252*a^3*b^3 - 99*a^2*b^4 + 30*a*b^5 + 3*b^6 + (9*a^6 + 68*a^5*b - 659*a^4*b^2 + 659*a^2*b^4 - 68*a*b^5 - 9*b^6)*e^(-2*d*x - 2*c) + (9*a^6 + 66*a^5*b - 905*a^4*b^2 + 1148*a^3*b^3 - 905*a^2*b^4 + 66*a*b^5 + 9*b^6)*e^(-4*d*x - 4

*c) + (3*a^6 + 28*a^5*b - 465*a^4*b^2 + 465*a^2*b^4 - 28*a*b^5 - 3*b^6)*e^(
-6*d*x - 6*c))/((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6
+ a^2*b^7 + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^
2*b^7)*e^(-2*d*x - 2*c) + 2*(3*a^7*b^2 + 7*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5
+ 7*a^3*b^6 + 3*a^2*b^7)*e^(-4*d*x - 4*c) + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*
b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^(-6*d*x - 6*c) + (a^7*b^2 + 5*a^6*
b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*e^(-8*d*x - 8*c))*d) -
3/128*(3*a^5 + 17*a^4*b + 14*a^3*b^2 - 14*a^2*b^3 - 17*a*b^4 - 3*b^5 + (3*
a^5 + 15*a^4*b - 98*a^3*b^2 - 98*a^2*b^3 + 15*a*b^4 + 3*b^5)*e^(6*d*x + 6*c
) + (9*a^5 + 27*a^4*b - 110*a^3*b^2 + 110*a^2*b^3 - 27*a*b^4 - 9*b^5)*e^(4*
d*x + 4*c) + (9*a^5 + 29*a^4*b - 86*a^3*b^2 - 86*a^2*b^3 + 29*a*b^4 + 9*b^5
)*e^(2*d*x + 2*c))/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6
+ (a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) +
4*(a^6*b^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c) + 2*(3*a^6*b
^2 + 4*a^5*b^3 + 2*a^4*b^4 + 4*a^3*b^5 + 3*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^
6*b^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) + 3/128*(3*a^5
+ 17*a^4*b + 14*a^3*b^2 - 14*a^2*b^3 - 17*a*b^4 - 3*b^5 + (9*a^5 + 29*a^4*
b - 86*a^3*b^2 - 86*a^2*b^3 + 29*a*b^4 + 9*b^5)*e^(-2*d*x - 2*c) + (9*a^5 +
27*a^4*b - 110*a^3*b^2 + 110*a^2*b^3 - 27*a*b^4 - 9*b^5)*e^(-4*d*x - 4*c)
+ (3*a^5 + 15*a^4*b - 98*a^3*b^2 - 98*a^2*b^3 + 15*a*b^4 + 3*b^5)*e^(-6*d*x
- 6*c))/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6 + 4*(a^6*b
^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^(-2*d*x - 2*c) + 2*(3*a^6*b^2 + 4*a
^5*b^3 + 2*a^4*b^4 + 4*a^3*b^5 + 3*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^6*b^2 +
2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^(-6*d*x - 6*c) + (a^6*b^2 + 4*a^5*b^3 +
6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6)*e^(-8*d*x - 8*c))*d) - 15/256*(3*a^4 + 8*
a^3*b + 10*a^2*b^2 + 8*a*b^3 + 3*b^4 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*e^
(6*d*x + 6*c) + (9*a^4 + 46*a^2*b^2 + 9*b^4)*e^(4*d*x + 4*c) + (9*a^4 + 2*a
^3*b - 2*a*b^3 - 9*b^4)*e^(2*d*x + 2*c))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
+ a^2*b^5 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*e^(8*d*x + 8*c) + 4
*(a^5*b^2 + a^4*b^3 - a^3*b^4 - a^2*b^5)*e^(6*d*x + 6*c) + 2*(3*a^5*b^2 + a
^4*b^3 + a^3*b^4 + 3*a^2*b^5)*e^(4*d*x + 4*c) + 4*(a^5*b^2 + a^4*b^3 - a^3*
b^4 - a^2*b^5)*e^(2*d*x + 2*c))*d) + 15/256*(3*a^4 + 8*a^3*b + 10*a^2*b^2 +
8*a*b^3 + 3*b^4 + (9*a^4 + 2*a^3*b - 2*a*b^3 - 9*b^4)*e^(-2*d*x - 2*c) + (
9*a^4 + 46*a^2*b^2 + 9*b^4)*e^(-4*d*x - 4*c) + 3*(a^4 + 2*a^3*b - 2*a*b^3 -
b^4)*e^(-6*d*x - 6*c))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5 + 4*(a^
5*b^2 + a^4*b^3 - a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5*b^2 + a^4*
b^3 + a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^5*b^2 + a^4*b^3 - a^3*b^
4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)
*e^(-8*d*x - 8*c))*d) + 5/64*(3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + (9*a^3 -
13*a^2*b - 13*a*b^2 + 9*b^3)*e^(-2*d*x - 2*c) + 3*(3*a^3 - 5*a^2*b + 5*a*b^
2 - 3*b^3)*e^(-4*d*x - 4*c) + (3*a^3 + a^2*b + a*b^2 + 3*b^3)*e^(-6*d*x - 6
*c))/((a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + 4*(a^4*b^2 - a^2*b^4)*e^(-2*d*x - 2*
c) + 2*(3*a^4*b^2 - 2*a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^4*b^2 -
a^2*b^4)*e^(-6*d*x - 6*c) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c
))*d) + 1/4*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b

```
)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/4*log(2*(a - b)*e^(-2*d*x - 2*c)
+ (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 9
/256*(a^2 + 2*a*b + b^2)*arctan(1/2*((a + b)*e^...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3603 vs. $2(130) = 260$.
time = 0.48, size = 7528, normalized size = 52.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^8 + 128*(a^2*b^2 + 2*
a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^2*b^2 + 2*a*b^3 + b^
4)*d*x*sinh(d*x + c)^8 - 4*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*
b^2 - b^4)*d*x)*cosh(d*x + c)^6 + 4*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh
(d*x + c)^2 - 3*a^4 - 13*a^3*b - a^2*b^2 + 9*a*b^3 + 16*(a^2*b^2 - b^4)*d*x
)*sinh(d*x + c)^6 + 8*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 -
3*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 4*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a
^2*b^2 - 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^2*b^2 + 2*a*b^3
+ b^4)*d*x*cosh(d*x + c)^4 - 9*a^4 - 21*a^3*b + 9*a^2*b^2 - 27*a*b^3 + 8*(3
*a^2*b^2 - 2*a*b^3 + 3*b^4)*d*x - 15*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3
- 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 12*a^4 - 60*a^
3*b - 84*a^2*b^2 - 36*a*b^3 + 16*(56*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x
 + c)^5 - 5*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)
*cosh(d*x + c)^3 - (9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2
- 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^2*b^2 + 2*a*
b^3 + b^4)*d*x - 4*(9*a^4 + 23*a^3*b - 13*a^2*b^2 - 27*a*b^3 - 16*(a^2*b^2
- b^4)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x
 + c)^6 - 15*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x
)*cosh(d*x + c)^4 - 9*a^4 - 23*a^3*b + 13*a^2*b^2 + 27*a*b^3 + 16*(a^2*b^2
- b^4)*d*x - 6*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2 - 2*
a*b^3 + 3*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4 + 16*a^3*b +
 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^8 + 8*(3*a^4 + 16*a^3*b + 38
*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 16*a
^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*sinh(d*x + c)^8 + 4*(3*a^4 + 10*a^3*
b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^6 + 4*(3*a^4 + 10*a^3*b +
 12*a^2*b^2 - 10*a*b^3 - 15*b^4 + 7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b
^3 + 15*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 16*a^3*b + 38
*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^3 + 3*(3*a^4 + 10*a^3*b + 12*a^
2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 + 24*a
^3*b + 34*a^2*b^2 + 45*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 16*a^3*b + 38*
a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^4 + 9*a^4 + 24*a^3*b + 34*a^2*b^
```

```
2 + 45*b^4 + 30*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b
^4 + 8*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)
^5 + 10*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^3
 + (9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^2 + 4
*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^6 + 1
5*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^4 + 3*a
^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4 + 3*(9*a^4 + 24*a^3*b + 34*a
^2*b^2 + 45*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 + 16*a^3*b +
38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^7 + 3*(3*a^4 + 10*a^3*b + 12*
a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^5 + (9*a^4 + 24*a^3*b + 34*a^2*b
^2 + 45*b^4)*cosh(d*x + c)^3 + (3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 -
15*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b
+ b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))
*sinh(d*x + c) - 4*((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x +
c)*sinh(d*x + c) + (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a
 + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*s
inh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 +
 a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c
))*sinh(d*x + c) + a + b)) + 8*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x +
 c)^7 - 3*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*c
osh(d*x + c)^5 - 2*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2
- 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c)^3 - (9*a^4 + 23*a^3*b - 13*a^2*b^2 -
27*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^2
+ 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^8 +
8*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5
*a*b^6 + b^7)*d*sinh(d*x + c)^8 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^
2*b^5 - 3*a*b^6 - b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a
^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + (a^5*b^2 + 3*a^4*b
^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d)*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(130) = 260.

time = 0.63, size = 408, normalized size = 2.83

$$-\frac{\frac{(3\,a^3+10\,a^2b+15\,ab^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)\sqrt{ab}}-\frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,(3\,a^4e^{(6\,dx+6\,c)}+13\,a^3be^{(6\,dx+6\,c)}+a^2b^2e^{(6\,dx+6\,c)}-9\,ab^3e^{(6\,dx+6\,c)}+9\,a^4e^{(4\,dx+4\,c)}+21\,a^3be^{(4\,dx+4\,c)}-9\,a^2b^2e^{(4\,dx+4\,c)}+27\,ab^3e^{(4\,dx+4\,c)}+9\,a^4e^{(2\,dx+2\,c)}+23\,a^3be^{(2\,dx+2\,c)}-13\,a^2b^2e^{(2\,dx+2\,c)}-27\,ab^3e^{(2\,dx+2\,c)}+3\,a^4+15\,a^3b+21\,a^2b^2+9\,ab^3)}{(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((3*a^3 + 10*a^2*b + 15*a*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*
d*x + 2*c) + a - b)/sqrt(a*b))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*sqrt(
a*b)) - 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(3*a^4*e^(6*d*x + 6
*c) + 13*a^3*b*e^(6*d*x + 6*c) + a^2*b^2*e^(6*d*x + 6*c) - 9*a*b^3*e^(6*d*x
 + 6*c) + 9*a^4*e^(4*d*x + 4*c) + 21*a^3*b*e^(4*d*x + 4*c) - 9*a^2*b^2*e^(4
*d*x + 4*c) + 27*a*b^3*e^(4*d*x + 4*c) + 9*a^4*e^(2*d*x + 2*c) + 23*a^3*b*e
^(2*d*x + 2*c) - 13*a^2*b^2*e^(2*d*x + 2*c) - 27*a*b^3*e^(2*d*x + 2*c) + 3*
a^4 + 15*a^3*b + 21*a^2*b^2 + 9*a*b^3)/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^
5)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*
d*x + 2*c) + a + b)^2))/d
```

Mupad [B]

time = 0.92, size = 2669, normalized size = 18.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^6/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh
(c + d*x)^3*(9*a*b + 5*a^2))/(8*b*(2*a*b + a^2 + b^2)) + (a*tanh(c + d*x)*(
7*a*b + 3*a^2))/(8*b^2*(2*a*b + a^2 + b^2)))/(a^2*d + b^2*d*tanh(c + d*x)^4
 + 2*a*b*d*tanh(c + d*x)^2) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^3) - (ata
n((((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4
 + 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 +
 4*a^3*b^4*d^2 + a^4*b^3*d^2)) + (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 393
6*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 80
0*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3
 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)) - (tanh(
c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^
11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5
*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d +
 a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*
d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d +
 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2)*1i)/(16*(
b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)) + ((-a*b^5)^(1/2)*((tanh(c +
```

```
d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b^3 + 190*a^4*b^2))
/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2))
 - (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d
^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2
)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5
*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)) + (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*
b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1
280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 -
256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d
^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^
(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a
*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2)*1i)/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*
d + 3*a*b^7*d)))/((120*a*b^4 + 51*a^4*b + 9*a^5 + 185*a^2*b^3 + 139*a^3*b^2
)/(32*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5
*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)) - ((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*
a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7
*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)) + (((224
*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280
*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^
9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*
a^5*b^4*d^3 + a^6*b^3*d^3)) - (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2
 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b
^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b
^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*
b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10
*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))
*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*
d)) + ((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*
b^4 + 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^
2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)) - (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 +
3936*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 +
 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*
d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)) + (ta
nh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a
*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*
a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*
d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b
^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*
d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2))/(16*(
b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d))))*(-a*b^5)^(1/2)*(10*a*b + 3*
a^2 + 15*b^2)*1i)/(8*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d))
```

3.191 $\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=109

$$\frac{\log(\cosh(c+dx))}{(a+b)^3d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d}-\frac{a^2}{4b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}+\frac{a(a+2b)}{2b^2(a+b)^2d\left(a+b\tanh\right.}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/4*a^2/b^2/(
a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/2*a*(a+2*b)/b^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2
)
```

Rubi [A]
time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{a^2}{4b^2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{a(a+2b)}{2b^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3}+\frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) - a^2/(4*b^2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) + (a*(a + 2*b))/(2*b^2
*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
```

```
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)}+\frac{a^2}{b(a+b)(a+bx)^3}-\frac{a(a+2b)}{b(a+b)^2(a+bx)^2}+\frac{b}{(a+b)^3(a+bx)}\right)\,dx, x, \text{t}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)^3d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d}-\frac{a^2}{4b^2(a+b)d\left(a+b\tanh^2(c\right.}$$

Mathematica [A]
time = 0.78, size = 91, normalized size = 0.83

$$-\frac{-4\log(\cosh(c+dx))-2\log\left(a+b\tanh^2(c+dx)\right)+\frac{a^2(a+b)^2}{b^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{2a(a+b)(a+2b)}{b^2\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
[Out] -1/4*(-4*Log[Cosh[c + d*x]] - 2*Log[a + b*Tanh[c + d*x]^2] + (a^2*(a + b)^2
)/(b^2*(a + b*Tanh[c + d*x]^2)^2) - (2*a*(a + b)*(a + 2*b))/(b^2*(a + b*Tan
h[c + d*x]^2)))/((a + b)^3*d)
```

Maple [A]
time = 0.72, size = 127, normalized size = 1.17

method	result
derivativedivides	$\frac{-\frac{-\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)+\frac{a^2\left(a^2+2ab+b^2\right)}{2b^2\left(a+b\left(\tanh^2(dx+c)\right)\right)^2}-\frac{a\left(a^2+3ab+2b^2\right)}{b^2\left(a+b\left(\tanh^2(dx+c)\right)\right)}}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}}{d}$
default	$\frac{-\frac{-\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)+\frac{a^2\left(a^2+2ab+b^2\right)}{2b^2\left(a+b\left(\tanh^2(dx+c)\right)\right)^2}-\frac{a\left(a^2+3ab+2b^2\right)}{b^2\left(a+b\left(\tanh^2(dx+c)\right)\right)}}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}}{d}$

risch	$-\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2c}{d(a^3+3a^2b+3a\,b^2+b^3)}+\frac{4\,\mathrm{e}^{2dx+2c}a\left(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}+a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+\cdots\right.}{\left(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}+2a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+a+b\right)^2(a+b)(a^2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^3*(-ln(a+b*tanh(d*x+c)^2)+1/2*a^2*(a^2+2*a*b+b^2)/b^2/(a+b*
tanh(d*x+c)^2)^2-a*(a^2+3*a*b+2*b^2)/b^2/(a+b*tanh(d*x+c)^2))-1/2/(a+b)^3*l
n(1+tanh(d*x+c))-1/2/(a+b)^3*ln(tanh(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(103) = 206$.
time = 0.33, size = 376, normalized size = 3.45

$$\frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}+\frac{4\left((a^2+ab)e^{(-2\,dx-2\,c)}+(a^2-2\,ab)e^{(-4\,dx-4\,c)}+(a^2+ab)e^{(-6\,dx-6\,c)}\right)}{\left(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5+4\,(a^5+3\,a^4b+2\,a^3b^2-2\,a^2b^3-3\,ab^4-b^5)e^{(-2\,dx-2\,c)}+2\,(3\,a^5+7\,a^4b+6\,a^3b^2+6\,a^2b^3+7\,ab^4+3\,b^5)e^{(-4\,dx-4\,c)}+4\,(a^5+3\,a^4b+2\,a^3b^2-2\,a^2b^3-3\,ab^4-b^5)e^{(-6\,dx-6\,c)}+(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)e^{(-8\,dx-8\,c)}\right)d}+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 4*((a^2 + a*b)*e^(-2*d*x -
2*c) + (a^2 - 2*a*b)*e^(-4*d*x - 4*c) + (a^2 + a*b)*e^(-6*d*x - 6*c))/((a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4*b + 2
*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7*a^4*b
 + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5 + 3*a
^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d) + 1/2*
log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. $2(103) = 206$.
time = 0.38, size = 2584, normalized size = 23.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8
 + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^
2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x - a^2 - a*b)*sinh(d*x + c)^6 + 16*
(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x - a^2 - a*b
)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 +
```

4*a*b)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + (
3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)
^2 - 2*a^2 + 4*a*b)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^5 + 10*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3*a^2 - 2*a
*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 +
2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 8*(7*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x - a^2 - a*b)*c
osh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 +
4*a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*s
inh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*
cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 +
15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*co
sh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*co
sh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 16*((a^2 + 2*a*b + b^2)*d*x*co
sh(d*x + c)^7 + 3*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^5 + ((3*a^2 -
2*a*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c)^3 + ((a^2 - b^2)*d*x - a
^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a
^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 +
10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 4*(a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 4*(7*(
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2
+ (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^
6 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*
x + c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2
- 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*
b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (3*a
^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b

```
^5)*d*cosh(d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*
a*b^4 + 3*b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
 3*a*b^4 - b^5)*d)*sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*d*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 +
 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3907 vs. $2(94) = 188$.
time = 114.45, size = 3907, normalized size = 35.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(c
 + d*x) + 1)/d - tanh(c + d*x)**4/(4*d) - tanh(c + d*x)**2/(2*d))/a**3, Eq(
b, 0)), (3*tanh(c + d*x)**4/(6*b**3*d*tanh(c + d*x)**6 - 18*b**3*d*tanh(c +
 d*x)**4 + 18*b**3*d*tanh(c + d*x)**2 - 6*b**3*d) - 3*tanh(c + d*x)**2/(6*b
**3*d*tanh(c + d*x)**6 - 18*b**3*d*tanh(c + d*x)**4 + 18*b**3*d*tanh(c + d*
x)**2 - 6*b**3*d) + 1/(6*b**3*d*tanh(c + d*x)**6 - 18*b**3*d*tanh(c + d*x)*
*4 + 18*b**3*d*tanh(c + d*x)**2 - 6*b**3*d), Eq(a, -b)), (x*tanh(c)**5/(a +
 b*tanh(c)**2)**3, Eq(d, 0)), (a**4/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c +
 d*x)**2 + 12*a**4*b**3*d + 4*a**3*b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d
*tanh(c + d*x)**2 + 12*a**3*b**4*d + 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a
**2*b**5*d*tanh(c + d*x)**2 + 4*a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4
+ 8*a*b**6*d*tanh(c + d*x)**2 + 4*b**7*d*tanh(c + d*x)**4) + 2*a**3*b*tanh(
c + d*x)**2/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*
d + 4*a**3*b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a
**3*b**4*d + 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)
**2 + 4*a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*
x)**2 + 4*b**7*d*tanh(c + d*x)**4) + 4*a**3*b/(4*a**5*b**2*d + 8*a**4*b**3*
d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4*a**3*b**4*d*tanh(c + d*x)**4 + 24*a
**3*b**4*d*tanh(c + d*x)**2 + 12*a**3*b**4*d + 12*a**2*b**5*d*tanh(c + d*x)
**4 + 24*a**2*b**5*d*tanh(c + d*x)**2 + 4*a**2*b**5*d + 12*a*b**6*d*tanh(c
+ d*x)**4 + 8*a*b**6*d*tanh(c + d*x)**2 + 4*b**7*d*tanh(c + d*x)**4) + 4*a*
*2*b**2*d*x/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*
d + 4*a**3*b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a
**3*b**4*d + 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)
**2 + 4*a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*
x)**2 + 4*b**7*d*tanh(c + d*x)**4) + 2*a**2*b**2*log(-sqrt(-a/b) + tanh(c +
```

```
d*x))/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4
*a**3*b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a**3*b
**4*d + 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)**2 +
 4*a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*x)**2
 + 4*b**7*d*tanh(c + d*x)**4) + 2*a**2*b**2*log(sqrt(-a/b) + tanh(c + d*x))
/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4*a**3*
b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a**3*b**4*d
+ 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)**2 + 4*a**
2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*x)**2 + 4*b
**7*d*tanh(c + d*x)**4) - 4*a**2*b**2*log(tanh(c + d*x) + 1)/(4*a**5*b**2*d
 + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4*a**3*b**4*d*tanh(c +
 d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a**3*b**4*d + 12*a**2*b**5*
d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)**2 + 4*a**2*b**5*d + 12*a
*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*x)**2 + 4*b**7*d*tanh(c +
d*x)**4) + 6*a**2*b**2*tanh(c + d*x)**2/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh
(c + d*x)**2 + 12*a**4*b**3*d + 4*a**3*b**4*d*tanh(c + d*x)**4 + 24*a**3*b*
*4*d*tanh(c + d*x)**2 + 12*a**3*b**4*d + 12*a**2*b**5*d*tanh(c + d*x)**4 +
24*a**2*b**5*d*tanh(c + d*x)**2 + 4*a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)
**4 + 8*a*b**6*d*tanh(c + d*x)**2 + 4*b**7*d*tanh(c + d*x)**4) + 3*a**2*b**
2/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4*a**3
*b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a**3*b**4*d
 + 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)**2 + 4*a*
*2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*x)**2 + 4*
b**7*d*tanh(c + d*x)**4) + 8*a*b**3*d*x*tanh(c + d*x)**2/(4*a**5*b**2*d + 8
*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4*a**3*b**4*d*tanh(c + d*x
)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a**3*b**4*d + 12*a**2*b**5*d*ta
nh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)**2 + 4*a**2*b**5*d + 12*a*b**
6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*x)**2 + 4*b**7*d*tanh(c + d*x)
**4) + 4*a*b**3*log(-sqrt(-a/b) + tanh(c + d*x))*tanh(c + d*x)**2/(4*a**5*b
**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4*a**3*b**4*d*tan
h(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a**3*b**4*d + 12*a**2*
b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)**2 + 4*a**2*b**5*d +
 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*x)**2 + 4*b**7*d*tanh
(c + d*x)**4) + 4*a*b**3*log(sqrt(-a/b) + tanh(c + d*x))*tanh(c + d*x)**2/(
4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4*a**3*b*
*4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a**3*b**4*d +
12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)**2 + 4*a**2*
b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*x)**2 + 4*b**
7*d*tanh(c + d*x)**4) - 8*a*b**3*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/(4
*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4*a**3*b**
4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(103) = 206$.

time = 0.59, size = 245, normalized size = 2.25

$$\frac{\frac{2 \log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2-4\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)-12\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)-4\,a+12\,b}{(a^2+2\,ab+b^2)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
+ e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2
*c))^2 - 4*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 12*b*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) - 4*a + 12*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)^2)
)/d
```

Mupad [B]
time = 0.83, size = 416, normalized size = 3.82

$$\frac{a^4+a^3\,b\,\left(2\,\mathrm{tanh}\left(c+d\,x\right)^2+4\right)-a\,b^3\,\left(-4\,\mathrm{tanh}\left(c+d\,x\right)^2+\mathrm{tanh}\left(c+d\,x\right)^2\,\mathrm{atan}\left(\frac{a\,\mathrm{tanh}\left(c+d\,x\right)^2\,1{}\mathrm{i}+b\,\mathrm{tanh}\left(c+d\,x\right)^2\,1{}\mathrm{i}}{2\,a-a\,\mathrm{tanh}\left(c+d\,x\right)^2+b\,\mathrm{tanh}\left(c+d\,x\right)^2}\right)\,8{}\mathrm{i}\right)+a^2\,b^2\,\left(6\,\mathrm{tanh}\left(c+d\,x\right)^2+3-\mathrm{atan}\left(\frac{a\,\mathrm{tanh}\left(c+d\,x\right)^2\,1{}\mathrm{i}+b\,\mathrm{tanh}\left(c+d\,x\right)^2\,1{}\mathrm{i}}{2\,a-a\,\mathrm{tanh}\left(c+d\,x\right)^2+b\,\mathrm{tanh}\left(c+d\,x\right)^2}\right)\,4{}\mathrm{i}\right)-b^4\,\mathrm{tanh}\left(c+d\,x\right)^4\,\mathrm{atan}\left(\frac{a\,\mathrm{tanh}\left(c+d\,x\right)^2\,1{}\mathrm{i}+b\,\mathrm{tanh}\left(c+d\,x\right)^2\,1{}\mathrm{i}}{2\,a-a\,\mathrm{tanh}\left(c+d\,x\right)^2+b\,\mathrm{tanh}\left(c+d\,x\right)^2}\right)\,4{}\mathrm{i}}{4\,d\,a^5\,b^2+8\,d\,a^4\,b^3\,\mathrm{tanh}\left(c+d\,x\right)^2+12\,d\,a^4\,b^3+4\,d\,a^3\,b^4\,\mathrm{tanh}\left(c+d\,x\right)^4+24\,d\,a^3\,b^4\,\mathrm{tanh}\left(c+d\,x\right)^2+12\,d\,a^3\,b^4+12\,d\,a^2\,b^5\,\mathrm{tanh}\left(c+d\,x\right)^4+24\,d\,a^2\,b^5\,\mathrm{tanh}\left(c+d\,x\right)^2+4\,d\,a^2\,b^5+12\,d\,a\,b^6\,\mathrm{tanh}\left(c+d\,x\right)^4+8\,d\,a\,b^6\,\mathrm{tanh}\left(c+d\,x\right)^2+4\,d\,b^7\,\mathrm{tanh}\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] (a^4 + a^3*b*(2*tanh(c + d*x)^2 + 4) - a*b^3*(tanh(c + d*x)^2*atan((a*tanh(
c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c +
 d*x)^2))*8i - 4*tanh(c + d*x)^2) + a^2*b^2*(6*tanh(c + d*x)^2 - atan((a*ta
nh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(
c + d*x)^2))*4i + 3) - b^4*tanh(c + d*x)^4*atan((a*tanh(c + d*x)^2*1i + b*t
anh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*4i)/(4*a^
2*b^5*d + 12*a^3*b^4*d + 12*a^4*b^3*d + 4*a^5*b^2*d + 4*b^7*d*tanh(c + d*x)
^4 + 24*a^2*b^5*d*tanh(c + d*x)^2 + 24*a^3*b^4*d*tanh(c + d*x)^2 + 8*a^4*b^
3*d*tanh(c + d*x)^2 + 12*a^2*b^5*d*tanh(c + d*x)^4 + 4*a^3*b^4*d*tanh(c + d
*x)^4 + 8*a*b^6*d*tanh(c + d*x)^2 + 12*a*b^6*d*tanh(c + d*x)^4)
```

3.192 $\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=137

$$\frac{x}{(a+b)^3}-\frac{\left(a^2+6ab-3b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{3/2}(a+b)^3d}+\frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}-\frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

```
[Out] x/(a+b)^3-1/8*(a^2+6*a*b-3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)
/(a+b)^3/d/a^(1/2)+1/4*a*tanh(d*x+c)/b/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/8*(a
+5*b)*tanh(d*x+c)/b/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 481, 541, 536, 212, 211}

$$-\frac{\left(a^2+6ab-3b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{3/2}d(a+b)^3}-\frac{(a+5b)\tanh(c+dx)}{8bd(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{a\tanh(c+dx)}{4bd(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{x}{(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]
])/(8*Sqrt[a]*b^(3/2)*(a + b)^3*d) + (a*Tanh[c + d*x])/(4*b*(a + b)*d*(a +
b*Tanh[c + d*x]^2)^2) - ((a + 5*b)*Tanh[c + d*x])/(8*b*(a + b)^2*d*(a + b*T
anh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+(-a-4b)x^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+d\right.}{4b(a+b)d}$$

$$= \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Su}}{}$$

$$= \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{Su}}{}$$

$$= \frac{x}{(a+b)^3} - \frac{(a^2+6ab-3b^2)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{3/2}(a+b)^3d} + \frac{a\tanh(c+d}{4b(a+b)d\left(a+b\tanh\right.}$$

Mathematica [A]
time = 0.80, size = 135, normalized size = 0.99

$$\frac{8(c+dx) - \frac{(a^2+6ab-3b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} + \frac{4a(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{(a-5b)(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a
]])/(Sqrt[a]*b^(3/2)) + (4*a*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Co
sh[2*(c + d*x)])^2 + ((a - 5*b)*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a +
 b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

Maple [A]
time = 0.90, size = 148, normalized size = 1.08

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\frac{1}{8}a^2+\frac{3}{4}ab+\frac{5}{8}b^2\right)\left(\tanh^3(dx+c)\right)-\frac{a\left(a^2-2ab-3b^2\right)\tanh(dx+c)}{8b}}{\left(a+b\left(\tanh^2(dx+c)\right)\right)^2}+\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{(a+b)^3}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-}{d}$

default	$\frac{-\frac{\frac{\left(\frac{1}{8}a^2+\frac{3}{4}ab+\frac{5}{8}b^2\right)\left(\tanh^3(dx+c)\right)-\frac{a\left(a^2-2ab-3b^2\right)\tanh(dx+c)}{8b}}{\left(a+b\left(\tanh^2(dx+c)\right)\right)^2}+\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{(a+b)^3}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3} \ldots}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{a^3e^{6dx+6c}-9a^2b\,e^{6dx+6c}-5a\,b^2e^{6dx+6c}+5b^3e^{6dx+6c}+3a^3e^{4dx+4c}-17a^2b\,e^{4dx+4c}+13a\,b^2e^{4dx} \ldots}{4b\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c} \ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^3*(((1/8*a^2+3/4*a*b+5/8*b^2)*tanh(d*x+c)^3-1/8*a*(a^2-2*a*b-
3*b^2)/b*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(a^2+6*a*b-3*b^2)/b/(a*b)^(
1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^3*ln(1+tanh(d*x+c))-1/2/(
a+b)^3*ln(tanh(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2432 vs. $2(123) = 246$.
time = 1.18, size = 2432, normalized size = 17.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/128*(a^4 + 24*a^3*b - 54*a^2*b^2 - 16*a*b^3 - 3*b^4)*arctan(1/2*((a + b)
*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*
b^4)*sqrt(a*b)*d) + 1/128*(a^4 + 24*a^3*b - 54*a^2*b^2 - 16*a*b^3 - 3*b^4)*
arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5*b + 3*a^4*b^
2 + 3*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 1/64*(a^5 - 33*a^4*b - 54*a^3*b^2 -
 2*a^2*b^3 + 21*a*b^4 + 3*b^5 + (a^5 - 71*a^4*b + 98*a^3*b^2 + 154*a^2*b^3
- 19*a*b^4 - 3*b^5)*e^(6*d*x + 6*c) + (3*a^5 - 171*a^4*b + 310*a^3*b^2 - 25
4*a^2*b^3 + 39*a*b^4 + 9*b^5)*e^(4*d*x + 4*c) + (3*a^5 - 133*a^4*b + 86*a^3
*b^2 + 190*a^2*b^3 - 41*a*b^4 - 9*b^5)*e^(2*d*x + 2*c))/((a^7*b + 5*a^6*b^2
 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6 + (a^7*b + 5*a^6*b^2 + 10*
a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^7*b + 3*
a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c) + 2*
(3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*e^(4*
d*x + 4*c) + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2
*b^6)*e^(2*d*x + 2*c))*d) + 1/64*(a^5 - 33*a^4*b - 54*a^3*b^2 - 2*a^2*b^3 +
 21*a*b^4 + 3*b^5 + (3*a^5 - 133*a^4*b + 86*a^3*b^2 + 190*a^2*b^3 - 41*a*b^
4 - 9*b^5)*e^(-2*d*x - 2*c) + (3*a^5 - 171*a^4*b + 310*a^3*b^2 - 254*a^2*b^
3 + 39*a*b^4 + 9*b^5)*e^(-4*d*x - 4*c) + (a^5 - 71*a^4*b + 98*a^3*b^2 + 154
*a^2*b^3 - 19*a*b^4 - 3*b^5)*e^(-6*d*x - 6*c))/((a^7*b + 5*a^6*b^2 + 10*a^5
```

```
*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6 + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3
- 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^(-2*d*x - 2*c) + 2*(3*a^7*b + 7*a^6*b^
2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^
7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^(-6*d*x -
6*c) + (a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*
e^(-8*d*x - 8*c))*d) - 1/16*(a^4 - 4*a^3*b - 14*a^2*b^2 - 12*a*b^3 - 3*b^4
+ (a^4 - 26*a^3*b - 20*a^2*b^2 + 10*a*b^3 + 3*b^4)*e^(6*d*x + 6*c) + (3*a^4
 - 52*a^3*b + 6*a^2*b^2 - 12*a*b^3 - 9*b^4)*e^(4*d*x + 4*c) + (3*a^4 - 30*a
^3*b - 28*a^2*b^2 + 14*a*b^3 + 9*b^4)*e^(2*d*x + 2*c))/((a^6*b + 4*a^5*b^2
+ 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*
b^4 + a^2*b^5)*e^(8*d*x + 8*c) + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5
)*e^(6*d*x + 6*c) + 2*(3*a^6*b + 4*a^5*b^2 + 2*a^4*b^3 + 4*a^3*b^4 + 3*a^2*
b^5)*e^(4*d*x + 4*c) + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5)*e^(2*d*x
 + 2*c))*d) + 1/16*(a^4 - 4*a^3*b - 14*a^2*b^2 - 12*a*b^3 - 3*b^4 + (3*a^4
- 30*a^3*b - 28*a^2*b^2 + 14*a*b^3 + 9*b^4)*e^(-2*d*x - 2*c) + (3*a^4 - 52*
a^3*b + 6*a^2*b^2 - 12*a*b^3 - 9*b^4)*e^(-4*d*x - 4*c) + (a^4 - 26*a^3*b -
20*a^2*b^2 + 10*a*b^3 + 3*b^4)*e^(-6*d*x - 6*c))/((a^6*b + 4*a^5*b^2 + 6*a^
4*b^3 + 4*a^3*b^4 + a^2*b^5 + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5)*e
^(-2*d*x - 2*c) + 2*(3*a^6*b + 4*a^5*b^2 + 2*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^
5)*e^(-4*d*x - 4*c) + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5)*e^(-6*d*x
 - 6*c) + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*e^(-8*d*x -
 8*c))*d) + 3/32*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3 + (3*a^3 + 13*a^2*b + a*b
^2 - 9*b^3)*e^(-2*d*x - 2*c) + (3*a^3 + 7*a^2*b - 3*a*b^2 + 9*b^3)*e^(-4*d*
x - 4*c) + (a^3 - a^2*b - 5*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4 + 4*(a^5*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-2
*d*x - 2*c) + 2*(3*a^5*b + a^4*b^2 + a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c)
+ 4*(a^5*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 3*a^4
*b^2 + 3*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) + 1/4*log((a + b)*e^(4*d*x
 + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*d) - 1/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a +
 b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/32*(a + 3*b)*arctan(1/2*((a + b
)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b*d) + 1/32*(a + 3*b)*
arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b*d
) + 3/64*(a - 3*b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/(sqrt(a*b)*a^2*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3727 vs. $2(123) = 246$.
time = 0.47, size = 7757, normalized size = 56.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

[Out] [1/16*(16*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^8 + 128*(a^3*b^2
+ 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^3*b^2 + 2*a^
2*b^3 + a*b^4)*d*x*sinh(d*x + c)^8 - 4*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a
b^4 - 16(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^6 - 4*(a^4*b - 9*a^3*b^2 - 5
*a^2*b^3 + 5*a*b^4 - 112*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^2
- 16*(a^3*b^2 - a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^3*b^2 + 2*a^2*b^3 +
a*b^4)*d*x*cosh(d*x + c)^3 - 3*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 -
16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*a^4*b + 12*a^3
*b^2 + 36*a^2*b^3 + 20*a*b^4 - 4*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*
b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^
3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^4 - 3*a^4*b + 17*a^3*b^2 - 13*
a^2*b^3 + 15*a*b^4 + 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x - 15*(a^4*b -
9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 16*(56*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^
5 - 5*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*
cosh(d*x + c)^3 - (3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*
b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^3*b^
2 + 2*a^2*b^3 + a*b^4)*d*x - 4*(3*a^4*b - 11*a^3*b^2 + a^2*b^3 + 15*a*b^4 -
16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^3*b^2 + 2*a^2*b^3 +
a*b^4)*d*x*cosh(d*x + c)^6 - 3*a^4*b + 11*a^3*b^2 - a^2*b^3 - 15*a*b^4 - 15
*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(
d*x + c)^4 + 16*(a^3*b^2 - a*b^4)*d*x - 6*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^
3 - 15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + ((a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^8 + 8*(
a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 +
8*a^3*b + 10*a^2*b^2 - 3*b^4)*sinh(d*x + c)^8 + 4*(a^4 + 6*a^3*b - 4*a^2*b^
2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b
^3 + 3*b^4 + 7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^2)*sinh(d
x + c)^6 + 8(7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^3 + 3*(
a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^4 + 2
(35(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^4 + 3*a^4 + 16*a^3*
b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4 + 30*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3
+ 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*a^3*b + 10*a^2*b^2 - 3
b^4 + 8(7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^5 + 10*(a^4
+ 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (3*a^4 + 16*a^3*
b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4
+ 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 + 8*a^
3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^6 + 15*(a^4 + 6*a^3*b - 4*a^2*b^2 -
6*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3
b^4 + 3(3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^7
+ 3*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + (3*a^4
+ 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^3 + (a^4 + 6*a^3
*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*

```
log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*c
osh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh
(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 +
(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(
a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(
-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*
x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*co
sh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(16*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*
d*x*cosh(d*x + c)^7 - 3*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*
b^2 - a*b^4)*d*x)*cosh(d*x + c)^5 - 2*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 -
15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^3 - (3*a^
4*b - 11*a^3*b^2 + a^2*b^3 + 15*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x
+ c))*sinh(d*x + c))/((a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^
2*b^6 + a*b^7)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10
*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 +
5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*sinh(d*x + c)^8
+ 4*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*cos
h(d*x + c)^6 + 4*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*
b^6 + a*b^7)*d*cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b
^5 - 3*a^2*b^6 - a*b^7)*d)*sinh(d*x + c)^6 + 2*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(123) = 246.
time = 0.58, size = 384, normalized size = 2.80

$$-\frac{\frac{(a^2+6\,ab-3\,b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3b+3\,a^2b^2+3\,ab^3+b^4)\sqrt{ab}}-\frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,(a^3e^{(6\,dx+6\,c)}-9\,a^2be^{(6\,dx+6\,c)}-5\,ab^2e^{(6\,dx+6\,c)}+5\,b^3e^{(6\,dx+6\,c)}+3\,a^3e^{(4\,dx+4\,c)}-17\,a^2be^{(4\,dx+4\,c)}+13\,ab^2e^{(4\,dx+4\,c)}-15\,b^3e^{(4\,dx+4\,c)}+3\,a^3e^{(2\,dx+2\,c)}-11\,a^2be^{(2\,dx+2\,c)}+ab^2e^{(2\,dx+2\,c)}+15\,b^3e^{(2\,dx+2\,c)}+a^3-3\,a^2b-9\,ab^2-5\,b^3)}{(a^3b+3\,a^2b^2+3\,ab^3+b^4)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((a^2 + 6*a*b - 3*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*
c) + a - b)/sqrt(a*b))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(a*b)) - 8*
(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a^3*e^(6*d*x + 6*c) - 9*a^2*
```

```
b*e^(6*d*x + 6*c) - 5*a*b^2*e^(6*d*x + 6*c) + 5*b^3*e^(6*d*x + 6*c) + 3*a^3
*e^(4*d*x + 4*c) - 17*a^2*b*e^(4*d*x + 4*c) + 13*a*b^2*e^(4*d*x + 4*c) - 15
*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) - 11*a^2*b*e^(2*d*x + 2*c) + a
*b^2*e^(2*d*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*b - 9*a*b^2 - 5
*b^3)/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x
+ 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [B]
time = 1.86, size = 2574, normalized size = 18.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - ((tanh
(c + d*x)^3*(a + 5*b))/(8*(2*a*b + a^2 + b^2)) - (a*tanh(c + d*x)*(a - 3*b)
)/(8*b*(2*a*b + a^2 + b^2)))/(a^2*d + b^2*d*tanh(c + d*x)^4 + 2*a*b*d*tanh(
c + d*x)^2) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^3) - (atan(((((tanh(c + d
*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(b^5*d^2 + 4*a*b
^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)) + (((96*b^9*d^2 + 544*
a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d^2 + 96*a^5*
b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b^6*d^3 + a^6
*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*b^2*d^3))
 - (tanh(c + d*x)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*(256*b^10*d^2 + 1280
*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6*d^2 - 2304*
a^5*b^5*d^2 - 1280*a^6*b^4*d^2 - 256*a^7*b^3*d^2))/(512*(3*a^2*b^5*d + 3*a^
3*b^4*d + a^4*b^3*d + a*b^6*d)*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b
^3*d^2 + 4*a^3*b^2*d^2)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2))/(16*(3*a^2*
b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))*(-a*b^3)^(1/2)*(6*a*b + a^2 -
3*b^2)*1i)/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)) + (((tanh
(c + d*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(b^5*d^2 +
 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)) - (((96*b^9*d^2
+ 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d^2 + 9
6*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b^6*d^3
 + a^6*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*b^2
*d^3)) + (tanh(c + d*x)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*(256*b^10*d^2
+ 1280*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6*d^2 -
 2304*a^5*b^5*d^2 - 1280*a^6*b^4*d^2 - 256*a^7*b^3*d^2))/(512*(3*a^2*b^5*d
+ 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6
*a^2*b^3*d^2 + 4*a^3*b^2*d^2)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2))/(16*(
3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))*(-a*b^3)^(1/2)*(6*a*b +
a^2 - 3*b^2)*1i)/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))/((
27*a*b^2 + 11*a^2*b + a^3 - 15*b^3)/(32*(b^7*d^3 + 6*a*b^6*d^3 + a^6*b*d^3
+ 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*b^2*d^3)) + (((t
```

anh(c + d*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)) + (((96*b^9*d^2 + 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d^2 + 96*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b^6*d^3 + a^6*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*b^2*d^3)) - (tanh(c + d*x)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*(256*b^10*d^2 + 1280*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6*d^2 - 2304*a^5*b^5*d^2 - 1280*a^6*b^4*d^2 - 256*a^7*b^3*d^2))/(512*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2))/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2))/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)) - (((tanh(c + d*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)) - (((96*b^9*d^2 + 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d^2 + 96*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b^6*d^3 + a^6*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*b^2*d^3)) + (tanh(c + d*x)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*(256*b^10*d^2 + 1280*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6*d^2 - 2304*a^5*b^5*d^2 - 1280*a^6*b^4*d^2 - 256*a^7*b^3*d^2))/(512*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2))/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2))/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d))))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*1i)/(8*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d))

3.193 $\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^3} \, dx$

Optimal. Leaf size=98

$$\frac{\log(\cosh(c+dx))}{(a+b)^3d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d}+\frac{a}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}-\frac{1}{2(a+b)^2d\left(a+b\tanh^2(\right.}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d+1/4*a/b/(a+b)
/d/(a+b*tanh(d*x+c)^2)^2-1/2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 78}

$$\frac{a}{4bd(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{1}{2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3}+\frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) + a/(4*b*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - 1/(2*(a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{a}{(a+b)(a+bx)^3} + \frac{b}{(a+b)^2(a+bx)^2} + \frac{b}{(a+b)^3(a+bx)}\right)\,dx, x, \text{ta}\right.}{2d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d} + \frac{a}{4b(a+b)d\left(a+b\tanh^2(c+\right.}$$

Mathematica [A]
time = 0.36, size = 80, normalized size = 0.82

$$\frac{4\log(\cosh(c+dx)) + 2\log\left(a+b\tanh^2(c+dx)\right) + \frac{a(a+b)^2}{b\left(a+b\tanh^2(c+dx)\right)^2} - \frac{2(a+b)}{a+b\tanh^2(c+dx)}}{4(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)^2)/(b*(a
+ b*Tanh[c + d*x]^2)^2) - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)^3
*d)
```

Maple [A]
time = 0.76, size = 115, normalized size = 1.17

method	result
derivativedivides	$\frac{-\frac{-\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)-\frac{a\left(a^2+2ab+b^2\right)}{2b\left(a+b\left(\tanh^2(dx+c)\right)\right)^2}-\frac{-a-b}{a+b\left(\tanh^2(dx+c)\right)}}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}}{d}$

default	$-\frac{-\frac{-\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)-\frac{a\left(a^2+2ab+b^2\right)}{2b\left(a+b\left(\tanh^2(dx+c)\right)\right)^2}-\frac{-a-b}{a+b\left(\tanh^2(dx+c)\right)}}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}}{d}$
risch	$-\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2c}{d(a^3+3a^2b+3a\,b^2+b^3)}+\frac{2(a^2\mathrm{e}^{4dx+4c}-b^2\mathrm{e}^{4dx+4c}+2a^2\mathrm{e}^{2dx+2c}-2ab\,\mathrm{e}^{2dx+2c}+2b^2\mathrm{e}^{2dx+2c}}{(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}+2a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+a+b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^3*(-ln(a+b*tanh(d*x+c)^2)-1/2*a*(a^2+2*a*b+b^2)/b/(a+b*tanh
(d*x+c)^2)^2-(-a-b)/(a+b*tanh(d*x+c)^2))-1/2/(a+b)^3*ln(1+tanh(d*x+c))-1/2/
(a+b)^3*ln(tanh(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(92) = 184$.
time = 0.34, size = 384, normalized size = 3.92

$$\frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}+\frac{2\left((a^2-b^2)e^{(-2\,dx-2\,c)}+2\,(a^2-ab+b^2)e^{(-4\,dx-4\,c)}+(a^2-b^2)e^{(-6\,dx-6\,c)}\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5+4\,(a^5+3\,a^4b+2\,a^3b^2-2\,a^2b^3-3\,ab^4-b^5)e^{(-2\,dx-2\,c)}+2\,(3\,a^5+7\,a^4b+6\,a^3b^2+6\,a^2b^3+7\,ab^4+3\,b^5)e^{(-4\,dx-4\,c)}+4\,(a^5+3\,a^4b+2\,a^3b^2-2\,a^2b^3-3\,ab^4-b^5)e^{(-6\,dx-6\,c)}+(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)e^{(-8\,dx-8\,c)})d}+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((a^2 - b^2)*e^(-2*d*x -
2*c) + 2*(a^2 - a*b + b^2)*e^(-4*d*x - 4*c) + (a^2 - b^2)*e^(-6*d*x - 6*c))
/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4
*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7
*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d)
+ 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(92) = 184$.
time = 0.41, size = 2611, normalized size = 26.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8
 + 4*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 + 4*(14*(a^2 + 2*a*b +
```

b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x - a^2 + b^2)*sinh(d*x + c)^6
+ 8*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 - b^2)*d*x - a^
2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x -
2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 15*(2*(a^2 - b^2)*d*x - a^2 + b
^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*(7*(a^2 +
2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(
d*x + c)^3 + ((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x + 4*(2*(a^2 - b^2)*d*x -
a^2 + b^2)*cosh(d*x + c)^2 + 4*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6
+ 15*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^4 + 2*(a^2 - b^2)*d*x +
6*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 -
a^2 + b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x
+ c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*
b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 30*(a^2
- b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2 -
2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x + c
)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 15*(a^2 - b^2)*cosh(d*x +
c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)
^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 3*(a^2 -
b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (a^2 - b^2
)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*si
nh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2)) + 8*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 3*(2*(a^2
- b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^5 + 2*((3*a^2 - 2*a*b + 3*b^2)*d*x -
2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^3 + (2*(a^2 - b^2)*d*x - a^2 + b^2)*c
osh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*
a*b^4 + b^5)*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
+ 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10*a^3
*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 4*(a^5 + 3*a^4*b + 2
*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*
a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^
5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^4 +
8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x +
c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*
x + c))*sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 +
5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2
*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 -
2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^4*b + 10*a^

```
3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5 + 3*a^4*b +
 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (3*a^5 + 7*a^4*
b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(
d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*c
osh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b
^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
b^5)*d)*sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^
4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*d*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 +
3*b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3403 vs. $2(82) = 164$.
time = 113.44, size = 3403, normalized size = 34.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (3*tanh(c + d
*x)**2/(12*b**3*d*tanh(c + d*x)**6 - 36*b**3*d*tanh(c + d*x)**4 + 36*b**3*d
*tanh(c + d*x)**2 - 12*b**3*d) - 1/(12*b**3*d*tanh(c + d*x)**6 - 36*b**3*d*
tanh(c + d*x)**4 + 36*b**3*d*tanh(c + d*x)**2 - 12*b**3*d), Eq(a, -b)), ((x
 - log(tanh(c + d*x) + 1)/d - tanh(c + d*x)**2/(2*d))/a**3, Eq(b, 0)), (x*t
anh(c)**3/(a + b*tanh(c)**2)**3, Eq(d, 0)), (a**3/(4*a**5*b*d + 8*a**4*b**2
*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*
a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x
)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c
 + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) + 4*a
**2*b*d*x/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4
*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b
**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 +
 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2
 + 4*b**6*d*tanh(c + d*x)**4) + 2*a**2*b*log(-sqrt(-a/b) + tanh(c + d*x))/(
4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*
d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*
a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**
4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d
*tanh(c + d*x)**4) + 2*a**2*b*log(sqrt(-a/b) + tanh(c + d*x))/(4*a**5*b*d +
 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d
*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*
```

```
tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b
**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*
x)**4) - 4*a**2*b*log(tanh(c + d*x) + 1)/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c
 + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3
*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24
*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**
4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) + 8*a*b**2*d*x
*tanh(c + d*x)**2/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b*
*2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 1
2*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d
*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c +
 d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) + 4*a*b**2*log(-sqrt(-a/b) + tanh(c +
 d*x))*tanh(c + d*x)**2/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a
**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)*
*2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh
(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*ta
nh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) + 4*a*b**2*log(sqrt(-a/b) + tan
h(c + d*x))*tanh(c + d*x)**2/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 +
 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c +
d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d
*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5
*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) - 8*a*b**2*log(tanh(c + d*
x) + 1)*tanh(c + d*x)**2/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*
a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)
**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tan
h(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*t
anh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) - 2*a*b**2*tanh(c + d*x)**2/(4
*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d
*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a
**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4
*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*
tanh(c + d*x)**4) - a*b**2/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 1
2*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*
x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*t
anh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d
*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) + 4*b**3*d*x*tanh(c + d*x)**
4/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b*
*3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d +
12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*
b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**
6*d*tanh(c + d*x)**4) + 2*b**3*log(-sqrt(-a/b) + tanh(c + d*x))*tanh(c + d*
x)**4/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**
3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*
d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a
**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(92) = 184$.
time = 0.55, size = 245, normalized size = 2.50

$$\frac{\frac{2\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^{2}+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^{2}+4\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)-4\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)-4\,a-4\,b}{\left(a^{2}+2\,ab+b^{2}\right)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right)^{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
+ e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2
*c))^2 + 4*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*b*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) - 4*a - 4*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2*c) + e
^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)^2))/
d
```

Mupad [B]
time = 1.81, size = 397, normalized size = 4.05

$$-\frac{-a^{3}+b^{3}\left(2\tanh(c+d\,x)^{2}+\tanh(c+d\,x)^{4}\,\mathrm{atan}\left(\frac{a\tanh(c+d\,x)^{2}\,1\mathrm{i}+b\tanh(c+d\,x)^{2}\,1\mathrm{i}}{2\,a-a\tanh(c+d\,x)^{2}+b\tanh(c+d\,x)^{2}}\right)\,4\mathrm{i}\right)+a\,b^{2}\left(2\tanh(c+d\,x)^{2}+1+\tanh(c+d\,x)^{2}\,\mathrm{atan}\left(\frac{a\tanh(c+d\,x)^{2}\,1\mathrm{i}+b\tanh(c+d\,x)^{2}\,1\mathrm{i}}{2\,a-a\tanh(c+d\,x)^{2}+b\tanh(c+d\,x)^{2}}\right)\,8\mathrm{i}\right)+a^{2}\,b\,\mathrm{atan}\left(\frac{a\tanh(c+d\,x)^{2}\,1\mathrm{i}+b\tanh(c+d\,x)^{2}\,1\mathrm{i}}{2\,a-a\tanh(c+d\,x)^{2}+b\tanh(c+d\,x)^{2}}\right)\,4\mathrm{i}}{4\,d\,a^{5}\,b+8\,d\,a^{4}\,b^{2}\tanh(c+d\,x)^{2}+12\,d\,a^{4}\,b^{2}+4\,d\,a^{3}\,b^{3}\tanh(c+d\,x)^{4}+24\,d\,a^{3}\,b^{3}\tanh(c+d\,x)^{2}+12\,d\,a^{3}\,b^{3}+12\,d\,a^{2}\,b^{4}\tanh(c+d\,x)^{4}+24\,d\,a^{2}\,b^{4}\tanh(c+d\,x)^{2}+4\,d\,a^{2}\,b^{4}+12\,d\,a\,b^{5}\tanh(c+d\,x)^{4}+8\,d\,a\,b^{5}\tanh(c+d\,x)^{2}+4\,d\,b^{6}\tanh(c+d\,x)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] -(b^3*(tanh(c + d*x)^4*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(
2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*4i + 2*tanh(c + d*x)^2) - a^3
 + a*b^2*(tanh(c + d*x)^2*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i
)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*8i + 2*tanh(c + d*x)^2 + 1
) + a^2*b*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(
c + d*x)^2 + b*tanh(c + d*x)^2))*4i)/(4*a^2*b^4*d + 12*a^3*b^3*d + 12*a^4*b
^2*d + 4*b^6*d*tanh(c + d*x)^4 + 4*a^5*b*d + 24*a^2*b^4*d*tanh(c + d*x)^2 +
 24*a^3*b^3*d*tanh(c + d*x)^2 + 8*a^4*b^2*d*tanh(c + d*x)^2 + 12*a^2*b^4*d*
tanh(c + d*x)^4 + 4*a^3*b^3*d*tanh(c + d*x)^4 + 8*a*b^5*d*tanh(c + d*x)^2 +
 12*a*b^5*d*tanh(c + d*x)^4)
```

3.194 $\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=137

$$\frac{x}{(a+b)^3}-\frac{(3a^2-6ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}\,(a+b)^3d}-\frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}-\frac{(3a-b)\,\mathrm{tanh}}{8a(a+b)^2d\,(a+b\,\mathrm{t}}$$

```
[Out] x/(a+b)^3-1/8*(3*a^2-6*a*b-b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)
/(a+b)^3/d/b^(1/2)-1/4*tanh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/8*(3*a-b
)*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 482, 541, 536, 212, 211}

$$-\frac{(3a^2-6ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}\,d(a+b)^3}-\frac{(3a-b)\tanh(c+dx)}{8ad(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\tanh(c+dx)}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{x}{(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 - ((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]
])/(8*a^(3/2)*Sqrt[b]*(a + b)^3*d) - Tanh[c + d*x]/(4*(a + b)*d*(a + b*Tanh
[c + d*x]^2)^2) - ((3*a - b)*Tanh[c + d*x])/(8*a*(a + b)^2*d*(a + b*Tanh[c
+ d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{1+3x^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+d\right.}{4(a+b)d}$$

$$= -\frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(3a-b)\tanh(c+dx)}{8a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} - \frac{\text{S}}{}$$

$$= -\frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(3a-b)\tanh(c+dx)}{8a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{S}}{}$$

$$= \frac{x}{(a+b)^3} - \frac{(3a^2-6ab-b^2)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}(a+b)^3d} - \frac{\tanh(c+dx}{4(a+b)d\left(a+b\tanh^2\right.}$$

Mathematica [A]
time = 0.83, size = 137, normalized size = 1.00

$$\frac{8(c+dx) + \frac{(-3a^2+6ab+b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} - \frac{4b(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2} - \frac{(5a-b)(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) + ((-3*a^2 + 6*a*b + b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[
a]])/(a^(3/2)*Sqrt[b]) - (4*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*C
osh[2*(c + d*x)])^2 - ((5*a - b)*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a
+ b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

Maple [A]
time = 0.86, size = 152, normalized size = 1.11

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3} - \frac{\frac{\frac{b(3a^2+2ab-b^2)(\tanh^3(dx+c))}{8a} + \left(\frac{5}{8}a^2+\frac{3}{4}ab+\frac{1}{8}b^2\right)\tanh(dx+c)}{\left(a+b\left(\tanh^2(dx+c)\right)\right)^2} + \frac{(3a^2-6ab-b^2)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{(a+b)^3} + \frac{1}{}}{d}$

default	$\dfrac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\frac{\frac{b\left(3a^2+2ab-b^2\right)\left(\tanh^3(dx+c)\right)}{8a}+\left(\frac{5}{8}a^2+\frac{3}{4}ab+\frac{1}{8}b^2\right)\tanh(dx+c)}{\left(a+b\left(\tanh^2(dx+c)\right)\right)^2}+\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{(a+b)^3}-}{d}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}+\frac{5a^3e^{6dx+6c}-5a^2b\,e^{6dx+6c}-9a\,b^2e^{6dx+6c}+b^3e^{6dx+6c}+15a^3e^{4dx+4c}-13a^2b\,e^{4dx+4c}+17a\,b^2e^{4d}}{4a\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/(a+b)^3*((1/8*b*(3*a^2+2*a*b-b^2)/a*t
anh(d*x+c)^3+(5/8*a^2+3/4*a*b+1/8*b^2)*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1
/8*(3*a^2-6*a*b-b^2)/a/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(
a+b)^3*ln(1+tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(123) = 246$.
time $= 0.82$, size $= 1472$, normalized size $= 10.74$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(3*a^3 - 21*a^2*b - 11*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(2*d*x +
2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d
) + 1/32*(3*a^3 - 21*a^2*b - 11*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*
x - 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*
b)*d) + 1/16*(5*a^4 - 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + (5*a^4 - 46*a^3*b - 4
0*a^2*b^2 + 14*a*b^3 + 3*b^4)*e^(6*d*x + 6*c) + (15*a^4 - 104*a^3*b + 58*a^
2*b^2 - 24*a*b^3 - 9*b^4)*e^(4*d*x + 4*c) + (15*a^4 - 58*a^3*b - 56*a^2*b^2
 + 26*a*b^3 + 9*b^4)*e^(2*d*x + 2*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4
*b^3 + 5*a^3*b^4 + a^2*b^5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a
^3*b^4 + a^2*b^5)*e^(8*d*x + 8*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^
3 - 3*a^3*b^4 - a^2*b^5)*e^(6*d*x + 6*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 +
 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(4*d*x + 4*c) + 4*(a^7 + 3*a^6*b + 2*
a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(2*d*x + 2*c))*d) - 1/16*(5*a^
4 - 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + (15*a^4 - 58*a^3*b - 56*a^2*b^2 + 26*a*
b^3 + 9*b^4)*e^(-2*d*x - 2*c) + (15*a^4 - 104*a^3*b + 58*a^2*b^2 - 24*a*b^3
 - 9*b^4)*e^(-4*d*x - 4*c) + (5*a^4 - 46*a^3*b - 40*a^2*b^2 + 14*a*b^3 + 3*
b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^
4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^
5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^
```

```
4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3
 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*
a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) - 1/8*(5*a^3 + 13*a^2*b
 + 11*a*b^2 + 3*b^3 + (15*a^3 + 13*a^2*b - 11*a*b^2 - 9*b^3)*e^(-2*d*x - 2*
c) + (15*a^3 - a^2*b + 9*a*b^2 + 9*b^3)*e^(-4*d*x - 4*c) + (5*a^3 - a^2*b -
 9*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3
 + a^2*b^4 + 4*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*e^(-2*d*x - 2*c) + 2*(
3*a^6 + 4*a^5*b + 2*a^4*b^2 + 4*a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(
a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*e^(-6*d*x - 6*c) + (a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) + 1/4*log((a + b)*e^(4*
d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d) - 1/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) +
a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 3/16*arctan(1/2*((a + b)*e^(-2
*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3744 vs. $2(123) = 246$.
time = 0.49, size = 7791, normalized size = 56.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 128*(a^4*b +
2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4*b + 2*a^3*
b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 + 4*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a
*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^6 + 4*(5*a^4*b - 5*a^3*b^2 -
 9*a^2*b^3 + a*b^4 + 112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2
+ 16*(a^4*b - a^2*b^3)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4*b + 2*a^3*b^2 + a
^2*b^3)*d*x*cosh(d*x + c)^3 + 3*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 +
16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 20*a^4*b + 36*a^
3*b^2 + 12*a^2*b^3 - 4*a*b^4 + 4*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*
b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^
4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 + 15*a^4*b - 13*a^3*b^2 + 17
*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x + 15*(5*a^4*b
 - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 16*(56*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^
5 + 5*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*
cosh(d*x + c)^3 + (15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*
b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^4*b
+ 2*a^3*b^2 + a^2*b^3)*d*x + 4*(15*a^4*b + a^3*b^2 - 11*a^2*b^3 + 3*a*b^4 +
 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^4*b + 2*a^3*b^2 + a^
2*b^3)*d*x*cosh(d*x + c)^6 + 15*a^4*b + a^3*b^2 - 11*a^2*b^3 + 3*a*b^4 + 15
*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(
```

d*x + c)^4 + 16*(a^4*b - a^2*b^3)*d*x + 6*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b
^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + ((3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^8 + 8*(
3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4
- 10*a^2*b^2 - 8*a*b^3 - b^4)*sinh(d*x + c)^8 + 4*(3*a^4 - 6*a^3*b - 4*a^2*
b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^6 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a
b^3 + b^4 + 7(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d
x + c)^6 + 8(7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(
3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 2
(35(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^4 + 9*a^4 - 24*a^3*
b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + 30*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b
^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 - 10*a^2*b^2 - 8*a*b^3 -
 b^4 + 8*(7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^5 + 10*(3*a^
4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^3 + (9*a^4 - 24*a^3*
b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^
4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^4 - 10
*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^6 + 15*(3*a^4 - 6*a^3*b - 4*a^2*b^2
 + 6*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 +
 b^4 + 3*(9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^7
 + 3*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^5 + (9*a^4
 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (3*a^4 - 6*a
^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*
log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*c
osh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh
(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 +
(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(
a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(
-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*
x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*co
sh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*
d*x*cosh(d*x + c)^7 + 3*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*
b - a^2*b^3)*d*x)*cosh(d*x + c)^5 + 2*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 -
 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^3 + (15*a
^4*b + a^3*b^2 - 11*a^2*b^3 + 3*a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x
+ c))*sinh(d*x + c))/((a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*
b^5 + a^2*b^6)*d*cosh(d*x + c)^8 + 8*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a
^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7*b + 5*
a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*d*sinh(d*x + c)^8
+ 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*d*cos
h(d*x + c)^6 + 4*(7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^
5 + a^2*b^6)*d*cosh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4

```
- 3*a^3*b^5 - a^2*b^6)*d)*sinh(d*x + c)^6 + 2*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(123) = 246.
time = 0.56, size = 388, normalized size = 2.83

$$-\frac{\frac{(3a^2-6ab-b^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab}}-\frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3}-\frac{2\left(5a^3e^{(6dx+6c)}-5a^2be^{(6dx+6c)}-9ab^2e^{(6dx+6c)}+b^3e^{(6dx+6c)}+15a^3e^{(4dx+4c)}-13a^2be^{(4dx+4c)}+17ab^2e^{(4dx+4c)}-3b^3e^{(4dx+4c)}+15a^3e^{(2dx+2c)}+a^2be^{(2dx+2c)}-11ab^2e^{(2dx+2c)}+3b^3e^{(2dx+2c)}+5a^3+9a^2b+3ab^2-b^3\right)}{(a^4+3a^3b+3a^2b^2+ab^3)\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((3*a^2 - 6*a*b - b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*
c) + a - b)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) - 8*
(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(5*a^3*e^(6*d*x + 6*c) - 5*a^
2*b*e^(6*d*x + 6*c) - 9*a*b^2*e^(6*d*x + 6*c) + b^3*e^(6*d*x + 6*c) + 15*a^
3*e^(4*d*x + 4*c) - 13*a^2*b*e^(4*d*x + 4*c) + 17*a*b^2*e^(4*d*x + 4*c) - 3
*b^3*e^(4*d*x + 4*c) + 15*a^3*e^(2*d*x + 2*c) + a^2*b*e^(2*d*x + 2*c) - 11*
a*b^2*e^(2*d*x + 2*c) + 3*b^3*e^(2*d*x + 2*c) + 5*a^3 + 9*a^2*b + 3*a*b^2 -
 b^3)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x
+ 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [B]
time = 3.47, size = 255, normalized size = 1.86

$$\frac{\frac{a^2x}{(a+b)(a^2+2ab+b^2)}-\frac{\tanh(c+dx)(5a+b)}{8d(a^2+2ab+b^2)}+\frac{b^2x\tanh(c+dx)^4}{a^3+3a^2b+3ab^2+b^3}+\frac{2abx\tanh(c+dx)^2}{a^3+3a^2b+3ab^2+b^3}-\frac{\tanh(c+dx)^3(3ab-b^2)}{8ad(a^2+2ab+b^2)}}{a^2+2ab\tanh(c+dx)^2+b^2\tanh(c+dx)^4}+\frac{\mathrm{atan}\left(\frac{b\tanh(c+dx)}{\sqrt{ab}}\right)(-3a^2+6ab+b^2)}{\sqrt{ab}\,(8a^4d+ab(24da^2+24dab+8db^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] ((a^2*x)/((a + b)*(2*a*b + a^2 + b^2)) - (tanh(c + d*x)*(5*a + b))/(8*d*(2*
a*b + a^2 + b^2)) + (b^2*x*tanh(c + d*x)^4)/(3*a*b^2 + 3*a^2*b + a^3 + b^3)
 + (2*a*b*x*tanh(c + d*x)^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*
x)^3*(3*a*b - b^2))/(8*a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*x)^4
 + 2*a*b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(6*a*b - 3
*a^2 + b^2))/((a*b)^(1/2)*(8*a^4*d + a*b*(24*a^2*d + 8*b^2*d + 24*a*b*d)))
```

3.195 $\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^3} \, dx$

Optimal. Leaf size=94

$$\frac{\log(\cosh(c+dx))}{(a+b)^3d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d}-\frac{1}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}-\frac{1}{2(a+b)^2d\left(a+b\tanh^2(c\right.}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/4/(a+b)/d/(
a+b*tanh(d*x+c)^2)^2-1/2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 455, 46}

$$-\frac{1}{2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{1}{4d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2d(a+b)^3}+\frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*
d) - 1/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - 1/(2*(a + b)^2*d*(a + b*Ta
nh[c + d*x]^2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)}+\frac{b}{(a+b)(a+bx)^3}+\frac{b}{(a+b)^2(a+bx)^2}+\frac{b}{(a+b)^3(a+bx)}\right)\,dx, x, \text{ta}\right.}{2d}$$
$$= \frac{\log(\cosh(c+dx))}{(a+b)^3d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d}-\frac{1}{4(a+b)d\left(a+b\tanh^2(c+\right.}$$

Mathematica [A]
time = 0.43, size = 77, normalized size = 0.82

$$\frac{4\log(\cosh(c+dx))+2\log\left(a+b\tanh^2(c+dx)\right)-\frac{(a+b)^2}{\left(a+b\tanh^2(c+dx)\right)^2}-\frac{2(a+b)}{a+b\tanh^2(c+dx)}}{4(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] - (a + b)^2/(a + b*Tan
h[c + d*x]^2)^2 - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)^3*d)
```

Maple [A]
time = 0.96, size = 116, normalized size = 1.23

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b\left(-\frac{a+b}{b\left(a+b\left(\tanh^2(dx+c)\right)\right)}+\frac{\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{b}-\frac{a^2+2ab+b^2}{2b\left(a+b\left(\tanh^2(dx+c)\right)\right)^2}\right)}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b\left(-\frac{a+b}{b\left(a+b\left(\tanh^2(dx+c)\right)\right)}+\frac{\ln\left(a+b\left(\tanh^2(dx+c)\right)\right)}{b}-\frac{a^2+2ab+b^2}{2b\left(a+b\left(\tanh^2(dx+c)\right)\right)^2}\right)}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}}{d}$

risch	$-\frac{x}{a^{3}+3a^{2}b+3a\,b^{2}+b^{3}}-\frac{2c}{d(a^{3}+3a^{2}b+3a\,b^{2}+b^{3})}-\frac{4\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-b\,e^{2dx+2c}+a+b\right)b\,e^{2dx+2c}}{\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)^{2}d(a+b)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)+1/2*b/(a+b)^3*(-(a+b)/b/(a+b*tanh(d*x+c
)^2)+1/b*ln(a+b*tanh(d*x+c)^2)-1/2*(a^2+2*a*b+b^2)/b/(a+b*tanh(d*x+c)^2)^2)
-1/2/(a+b)^3*ln(1+tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(88) = 176$.
time = 0.33, size = 378, normalized size = 4.02

$$\frac{dx+c}{(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})d}-\frac{4\left((ab+b^{2})e^{(-2\,dx-2\,c)}+(2\,ab-b^{2})e^{(-4\,dx-4\,c)}+(ab+b^{2})e^{(-6\,dx-6\,c)}\right)}{(a^{5}+5\,a^{4}b+10\,a^{3}b^{2}+10\,a^{2}b^{3}+5\,ab^{4}+b^{5}+4\,(a^{5}+3\,a^{4}b+2\,a^{3}b^{2}-2\,a^{2}b^{3}-3\,ab^{4}-b^{5})e^{(-2\,dx-2\,c)}+2\,(3\,a^{5}+7\,a^{4}b+6\,a^{3}b^{2}+6\,a^{2}b^{3}+7\,ab^{4}+3\,b^{5})e^{(-4\,dx-4\,c)}+4\,(a^{5}+3\,a^{4}b+2\,a^{3}b^{2}-2\,a^{2}b^{3}-3\,ab^{4}-b^{5})e^{(-6\,dx-6\,c)}+(a^{5}+5\,a^{4}b+10\,a^{3}b^{2}+10\,a^{2}b^{3}+5\,ab^{4}+b^{5})e^{(-8\,dx-8\,c)})d}+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 4*((a*b + b^2)*e^(-2*d*x -
2*c) + (2*a*b - b^2)*e^(-4*d*x - 4*c) + (a*b + b^2)*e^(-6*d*x - 6*c))/((a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4*b + 2
*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7*a^4*b
 + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5 + 3*a
^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d) + 1/2*
log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2554 vs. $2(88) = 176$.
time = 0.41, size = 2554, normalized size = 27.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8
 + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^
2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x + a*b + b^2)*sinh(d*x + c)^6 + 16*
(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x + a*b + b^2
)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b -
```

```
2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + (
3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)
^2 + 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^5 + 10*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^3 + ((3*a^2 - 2*a
*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 +
2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^2 + 8*(7*(
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x + a*b + b^2)*c
osh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b -
 2*b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*s
inh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*
cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 +
15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*co
sh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*co
sh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 16*((a^2 + 2*a*b + b^2)*d*x*co
sh(d*x + c)^7 + 3*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^5 + ((3*a^2 -
 2*a*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c)^3 + ((a^2 - b^2)*d*x + a
*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a
^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 4*(a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 4*(7*(
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2
+ (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^
6 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*
x + c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2
 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*
b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (3*a
^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b
```

```
^5)*d*cosh(d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*
a*b^4 + 3*b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
 3*a*b^4 - b^5)*d)*sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*d*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 +
 7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3412 vs. $2(80) = 160$.
time = 113.17, size = 3412, normalized size = 36.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**5, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tan
h(c + d*x) + 1)/d)/a**3, Eq(b, 0)), (1/(6*b**3*d*tanh(c + d*x)**6 - 18*b**3
*d*tanh(c + d*x)**4 + 18*b**3*d*tanh(c + d*x)**2 - 6*b**3*d), Eq(a, -b)), (
x*tanh(c)/(a + b*tanh(c)**2)**3, Eq(d, 0)), (4*a**2*d*x/(4*a**5*d + 8*a**4*
b*d*tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a*
*3*b**2*d*tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)*
*4 + 24*a**2*b**3*d*tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c +
 d*x)**4 + 8*a*b**4*d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4) + 2*a**
2*log(-sqrt(-a/b) + tanh(c + d*x))/(4*a**5*d + 8*a**4*b*d*tanh(c + d*x)**2
+ 12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**3*b**2*d*tanh(c + d*
x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)**4 + 24*a**2*b**3*d*t
anh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c + d*x)**4 + 8*a*b**4*d
*tanh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4) + 2*a**2*log(sqrt(-a/b) + ta
nh(c + d*x))/(4*a**5*d + 8*a**4*b*d*tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3
*b**2*d*tanh(c + d*x)**4 + 24*a**3*b**2*d*tanh(c + d*x)**2 + 12*a**3*b**2*d
 + 12*a**2*b**3*d*tanh(c + d*x)**4 + 24*a**2*b**3*d*tanh(c + d*x)**2 + 4*a*
*2*b**3*d + 12*a*b**4*d*tanh(c + d*x)**4 + 8*a*b**4*d*tanh(c + d*x)**2 + 4*
b**5*d*tanh(c + d*x)**4) - 4*a**2*log(tanh(c + d*x) + 1)/(4*a**5*d + 8*a**4
*b*d*tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a
**3*b**2*d*tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)
**4 + 24*a**2*b**3*d*tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c
+ d*x)**4 + 8*a*b**4*d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4) - 3*a*
*2/(4*a**5*d + 8*a**4*b*d*tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*ta
nh(c + d*x)**4 + 24*a**3*b**2*d*tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2
*b**3*d*tanh(c + d*x)**4 + 24*a**2*b**3*d*tanh(c + d*x)**2 + 4*a**2*b**3*d
+ 12*a*b**4*d*tanh(c + d*x)**4 + 8*a*b**4*d*tanh(c + d*x)**2 + 4*b**5*d*tan
```

```
h(c + d*x)**4) + 8*a*b*d*x*tanh(c + d*x)**2/(4*a**5*d + 8*a**4*b*d*tanh(c +
 d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**3*b**2*d*ta
nh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)**4 + 24*a**2
*b**3*d*tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c + d*x)**4 + 8
*a*b**4*d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4) + 4*a*b*log(-sqrt(-
a/b) + tanh(c + d*x))*tanh(c + d*x)**2/(4*a**5*d + 8*a**4*b*d*tanh(c + d*x)
**2 + 12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**3*b**2*d*tanh(c
+ d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)**4 + 24*a**2*b**3
*d*tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c + d*x)**4 + 8*a*b*
*4*d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4) + 4*a*b*log(sqrt(-a/b) +
 tanh(c + d*x))*tanh(c + d*x)**2/(4*a**5*d + 8*a**4*b*d*tanh(c + d*x)**2 +
12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**3*b**2*d*tanh(c + d*x)
**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)**4 + 24*a**2*b**3*d*tan
h(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c + d*x)**4 + 8*a*b**4*d*t
anh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4) - 8*a*b*log(tanh(c + d*x) + 1)
*tanh(c + d*x)**2/(4*a**5*d + 8*a**4*b*d*tanh(c + d*x)**2 + 12*a**4*b*d + 4
*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**3*b**2*d*tanh(c + d*x)**2 + 12*a**3*b
**2*d + 12*a**2*b**3*d*tanh(c + d*x)**4 + 24*a**2*b**3*d*tanh(c + d*x)**2 +
 4*a**2*b**3*d + 12*a*b**4*d*tanh(c + d*x)**4 + 8*a*b**4*d*tanh(c + d*x)**2
 + 4*b**5*d*tanh(c + d*x)**4) - 2*a*b*tanh(c + d*x)**2/(4*a**5*d + 8*a**4*b
*d*tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**
3*b**2*d*tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)**
4 + 24*a**2*b**3*d*tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c +
d*x)**4 + 8*a*b**4*d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4) - 4*a*b/
(4*a**5*d + 8*a**4*b*d*tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*tanh(
c + d*x)**4 + 24*a**3*b**2*d*tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b*
*3*d*tanh(c + d*x)**4 + 24*a**2*b**3*d*tanh(c + d*x)**2 + 4*a**2*b**3*d + 1
2*a*b**4*d*tanh(c + d*x)**4 + 8*a*b**4*d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c
 + d*x)**4) + 4*b**2*d*x*tanh(c + d*x)**4/(4*a**5*d + 8*a**4*b*d*tanh(c + d
*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**3*b**2*d*tanh
(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)**4 + 24*a**2*b
**3*d*tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c + d*x)**4 + 8*a
*b**4*d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4) + 2*b**2*log(-sqrt(-a
/b) + tanh(c + d*x))*tanh(c + d*x)**4/(4*a**5*d + 8*a**4*b*d*tanh(c + d*x)*
*2 + 12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**3*b**2*d*tanh(c +
 d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)**4 + 24*a**2*b**3*
d*tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c + d*x)**4 + 8*a*b**
4*d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4) + 2*b**2*log(sqrt(-a/b) +
 tanh(c + d*x))*tanh(c + d*x)**4/(4*a**5*d + 8*a**4*b*d*tanh(c + d*x)**2 +
12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**3*b**2*d*tanh(c + d*x)
**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + ...
```

<u>**Giac**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(88) = 176.

time = 0.53, size = 245, normalized size = 2.61

$$\frac{\frac{2\,\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+12\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+4\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+12\,a-4\,b}{(a^2+2\,ab+b^2)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
+ e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2
*c))^2 + 12*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*b*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) + 12*a - 4*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)^2)
)/d
```

Mupad [B]
time = 2.25, size = 235, normalized size = 2.50

$$\frac{\ln\left(b\tanh(c+d\,x)^2+a\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3}-\frac{\ln\left(1-\tanh(c+d\,x)\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3}-\frac{\ln\left(\tanh(c+d\,x)+1\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3}+\frac{\frac{\tanh(c+d\,x)^4\,\left(\frac{b^3}{4}+\frac{3\,a\,b^2}{4}\right)}{a^2\,d\,(a^2+2\,a\,b+b^2)}+\frac{\tanh(c+d\,x)^2\,\left(\frac{b^2}{2}+a\,b\right)}{a\,d\,(a^2+2\,a\,b+b^2)}}{a^2+2\,a\,b\tanh\left(c+d\,x\right)^2+b^2\tanh\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] log(a + b*tanh(c + d*x)^2)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - lo
g(1 - tanh(c + d*x))/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - log(tanh
(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh(c + d*x
)^4*((3*a*b^2)/4 + b^3/4))/(a^2*d*(2*a*b + a^2 + b^2)) + (tanh(c + d*x)^2*(
a*b + b^2/2))/(a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*x)^4 + 2*a*b
*tanh(c + d*x)^2)
```

3.196 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{x}{(a+b)^3}+\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^3d}+\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}+\frac{b(7a-}{8a^2(a+b)^2}$$

```
[Out] x/(a+b)^3+1/8*(15*a^2+10*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(
1/2)/a^(5/2)/(a+b)^3/d+1/4*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/
8*b*(7*a+3*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3742, 425, 541, 536, 212, 211}

$$\frac{b(7a+3b)\tanh(c+dx)}{8a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^3}+\frac{b\tanh(c+dx)}{4ad(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{x}{(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-3),x]
```

```
[Out] x/(a + b)^3 + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^3*d) + (b*Tanh[c + d*x])/(4*a*(a + b)*d*(
a + b*Tanh[c + d*x]^2)^2) + (b*(7*a + 3*b)*Tanh[c + d*x])/(8*a^2*(a + b)^2*
d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{b-4(a+b)+3bx^2}{(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+d\right.}{4a(a+b)d}$$

$$= \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+3b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{S}}{}$$

$$= \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+3b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{S}}{}$$

$$= \frac{x}{(a+b)^3} + \frac{\sqrt{b}\,(15a^2+10ab+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^3d} + \frac{b\tan}{4a(a+b)d\,(a}$$

Mathematica [A]

time = 0.24, size = 147, normalized size = 1.04

$$\frac{\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} - 4\log(1-\tanh(c+dx)) + 4\log(1+\tanh(c+dx)) + \frac{2b(a+b)^2\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(a+b)(7a+3b)\tanh(c+dx)}{a^2\left(a+b\tanh^2(c+dx)\right)}}{8(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-3),x]
```

```
[Out] ((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]
)/a^(5/2) - 4*Log[1 - Tanh[c + d*x]] + 4*Log[1 + Tanh[c + d*x]] + (2*b*(a +
 b)^2*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^2) + (b*(a + b)*(7*a + 3*b)
*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)))/(8*(a + b)^3*d)
```

Maple [A]

time = 1.27, size = 156, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3} + \frac{b\left(\frac{\frac{b\left(7a^2+10ab+3b^2\right)\left(\tanh^3(dx+c)\right)}{8a^2} + \frac{\left(9a^2+14ab+5b^2\right)\tanh(dx+c)}{8a}}{\left(a+b\left(\tanh^2(dx+c)\right)\right)^2} + \frac{\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right.}{(a+b)^3}}{d}$

default	$\frac{\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}+\frac{b\left(\frac{\frac{b(7a^2+10ab+3b^2)(\tanh^3(dx+c))}{8a^2}+\frac{(9a^2+14ab+5b^2)\tanh(dx+c)}{8a}}{(a+b(\tanh^2(dx+c)))^2}+\frac{(15a^2+10ab+3b^2)\arctan\left(\frac{b\tanh(dx+}{\sqrt{ab}}\right.}{8a^2\sqrt{ab}}\right.}{(a+b)^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{(9a^3e^{6dx+6c}-a^2be^{6dx+6c}-13ab^2e^{6dx+6c}-3b^3e^{6dx+6c}+27a^3e^{4dx+4c}-9a^2be^{4dx+4c}+21ab^2e^4}{4(ae^{4dx+4c}+be^{4dx+4c}+2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a+b)^3*ln(1+tanh(d*x+c))+1/(a+b)^3*b*((1/8*b*(7*a^2+10*a*b+3*b^2)
/a^2*tanh(d*x+c)^3+1/8*(9*a^2+14*a*b+5*b^2)/a*tanh(d*x+c))/(a+b*tanh(d*x+c)
^2)^2+1/8*(15*a^2+10*a*b+3*b^2)/a^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^
(1/2)))-1/2/(a+b)^3*ln(tanh(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(128) = 256$.
time = 0.60, size = 507, normalized size = 3.57

$$-\frac{(15a^2b+10ab^2+3b^3)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^5+3a^4b+3a^3b^2+a^2b^3)\sqrt{ab}d}+\frac{9a^3b+21a^2b^2+15ab^3+3b^4+(27a^3b+13a^2b^2-23ab^3-9b^4)e^{(-2dx-2c)}+3(9a^3b-3a^2b^2+7ab^3+3b^4)e^{(-4dx-4c)}+(9a^3b-a^2b^2-13ab^3-3b^4)e^{(-6dx-6c)}}{4(a^7+5a^6b+10a^5b^2+10a^4b^3+5a^3b^4+a^2b^5+4(a^7+3a^6b+2a^5b^2-2a^4b^3-3a^3b^4-a^2b^5)e^{(-2dx-2c)}+2(3a^7+7a^6b+6a^5b^2+6a^4b^3+7a^3b^4+3a^2b^5)e^{(-4dx-4c)}+4(a^7+3a^6b+2a^5b^2-2a^4b^3-3a^3b^4-a^2b^5)e^{(-6dx-6c)}+(a^7+5a^6b+10a^5b^2+10a^4b^3+5a^3b^4+a^2b^5)e^{(-8dx-8c)})d}+\frac{dx+c}{(a^3+3a^2b+3ab^2+b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a
 - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/4*
(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^2 - 23*a*b^
3 - 9*b^4)*e^(-2*d*x - 2*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(
-4*d*x - 4*c) + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x - 6*c))/((
a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*
a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(
3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x
- 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^
(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2
*b^5)*e^(-8*d*x - 8*c))*d) + (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3587 vs. $2(128) = 256$.
time = 0.45, size = 7496, normalized size = 52.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^8 + 128*(a^4 + 2*a^3*
b + a^2*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4 + 2*a^3*b + a^2*b^
2)*d*x*sinh(d*x + c)^8 - 4*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4
- a^2*b^2)*d*x)*cosh(d*x + c)^6 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh
(d*x + c)^2 - 9*a^3*b + a^2*b^2 + 13*a*b^3 + 3*b^4 + 16*(a^4 - a^2*b^2)*d*x
)*sinh(d*x + c)^6 + 8*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^3 -
3*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 4*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a
^4 - 2*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4 + 2*a^3*b + a^
2*b^2)*d*x*cosh(d*x + c)^4 - 27*a^3*b + 9*a^2*b^2 - 21*a*b^3 - 9*b^4 + 8*(3
*a^4 - 2*a^3*b + 3*a^2*b^2)*d*x - 15*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4
- 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 36*a^3*b - 84*
a^2*b^2 - 60*a*b^3 - 12*b^4 + 16*(56*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x
 + c)^5 - 5*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)
*cosh(d*x + c)^3 - (27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*
a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^4 + 2*a^3*b
+ a^2*b^2)*d*x - 4*(27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4 - 16*(a^4 - a^
2*b^2)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x
 + c)^6 - 15*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x
)*cosh(d*x + c)^4 - 27*a^3*b - 13*a^2*b^2 + 23*a*b^3 + 9*b^4 + 16*(a^4 - a^
2*b^2)*d*x - 6*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*
b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 40*a^3*b
+ 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(15*a^4 + 40*a^3*b + 3
8*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 40*
a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(15*a^4 + 10*a^3
*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(15*a^4 + 10*a^3*b
- 12*a^2*b^2 - 10*a*b^3 - 3*b^4 + 7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*
b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 40*a^3*b + 3
8*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(15*a^4 + 10*a^3*b - 12*a
^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 34*
a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 40*a^3*b + 38
*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 45*a^4 + 34*a^2*b^2 + 24*a*b
^3 + 9*b^4 + 30*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b
^4 + 8*(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)
^5 + 10*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^3
 + (45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4
*(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 1
5*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 15*
a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4 + 3*(45*a^4 + 34*a^2*b^2 + 2
4*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^4 + 40*a^3*b +
 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(15*a^4 + 10*a^3*b - 12
```

```
*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (45*a^4 + 34*a^2*b^2 + 24*a*
b^3 + 9*b^4)*cosh(d*x + c)^3 + (15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 -
 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b
+ b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))
*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x +
c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a
 + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*s
inh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 +
 a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c
))*sinh(d*x + c) + a + b)) + 8*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x +
 c)^7 - 3*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*c
osh(d*x + c)^5 - 2*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*
a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^3 - (27*a^3*b + 13*a^2*b^2 - 23*a*b^3
 - 9*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 5*
a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 +
8*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b
^4 + a^2*b^5)*d*sinh(d*x + c)^8 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3
- 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 5*a^6*b + 10*a^5*b^2
 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 + (a^7 + 3*a^6*b + 2
*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d)*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(128) = 256.
time = 0.45, size = 409, normalized size = 2.88

$$\frac{\frac{(15a^2b+10ab^2+3b^3)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^5+3a^4b+3a^3b^2+a^2b^3)\sqrt{ab}}+\frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3}-\frac{2\left(9a^3be^{(6dx+6c)}-a^2b^2e^{(6dx+6c)}-13ab^3e^{(6dx+6c)}-3b^4e^{(6dx+6c)}+27a^3be^{(4dx+4c)}-9a^2b^2e^{(4dx+4c)}+21ab^3e^{(4dx+4c)}+9b^4e^{(4dx+4c)}+27a^3be^{(2dx+2c)}+13a^2b^2e^{(2dx+2c)}-23ab^3e^{(2dx+2c)}-9b^4e^{(2dx+2c)}+9a^3b+21a^2b^2+15ab^3+3b^4\right)}{(a^5+3a^4b+3a^3b^2+a^2b^3)\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d
*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a
```

```
*b)) + 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(9*a^3*b*e^(6*d*x +
6*c) - a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e^(6*d*x + 6*c) - 3*b^4*e^(6*d*x
+ 6*c) + 27*a^3*b*e^(4*d*x + 4*c) - 9*a^2*b^2*e^(4*d*x + 4*c) + 21*a*b^3*e^
(4*d*x + 4*c) + 9*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 13*a^2*b
^2*e^(2*d*x + 2*c) - 23*a*b^3*e^(2*d*x + 2*c) - 9*b^4*e^(2*d*x + 2*c) + 9*a
^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3
)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d
*x + 2*c) + a + b)^2))/d
```

Mupad [B]
time = 0.90, size = 260, normalized size = 1.83

$$\frac{\ln\left(\tanh(c+d\,x)+1\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3}-\frac{\ln\left(1-\tanh(c+d\,x)\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3}+\frac{\frac{\tanh(c+d\,x)^3\,\left(\frac{3\,b^3}{8}+\frac{7\,a\,b^2}{8}\right)}{a^2\,d\,\left(a^2+2\,a\,b+b^2\right)}+\frac{\tanh(c+d\,x)\,\left(5\,b^2+9\,a\,b\right)}{8\,a\,d\,\left(a^2+2\,a\,b+b^2\right)}}{a^2+2\,a\,b\tanh\left(c+d\,x\right)^2+b^2\tanh\left(c+d\,x\right)^4}+\frac{\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(15\,a^2\,b+10\,a\,b^2+3\,b^3\right)}{\sqrt{a\,b}\,\left(8\,a^5\,d+a\,b\,\left(24\,a^3\,d+a\,b\,\left(24\,a\,d+8\,b\,d\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - log(1
- tanh(c + d*x))/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh(c + d
*x)^3*((7*a*b^2)/8 + (3*b^3)/8))/(a^2*d*(2*a*b + a^2 + b^2)) + (tanh(c + d*
x)*(9*a*b + 5*b^2))/(8*a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*x)^4
 + 2*a*b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(10*a*b^2
+ 15*a^2*b + 3*b^3))/((a*b)^(1/2)*(8*a^5*d + a*b*(24*a^3*d + a*b*(24*a*d +
8*b*d))))
```

3.197 $\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=138

$$\frac{\log(\cosh(c+dx))}{(a+b)^3d}+\frac{\log(\tanh(c+dx))}{a^3d}-\frac{b(3a^2+3ab+b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3(a+b)^3d}+\frac{b}{4a(a+b)d\left(a+b\tanh\right.}$$

```
[Out] ln(cosh(d*x+c))/(a+b)^3/d+ln(tanh(d*x+c))/a^3/d-1/2*b*(3*a^2+3*a*b+b^2)*ln(
a+b*tanh(d*x+c)^2)/a^3/(a+b)^3/d+1/4*b/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/2*
b*(2*a+b)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 457, 84}

$$\frac{\log(\tanh(c+dx))}{a^3d}+\frac{b(2a+b)}{2a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{b(3a^2+3ab+b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3d(a+b)^3}+\frac{b}{4ad(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[Tanh[c + d*x]]/(a^3*d) - (b*(3*a^2 +
 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2])/(2*a^3*(a + b)^3*d) + b/(4*a*(a +
 b)*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(2*a + b))/(2*a^2*(a + b)^2*d*(a + b*
Tanh[c + d*x]^2))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{1}{a^3x} - \frac{b^2}{a(a+b)(a+bx)^3} - \frac{b^2(2a+b)}{a^2(a+b)^2(a+bx)^2} - \frac{b^2(3a^2+3ab+b^2)}{a^3(a+b)^3(a+bx)}\right.\right.}{2d}$$

$$= \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log(\tanh(c+dx))}{a^3d} - \frac{b(3a^2+3ab+b^2)\log\left(a+b\tanh^2(\right.}{2a^3(a+b)^3d}$$

Mathematica [A]
time = 1.25, size = 117, normalized size = 0.85

$$\frac{\frac{4\log(\cosh(c+dx))}{(a+b)^3} + \frac{4\log(\tanh(c+dx)) + \frac{b\left(-2\left(3a^2+3ab+b^2\right)\log\left(a+b\tanh^2(c+dx)\right) + \frac{a(a+b)\left(a(5a+3b)+2b(2a+b)\tanh^2(c+dx)\right)}{\left(a+b\tanh^2(c+dx)\right)^2}\right)}{(a+b)^3}}{a^3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
[Out] ((4*Log[Cosh[c + d*x]])/(a + b)^3 + (4*Log[Tanh[c + d*x]] + (b*(-2*(3*a^2 +
 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)*(a*(5*a + 3*b) + 2*b*
(2*a + b)*Tanh[c + d*x]^2))/(a + b*Tanh[c + d*x]^2)^2))/(a + b)^3)/a^3)/(4*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(132) = 264$.
time = 3.26, size = 273, normalized size = 1.98

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3} - \frac{b\left(\frac{2\left(3a^2+5ab+2b^2\right)ab\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(3a^3+10a^2b+10a\,b^2+3b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(3a^2+5ab+2b^2\right)ab\left(\mathrm{t}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}\right.}{(a+b)^3a}}{}$

default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{b\left(\frac{2\left(3a^2+5ab+2b^2\right)ab\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(3a^3+10a^2b+10a\,b^2+3b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(3a^2+5ab+2b^2\right)ab}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}\right.}{(a+b)}}{}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2x}{a^3}-\frac{2c}{a^3d}+\frac{6bx}{a(a^3+3a^2b+3a\,b^2+b^3)}+\frac{6bc}{ad(a^3+3a^2b+3a\,b^2+b^3)}+\frac{6b^2x}{a^2(a^3+3a^2b+3a\,b^2+b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^3*ln(tanh(1/2*d*x+1/2*c))-b/(a+b)^3/a^3*((2*(3*a^2+5*a*b+2*b^2)*a*
b*tanh(1/2*d*x+1/2*c)^6+4*b*(3*a^3+10*a^2*b+10*a*b^2+3*b^3)*tanh(1/2*d*x+1/
2*c)^4+2*(3*a^2+5*a*b+2*b^2)*a*b*tanh(1/2*d*x+1/2*c)^2)/(a*tanh(1/2*d*x+1/2
*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/2*(3*a^2+3
*a*b+b^2)*ln(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2
*d*x+1/2*c)^2+a))-1/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/(a+b)^3*ln(tanh(1/2
*d*x+1/2*c)-1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(132) = 264$.
time = 0.33, size = 498, normalized size = 3.61

$$-\frac{(3a^2b+3ab^2+b^3)\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a^6+3a^5b+3a^4b^2+a^3b^3)d}+\frac{dx+c}{(a^3+3a^2b+3ab^2+b^3)d}+\frac{2\left((3a^2b^2+4ab^3+b^4)e^{(-2dx-2c)}+2(3a^2b^2-ab^3-b^4)e^{(-4dx-4c)}+(3a^2b^2+4ab^3+b^4)e^{(-6dx-6c)}\right)}{\left(a^7+5a^6b+10a^5b^2+10a^4b^3+5a^3b^4+a^2b^5+4(a^7+3a^6b+2a^5b^2-2a^4b^3-3a^3b^4-a^2b^5)e^{(-2dx-2c)}+2(3a^7+7a^6b+6a^5b^2+6a^4b^3+7a^3b^4+3a^2b^5)e^{(-4dx-4c)}+4(a^7+3a^6b+2a^5b^2-2a^4b^3-3a^3b^4-a^2b^5)e^{(-6dx-6c)}+(a^7+5a^6b+10a^5b^2+10a^4b^3+5a^3b^4+a^2b^5)e^{(-8dx-8c)}\right)d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{a^3d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(
-4*d*x - 4*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + (d*x + c
)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((3*a^2*b^2 + 4*a*b^3 + b^4)*e^(-
2*d*x - 2*c) + 2*(3*a^2*b^2 - a*b^3 - b^4)*e^(-4*d*x - 4*c) + (3*a^2*b^2 +
4*a*b^3 + b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3
+ 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^
4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3
+ 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 -
2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5
*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + log(e^(-d*x
 - c) + 1)/(a^3*d) + log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4800 vs. $2(132) = 264$.
time = 0.67, size = 4800, normalized size = 34.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
[Out] -1/2*(2*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^8 + 16*(a^5 + 2*a^4*b +
 a^3*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^5 + 2*a^4*b + a^3*b^2)*d
*x*sinh(d*x + c)^8 - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d
*x)*cosh(d*x + c)^6 - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 14*(a^5 + 2*a^4*b
+ a^3*b^2)*d*x*cosh(d*x + c)^2 - 2*(a^5 - a^3*b^2)*d*x)*sinh(d*x + c)^6 + 8
*(14*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^3 - 3*(3*a^3*b^2 + 4*a^2*b
^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(6*a
^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x)*cosh(d*x
+ c)^4 + 4*(35*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^4 - 6*a^3*b^2 +
2*a^2*b^3 + 2*a*b^4 + (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x - 15*(3*a^3*b^2 + 4
*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4
+ 16*(7*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^5 - 5*(3*a^3*b^2 + 4*a^
2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^3 - (6*a^3*b^2 - 2*a^2
*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
 + c)^3 + 2*(a^5 + 2*a^4*b + a^3*b^2)*d*x - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^
4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^2 + 4*(14*(a^5 + 2*a^4*b + a^3*b^2
)*d*x*cosh(d*x + c)^6 - 3*a^3*b^2 - 4*a^2*b^3 - a*b^4 - 15*(3*a^3*b^2 + 4*a
^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^4 + 2*(a^5 - a^3*b^2)
*d*x - 6*(6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d
*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 +
 5*a*b^4 + b^5)*cosh(d*x + c)^8 + 8*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4*b + 9*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*sinh(d*x + c)^8 + 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4 - b^5)*cosh(d*x + c)^6 + 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b
^4 - b^5 + 7*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x +
c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
b^5)*cosh(d*x + c)^3 + 3*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*
cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5 + 2*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c
)^4 + 2*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(3*a^4*b +
9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^4 + 30*(3*a^4*b + 3*a
^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7
*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^5 + 10*(3
*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^3 + (9*a^4*b
+ 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^2 + 4*(7
*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^6 + 3*a^4
*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(3*a^4*b + 3*a^3*b^2 - 2*a^
2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^4 + 3*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3
 + 7*a*b^4 + 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4*b + 9*a^3*
b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^7 + 3*(3*a^4*b + 3*a^3*b^2
- 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^5 + (9*a^4*b + 3*a^3*b^2 + 6*a^2
*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c)^3 + (3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3
```

```
- 3*a*b^4 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)
^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^
2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10*
a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(d*x + c)^8 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^6 + 4*(a^5 + 3*a^4*b +
 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^5 + 5*
a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^3 + 3*(a^5 +
 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c))*sinh(d*x +
 c)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 2*(3*a^5
+ 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(d*x + c)^4 + 2*(3
*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^4 + 30*(a^5 + 3*
a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(
d*x + c)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cos
h(d*x + c)^3 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
 3*a*b^4 - b^5)*cosh(d*x + c)^2 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*cosh(d*x + c)^6 + a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b
^3 - 3*a*b^4 - b^5 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
b^5)*cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(132) = 264.
time = 0.59, size = 295, normalized size = 2.14

$$-\frac{\frac{\left(3a^2b+3ab^2+b^3\right)\log\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}{a^6+3a^5b+3a^4b^2+a^3b^3}+\frac{2(dx+c)}{a^3+3a^2b+3ab^2+b^3}-\frac{2\log\left(\left|e^{(2dx+2c)}-1\right|\right)}{a^3}-\frac{4\left(\left(3a^2b^2+ab^3\right)e^{(6dx+6c)}+\left(3a^2b^2+ab^3\right)e^{(2dx+2c)}+\frac{2\left(3a^3b^2-a^2b^3-ab^4\right)e^{(4dx+4c)}}{a+b}\right)}{\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)^2(a+b)^2a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*((3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) +
 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^6 + 3*a^5*b + 3*a^4*
b^2 + a^3*b^3) + 2*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*log(abs(e^
(2*d*x + 2*c) - 1))/a^3 - 4*((3*a^2*b^2 + a*b^3)*e^(6*d*x + 6*c) + (3*a^2*b
^2 + a*b^3)*e^(2*d*x + 2*c) + 2*(3*a^3*b^2 - a^2*b^3 - a*b^4)*e^(4*d*x + 4*
c)/(a + b))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
 2*b*e^(2*d*x + 2*c) + a + b)^2*(a + b)^2*a^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c+d\,x)}{{\left(b\tanh\left(c+d\,x\right)}^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

3.198 $\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=178

$$\frac{x}{(a+b)^3}-\frac{b^{3/2}(35a^2+42ab+15b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^3d}-\frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3(a+b)^2d}+\frac{}{4a(a+b)}$$

```
[Out] x/(a+b)^3-1/8*b^(3/2)*(35*a^2+42*a*b+15*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(
1/2))/a^(7/2)/(a+b)^3/d-1/8*(8*a^2+27*a*b+15*b^2)*coth(d*x+c)/a^3/(a+b)^2/d
+1/4*b*coth(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(9*a+5*b)*coth(d*x
+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.21, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3751, 483, 593, 597, 536, 212, 211}

$$\frac{b(9a+5b)\coth(c+dx)}{8a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{b^{3/2}(35a^2+42ab+15b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a+b)^3}-\frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3d(a+b)^2}+\frac{b\coth(c+dx)}{4ad(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{x}{(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 - (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(7/2)*(a + b)^3*d) - ((8*a^2 + 27*a*b + 15*b^2)*Coth[c
 + d*x])/(8*a^3*(a + b)^2*d) + (b*Coth[c + d*x])/(4*a*(a + b)*d*(a + b*Tanh
[c + d*x]^2)^2) + (b*(9*a + 5*b)*Coth[c + d*x])/(8*a^2*(a + b)^2*d*(a + b*T
anh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
```

```
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a-5b+5bx^2}{x^2(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c\right.}{4a(a+b)d}$$

$$= \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(9a+5b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} +$$

$$= -\frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3(a+b)^2d} + \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} +$$

$$= -\frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3(a+b)^2d} + \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} +$$

$$= \frac{x}{(a+b)^3} - \frac{b^{3/2}(35a^2+42ab+15b^2)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^3d} - \frac{(8a^2+27ab}{8a}$$

Mathematica [A]
time = 4.27, size = 166, normalized size = 0.93

$$-\frac{-\frac{8(c+dx)}{(a+b)^3} + \frac{b^{3/2}(35a^2+42ab+15b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^3} + \frac{8\coth(c+dx)}{a^3} + \frac{4b^3\sinh(2(c+dx))}{a^2(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{b^2(13a+7b)\sinh(2(c+dx))}{a^3(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*((-8*(c + d*x))/(a + b)^3 + (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan
[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^3) + (8*Coth[c + d*x])/
a^3 + (4*b^3*Sinh[2*(c + d*x)])/(a^2*(a + b)^2*(a - b + (a + b)*Cosh[2*(c +
 d*x)])^2) + (b^2*(13*a + 7*b)*Sinh[2*(c + d*x)])/(a^3*(a + b)^2*(a - b + (
a + b)*Cosh[2*(c + d*x)])))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(162) = 324$.
time = 2.92, size = 428, normalized size = 2.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^3*tanh(1/2*d*x+1/2*c)+2*b^2/(a+b)^3/a^3*((-1/8*a*(13*a^2+22*a*b
+9*b^2)*tanh(1/2*d*x+1/2*c)^7+(-39/8*a^3-55/4*a^2*b-99/8*a*b^2-7/2*b^3)*tan
h(1/2*d*x+1/2*c)^5+(-39/8*a^3-55/4*a^2*b-99/8*a*b^2-7/2*b^3)*tanh(1/2*d*x+1
/2*c)^3+(-13/8*a^3-11/4*a^2*b-9/8*a*b^2)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d
*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8*(3
5*a^2+42*a*b+15*b^2)*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/(b*(a+b))^(1/2)/((2*(
b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^
(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b*(a+b))^(1/2)/((2*(b*(
a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2
)+a+2*b)*a)^(1/2))))+1/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/a^3/tanh(1/2*d
*x+1/2*c)-1/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1944 vs. $2(162) = 324$.
time = 0.88, size = 1944, normalized size = 10.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(3*a^2*b + 3*a*b^2 + b^3)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2
*d*x + 2*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 1/4*(3*a^2
*b + 3*a*b^2 + b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*
c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 1/32*(15*a^3*b - 25
*a^2*b^2 - 39*a*b^3 - 15*b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/
sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a*b)*d) - 1/32*(15*a
^3*b - 25*a^2*b^2 - 39*a*b^3 - 15*b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c)
 + a - b)/sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a*b)*d) +
1/16*(8*a^5 + 31*a^4*b + 72*a^3*b^2 + 98*a^2*b^3 + 64*a*b^4 + 15*b^5 + (8*a
^5 + 49*a^4*b + 18*a^3*b^2 + 38*a*b^4 + 15*b^5)*e^(8*d*x + 8*c) + 2*(16*a^5
 + 57*a^4*b - 9*a^3*b^2 + 37*a^2*b^3 - 39*a*b^4 - 30*b^5)*e^(6*d*x + 6*c) +
 2*(24*a^5 + 56*a^4*b + 83*a^3*b^2 - 37*a^2*b^3 + 53*a*b^4 + 45*b^5)*e^(4*d
*x + 4*c) + 2*(16*a^5 + 39*a^4*b + 73*a^3*b^2 + 15*a^2*b^3 - 65*a*b^4 - 30*
b^5)*e^(2*d*x + 2*c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4
 + a^3*b^5 - (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5
)*e^(10*d*x + 10*c) - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^
4 - 5*a^3*b^5)*e^(8*d*x + 8*c) - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 +
13*a^4*b^4 + 5*a^3*b^5)*e^(6*d*x + 6*c) + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a
^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*e^(4*d*x + 4*c) + (3*a^8 + 7*a^7*b - 2*a^6
*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*e^(2*d*x + 2*c))*d) - 1/16*(8*a
^5 + 31*a^4*b + 72*a^3*b^2 + 98*a^2*b^3 + 64*a*b^4 + 15*b^5 + 2*(16*a^5 + 3
9*a^4*b + 73*a^3*b^2 + 15*a^2*b^3 - 65*a*b^4 - 30*b^5)*e^(-2*d*x - 2*c) + 2
*(24*a^5 + 56*a^4*b + 83*a^3*b^2 - 37*a^2*b^3 + 53*a*b^4 + 45*b^5)*e^(-4*d*
x - 4*c) + 2*(16*a^5 + 57*a^4*b - 9*a^3*b^2 + 37*a^2*b^3 - 39*a*b^4 - 30*b^
```

```
5)*e^(-6*d*x - 6*c) + (8*a^5 + 49*a^4*b + 18*a^3*b^2 + 38*a*b^4 + 15*b^5)*e
^(-8*d*x - 8*c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^
3*b^5 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)
*e^(-2*d*x - 2*c) + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 +
5*a^3*b^5)*e^(-4*d*x - 4*c) - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*
a^4*b^4 + 5*a^3*b^5)*e^(-6*d*x - 6*c) - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a
^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*e^(-8*d*x - 8*c) - (a^8 + 5*a^7*b + 10*a^6
*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*e^(-10*d*x - 10*c))*d) - 1/8*(8*a^
4 + 41*a^3*b + 73*a^2*b^2 + 55*a*b^3 + 15*b^4 + 2*(16*a^4 + 41*a^3*b - 55*a
*b^3 - 30*b^4)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b
^3 + 45*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*e
^(-6*d*x - 6*c) + (8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*e^(-8
*d*x - 8*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (3*a^7 + 4
*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(a^7 + 2*
a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-4*d*x - 4*c) - 2*(a^7 + 2*a^5*b^2 + 8*
a^4*b^3 + 5*a^3*b^4)*e^(-6*d*x - 6*c) - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a
^4*b^3 - 5*a^3*b^4)*e^(-8*d*x - 8*c) - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b
^3 + a^3*b^4)*e^(-10*d*x - 10*c))*d) + 15/16*b*arctan(1/2*((a + b)*e^(-2*d*
x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d) + 1/2*log(e^(2*d*x + 2*c) -
1)/(a^3*d) - 1/2*log(e^(-2*d*x - 2*c) - 1)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5772 vs. $2(162) = 324$.
time = 0.56, size = 11865, normalized size = 66.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^10 + 160*(a^5 + 2*a^4
*b + a^3*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 16*(a^5 + 2*a^4*b + a^3*b
^2)*d*x*sinh(d*x + c)^10 - 4*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 +
57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^8 -
4*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 180*(a^
5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^2 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2
)*d*x)*sinh(d*x + c)^8 + 32*(60*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)
^3 - (8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3
*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 8*(16*a^5
 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*
b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^6 + 8*(420*(a^5 + 2*a^4*b + a^3*b^2)*d*x*
cosh(d*x + c)^4 - 16*a^5 - 48*a^4*b - 19*a^3*b^2 + 28*a^2*b^3 + 69*a*b^4 +
30*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x - 14*(8*a^5 + 40*a^4*b + 67*a^3*
b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)
*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(252*(a^5 + 2*a^4*b + a^3*b^2)*d*x*c
```

osh(d*x + c)^5 - 14*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^3 - 3*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 32*a^5 - 160*a^4*b - 372*a^3*b^2 - 452*a^2*b^3 - 268*a*b^4 - 60*b^5 - 8*(24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^4 + 8*(420*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^6 - 24*a^5 - 56*a^4*b - 48*a^3*b^2 - 33*a^2*b^3 - 86*a*b^4 - 45*b^5 - 35*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^4 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x - 15*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(60*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^7 - 7*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^5 - 5*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^3 - (24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(a^5 + 2*a^4*b + a^3*b^2)*d*x - 8*(16*a^5 + 48*a^4*b + 45*a^3*b^2 - 36*a^2*b^3 - 79*a*b^4 - 30*b^5 + 2*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^2 + 8*(90*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^8 - 14*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^6 - 16*a^5 - 48*a^4*b - 45*a^3*b^2 + 36*a^2*b^3 + 79*a*b^4 + 30*b^5 - 15*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^4 - 2*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x - 6*(24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^10 + 10*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)*sinh(d*x + c)^9 + (35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*sinh(d*x + c)^10 + (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c)^8 + (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5 + 45*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^3 + (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c)^6 + 2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5 + 105*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^4 + 14*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^5 + 14*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*cosh(d*x + c)^3 + 3*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 35*a^4*b - 112*a^3*b^2 - 134*a^2*b^3 - 72*a*b^4 - 15*b^5 - 2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a

```
*b^4 + 75*b^5)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b
^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^6 - 35*a^4*b + 28*a^3*b^2 - 106*a^2*b
^3 - 180*a*b^4 - 75*b^5 + 35*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*
b^4 - 75*b^5)*cosh(d*x + c)^4 + 15*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 1
80*a*b^4 + 75*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 112
*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*cosh(d*x + c)^7 + 7*(105*a^4*b
+ 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(162) = 324.
time = 0.59, size = 437, normalized size = 2.46

$$-\frac{\frac{\left(35\,a^2b^2+42\,ab^3+15\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\sqrt{ab}}-\frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,\left(13\,a^3b^2e^{(6\,dx+6\,c)}+3\,a^2b^3e^{(6\,dx+6\,c)}-17\,ab^4e^{(6\,dx+6\,c)}-7\,b^5e^{(6\,dx+6\,c)}+39\,a^3b^2e^{(4\,dx+4\,c)}-5\,a^2b^3e^{(4\,dx+4\,c)}+25\,ab^4e^{(4\,dx+4\,c)}+21\,b^5e^{(4\,dx+4\,c)}+39\,a^3b^2e^{(2\,dx+2\,c)}+25\,a^2b^3e^{(2\,dx+2\,c)}-35\,ab^4e^{(2\,dx+2\,c)}-21\,b^5e^{(2\,dx+2\,c)}+13\,a^3b^2+33\,a^2b^3+27\,ab^4+7\,b^5\right)}{\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)^2}+\frac{16}{a^3\left(e^{(2\,dx+2\,c)}-1\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
[Out] -1/8*((35*a^2*b^2 + 42*a*b^3 + 15*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^
(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sq
rt(a*b)) - 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(13*a^3*b^2*e^(6
*d*x + 6*c) + 3*a^2*b^3*e^(6*d*x + 6*c) - 17*a*b^4*e^(6*d*x + 6*c) - 7*b^5*
e^(6*d*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) - 5*a^2*b^3*e^(4*d*x + 4*c) +
25*a*b^4*e^(4*d*x + 4*c) + 21*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2
*c) + 25*a^2*b^3*e^(2*d*x + 2*c) - 35*a*b^4*e^(2*d*x + 2*c) - 21*b^5*e^(2*d
*x + 2*c) + 13*a^3*b^2 + 33*a^2*b^3 + 27*a*b^4 + 7*b^5)/((a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) + 16/(a^3*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{coth}\left(c+d\,x\right)}^2}{{\left(b\,{\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
[Out] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

3.199 $\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=171

$$-\frac{\coth^2(c+dx)}{2a^3d}+\frac{\log(\cosh(c+dx))}{(a+b)^3d}+\frac{(a-3b)\log(\tanh(c+dx))}{a^4d}+\frac{b^2(6a^2+8ab+3b^2)\log\left(a+b\tanh^2(c+d\right.}{2a^4(a+b)^3d}$$

```
[Out] -1/2*coth(d*x+c)^2/a^3/d+ln(cosh(d*x+c))/(a+b)^3/d+(a-3*b)*ln(tanh(d*x+c))/
a^4/d+1/2*b^2*(6*a^2+8*a*b+3*b^2)*ln(a+b*tanh(d*x+c)^2)/a^4/(a+b)^3/d-1/4*b
^2/a^2/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/d/(a+b*t
anh(d*x+c)^2)
```

Rubi [A]
time = 0.19, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$,
Rules used = {3751, 457, 90}

$$\frac{(a-3b)\log(\tanh(c+dx))}{a^4d}-\frac{b^2(3a+2b)}{2a^3d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\coth^2(c+dx)}{2a^3d}-\frac{b^2}{4a^2d(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{b^2(6a^2+8ab+3b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^4d(a+b)^3}+\frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*Coth[c + d*x]^2/(a^3*d) + Log[Cosh[c + d*x]]/((a + b)^3*d) + ((a - 3*b
)*Log[Tanh[c + d*x]])/(a^4*d) + (b^2*(6*a^2 + 8*a*b + 3*b^2)*Log[a + b*Tanh
[c + d*x]^2])/(2*a^4*(a + b)^3*d) - b^2/(4*a^2*(a + b)*d*(a + b*Tanh[c + d*
x]^2)^2) - (b^2*(3*a + 2*b))/(2*a^3*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)^3}\,dx, x, \tanh^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)}+\frac{1}{a^3x^2}+\frac{a-3b}{a^4x}+\frac{b^3}{a^2(a+b)(a+bx)^3}+\frac{b^3(3a+2b)}{a^3(a+b)^2(a+bx)^2}+\frac{b^3(}{a^4}\right.\right.}{2d}\\ &= -\frac{\coth^2(c+dx)}{2a^3d}+\frac{\log(\cosh(c+dx))}{(a+b)^3d}+\frac{(a-3b)\log(\tanh(c+dx))}{a^4d}+\frac{b^2(6a}{}\end{aligned}$$

Mathematica [A]
time = 1.28, size = 138, normalized size = 0.81

$$-\frac{\frac{\coth^2(c+dx)}{a^3}+\frac{b^4}{2a^4(a+b)\left(b+a\coth^2(c+dx)\right)^2}-\frac{b^3(4a+3b)}{a^4(a+b)^2\left(b+a\coth^2(c+dx)\right)}-\frac{b^2(6a^2+8ab+3b^2)\log\left(b+a\coth^2(c+dx)\right)}{a^4(a+b)^3}-\frac{2\log(\sinh(c+dx))}{(a+b)^3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/2*(Coth[c + d*x]^2/a^3 + b^4/(2*a^4*(a + b)*(b + a*Coth[c + d*x]^2)^2) -
 (b^3*(4*a + 3*b))/(a^4*(a + b)^2*(b + a*Coth[c + d*x]^2)) - (b^2*(6*a^2 +
8*a*b + 3*b^2)*Log[b + a*Coth[c + d*x]^2])/(a^4*(a + b)^3) - (2*Log[Sinh[c
+ d*x]])/(a + b)^3)/d
```

Maple [A]
time = 3.27, size = 316, normalized size = 1.85 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*tanh(1/2*d*x+1/2*c)^2/a^3-1/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)+b^2
/a^4/(a+b)^3*((2*(4*a^2+7*a*b+3*b^2)*a*b*tanh(1/2*d*x+1/2*c)^6+4*b*(4*a^3+1
4*a^2*b+15*a*b^2+5*b^3)*tanh(1/2*d*x+1/2*c)^4+2*(4*a^2+7*a*b+3*b^2)*a*b*tan
h(1/2*d*x+1/2*c)^2)/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*
```

```
tanh(1/2*d*x+1/2*c)^2+a)^2+1/2*(6*a^2+8*a*b+3*b^2)*ln(a*tanh(1/2*d*x+1/2*c)
^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a))-1/8/a^3/tanh(1/2
*d*x+1/2*c)^2+1/4/a^4*(4*a-12*b)*ln(tanh(1/2*d*x+1/2*c))-1/(a+b)^3*ln(tanh(
1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(163) = 326$.
time = 0.36, size = 770, normalized size = 4.50

$$\frac{(6a^2b^2+8ab^3+3b^4)\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a^7+3a^6b+3a^5b^2+a^4b^3)d}+\frac{dx+c}{(a^3+3a^2b+3ab^2+b^3)d}-\frac{2\left((a^5+5a^4b+10a^3b^2+14a^2b^3+11ab^4+3b^5)e^{(-2dx-2c)}+2(2a^5+6a^4b+4a^3b^2-4a^2b^3-13ab^4-6b^5)e^{(-4dx-4c)}+2(3a^5+7a^4b+6a^3b^2+2a^2b^3+15ab^4+9b^5)e^{(-6dx-6c)}+2(2a^5+6a^4b+4a^3b^2-4a^2b^3-13ab^4-6b^5)e^{(-8dx-8c)}+(a^5+5a^4b+10a^3b^2+14a^2b^3+11ab^4+3b^5)e^{(-10dx-10c)}\right)}{\left(a^8+5a^7b+10a^6b^2+10a^5b^3+5a^4b^4+a^3b^5+2(a^8+a^7b-6a^6b^2-14a^5b^3-11a^4b^4-3a^3b^5)e^{(-2dx-2c)}-(a^8+5a^7b-6a^6b^2-38a^5b^3-43a^4b^4-15a^3b^5)e^{(-4dx-4c)}-4(a^8+a^7b+2a^6b^2+10a^5b^3+13a^4b^4+5a^3b^5)e^{(-6dx-6c)}-(a^8+5a^7b-6a^6b^2-38a^5b^3-43a^4b^4-15a^3b^5)e^{(-8dx-8c)}+2(a^8+a^7b-6a^6b^2-14a^5b^3-11a^4b^4-3a^3b^5)e^{(-10dx-10c)}+(a^8+5a^7b+10a^6b^2+10a^5b^3+5a^4b^4+a^3b^5)e^{(-12dx-12c)}\right)d}+\frac{(a-3b)\log\left(e^{(-dx-c)}+1\right)}{a^4d}+\frac{(a-3b)\log\left(e^{(-dx-c)}-1\right)}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*
e^(-4*d*x - 4*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d) + (d*x
+ c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^5 + 5*a^4*b + 10*a^3*b^2 +
 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-2*d*x - 2*c) + 2*(2*a^5 + 6*a^4*b + 4*a
^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-4*d*x - 4*c) + 2*(3*a^5 + 7*a^4*
b + 6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(-6*d*x - 6*c) + 2*(2*a^5 +
 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-8*d*x - 8*c) + (a^
5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-10*d*x - 10*c
))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5 + 2*(a^8
 + a^7*b - 6*a^6*b^2 - 14*a^5*b^3 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-2*d*x - 2*c
) - (a^8 + 5*a^7*b - 6*a^6*b^2 - 38*a^5*b^3 - 43*a^4*b^4 - 15*a^3*b^5)*e^(-
4*d*x - 4*c) - 4*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3
*b^5)*e^(-6*d*x - 6*c) - (a^8 + 5*a^7*b - 6*a^6*b^2 - 38*a^5*b^3 - 43*a^4*b
^4 - 15*a^3*b^5)*e^(-8*d*x - 8*c) + 2*(a^8 + a^7*b - 6*a^6*b^2 - 14*a^5*b^3
 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-10*d*x - 10*c) + (a^8 + 5*a^7*b + 10*a^6*b^2
 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*e^(-12*d*x - 12*c))*d) + (a - 3*b)*log
(e^(-d*x - c) + 1)/(a^4*d) + (a - 3*b)*log(e^(-d*x - c) - 1)/(a^4*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10720 vs. $2(163) = 326$.
time = 0.77, size = 10720, normalized size = 62.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^12 + 24*(a^6 + 2*a^5*b
+ a^4*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^6 + 2*a^5*b + a^4*b^2)
*d*x*sinh(d*x + c)^12 + 4*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2
*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^10 + 4*(a^6
 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + 33*(a^6 + 2*a
```

^5*b + a^4*b^2)*d*x*cosh(d*x + c)^2 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*sinh
(d*x + c)^10 + 40*(11*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^3 + (a^6
+ 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b
- 3*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(8*a^6 + 24*a^5*b + 1
6*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^
2)*d*x)*cosh(d*x + c)^8 + 2*(495*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c
)^4 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 -
(a^6 + 2*a^5*b - 15*a^4*b^2)*d*x + 90*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*
b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)
^2)*sinh(d*x + c)^8 + 16*(99*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^5
+ 30*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6
- 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4*b
^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)
*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 2*a^3*b^
3 + 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d*x + c)^6
+ 8*(231*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^6 + 3*a^6 + 7*a^5*b +
6*a^4*b^2 + 2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 + 105*(a^6 + 5*a^5*b + 10*a^4
*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)
*cosh(d*x + c)^4 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x + 7*(8*a^6 + 24*a^5*b +
16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b
^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(99*(a^6 + 2*a^5*b + a^4*b^2
)*d*x*cosh(d*x + c)^7 + 63*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^
2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^5 + 7*(8*a
^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*
a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^3 + 3*(3*a^6 + 7*a^5*b + 6*a^4*b^2 +
2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 -
52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^4 +
2*(495*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^8 + 420*(a^6 + 5*a^5*b
+ 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b
^2)*d*x)*cosh(d*x + c)^6 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*
a^2*b^4 - 24*a*b^5 + 70*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^
2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^4 - (a^6
+ 2*a^5*b - 15*a^4*b^2)*d*x + 60*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 2*a^3*b^3
+ 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(55*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^9 + 60*
(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*
a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^7 + 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2
- 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*c
osh(d*x + c)^5 + 20*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 2*a^3*b^3 + 15*a^2*b^4 +
9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a
^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 1
5*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^6 + 2*a^5*b + a^4*b^2
)*d*x + 4*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 +
(a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^6 + 2*a^5*b +

```
a^4*b^2)*d*x*cosh(d*x + c)^10 + 45*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3
 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^8
+ 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 -
(a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^6 + a^6 + 5*a^5*b + 10*a^4*
b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + 30*(3*a^6 + 7*a^5*b + 6*a^4*b^2 +
 2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d
*x + c)^4 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x + 3*(8*a^6 + 24*a^5*b + 16*a^4*
b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4
+ 14*a*b^5 + 3*b^6)*cosh(d*x + c)^12 + 12*(6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*
b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)*sinh(d*x + c)^11 + (6*a^4*b^2 + 20*a^
3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*sinh(d*x + c)^12 + 2*(6*a^4*b^2 - 4*
a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^10 + 2*(6*a^4*b^2 -
4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6 + 33*(6*a^4*b^2 + 20*a^3*b^3 + 25
*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(6*
a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^3 + (6*
a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(163) = 326.
time = 0.64, size = 474, normalized size = 2.77

$$\frac{\frac{(6a^2b^2+8ab^3+3b^4)\log\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}{a^7+3a^6b+3a^5b^2+a^4b^3}-\frac{2(dx+c)}{a^3+3a^2b+3ab^2+b^3}+\frac{2(a-3b)\log\left(\left|e^{(2dx+2c)}-1\right|\right)}{a^4}-\frac{4\left((a^5+5a^4b+10a^3b^2+14a^2b^3+11ab^4+3b^5)e^{(10dx+10c)}+2(2a^5+6a^4b+4a^3b^2-4a^2b^3-13ab^4-6b^5)e^{(8dx+8c)}+2(3a^5+7a^4b+6a^3b^2+2a^2b^3+15ab^4+9b^5)e^{(6dx+6c)}+2(2a^5+6a^4b+4a^3b^2-4a^2b^3-13ab^4-6b^5)e^{(4dx+4c)}+(a^5+5a^4b+10a^3b^2+14a^2b^3+11ab^4+3b^5)e^{(2dx+2c)}\right)}{\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)^2(a+b)^3a^3\left(e^{(2dx+2c)}-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*((6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c
) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^7 + 3*a^6*b + 3*a
^5*b^2 + a^4*b^3) - 2*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a - 3*
b)*log(abs(e^(2*d*x + 2*c) - 1))/a^4 - 4*((a^5 + 5*a^4*b + 10*a^3*b^2 + 14*
a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(10*d*x + 10*c) + 2*(2*a^5 + 6*a^4*b + 4*a^3*
b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(8*d*x + 8*c) + 2*(3*a^5 + 7*a^4*b +
6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(6*d*x + 6*c) + 2*(2*a^5 + 6*a^
```

```
4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(4*d*x + 4*c) + (a^5 + 5*
a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/((a*e^
(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*
c) + a + b)^2*(a + b)^3*a^3*(e^(2*d*x + 2*c) - 1)^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)
```

3.200 $\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=228

$$\frac{x}{(a+b)^3}+\frac{b^{5/2}(63a^2+90ab+35b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a+b)^3d}-\frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4(a+b)^2d}-\frac{(8\cdots}{}$$

```
[Out] x/(a+b)^3+1/8*b^(5/2)*(63*a^2+90*a*b+35*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(
1/2))/a^(9/2)/(a+b)^3/d-1/8*(8*a^3-8*a^2*b-55*a*b^2-35*b^3)*coth(d*x+c)/a^4
/(a+b)^2/d-1/24*(8*a^2+55*a*b+35*b^2)*coth(d*x+c)^3/a^3/(a+b)^2/d+1/4*b*cot
h(d*x+c)^3/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(11*a+7*b)*coth(d*x+c)^3/a
^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.26, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3751, 483, 593, 597, 536, 212, 211}

$$\frac{b(11a+7b)\coth^3(c+dx)}{8a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)}+\frac{b^{5/2}(63a^2+90ab+35b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a+b)^3}-\frac{(8a^2+55ab+35b^2)\coth^3(c+dx)}{24a^3d(a+b)^2}-\frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4d(a+b)^2}+\frac{b\coth^3(c+dx)}{4ad(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{x}{(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] x/(a + b)^3 + (b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(9/2)*(a + b)^3*d) - ((8*a^3 - 8*a^2*b - 55*a*b^2 - 35
*b^3)*Coth[c + d*x])/(8*a^4*(a + b)^2*d) - ((8*a^2 + 55*a*b + 35*b^2)*Coth[
c + d*x]^3)/(24*a^3*(a + b)^2*d) + (b*Coth[c + d*x]^3)/(4*a*(a + b)*d*(a +
b*Tanh[c + d*x]^2)^2) + (b*(11*a + 7*b)*Coth[c + d*x]^3)/(8*a^2*(a + b)^2*d
*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
```

```
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a-7b+7bx^2}{x^4(1-x^2)(a+bx^2)^2}\,dx, x, \tanh(c+\right.}{4a(a+b)d}$$

$$= \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(11a+7b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\text{S}}{}$$

$$= -\frac{(8a^2+55ab+35b^2)\coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} +$$

$$= -\frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4(a+b)^2d} - \frac{(8a^2+55ab+35b^2)\coth^3(c+}{24a^3(a+b)^2d}$$

$$= -\frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4(a+b)^2d} - \frac{(8a^2+55ab+35b^2)\coth^3(c+}{24a^3(a+b)^2d}$$

$$= \frac{x}{(a+b)^3} + \frac{b^{5/2}(63a^2+90ab+35b^2)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a+b)^3d} - \frac{(8a^3-8a^2b-}{}$$

Mathematica [A]
time = 2.52, size = 194, normalized size = 0.85

$$\frac{\frac{24(c+dx)}{(a+b)^3} + \frac{3b^{5/2}(63a^2+90ab+35b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{9/2}(a+b)^3} + \frac{8(-4a+9b)\coth(c+dx)}{a^4} - \frac{8\coth(c+dx)\text{csch}^2(c+dx)}{a^3} + \frac{12b^4\sinh(2(c+dx))}{a^3(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{3b^3(17a+11b)\sinh(2(c+dx))}{a^4(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] ((24*(c + d*x))/(a + b)^3 + (3*b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(S
qrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(9/2)*(a + b)^3) + (8*(-4*a + 9*b)*Coth[
c + d*x])/a^4 - (8*Coth[c + d*x]*Csch[c + d*x]^2)/a^3 + (12*b^4*Sinh[2*(c +
 d*x)])/(a^3*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (3*b^3*(17*
a + 11*b)*Sinh[2*(c + d*x)])/(a^4*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*
x)])))/(24*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(210) = 420.
time = 3.17, size = 481, normalized size = 2.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/8/a^4*(1/3*a*tanh(1/2*d*x+1/2*c)^3+5*a*tanh(1/2*d*x+1/2*c)-12*b*tan h(1/2*d*x+1/2*c))+1/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/24/a^3/tanh(1/2*d*x +1/2*c)^3-1/8*(5*a-12*b)/a^4/tanh(1/2*d*x+1/2*c)-2*b^3/(a+b)^3/a^4*((-1/8*a *(17*a^2+30*a*b+13*b^2)*tanh(1/2*d*x+1/2*c)^7+(-51/8*a^3-75/4*a^2*b-143/8*a *b^2-11/2*b^3)*tanh(1/2*d*x+1/2*c)^5+(-51/8*a^3-75/4*a^2*b-143/8*a*b^2-11/2 *b^3)*tanh(1/2*d*x+1/2*c)^3+(-17/8*a^3-15/4*a^2*b-13/8*a*b^2)*tanh(1/2*d*x+ 1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x +1/2*c)^2+a)^2+1/8*(63*a^2+90*a*b+35*b^2)*a*(-1/2*(-a+(b*(a+b))^(1/2)-b)/a/ (b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+ 1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2*(a+(b*(a+b))^(1/2)+b)/a/(b* (a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2* c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))))-1/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)- 1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4285 vs. $2(210) = 420$.
time = 1.35, size = 4285, normalized size = 18.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/8*(3*a^3*b - 3*a^2*b^2 - 7*a*b^3 - 3*b^4)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d) + 1/8*(3*a^3*b - 3*a^2*b^2 - 7*a*b^3 - 3*b^4)*log(2*(a - b)*e^(-2*d*x - 2 *c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b ^3)*d) + 1/128*(15*a^4*b - 200*a^3*b^2 - 186*a^2*b^3 + 35*b^5)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a ^4*b^3)*sqrt(a*b)*d) - 1/128*(15*a^4*b - 200*a^3*b^2 - 186*a^2*b^3 + 35*b^5)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt(a*b)*d) + 1/192*(176*a^6 + 781*a^5*b + 1571*a^4 *b^2 + 1538*a^3*b^3 + 502*a^2*b^4 - 175*a*b^5 - 105*b^6 + 3*(96*a^6 + 465*a ^5*b + 665*a^4*b^2 + 706*a^3*b^3 + 506*a^2*b^4 + 61*a*b^5 - 35*b^6)*e^(12*d *x + 12*c) + 6*(120*a^6 + 192*a^5*b - 315*a^4*b^2 - 728*a^3*b^3 - 1070*a^2* b^4 - 240*a*b^5 + 105*b^6)*e^(10*d*x + 10*c) + (176*a^6 - 281*a^5*b + 3509* a^4*b^2 + 3950*a^3*b^3 + 12226*a^2*b^4 + 3755*a*b^5 - 1575*b^6)*e^(8*d*x + 8*c) - 4*(184*a^6 + 48*a^5*b + 473*a^4*b^2 + 970*a^3*b^3 + 3684*a^2*b^4 + 1 070*a*b^5 - 525*b^6)*e^(6*d*x + 6*c) - (384*a^6 + 1127*a^5*b - 861*a^4*b^2 - 7146*a^3*b^3 - 11386*a^2*b^4 - 1965*a*b^5 + 1575*b^6)*e^(4*d*x + 4*c) + 2 *(136*a^6 - 96*a^5*b - 1309*a^4*b^2 - 2996*a^3*b^3 - 2238*a^2*b^4 - 4*a*b^5 + 315*b^6)*e^(2*d*x + 2*c))/((a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5* a^5*b^4 + a^4*b^5 - (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*e^(14*d*x + 14*c) - (a^9 - 3*a^8*b - 22*a^7*b^2 - 38*a^6*b^3 - 27*a^5*b^4 - 7*a^4*b^5)*e^(12*d*x + 12*c) + (3*a^9 + 7*a^8*b - 18*a^7*b^2 - 66*a^6*b^3 - 65*a^5*b^4 - 21*a^4*b^5)*e^(10*d*x + 10*c) + (3*a^9 - a^8*b + 14*a^7*b^2 + 78*a^6*b^3 + 95*a^5*b^4 + 35*a^4*b^5)*e^(8*d*x + 8*c) - (3*a^9 - a^8*b + 14*a^7*b^2 + 78*a^6*b^3 + 95*a^5*b^4 + 35*a^4*b^5)*e^(6*d*x + 6*c) - (3*a^9 + 7*a^8*b - 18*a^7*b^2 - 66*a^6*b^3 - 65*a^5*b^4 - 21*a^4*b^5)*e^(4*d*x + 4*c) + (a^9 - 3*a^8*b - 22*a^7*b^2 - 38*a^6*b^3 - 27*a^5*b^4 - 7*a^4*b^5)*e^(2*d*x + 2*c))*d) - 1/192*(176*a^6 + 781*a^5*b + 1571*a^4*b^2 + 1538*a^3*b^3 + 502*a^2*b^4 - 175*a*b^5 - 105*b^6 + 2*(136*a^6 - 96*a^5*b - 1309*a^4*b^2 - 2996*a^3*b^3 - 2238*a^2*b^4 - 4*a*b^5 + 315*b^6)*e^(-2*d*x - 2*c) - (384*a^6 + 1127*a^5*b - 861*a^4*b^2 - 7146*a^3*b^3 - 11386*a^2*b^4 - 1965*a*b^5 + 1575*b^6)*e^(-4*d*x - 4*c) - 4*(184*a^6 + 48*a^5*b + 473*a^4*b^2 + 970*a^3*b^3 + 3684*a^2*b^4 + 1070*a*b^5 - 525*b^6)*e^(-6*d*x - 6*c) + (176*a^6 - 281*a^5*b + 3509*a^4*b^2 + 3950*a^3*b^3 + 12226*a^2*b^4 + 3755*a*b^5 - 1575*b^6)*e^(-8*d*x - 8*c) + 6*(120*a^6 + 192*a^5*b - 315*a^4*b^2 - 728*a^3*b^3 - 1070*a^2*b^4 - 240*a*b^5 + 105*b^6)*e^(-10*d*x - 10*c) + 3*(96*a^6 + 465*a^5*b + 665*a^4*b^2 + 706*a^3*b^3 + 506*a^2*b^4 + 61*a*b^5 - 35*b^6)*e^(-12*d*x - 12*c))/((a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5 + (a^9 - 3*a^8*b - 22*a^7*b^2 - 38*a^6*b^3 - 27*a^5*b^4 - 7*a^4*b^5)*e^(-2*d*x - 2*c) - (3*a^9 + 7*a^8*b - 18*a^7*b^2 - 66*a^6*b^3 - 65*a^5*b^4 - 21*a^4*b^5)*e^(-4*d*x - 4*c) - (3*a^9 - a^8*b + 14*a^7*b^2 + 78*a^6*b^3 + 95*a^5*b^4 + 35*a^4*b^5)*e^(-6*d*x - 6*c) + (3*a^9 - a^8*b + 14*a^7*b^2 + 78*a^6*b^3 + 95*a^5*b^4 + 35*a^4*b^5)*e^(-8*d*x - 8*c) + (3*a^9 + 7*a^8*b - 18*a^7*b^2 - 66*a^6*b^3 - 65*a^5*b^4 - 21*a^4*b^5)*e^(-10*d*x - 10*c) - (a^9 - 3*a^8*b - 22*a^7*b^2 - 38*a^6*b^3 - 27*a^5*b^4 - 7*a^4*b^5)*e^(-12*d*x - 12*c) - (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*e^(-14*d*x - 14*c))*d) + 1/48*(32*a^5 + 83*a^4*b - 60*a^3*b^2 - 346*a^2*b^3 - 340*a*b^4 - 105*b^5 + 3*(32*a^5 + 95*a^4*b + 154*a^3*b^2 + 84*a^2*b^3 - 42*a*b^4 - 35*b^5)*e^(12*d*x + 12*c) + 6*(48*a^5 + 40*a^4*b - 117*a^3*b^2 - 201*a^2*b^3 + 45*a*b^4 + 105*b^5)*e^(10*d*x + 10*c) + (224*a^5 + 281*a^4*b + 384*a^3*b^2 + 2318*a^2*b^3 - 160*a*b^4 - 1575*b^5)*e^(8*d*x + 8*c) - 4*(16*a^5 - 136*a^4*b - 9*a^3*b^2 + 697*a^2*b^3 - 115*a*b^4 - 525*b^5)*e^(6*d*x + 6*c) - (96*a^5 + 137*a^4*b - 1262*a^3*b^2 - 1840*a^2*b^3 + 1230*a*b^4 + 1575*b^5)*e^(4*d*x + 4*c) + 2*(16*a^5 - 136*a^4*b - 435*a^3*b^2 - 35*a^2*b^3 + 563*a*b^4 + 315*b^5)*e^(2*d*x + 2*c))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4 - (a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*e^(14*d*x + 14*c) - (a^8 - 4*a^7*b - 18*a^6*b^2 - 20*a^5*b^3 - 7*a^4*b^4)*e^(12*d*x + 12*c) + (3*a^8 + 4*a^7*b - 22*a^6*b^2 - 44*a^5*b^3 - 21*a^4*b^4)*e^(10*d*x + 10*c) + (3*a^8 - 4*a^7*b + 18*a^6*b^2 + 60*a^5*b^3 + 35*a^4*b^4)*e^(8*d*x + 8*c) - (3*a^8 - 4*a^7*b + 18*a^6*b^2 + 60*a^5*b^3 + 35*a^4*b^4)*e^(6*d*x + 6*c) - (3*a^8 + 4*a^7*b - 22*a^6*b^2 - 44*a^5*b^3 - 21*a^4*b^4)*e^(4*d*x + 4*c) + (a^8 - 4*a^7*b - 18*a^6*b^2 - 20*a^5*b^3 - 7*a^4*b^4)*e^(2*d*x + 2*c))*d) - 1/48*(32*a^5 + 83*a^4*b - 60*a^3*b^2 - 346*a^2*b^3 - 340*a*b^4 - 105*b^5 + 2*(16*a^5 - 136*a^4*b - 435*a^3*b^2 - 35*a^2*b^3

```
 + 563*a*b^4 + 315*b^5)*e^(-2*d*x - 2*c) - (96*a^5 + 137*a^4*b - 1262*a^3*b
^2 - 1840*a^2*b^3 + 1230*a*b^4 + 1575*b^5)*e^(-4*d*x - 4*c) - 4*(16*a^5 - 1
36*a^4*b - 9*a^3*b^2 + 697*a^2*b^3 - 115*a*b^4 ...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10858 vs. $2(210) = 420$.
time = 0.58, size = 22038, normalized size = 96.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(48*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^14 + 672*(a^6 + 2*a^5
*b + a^4*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^13 + 48*(a^6 + 2*a^5*b + a^4*
b^2)*d*x*sinh(d*x + c)^14 - 12*(16*a^6 + 56*a^5*b + 40*a^4*b^2 - 63*a^3*b^3
 - 153*a^2*b^4 - 125*a*b^5 - 35*b^6 - 4*(a^6 - 6*a^5*b - 7*a^4*b^2)*d*x)*co
sh(d*x + c)^12 - 12*(16*a^6 + 56*a^5*b + 40*a^4*b^2 - 63*a^3*b^3 - 153*a^2*
b^4 - 125*a*b^5 - 35*b^6 - 364*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^
2 - 4*(a^6 - 6*a^5*b - 7*a^4*b^2)*d*x)*sinh(d*x + c)^12 + 48*(364*(a^6 + 2*
a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^3 - 3*(16*a^6 + 56*a^5*b + 40*a^4*b^2 -
63*a^3*b^3 - 153*a^2*b^4 - 125*a*b^5 - 35*b^6 - 4*(a^6 - 6*a^5*b - 7*a^4*b^
2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^11 - 24*(24*a^6 + 32*a^5*b - 40*a^4*b^
2 + 189*a^2*b^4 + 270*a*b^5 + 105*b^6 + 2*(3*a^6 - 2*a^5*b - 21*a^4*b^2)*d*
x)*cosh(d*x + c)^10 + 24*(2002*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^
4 - 24*a^6 - 32*a^5*b + 40*a^4*b^2 - 189*a^2*b^4 - 270*a*b^5 - 105*b^6 - 2*
(3*a^6 - 2*a^5*b - 21*a^4*b^2)*d*x - 33*(16*a^6 + 56*a^5*b + 40*a^4*b^2 - 6
3*a^3*b^3 - 153*a^2*b^4 - 125*a*b^5 - 35*b^6 - 4*(a^6 - 6*a^5*b - 7*a^4*b^2
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 48*(2002*(a^6 + 2*a^5*b + a^4*b^
2)*d*x*cosh(d*x + c)^5 - 55*(16*a^6 + 56*a^5*b + 40*a^4*b^2 - 63*a^3*b^3 -
153*a^2*b^4 - 125*a*b^5 - 35*b^6 - 4*(a^6 - 6*a^5*b - 7*a^4*b^2)*d*x)*cosh(
d*x + c)^3 - 5*(24*a^6 + 32*a^5*b - 40*a^4*b^2 + 189*a^2*b^4 + 270*a*b^5 +
105*b^6 + 2*(3*a^6 - 2*a^5*b - 21*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^9 - 4*(128*a^6 + 328*a^5*b + 872*a^4*b^2 + 695*a^3*b^3 - 1145*a^2*b^4 - 3
175*a*b^5 - 1575*b^6 + 12*(3*a^6 - 10*a^5*b + 35*a^4*b^2)*d*x)*cosh(d*x + c
)^8 + 4*(36036*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^6 - 128*a^6 - 32
8*a^5*b - 872*a^4*b^2 - 695*a^3*b^3 + 1145*a^2*b^4 + 3175*a*b^5 + 1575*b^6
 - 1485*(16*a^6 + 56*a^5*b + 40*a^4*b^2 - 63*a^3*b^3 - 153*a^2*b^4 - 125*a*b
^5 - 35*b^6 - 4*(a^6 - 6*a^5*b - 7*a^4*b^2)*d*x)*cosh(d*x + c)^4 - 12*(3*a^
6 - 10*a^5*b + 35*a^4*b^2)*d*x - 270*(24*a^6 + 32*a^5*b - 40*a^4*b^2 + 189*
a^2*b^4 + 270*a*b^5 + 105*b^6 + 2*(3*a^6 - 2*a^5*b - 21*a^4*b^2)*d*x)*cosh(
d*x + c)^2)*sinh(d*x + c)^8 + 32*(5148*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d
*x + c)^7 - 297*(16*a^6 + 56*a^5*b + 40*a^4*b^2 - 63*a^3*b^3 - 153*a^2*b^4
 - 125*a*b^5 - 35*b^6 - 4*(a^6 - 6*a^5*b - 7*a^4*b^2)*d*x)*cosh(d*x + c)^5 -
 90*(24*a^6 + 32*a^5*b - 40*a^4*b^2 + 189*a^2*b^4 + 270*a*b^5 + 105*b^6 + 2
```

```
*(3*a^6 - 2*a^5*b - 21*a^4*b^2)*d*x)*cosh(d*x + c)^3 - (128*a^6 + 328*a^5*b
 + 872*a^4*b^2 + 695*a^3*b^3 - 1145*a^2*b^4 - 3175*a*b^5 - 1575*b^6 + 12*(3
*a^6 - 10*a^5*b + 35*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 16*(8*a
^6 + 144*a^5*b + 88*a^4*b^2 - 160*a^3*b^3 + 255*a^2*b^4 + 1000*a*b^5 + 525*
b^6 - 3*(3*a^6 - 10*a^5*b + 35*a^4*b^2)*d*x)*cosh(d*x + c)^6 + 16*(9009*(a^
6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^8 - 693*(16*a^6 + 56*a^5*b + 40*a^
4*b^2 - 63*a^3*b^3 - 153*a^2*b^4 - 125*a*b^5 - 35*b^6 - 4*(a^6 - 6*a^5*b -
7*a^4*b^2)*d*x)*cosh(d*x + c)^6 - 8*a^6 - 144*a^5*b - 88*a^4*b^2 + 160*a^3*
b^3 - 255*a^2*b^4 - 1000*a*b^5 - 525*b^6 - 315*(24*a^6 + 32*a^5*b - 40*a^4*
b^2 + 189*a^2*b^4 + 270*a*b^5 + 105*b^6 + 2*(3*a^6 - 2*a^5*b - 21*a^4*b^2)*
d*x)*cosh(d*x + c)^4 + 3*(3*a^6 - 10*a^5*b + 35*a^4*b^2)*d*x - 7*(128*a^6 +
 328*a^5*b + 872*a^4*b^2 + 695*a^3*b^3 - 1145*a^2*b^4 - 3175*a*b^5 - 1575*b
^6 + 12*(3*a^6 - 10*a^5*b + 35*a^4*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)
^6 - 128*a^6 - 352*a^5*b + 160*a^4*b^2 + 1804*a^3*b^3 + 2780*a^2*b^4 + 1780
*a*b^5 + 420*b^6 + 32*(3003*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^9 -
 297*(16*a^6 + 56*a^5*b + 40*a^4*b^2 - 63*a^3*b^3 - 153*a^2*b^4 - 125*a*b^5
 - 35*b^6 - 4*(a^6 - 6*a^5*b - 7*a^4*b^2)*d*x)*cosh(d*x + c)^7 - 189*(24*a^
6 + 32*a^5*b - 40*a^4*b^2 + 189*a^2*b^4 + 270*a*b^5 + 105*b^6 + 2*(3*a^6 -
2*a^5*b - 21*a^4*b^2)*d*x)*cosh(d*x + c)^5 - 7*(128*a^6 + 328*a^5*b + 872*a
^4*b^2 + 695*a^3*b^3 - 1145*a^2*b^4 - 3175*a*b^5 - 1575*b^6 + 12*(3*a^6 - 1
0*a^5*b + 35*a^4*b^2)*d*x)*cosh(d*x + c)^3 - 3*(8*a^6 + 144*a^5*b + 88*a^4*
b^2 - 160*a^3*b^3 + 255*a^2*b^4 + 1000*a*b^5 + 525*b^6 - 3*(3*a^6 - 10*a^5*
b + 35*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(48*a^6 + 184*a^5*b
 + 840*a^4*b^2 + 969*a^3*b^3 - 1541*a^2*b^4 - 3525*a*b^5 - 1575*b^6 - 12*(3
*a^6 - 2*a^5*b - 21*a^4*b^2)*d*x)*cosh(d*x + c)^4 + 4*(12012*(a^6 + 2*a^5*b
 + a^4*b^2)*d*x*cosh(d*x + c)^10 - 1485*(16*a^6 + 56*a^5*b + 40*a^4*b^2 - 6
3*a^3*b^3 - 153*a^2*b^4 - 125*a*b^5 - 35*b^6 - 4*(a^6 - 6*a^5*b - 7*a^4*b^2
)*d*x)*cosh(d*x + c)^8 - 1260*(24*a^6 + 32*a^5*b - 40*a^4*b^2 + 189*a^2*b^4
 + 270*a*b^5 + 105*b^6 + 2*(3*a^6 - 2*a^5*b - 21*a^4*b^2)*d*x)*cosh(d*x + c
)^6 - 48*a^6 - 184*a^5*b - 840*a^4*b^2 - 969*a^3*b^3 + 1541*a^2*b^4 + 3525*
a*b^5 + 1575*b^6 - 70*(128*a^6 + 328*a^5*b + 872*a^4*b^2 + 695*a^3*b^3 - 11
45*a^2*b^4 - 3175*a*b^5 - 1575*b^6 + 12*(3*a^6 - 10*a^5*b + 35*a^4*b^2)*d*x
)*cosh(d*x + c)^4 + 12*(3*a^6 - 2*a^5*b - 21*a^4*b^2)*d*x - 60*(8*a^6 + 144
*a^5*b + 88*a^4*b^2 - 160*a^3*b^3 + 255*a^2*b^4 + 1000*a*b^5 + 525*b^6 - 3*
(3*a^6 - 10*a^5*b + 35*a^4*b^2)*d*x)*cosh(d*x +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(210) = 420.
time = 0.68, size = 493, normalized size = 2.16

$$\frac{\frac{3\,(63\,a^2b^3+90\,ab^4+35\,b^5)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{ab}}+\frac{24\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{6\,(17\,a^3b^3e^{(6\,dx+6\,c)}+7\,a^2b^4e^{(6\,dx+6\,c)}-21\,ab^5e^{(6\,dx+6\,c)}-11\,b^6e^{(6\,dx+6\,c)}+51\,a^3b^3e^{(4\,dx+4\,c)}-a^2b^4e^{(4\,dx+4\,c)}+29\,ab^5e^{(4\,dx+4\,c)}+33\,b^6e^{(4\,dx+4\,c)}+51\,a^3b^3e^{(2\,dx+2\,c)}+37\,a^2b^4e^{(2\,dx+2\,c)}-47\,ab^5e^{(2\,dx+2\,c)}-33\,b^6e^{(2\,dx+2\,c)}+17\,a^3b^3+45\,a^2b^4+39\,ab^5+11\,b^6)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b)^2}-\frac{16\,(6\,ae^{(4\,dx+4\,c)}-9\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+18\,be^{(2\,dx+2\,c)}+4\,a-9\,b)}{a^4(e^{(2\,dx+2\,c)}-1)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(63*a^2*b^3 + 90*a*b^4 + 35*b^5)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*
e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*
sqrt(a*b)) + 24*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 6*(17*a^3*b^3*e
^(6*d*x + 6*c) + 7*a^2*b^4*e^(6*d*x + 6*c) - 21*a*b^5*e^(6*d*x + 6*c) - 11*
b^6*e^(6*d*x + 6*c) + 51*a^3*b^3*e^(4*d*x + 4*c) - a^2*b^4*e^(4*d*x + 4*c)
+ 29*a*b^5*e^(4*d*x + 4*c) + 33*b^6*e^(4*d*x + 4*c) + 51*a^3*b^3*e^(2*d*x +
 2*c) + 37*a^2*b^4*e^(2*d*x + 2*c) - 47*a*b^5*e^(2*d*x + 2*c) - 33*b^6*e^(2
*d*x + 2*c) + 17*a^3*b^3 + 45*a^2*b^4 + 39*a*b^5 + 11*b^6)/((a^7 + 3*a^6*b
+ 3*a^5*b^2 + a^4*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) - 16*(6*a*e^(4*d*x + 4*c) - 9*b*
e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 18*b*e^(2*d*x + 2*c) + 4*a - 9*b)/(
a^4*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^4}{{\left(b\,\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)
```

3.201 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=201

$$\frac{x}{(a+b)^4}+\frac{\sqrt{b}\,(35a^3+35a^2b+21ab^2+5b^3)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^4d}+\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3}+\text{—}\;24$$

```
[Out] x/(a+b)^4+1/16*(35*a^3+35*a^2*b+21*a*b^2+5*b^3)*arctan(b^(1/2)*tanh(d*x+c)/
a^(1/2))*b^(1/2)/a^(7/2)/(a+b)^4/d+1/6*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*
x+c)^2)^3+1/24*b*(11*a+5*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2
+1/16*b*(19*a^2+16*a*b+5*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.20, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3742, 425, 541, 536, 212, 211}

$$\frac{b(11a+5b)\tanh(c+dx)}{24a^2d(a+b)^2\left(a+b\tanh^2(c+dx)\right)^2}+\frac{b(19a^2+16ab+5b^2)\tanh(c+dx)}{16a^3d(a+b)^3\left(a+b\tanh^2(c+dx)\right)}+\frac{\sqrt{b}\,(35a^3+35a^2b+21ab^2+5b^3)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}d(a+b)^4}+\frac{b\tanh(c+dx)}{6ad(a+b)\left(a+b\tanh^2(c+dx)\right)^3}+\frac{x}{(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^2)^(-4),x]
```

```
[Out] x/(a + b)^4 + (Sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a]])/(16*a^(7/2)*(a + b)^4*d) + (b*Tanh[c + d*x])/(6
*a*(a + b)*d*(a + b*Tanh[c + d*x]^2)^3) + (b*(11*a + 5*b)*Tanh[c + d*x])/(2
4*a^2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(19*a^2 + 16*a*b + 5*b^2)
*Tanh[c + d*x])/(16*a^3*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} - \frac{\text{Subst}\left(\int \frac{b-6(a+b)+5bx^2}{(1-x^2)(a+bx^2)^3}\,dx, x, \tanh(c+d\right.}{6a(a+b)d}$$

$$= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} +$$

$$= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} +$$

$$= \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} +$$

$$= \frac{x}{(a+b)^4} + \frac{\sqrt{b}\,(35a^3+35a^2b+21ab^2+5b^3)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^4d} + \frac{}{6a(a}$$

Mathematica [A]
time = 0.44, size = 203, normalized size = 1.01

$$\frac{\frac{3\sqrt{b}\,(35a^3+35a^2b+21ab^2+5b^3)\,\text{ArcTan}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}} - 24\log(1-\tanh(c+dx)) + 24\log(1+\tanh(c+dx)) + \frac{8b(a+b)^3\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)^3} + \frac{2b(a+b)^2(11a+5b)\tanh(c+dx)}{a^2\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3b(a+b)(19a^2+16ab+5b^2)\tanh(c+dx)}{a^3\left(a+b\tanh^2(c+dx)\right)}}{48(a+b)^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-4),x]
```

```
[Out] ((3*Sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[b]*Tanh[c +
 d*x])/Sqrt[a]])/a^(7/2) - 24*Log[1 - Tanh[c + d*x]] + 24*Log[1 + Tanh[c +
d*x]] + (8*b*(a + b)^3*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^3) + (2*b*
(a + b)^2*(11*a + 5*b)*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)^2) + (3*
b*(a + b)*(19*a^2 + 16*a*b + 5*b^2)*Tanh[c + d*x])/(a^3*(a + b*Tanh[c + d*x
]^2)))/(48*(a + b)^4*d)
```

Maple [A]
time = 1.28, size = 219, normalized size = 1.09

method	result

derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^4}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^4}+\frac{b\left(\frac{\frac{b^2\left(19a^3+35a^2b+21a\,b^2+5b^3\right)\left(\tanh^5(dx+c)\right)}{16a^3}+\frac{b\left(17a^3+33a^2b+21a\,b^2+5b^3\right)\left(\tanh^3\right.}{6a^2}}{\left(a+b\left(\tanh^2(dx+c)\right)\right)^3}\right.}{}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^4}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^4}+\frac{b\left(\frac{\frac{b^2\left(19a^3+35a^2b+21a\,b^2+5b^3\right)\left(\tanh^5(dx+c)\right)}{16a^3}+\frac{b\left(17a^3+33a^2b+21a\,b^2+5b^3\right)\left(\tanh^3\right.}{6a^2}}{\left(a+b\left(\tanh^2(dx+c)\right)\right)^3}\right.}{}}{d}$
risch	$\frac{x}{a^4+4a^3b+6a^2b^2+4a\,b^3+b^4}-\frac{b\left(450a^3b^2+319a^4b+87a^5+435a^5\mathrm{e}^{2dx+2c}+306a^2b^3+15b^5+103a\,b^4-246a^2b^3\mathrm{e}^{10dx+10c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^4*ln(tanh(d*x+c)-1)+1/2/(a+b)^4*ln(1+tanh(d*x+c))+1/(a+b)^4
*b*((1/16*b^2*(19*a^3+35*a^2*b+21*a*b^2+5*b^3)/a^3*tanh(d*x+c)^5+1/6*b*(17*
a^3+33*a^2*b+21*a*b^2+5*b^3)/a^2*tanh(d*x+c)^3+1/16*(29*a^3+61*a^2*b+43*a*b
^2+11*b^3)/a*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^3+1/16*(35*a^3+35*a^2*b+21*a*
b^2+5*b^3)/a^3/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(185) = 370$.
time = 0.77, size = 925, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] -1/16*(35*a^3*b + 35*a^2*b^2 + 21*a*b^3 + 5*b^4)*arctan(1/2*((a + b)*e^(-2*
d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^
3*b^4)*sqrt(a*b)*d) + 1/24*(87*a^5*b + 319*a^4*b^2 + 450*a^3*b^3 + 306*a^2*
b^4 + 103*a*b^5 + 15*b^6 + 3*(145*a^5*b + 267*a^4*b^2 + 34*a^3*b^3 - 178*a^
2*b^4 - 115*a*b^5 - 25*b^6)*e^(-2*d*x - 2*c) + 6*(145*a^5*b + 93*a^4*b^2 -
6*a^3*b^3 + 106*a^2*b^4 + 85*a*b^5 + 25*b^6)*e^(-4*d*x - 4*c) + 2*(435*a^5*
b + 29*a^4*b^2 + 162*a^3*b^3 - 306*a^2*b^4 - 245*a*b^5 - 75*b^6)*e^(-6*d*x
- 6*c) + 3*(145*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 150*a^2*b^4 + 105*a*b^5 +
 25*b^6)*e^(-8*d*x - 8*c) + 3*(29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3 - 82*a^2*
b^4 - 31*a*b^5 - 5*b^6)*e^(-10*d*x - 10*c))/((a^10 + 7*a^9*b + 21*a^8*b^2 +
 35*a^7*b^3 + 35*a^6*b^4 + 21*a^5*b^5 + 7*a^4*b^6 + a^3*b^7 + 6*(a^10 + 5*a
^9*b + 9*a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - 9*a^5*b^5 - 5*a^4*b^6 - a^3*b^7)
```

```
*e^(-2*d*x - 2*c) + 3*(5*a^10 + 19*a^9*b + 25*a^8*b^2 + 15*a^7*b^3 + 15*a^6
*b^4 + 25*a^5*b^5 + 19*a^4*b^6 + 5*a^3*b^7)*e^(-4*d*x - 4*c) + 4*(5*a^10 +
17*a^9*b + 21*a^8*b^2 + 9*a^7*b^3 - 9*a^6*b^4 - 21*a^5*b^5 - 17*a^4*b^6 - 5
*a^3*b^7)*e^(-6*d*x - 6*c) + 3*(5*a^10 + 19*a^9*b + 25*a^8*b^2 + 15*a^7*b^3
 + 15*a^6*b^4 + 25*a^5*b^5 + 19*a^4*b^6 + 5*a^3*b^7)*e^(-8*d*x - 8*c) + 6*(
a^10 + 5*a^9*b + 9*a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - 9*a^5*b^5 - 5*a^4*b^6
- a^3*b^7)*e^(-10*d*x - 10*c) + (a^10 + 7*a^9*b + 21*a^8*b^2 + 35*a^7*b^3 +
 35*a^6*b^4 + 21*a^5*b^5 + 7*a^4*b^6 + a^3*b^7)*e^(-12*d*x - 12*c))*d) + (d
*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9849 vs. $2(185) = 370$.
time = 0.59, size = 20020, normalized size = 99.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(96*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x + c)^12 + 1152
*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 +
 96*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*sinh(d*x + c)^12 - 12*(29*a^5
*b + 23*a^4*b^2 - 62*a^3*b^3 - 82*a^2*b^4 - 31*a*b^5 - 5*b^6 - 48*(a^6 + a^
5*b - a^4*b^2 - a^3*b^3)*d*x)*cosh(d*x + c)^10 - 12*(29*a^5*b + 23*a^4*b^2
- 62*a^3*b^3 - 82*a^2*b^4 - 31*a*b^5 - 5*b^6 - 528*(a^6 + 3*a^5*b + 3*a^4*b
^2 + a^3*b^3)*d*x*cosh(d*x + c)^2 - 48*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*
x)*sinh(d*x + c)^10 + 120*(176*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*co
sh(d*x + c)^3 - (29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3 - 82*a^2*b^4 - 31*a*b^5
 - 5*b^6 - 48*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*x)*cosh(d*x + c))*sinh(d*
x + c)^9 - 12*(145*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 150*a^2*b^4 + 105*a*b^
5 + 25*b^6 - 24*(5*a^6 - a^5*b - a^4*b^2 + 5*a^3*b^3)*d*x)*cosh(d*x + c)^8
+ 12*(3960*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x + c)^4 - 145*
a^5*b - 17*a^4*b^2 + 58*a^3*b^3 - 150*a^2*b^4 - 105*a*b^5 - 25*b^6 + 24*(5*
a^6 - a^5*b - a^4*b^2 + 5*a^3*b^3)*d*x - 45*(29*a^5*b + 23*a^4*b^2 - 62*a^3
*b^3 - 82*a^2*b^4 - 31*a*b^5 - 5*b^6 - 48*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)
*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 96*(792*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*d*x*cosh(d*x + c)^5 - 15*(29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3 -
82*a^2*b^4 - 31*a*b^5 - 5*b^6 - 48*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*x)*c
osh(d*x + c)^3 - (145*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 150*a^2*b^4 + 105*a
*b^5 + 25*b^6 - 24*(5*a^6 - a^5*b - a^4*b^2 + 5*a^3*b^3)*d*x)*cosh(d*x + c)
)*sinh(d*x + c)^7 - 8*(435*a^5*b + 29*a^4*b^2 + 162*a^3*b^3 - 306*a^2*b^4 -
 245*a*b^5 - 75*b^6 - 48*(5*a^6 - 3*a^5*b + 3*a^4*b^2 - 5*a^3*b^3)*d*x)*cos
h(d*x + c)^6 + 8*(11088*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x
+ c)^6 - 435*a^5*b - 29*a^4*b^2 - 162*a^3*b^3 + 306*a^2*b^4 + 245*a*b^5 + 7
5*b^6 - 315*(29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3 - 82*a^2*b^4 - 31*a*b^5 - 5
```

```
*b^6 - 48*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*x)*cosh(d*x + c)^4 + 48*(5*a^
6 - 3*a^5*b + 3*a^4*b^2 - 5*a^3*b^3)*d*x - 42*(145*a^5*b + 17*a^4*b^2 - 58*
a^3*b^3 + 150*a^2*b^4 + 105*a*b^5 + 25*b^6 - 24*(5*a^6 - a^5*b - a^4*b^2 +
5*a^3*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 348*a^5*b - 1276*a^4*b^2
 - 1800*a^3*b^3 - 1224*a^2*b^4 - 412*a*b^5 - 60*b^6 + 48*(1584*(a^6 + 3*a^5
*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x + c)^7 - 63*(29*a^5*b + 23*a^4*b^2 -
 62*a^3*b^3 - 82*a^2*b^4 - 31*a*b^5 - 5*b^6 - 48*(a^6 + a^5*b - a^4*b^2 - a
^3*b^3)*d*x)*cosh(d*x + c)^5 - 14*(145*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 15
0*a^2*b^4 + 105*a*b^5 + 25*b^6 - 24*(5*a^6 - a^5*b - a^4*b^2 + 5*a^3*b^3)*d
*x)*cosh(d*x + c)^3 - (435*a^5*b + 29*a^4*b^2 + 162*a^3*b^3 - 306*a^2*b^4 -
 245*a*b^5 - 75*b^6 - 48*(5*a^6 - 3*a^5*b + 3*a^4*b^2 - 5*a^3*b^3)*d*x)*cos
h(d*x + c))*sinh(d*x + c)^5 - 24*(145*a^5*b + 93*a^4*b^2 - 6*a^3*b^3 + 106*
a^2*b^4 + 85*a*b^5 + 25*b^6 - 12*(5*a^6 - a^5*b - a^4*b^2 + 5*a^3*b^3)*d*x)
*cosh(d*x + c)^4 + 24*(1980*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(
d*x + c)^8 - 105*(29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3 - 82*a^2*b^4 - 31*a*b^
5 - 5*b^6 - 48*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*x)*cosh(d*x + c)^6 - 145
*a^5*b - 93*a^4*b^2 + 6*a^3*b^3 - 106*a^2*b^4 - 85*a*b^5 - 25*b^6 - 35*(145
*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 150*a^2*b^4 + 105*a*b^5 + 25*b^6 - 24*(5
*a^6 - a^5*b - a^4*b^2 + 5*a^3*b^3)*d*x)*cosh(d*x + c)^4 + 12*(5*a^6 - a^5*
b - a^4*b^2 + 5*a^3*b^3)*d*x - 5*(435*a^5*b + 29*a^4*b^2 + 162*a^3*b^3 - 30
6*a^2*b^4 - 245*a*b^5 - 75*b^6 - 48*(5*a^6 - 3*a^5*b + 3*a^4*b^2 - 5*a^3*b^
3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(660*(a^6 + 3*a^5*b + 3*a^4*b
^2 + a^3*b^3)*d*x*cosh(d*x + c)^9 - 45*(29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3
 - 82*a^2*b^4 - 31*a*b^5 - 5*b^6 - 48*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*x)
*cosh(d*x + c)^7 - 21*(145*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 150*a^2*b^4 +
105*a*b^5 + 25*b^6 - 24*(5*a^6 - a^5*b - a^4*b^2 + 5*a^3*b^3)*d*x)*cosh(d*x
 + c)^5 - 5*(435*a^5*b + 29*a^4*b^2 + 162*a^3*b^3 - 306*a^2*b^4 - 245*a*b^5
 - 75*b^6 - 48*(5*a^6 - 3*a^5*b + 3*a^4*b^2 - 5*a^3*b^3)*d*x)*cosh(d*x + c)
^3 - 3*(145*a^5*b + 93*a^4*b^2 - 6*a^3*b^3 + 106*a^2*b^4 + 85*a*b^5 + 25*b^
6 - 12*(5*a^6 - a^5*b - a^4*b^2 + 5*a^3*b^3)*d*x)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 96*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x - 12*(145*a^5*b + 267*
a^4*b^2 + 34*a^3*b^3 - 178*a^2*b^4 - 115*a*b^5 - 25*b^6 - 48*(a^6 + a^5*b -
 a^4*b^2 - a^3*b^3)*d*x)*cosh(d*x + c)^2 + 12*(528*(a^6 + 3*a^5*b + 3*a^4*b
^2 + a^3*b^3)*d*x*cosh(d*x + c)^10 - 45*(29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3
 - 82*a^2*b^4 - 31*a*b^5 - 5*b^6 - 48*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*x
)*cosh(d*x + c)^8 - 28*(145*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 150*a^2*b^4 +
 105*a*b^5 + 25*b^6 - 24*(5*a^6 - a^5*b - a^4*b^2 + 5*a^3*b^3)*d*x)*cosh(d*
x + c)^6 - 145*a^5*b - 267*a^4*b^2 - 34*a^3*b^3 + 178*a^2*b^4 + 115*a*b^5 +
 25*b^6 - 10*(435*a^5*b + 29*a^4*b^2 + 162*a^3*b^3 - 306*a^2*b^4 - 245*a*b^
5 - 75*b^6 - 48*(5*a^6 - 3*a^5*b + 3*a^4*b^2 - 5*a^3*b^3)*d*x)*cosh(d*x + c
)^4 + 48*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*x ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 750 vs. 2(185) = 370.
time = 0.50, size = 750, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(35*a^3*b + 35*a^2*b^2 + 21*a*b^3 + 5*b^4)*arctan(1/2*(a*e^(2*d*x +
 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*sqrt(a*b)) + 48*(d*x + c)/(a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4) - 2*(87*a^5*b*e^(10*d*x + 10*c) + 69*a^4*b^2*e^(10*d*x + 1
0*c) - 186*a^3*b^3*e^(10*d*x + 10*c) - 246*a^2*b^4*e^(10*d*x + 10*c) - 93*a
*b^5*e^(10*d*x + 10*c) - 15*b^6*e^(10*d*x + 10*c) + 435*a^5*b*e^(8*d*x + 8*
c) + 51*a^4*b^2*e^(8*d*x + 8*c) - 174*a^3*b^3*e^(8*d*x + 8*c) + 450*a^2*b^4
*e^(8*d*x + 8*c) + 315*a*b^5*e^(8*d*x + 8*c) + 75*b^6*e^(8*d*x + 8*c) + 870
*a^5*b*e^(6*d*x + 6*c) + 58*a^4*b^2*e^(6*d*x + 6*c) + 324*a^3*b^3*e^(6*d*x
+ 6*c) - 612*a^2*b^4*e^(6*d*x + 6*c) - 490*a*b^5*e^(6*d*x + 6*c) - 150*b^6*
e^(6*d*x + 6*c) + 870*a^5*b*e^(4*d*x + 4*c) + 558*a^4*b^2*e^(4*d*x + 4*c) -
 36*a^3*b^3*e^(4*d*x + 4*c) + 636*a^2*b^4*e^(4*d*x + 4*c) + 510*a*b^5*e^(4*
d*x + 4*c) + 150*b^6*e^(4*d*x + 4*c) + 435*a^5*b*e^(2*d*x + 2*c) + 801*a^4*
b^2*e^(2*d*x + 2*c) + 102*a^3*b^3*e^(2*d*x + 2*c) - 534*a^2*b^4*e^(2*d*x +
2*c) - 345*a*b^5*e^(2*d*x + 2*c) - 75*b^6*e^(2*d*x + 2*c) + 87*a^5*b + 319*
a^4*b^2 + 450*a^3*b^3 + 306*a^2*b^4 + 103*a*b^5 + 15*b^6)/((a^7 + 4*a^6*b +
 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) +
2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^3))/d
```

Mupad [B]
time = 1.38, size = 2500, normalized size = 12.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tanh(c + d*x)^2)^4,x)
```

```
[Out] log(tanh(c + d*x) + 1)/(2*a^4*d + 2*b^4*d + 12*a^2*b^2*d + 8*a*b^3*d + 8*a^
3*b*d) + ((tanh(c + d*x)^3*(16*a*b^3 + 5*b^4 + 17*a^2*b^2))/(6*a^2*(3*a*b^2
 + 3*a^2*b + a^3 + b^3)) + (tanh(c + d*x)*(32*a*b^2 + 29*a^2*b + 11*b^3))/(
16*a*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (b^2*tanh(c + d*x)^5*(16*a*b^2 + 19
```

*a^2*b + 5*b^3))/(16*a^2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))/(a^3*d + b^3
*d*tanh(c + d*x)^6 + 3*a^2*b*d*tanh(c + d*x)^2 + 3*a*b^2*d*tanh(c + d*x)^4)
 - log(tanh(c + d*x) - 1)/(2*d*(a + b)^4) - (atan((((-a^7*b)^(1/2)*((tanh(c
 + d*x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2
450*a^5*b^4 + 1481*a^6*b^3))/(128*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 +
6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)) + ((((5
*a^3*b^13*d^2)/4 + 14*a^4*b^12*d^2 + (287*a^5*b^11*d^2)/4 + 224*a^6*b^10*d^
2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/2 + 668*a^10
*b^6*d^2 + (1561*a^11*b^5*d^2)/4 + 154*a^12*b^4*d^2 + (147*a^13*b^3*d^2)/4
+ 4*a^14*b^2*d^2)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 +
36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^11*b^4*d^3 + 84*
a^12*b^3*d^3 + 36*a^13*b^2*d^3) - (tanh(c + d*x)*(-a^7*b)^(1/2)*(21*a*b^2 +
 35*a^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^11*d^2 + 7168*a^7*b^10*d^2 + 20480*
a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^10*b^7*d^2 - 14336*a^11*b^6*d^2 -
 28672*a^12*b^5*d^2 - 20480*a^13*b^4*d^2 - 7168*a^14*b^3*d^2 - 1024*a^15*b^
2*d^2))/(4096*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)
*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 +
20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)))*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 3
5*a^3 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^1
0*b*d)))*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*1i)/(32*(a^11*d + a^7*b^4*d
 + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)) + ((-a^7*b)^(1/2)*((tanh(c + d*
x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2450*a
^5*b^4 + 1481*a^6*b^3))/(128*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7
*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)) - ((((5*a^3*
b^13*d^2)/4 + 14*a^4*b^12*d^2 + (287*a^5*b^11*d^2)/4 + 224*a^6*b^10*d^2 + (
953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/2 + 668*a^10*b^6*
d^2 + (1561*a^11*b^5*d^2)/4 + 154*a^12*b^4*d^2 + (147*a^13*b^3*d^2)/4 + 4*a
^14*b^2*d^2)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^
8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^11*b^4*d^3 + 84*a^12*
b^3*d^3 + 36*a^13*b^2*d^3) + (tanh(c + d*x)*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a
^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^11*d^2 + 7168*a^7*b^10*d^2 + 20480*a^8*b
^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^10*b^7*d^2 - 14336*a^11*b^6*d^2 - 2867
2*a^12*b^5*d^2 - 20480*a^13*b^4*d^2 - 7168*a^14*b^3*d^2 - 1024*a^15*b^2*d^2
))/(4096*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)*(a^1
2*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^
9*b^3*d^2 + 15*a^10*b^2*d^2)))*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3
 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d
)))*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*1i)/(32*(a^11*d + a^7*b^4*d + 4*
a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)))/((((185*a*b^7)/128 + (25*b^8)/128 +
(303*a^2*b^6)/64 + (567*a^3*b^5)/64 + (1225*a^4*b^4)/128 + (665*a^5*b^3)/12
8)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^8*b^7*d^3
+ 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^11*b^4*d^3 + 84*a^12*b^3*d^3 +
36*a^13*b^2*d^3) + ((-a^7*b)^(1/2)*((tanh(c + d*x)*(210*a*b^8 + 25*b^9 + 79
1*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2450*a^5*b^4 + 1481*a^6*b^3))/(12
8*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 +

20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)) + ((((5*a^3*b^13*d^2)/4 + 14*a^4*b^12*d^2 + (287*a^5*b^11*d^2)/4 + 224*a^6*b^10*d^2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/2 + 668*a^10*b^6*d^2 + (1561*a^11*b^5*d^2)/4 + 154*a^12*b^4*d^2 + (147*a^13*b^3*d^2)/4 + 4*a^14*b^2*d^2)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^11*b^4*d^3 + 84*a^12*b^3*d^3 + 36*a^13*b^2*d^3) - (tanh(c + d*x)*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^11*d^2 + 7168*a^7*b^10*d^2 + 20480*a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^10*b^7*d^2 - 14336*a^11*b^6*d^2 - 28672*a^12*b^5*d^2 - 20480*a^13*b^4*d^2 - 7168*a^14*b^3*d^2 - 1024*a^15*b^2*d^2))/(4096*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2)))*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)))*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)) - ((-a^7*b)^(1/2)*((tanh(c + d*x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2450*a^5*b^4 + 1481*a^6*b^3))/(128*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^...

3.202 $\int \sqrt{1-\tanh^2(x)}\, dx$

Optimal. Leaf size=3

$$\text{ArcSin}(\tanh(x))$$

```
[Out] arcsin(tanh(x))
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3738, 4207, 222}

$$\text{ArcSin}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Tanh[x]^2],x]
```

```
[Out] ArcSin[Tanh[x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-\tanh^2(x)}\, dx &= \int \sqrt{\operatorname{sech}^2(x)}\, dx \\
&= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\, dx, x, \tanh(x)\right) \\
&= \sin^{-1}(\tanh(x))
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(3) = 6.
time = 0.01, size = 19, normalized size = 6.33

$$2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x)\sqrt{\text{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Tanh[x]^2],x]
[Out] 2*ArcTan[Tanh[x/2]]*Cosh[x]*Sqrt[Sech[x]^2]
```

Maple [A]
time = 0.47, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\arcsin(\tanh(x))$	4
default	$\arcsin(\tanh(x))$	4
risch	$i\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x+i)-i\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x-i)$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(tanh(x))
```

Maxima [A]
time = 0.51, size = 5, normalized size = 1.67

$$2\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] 2*arctan(e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(3) = 6.
time = 0.35, size = 8, normalized size = 2.67

$$2\arctan(\cosh(x)+\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-\tanh^2{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(1 - tanh(x)**2), x)
```

Giac [A]
time = 0.41, size = 5, normalized size = 1.67

$$2\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] 2*arctan(e^x)
```

Mupad [B]
time = 0.04, size = 3, normalized size = 1.00

$$\mathrm{asin}(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - tanh(x)^2)^(1/2),x)
[Out] asin(tanh(x))
```

3.203 $\int \sqrt{-1+\tanh^2(x)}\, dx$

Optimal. Leaf size=16

$$-\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\mathrm{sech}^2(x)}}\right)$$

```
[Out] -arctanh(tanh(x)/(-sech(x)^2)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3738, 4207, 223, 212}

$$-\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\mathrm{sech}^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Tanh[x]^2],x]
```

```
[Out] -ArcTanh[Tanh[x]/Sqrt[-Sech[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1+\tanh^2(x)}\,dx &= \int \sqrt{-\operatorname{sech}^2(x)}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, \tanh(x)\right)\\ &= -\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)\\ &= -\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.31

$$2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x)\sqrt{-\operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Tanh[x]^2],x]
[Out] 2*ArcTan[Tanh[x/2]]*Cosh[x]*Sqrt[-Sech[x]^2]
```

Maple [A]
time = 0.74, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$-\ln\left(\tanh(x)+\sqrt{-1+\tanh^2(x)}\right)$	15
default	$-\ln\left(\tanh(x)+\sqrt{-1+\tanh^2(x)}\right)$	15
risch	$i\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x+i)-i\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x-i)$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln(tanh(x)+(-1+tanh(x)^2)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 5, normalized size = 0.31

$$2i\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*arctan(e^x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tanh^{2}(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(x)**2 - 1), x)
```

Giac [A]
time = 0.42, size = 1, normalized size = 0.06

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 0
```

Mupad [B]
time = 0.25, size = 14, normalized size = 0.88

$$-\ln\left(\tanh(x) + \sqrt{\tanh(x)^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)^2 - 1)^(1/2),x)
```

```
[Out] -log(tanh(x) + (tanh(x)^2 - 1)^(1/2))
```

3.204 $\int \left(1-\tanh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=22

$$\frac{1}{2}\text{ArcSin}(\tanh(x))+\frac{1}{2}\sqrt{\text{sech}^2(x)}\,\tanh(x)$$

[Out] 1/2*arcsin(tanh(x))+1/2*(sech(x)^2)^(1/2)*tanh(x)

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3738, 4207, 201, 222}

$$\frac{1}{2}\text{ArcSin}(\tanh(x))+\frac{1}{2}\tanh(x)\sqrt{\text{sech}^2(x)}$$

Antiderivative was successfully verified.

[In] Int[(1 - Tanh[x]^2)^(3/2),x]

[Out] ArcSin[Tanh[x]]/2 + (Sqrt[Sech[x]^2]*Tanh[x])/2

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(1-\tanh^2(x)\right)^{3/2}\,dx &= \int \operatorname{sech}^2(x)^{3/2}\,dx \\
&= \text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tanh(x)\right) \\
&= \frac{1}{2}\sqrt{\operatorname{sech}^2(x)}\,\tanh(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(x)\right) \\
&= \frac{1}{2}\sin^{-1}(\tanh(x))+\frac{1}{2}\sqrt{\operatorname{sech}^2(x)}\,\tanh(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.32

$$\frac{\operatorname{sech}(x)\left(2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\operatorname{sech}(x)\tanh(x)\right)}{2\sqrt{\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Tanh[x]^2)^(3/2),x]
[Out] (Sech[x]*(2*ArcTan[Tanh[x/2]] + Sech[x]*Tanh[x]))/(2*Sqrt[Sech[x]^2])
```

Maple [A]
time = 0.38, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\tanh(x)\sqrt{1-\left(\tanh^2(x)\right)}}{2}+\frac{\arcsin(\tanh(x))}{2}$	21
default	$\frac{\tanh(x)\sqrt{1-\left(\tanh^2(x)\right)}}{2}+\frac{\arcsin(\tanh(x))}{2}$	21
risch	$\frac{\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,(e^{2x}-1)}{1+e^{2x}}+\frac{i\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x+i)}{2}-\frac{i\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x-i)}{2}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*tanh(x)*(1-tanh(x)^2)^(1/2)+1/2*arcsin(tanh(x))
```

Maxima [A]
time = 0.49, size = 28, normalized size = 1.27

$$\frac{e^{(3\,x)}-e^{x}}{e^{(4\,x)}+2\,e^{(2\,x)}+1}+\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (e^(3*x) - e^x)/(e^(4*x) + 2*e^(2*x) + 1) + arctan(e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(16) = 32$.
time = 0.34, size = 140, normalized size = 6.36

$$\frac{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 1\right)\sinh(x)^2 + 2\cosh(x)^2 + 4\left(\cosh(x)^3 + \cosh(x)\right)\sinh(x) + 1\right)\arctan\left(\cosh(x) + \sinh(x)\right) + \left(3\cosh(x)^2 - 1\right)\sinh(x) - \cosh(x)}{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 1\right)\sinh(x)^2 + 2\cosh(x)^2 + 4\left(\cosh(x)^3 + \cosh(x)\right)\sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh(
x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)
^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)^2 - 1)*si
nh(x) - cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x
)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1 - \tanh^2(x)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((1 - tanh(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(16) = 32$.
time = 0.40, size = 45, normalized size = 2.05

$$\frac{1}{4}\pi - \frac{e^{(-x)} - e^x}{\left(e^{(-x)} - e^x\right)^2 + 4} + \frac{1}{2}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*pi - (e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 1/2*arctan(1/2*(e^(2*x) -
1)*e^(-x))
```

Mupad [B]
time = 0.10, size = 20, normalized size = 0.91

$$\frac{\mathrm{asin}(\tanh(x))}{2} + \frac{\tanh(x)\,\sqrt{1 - \tanh(x)^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - tanh(x)^2)^(3/2),x)
[Out] asin(tanh(x))/2 + (tanh(x)*(1 - tanh(x)^2)^(1/2))/2
```

3.205 $\int \left(-1+\tanh^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=35

$$\frac{1}{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\mathrm{sech}^2(x)}}\right)-\frac{1}{2}\sqrt{-\mathrm{sech}^2(x)}\,\tanh(x)$$

```
[Out] 1/2*arctanh(tanh(x)/(-sech(x)^2)^(1/2))-1/2*(-sech(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3738, 4207, 201, 223, 212}

$$\frac{1}{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\mathrm{sech}^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{-\mathrm{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[-Sech[x]^2]]/2 - (Sqrt[-Sech[x]^2]*Tanh[x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(-1+\tanh^2(x)\right)^{3/2} dx &= \int \left(-\operatorname{sech}^2(x)\right)^{3/2} dx\\ &= -\text{Subst}\left(\int \sqrt{-1+x^2}\, dx, x, \tanh(x)\right)\\ &= -\frac{1}{2}\sqrt{-\operatorname{sech}^2(x)}\tanh(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\, dx, x, \tanh(x)\right)\\ &= -\frac{1}{2}\sqrt{-\operatorname{sech}^2(x)}\tanh(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\, dx, x, \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)\\ &= \frac{1}{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)-\frac{1}{2}\sqrt{-\operatorname{sech}^2(x)}\tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.80

$$-\frac{1}{2}\sqrt{-\operatorname{sech}^2(x)}\left(2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x)+\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Tanh[x]^2)^(3/2),x]
[Out] -1/2*(Sqrt[-Sech[x]^2]*(2*ArcTan[Tanh[x/2]]*Cosh[x] + Tanh[x]))
```

Maple [A]
time = 0.69, size = 28, normalized size = 0.80

method	result
derivativedivides	$-\frac{\tanh(x)\sqrt{-1+\tanh^2(x)}}{2}+\frac{\ln\left(\tanh(x)+\sqrt{-1+\tanh^2(x)}\right)}{2}$

default	$-\frac{\tanh(x)\sqrt{-1+\tanh^2(x)}}{2}+\frac{\ln\left(\tanh(x)+\sqrt{-1+\tanh^2(x)}\right)}{2}$
risch	$-\frac{\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,(e^{2x}-1)}{1+e^{2x}}-\frac{i\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x+i)}{2}+\frac{i\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x-i)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*tanh(x)*(-1+tanh(x)^2)^(1/2)+1/2*ln(tanh(x)+(-1+tanh(x)^2)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 32, normalized size = 0.91

$$\frac{-i\,e^{(3\,x)}+i\,e^{x}}{e^{(4\,x)}+2\,e^{(2\,x)}+1}-i\,\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] (-I*e^(3*x) + I*e^x)/(e^(4*x) + 2*e^(2*x) + 1) - I*arctan(e^x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\tanh^2(x)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)**2)**(3/2),x)
[Out] Integral((tanh(x)**2 - 1)**(3/2), x)
```

Giac [A]
time = 0.42, size = 41, normalized size = 1.17

$$\frac{\sqrt{-e^{(2\,x)}}+\frac{1}{\sqrt{-e^{(2\,x)}}}}{\left(\sqrt{-e^{(2\,x)}}+\frac{1}{\sqrt{-e^{(2\,x)}}}\right)^{2}-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(-e^(2*x)) + 1/sqrt(-e^(2*x)))/((sqrt(-e^(2*x)) + 1/sqrt(-e^(2*x)))^2
- 4)
```

Mupad [B]
time = 1.17, size = 27, normalized size = 0.77

$$\frac{\ln\left(\tanh(x)+\sqrt{\tanh\left(x\right)^2-1}\right)}{2}-\frac{\tanh(x)\ \sqrt{\tanh\left(x\right)^2-1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)^2 - 1)^(3/2),x)
```

```
[Out] log(tanh(x) + (tanh(x)^2 - 1)^(1/2))/2 - (tanh(x)*(tanh(x)^2 - 1)^(1/2))/2
```

3.206 $\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx$

Optimal. Leaf size=11

$$\frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

[Out] tanh(x)/(sech(x)^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3738, 4207, 197}

$$\frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - Tanh[x]^2],x]

[Out] Tanh[x]/Sqrt[Sech[x]^2]

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}^2(x)}}\,dx$$
$$= \operatorname{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Tanh[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[Sech[x]^2]
```

Maple [A]
time = 0.30, size = 14, normalized size = 1.27

method	result	size
derivativedivides	$\frac{\tanh(x)}{\sqrt{1-\left(\tanh^2(x)\right)}}$	14
default	$\frac{\tanh(x)}{\sqrt{1-\left(\tanh^2(x)\right)}}$	14
risch	$\frac{e^{2x}}{2\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}-\frac{1}{2(1+e^{2x})\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1-tanh(x)^2)^(1/2)*tanh(x)
```

Maxima [A]
time = 0.48, size = 11, normalized size = 1.00

$$-\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

Fricas [A]
time = 0.38, size = 2, normalized size = 0.18

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sinh(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - tanh(x)**2), x)
```

Giac [A]
time = 0.41, size = 11, normalized size = 1.00

$$-\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

Mupad [B]
time = 0.14, size = 2, normalized size = 0.18

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - tanh(x)^2)^(1/2),x)
```

```
[Out] sinh(x)
```

3.207 $\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

```
[Out] tanh(x)/(-sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3738, 4207, 197}

$$\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Tanh[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[-Sech[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\operatorname{sech}^2(x)}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{(-1+x^2)^{3/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Tanh[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[-Sech[x]^2]
```

Maple [A]
time = 0.61, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\tanh(x)}{\sqrt{-1+\tanh^2(x)}}$	12
default	$\frac{\tanh(x)}{\sqrt{-1+\tanh^2(x)}}$	12
risch	$\frac{e^{2x}}{2\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})} - \frac{1}{2(1+e^{2x})\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] tanh(x)/(-1+tanh(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.47, size = 25, normalized size = 1.92

$$-\frac{e^{(-2x)}}{2\sqrt{-e^{(-2x)}}} + \frac{1}{2\sqrt{-e^{(-2x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*e^(-2*x)/sqrt(-e^(-2*x)) + 1/2/sqrt(-e^(-2*x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [A]
time = 0.22, size = 12, normalized size = 0.92

$$\frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+tanh(x)**2)**(1/2),x)
[Out] tanh(x)/sqrt(tanh(x)**2 - 1)
```

Giac [A]
time = 0.40, size = 21, normalized size = 1.62

$$-\frac{1}{2}\sqrt{-e^{(2x)}}-\frac{1}{2\sqrt{-e^{(2x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-e^(2*x)) - 1/2/sqrt(-e^(2*x))
```

Mupad [B]
time = 0.09, size = 14, normalized size = 1.08

$$-\frac{\sinh(2x)\sqrt{-\frac{1}{\cosh(x)^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tanh(x)^2 - 1)^(1/2),x)
[Out] -(sinh(2*x)*(-1/cosh(x)^2)^(1/2))/2
```

3.208 $\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=87

$$\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}+\frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2
)+1/3*(a-b)*(a+b*tanh(x)^2)^(3/2)/b^2-1/5*(a+b*tanh(x)^2)^(5/2)/b^2
```

Rubi [A]
time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 457, 90, 52, 65, 214}

$$-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}+\frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}-\sqrt{a+b\tanh^2(x)}+\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2] + ((a - b)*(a + b*Tanh[x]^2)^(3/2))/(3*b^2) - (a + b*Tanh[x]^2)^(5/2)/(
5*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{x^5\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(a-b)\sqrt{a+bx}}{b} + \frac{\sqrt{a+bx}}{1-x} - \frac{(a+bx)^{3/2}}{b}\right)\,dx, x, \tanh$$
$$= \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2} + \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+}}{1-}\right.$$
$$= -\sqrt{a+b\tanh^2(x)} + \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\tanh^2(x)\right)^{5/}}{5b^2}$$
$$= -\sqrt{a+b\tanh^2(x)} + \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\tanh^2(x)\right)^{5/}}{5b^2}$$
$$= \sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)} + \frac{(a-b)\,(a}{}$$

Mathematica [A]
time = 0.35, size = 85, normalized size = 0.98

$$\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) + \frac{\sqrt{a+b\tanh^2(x)}\,\left(2a^2-5ab-15b^2-b(a+5b)\tanh^2(x)-3b^2\tanh^4(x)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] + (Sqrt[a + b*Tanh[x
]^2]*(2*a^2 - 5*a*b - 15*b^2 - b*(a + 5*b)*Tanh[x]^2 - 3*b^2*Tanh[x]^4))/(1
5*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(71) = 142.
time = 0.93, size = 288, normalized size = 3.31

method	result

derivativedivides	$-\frac{(\tanh^2(x))(a+b(\tanh^2(x)))^{\frac{3}{2}}}{5b}+\frac{2a(a+b(\tanh^2(x)))^{\frac{3}{2}}}{15b^2}-\frac{(a+b(\tanh^2(x)))^{\frac{3}{2}}}{3b}-\frac{\sqrt{b(\tanh(x)-1)^2+2b}}{2}$
default	$-\frac{(\tanh^2(x))(a+b(\tanh^2(x)))^{\frac{3}{2}}}{5b}+\frac{2a(a+b(\tanh^2(x)))^{\frac{3}{2}}}{15b^2}-\frac{(a+b(\tanh^2(x)))^{\frac{3}{2}}}{3b}-\frac{\sqrt{b(\tanh(x)-1)^2+2b}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*tanh(x)^2*(a+b*tanh(x)^2)^(3/2)/b+2/15*a/b^2*(a+b*tanh(x)^2)^(3/2)-1/3
*(a+b*tanh(x)^2)^(3/2)/b-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/
2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b
)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh
(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2*(b*(1+tanh(x))^2-2*b*
(1+tanh(x))+a+b)^(1/2)+1/2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(
x))^2-2*b*(1+tanh(x))+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x
))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1982 vs. $2(71) = 142$.
time = 0.59, size = 4529, normalized size = 52.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b
^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*
(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*c
osh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2
```

*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh
(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cos
h(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*
b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^
2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5
+ 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*co
sh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(
2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*s
inh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(
x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x
)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)
^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3
+ 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3
 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15
*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a
*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*s
inh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh
(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^
2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*
sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a
^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*
(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b
 - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh
(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)
^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(b^2*c
osh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5
*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3
 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*
sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*
b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*c
osh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*c
osh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 +
28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b
^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^
3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*c
osh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)
^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sin
h(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((2*
a^2 - 6*a*b - 23*b^2)*cosh(x)^8 + 8*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)*sinh(x
)^7 + (2*a^2 - 6*a*b - 23*b^2)*sinh(x)^8 + 4*(2*a^2 - 5*a*b - 12*b^2)*cosh(
x)^6 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*si
nh(x)^6 + 8*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^3 + 3*(2*a^2 - 5*a*b - 12*b

```
^2)*cosh(x))*sinh(x)^5 + 2*(6*a^2 - 14*a*b - 49*b^2)*cosh(x)^4 + 2*(35*(2*a
^2 - 6*a*b - 23*b^2)*cosh(x)^4 + 30*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^2 + 6*
a^2 - 14*a*b - 49*b^2)*sinh(x)^4 + 8*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^5
+ 10*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^3 + (6*a^2 - 14*a*b - 49*b^2)*cosh(x)
)*sinh(x)^3 + 4*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^2 + 4*(7*(2*a^2 - 6*a*b -
23*b^2)*cosh(x)^6 + 15*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^4 + 3*(6*a^2 - 14*a
*b - 49*b^2)*cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*sinh(x)^2 + 2*a^2 - 6*a*b
- 23*b^2 + 8*((2*a^2 - 6*a*b - 23*b^2)*cosh(x)^7 + 3*(2*a^2 - 5*a*b - 12*b^
2)*cosh(x)^5 + (6*a^2 - 14*a*b - 49*b^2)*cosh(x)^3 + (2*a^2 - 5*a*b - 12*b^
2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^10 + 10*b^2*cosh(
x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)
*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^
7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(
x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 +
 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^
4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)
^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^...
```

Sympy [**A**]
time = 3.80, size = 97, normalized size = 1.11

$$-\frac{2\left(\frac{b^3\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^3(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}{10}+\frac{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\left(-\frac{ab}{2}+\frac{b^2}{2}\right)}{3}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**5,x)
```

```
[Out] -2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(sqrt(a + b*tanh(x)**2
)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(5/2)/10 + (a + b*
tanh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**3
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 980 vs. 2(71) = 142.
time = 1.10, size = 980, normalized size = 11.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) - 4/15*(15*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^9*(2*a + 3*b) + 15*(sq
rt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b))^8*(10*a + 9*b)*sqrt(a + b) + 20*(18*a^2 + 23*a*b + b^2)*(sqrt(a +
 b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b))^7 + 20*(30*a^2 - 7*a*b - 65*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) + 2*(330*a^
3 - 705*a^2*b - 1480*a*b^2 + 19*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + 10*(18*a^3 - 279*a^2*
b + 68*a*b^2 + 349*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) - 20*(30*a^4 + 81*a^3*b
- 149*a^2*b^2 - 245*a*b^3 + 19*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - 20*(42*a^4 - 33*a^3*b
- 139*a^2*b^2 + 69*a*b^3 + 325*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - 5*(90*a^5
- 121*a^4*b - 184*a^3*b^2 + 658*a^2*b^3 + 166*a*b^4 - 1233*b^5)*(sqrt(a + b
)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
) - (90*a^5 - 215*a^4*b + 240*a^3*b^2 + 638*a^2*b^3 - 2034*a*b^4 + 1713*b^5
)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^5
```

Mupad [B]
time = 9.60, size = 119, normalized size = 1.37

$$-\frac{\left(b\tanh(x)^2+a\right)^{5/2}}{5\,b^2}-2\,\mathrm{atan}\left(\frac{2\,\sqrt{b\tanh\left(x\right)^2+a}\,\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\,\sqrt{-\frac{a}{4}-\frac{b}{4}}-\sqrt{b\tanh\left(x\right)^2+a}\,\left(\left(a+b\right)\,\left(\frac{a+b}{b^2}-\frac{2\,a}{b^2}\right)+\frac{a^2}{b^2}\right)-\left(\frac{a+b}{3\,b^2}-\frac{2\,a}{3\,b^2}\right)\,\left(b\tanh(x)^2+a\right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] - (a + b*tanh(x)^2)^(5/2)/(5*b^2) - 2*atan((2*(a + b*tanh(x)^2)^(1/2)*(- a/
4 - b/4)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2) - (a + b*tanh(x)^2)^(1/2)*((a
+ b)*((a + b)/b^2 - (2*a)/b^2) + a^2/b^2) - ((a + b)/(3*b^2) - (2*a)/(3*b^2
))*(a + b*tanh(x)^2)^(3/2)
```

3.209 $\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=121

$$\frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{(a+4b)\tanh(x)\sqrt{\cdot}}{8b}$$

```
[Out] 1/8*(a^2-4*a*b-8*b^2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2
)+arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/8*(a+4*b
)*(a+b*tanh(x)^2)^(1/2)*tanh(x)/b-1/4*(a+b*tanh(x)^2)^(1/2)*tanh(x)^3
```

Rubi [A]
time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3751, 489, 596, 537, 223, 212, 385}

$$\frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}}-\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(8
*b^(3/2)) + Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]
] - ((a + 4*b)*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/(8*b) - (Tanh[x]^3*Sqrt[a + b
*Tanh[x]^2])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{x^2(3a+(a+4b)x^2)}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \text{t}\right.$$

$$= -\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)} + \text{—} \quad \text{Su}$$

$$= -\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)} + (a$$

$$= -\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)} + (a$$

$$= \frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 4.67, size = 242, normalized size = 2.00

$$\frac{\left(4\sqrt{2}a(a+4b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)-32\sqrt{2}ab\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\Pi\left(\frac{b}{a+b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)-(3a^2+9ab+2b^2+4(a^2+4ab-2b^2)\cosh(2x)+(a^2+7ab+6b^2)\cosh(4x))\text{sech}^4(x)\right)\tanh(x)}{32\sqrt{2}b\sqrt{(a-b+(a+b)\cosh(2x))\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((4*Sqrt[2]*a*(a + 4*b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Ell
ipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]
- 32*Sqrt[2]*a*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi
[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]]
, 1] - (3*a^2 + 9*a*b + 2*b^2 + 4*(a^2 + 4*a*b - 2*b^2)*Cosh[2*x] + (a^2 +
7*a*b + 6*b^2)*Cosh[4*x])*Sech[x]^4)*Tanh[x])/(32*Sqrt[2]*b*Sqrt[(a - b + (
a + b)*Cosh[2*x])*Sech[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(99) = 198.
time = 0.80, size = 338, normalized size = 2.79

method	result
derivativedivides	$-\frac{\tanh(x)(a+b(\tanh^2(x)))^{\frac{3}{2}}}{4b}+\frac{a\left(\frac{\sqrt{a+b(\tanh^2(x))}\tanh(x)}{2}+\frac{a\ln\left(\sqrt{b}\tanh(x)+\sqrt{a+b(\tanh^2(x}{2\sqrt{b}}\right.}{4b}$
default	$-\frac{\tanh(x)(a+b(\tanh^2(x)))^{\frac{3}{2}}}{4b}+\frac{a\left(\frac{\sqrt{a+b(\tanh^2(x))}\tanh(x)}{2}+\frac{a\ln\left(\sqrt{b}\tanh(x)+\sqrt{a+b(\tanh^2(x}{2\sqrt{b}}\right.}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)/b+1/4*a/b*(1/2*(a+b*tanh(x)^2)^(1/2)*tan
h(x)+1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2)))-1/2*(a+b*tanh
(x)^2)^(1/2)*tanh(x)-1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2)
)-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x
)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/2
)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1
)+a+b)^(1/2))/(tanh(x)-1))+1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-
1/2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a
+b)^(1/2))-1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+
tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. 2(99) = 198.
time = 0.70, size = 9360, normalized size = 77.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*c
osh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x
)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh
(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 +
9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5
 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*
cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2
*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)
*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sin
h(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh
(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(
x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3
- a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b
^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 -
15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a
*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)
*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*si
nh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*
b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2
)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a
^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b +
 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(co
sh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(
x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((a^2 -
 4*a*b - 8*b^2)*cosh(x)^8 + 8*(a^2 - 4*a*b - 8*b^2)*cosh(x)*sinh(x)^7 + (a^
2 - 4*a*b - 8*b^2)*sinh(x)^8 + 4*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 4*(7*(a^
2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*sinh(x)^6 + 8*(7*(a^2 -
 4*a*b - 8*b^2)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x)^5 + 6*
(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 2*(35*(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 3
0*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + 3*a^2 - 12*a*b - 24*b^2)*sinh(x)^4 + 8*
(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 10*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + 3
*(a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - 4*a*b - 8*b^2)*cosh(x)
^2 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 15*(a^2 - 4*a*b - 8*b^2)*cosh(x
)^4 + 9*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*sinh(x)^2 +
a^2 - 4*a*b - 8*b^2 + 8*((a^2 - 4*a*b - 8*b^2)*cosh(x)^7 + 3*(a^2 - 4*a*b -
 8*b^2)*cosh(x)^5 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + (a^2 - 4*a*b - 8*b^
2)*cosh(x))*sinh(x))*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x
)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*
cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (
a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sin
```

```
h(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(
x))*sinh(x) + 1)) + 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x
)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)
^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 +
30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5
+ 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2
*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*
b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cos
h(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a +
b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 2*sqrt(2)*((a*b + 6*b^2)*cosh(x)^6 + 6*(a*b + 6*b^2)*cosh(x)
*sinh(x)^5 + (a*b + 6*b^2)*sinh(x)^6 + (a*b - 2*b^2)*cosh(x)^4 + (15*(a*b +
 6*b^2)*cosh(x)^2 + a*b - 2*b^2)*sinh(x)^4 + 4*(5*(a*b + 6*b^2)*cosh(x)^3 +
 (a*b - 2*b^2)*cosh(x))*sinh(x)^3 - (a*b - 2*b^2)*cosh(x)^2 + (15*(a*b + 6*
b^2)*cosh(x)^4 + 6*(a*b - 2*b^2)*cosh(x)^2 - a*b + 2*b^2)*sinh(x)^2 - a*b -
 6*b^2 + 2*(3*(a*b + 6*b^2)*cosh(x)^5 + 2*(a*b - 2*b^2)*cosh(x)^3 - (a*b -
2*b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8 + 8*b^2*cos
h(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tanh^2(x)}\,\tanh^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**4,x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(99) = 198.
time = 1.00, size = 938, normalized size = 7.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
```

```
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) + 1/4*(a^2 - 4*a*b - 8*b^2)*arctan(-1/2*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b) - 1/2*((a^2 + 12*a*b + 16*b^2)*
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^7 + (7*a^2 + 52*a*b + 16*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^
(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) + (21
*a^3 + 109*a^2*b + 28*a*b^2 - 48*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + (35*a^3 + 115*a^2*b
- 156*a*b^2 - 176*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + (35*a^4 + 130*a^3*b - 3
17*a^2*b^2 - 156*a*b^3 + 304*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 + (21*a^4 + 94*a^3*b - 379
*a^2*b^2 + 476*a*b^3 + 48*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^
(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + (7*a^5 + 53*a^4
*b - 135*a^3*b^2 + 271*a^2*b^3 - 140*a*b^4 - 272*b^5)*(sqrt(a + b)*e^(2*x)
- sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (a^5 +
 11*a^4*b - 17*a^3*b^2 + 65*a^2*b^3 - 116*a*b^4 + 112*b^5)*sqrt(a + b))/(((
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x
) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^4*b)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^4 \sqrt{b\tanh(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^4*(a + b*tanh(x)^2)^(1/2), x)
```

3.210 $\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=63

$$\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2
)-1/3*(a+b*tanh(x)^2)^(3/2)/b
```

Rubi [A]
time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 457, 81, 52, 65, 214}

$$-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b}-\sqrt{a+b\tanh^2(x)}+\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2] - (a + b*Tanh[x]^2)^(3/2)/(3*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{x^3\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b} + \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right)$$

$$= -\sqrt{a+b\tanh^2(x)} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b} + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{}{(1-x)}\right.$$

$$= -\sqrt{a+b\tanh^2(x)} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx\right.}{b}$$

$$= \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)} - \frac{(a+b\tan}{3}$$

Mathematica [A]
time = 0.12, size = 60, normalized size = 0.95

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{\sqrt{a+b\tanh^2(x)}\left(a+3b+b\tanh^2(x)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh[x
]^2]*(a + 3*b + b*Tanh[x]^2))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(51) = 102$.
time = 0.68, size = 253, normalized size = 4.02

method	result
derivativedivides	$-\frac{\left(a+b\left(\tanh^2(x)\right)\right)^{\frac{3}{2}}}{3b} - \frac{\sqrt{b\left(\tanh(x)-1\right)^2+2b\left(\tanh(x)-1\right)+a+b}}{2} - \frac{\sqrt{b}\,\ln\left(\frac{b(\tanh(x)-1}{\sqrt{b}}\right.}{}$

default	$-\frac{(a+b(\tanh^2(x)))^{\frac{3}{2}}}{3b}-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-\frac{\sqrt{b}\ \ln\left(\frac{b(\tanh(x)-1)+}{\sqrt{b}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a+b*tanh(x)^2)^(3/2)/b-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2
)-1/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)
+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(
tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2*(b*(1+tanh(x))^2-
2*b*(1+tanh(x))+a+b)^(1/2)+1/2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+t
anh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+ta
nh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(51) = 102$.
time = 0.52, size = 2329, normalized size = 36.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4
 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x
)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(
b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(((a^3
 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sin
h(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*co
sh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a
^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x
```

```
)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)
^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3
*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*co
sh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 +
 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a
^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2
 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2
+ 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2
*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*c
osh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a
^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*si
nh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
 + 3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3
*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3
+ 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*co
sh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(-((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^
2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*co
sh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2)) - 4*sqrt(2)*((a + 4*b)*cosh(x)^4 + 4*(a + 4*b)*cosh(x)*sinh(x)^3 +
(a + 4*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*(a + 4*b)*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*((a + 4*b)*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a
+ 4*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sin
h(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3
 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^
2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)
, -1/6*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^
4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(
x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*
(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(
sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a -
 b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2
+ a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*
(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 3*(b*cosh(
x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)
^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)
^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b
```

```
*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*((a + 4*b)*cosh(x)^
4 + 4*(a + 4*b)*cosh(x)*sinh(x)^3 + (a + 4*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(
x)^2 + 2*(3*(a + 4*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 4*b)*cosh(x)
^3 + (a + 2*b)*cosh(x))*sinh(x) + a + 4*b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(
x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)
^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)
^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh...
```

Sympy [**A**]
time = 2.49, size = 71, normalized size = 1.13

$$-\frac{2\left(\frac{b^{2}\sqrt{a+b\tanh^{2}(x)}}{2}+\frac{b^{2}(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^{2}(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^{2}(x)\right)^{\frac{3}{2}}}{6}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**3,x)
```

```
[Out] -2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a + b*tanh(x)**2
)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/2)/6)/b**2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. 2(51) = 102.
time = 0.82, size = 630, normalized size = 10.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) - 4/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
```

```
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5*(a + 2*b) + 3*(sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
 b))^4*(3*a + 2*b)*sqrt(a + b) + 2*(3*a^2 - 3*a*b - 10*b^2)*(sqrt(a + b)*e^
(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3
- 6*(a^2 + 3*a*b + 6*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - 3*(3*a^3 + 4*a^2*b -
 9*a*b^2 - 26*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b)) - (3*a^3 - 17*a*b^2 + 34*b^3)*sqrt(a + b))/
((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^3
```

Mupad [B]
time = 3.47, size = 66, normalized size = 1.05

$$-\sqrt{b\tanh\left(x\right)^2+a}-\frac{{\left(b\,\mathrm{tanh}(x)^2+a\right)}^{3/2}}{3\,b}-2\,\mathrm{atan}\left(\frac{2\,\sqrt{b\tanh\left(x\right)^2+a}\,\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\,\sqrt{-\frac{a}{4}-\frac{b}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] - (a + b*tanh(x)^2)^(1/2) - (a + b*tanh(x)^2)^(3/2)/(3*b) - 2*atan((2*(a +
b*tanh(x)^2)^(1/2)*(- a/4 - b/4)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2)
```

3.211 $\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] -1/2*(a+2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)+arctanh
((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/2*(a+b*tanh(x)^2)
^(1/2)*tanh(x)
```

Rubi [A]
time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 489, 537, 223, 212, 385}

$$-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}}+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -1/2*((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b] +
 Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - (Tanh[x
]*Sqrt[a + b*Tanh[x]^2])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a+(a+2b)x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \text{tan}$$

$$= -\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}(-a-2b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \text{t}$$

$$= -\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}(-a-2b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a}}$$

$$= -\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\text{ta}}{\sqrt{a+b\tan}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.36, size = 193, normalized size = 2.27

$$\frac{\left(\sqrt{2}\,a\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)-2\sqrt{2}\,a\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\,\Pi\left(\frac{b}{a+b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)-(a-b+(a+b)\cosh(2x))\text{sech}^2(x)\right)\tanh(x)}{2\sqrt{2}\sqrt{(a-b+(a+b)\cosh(2x))\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcSi
n[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2]*
a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b), Arc
Sin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - (a - b +
 (a + b)*Cosh[2*x])*Sech[x]^2)*Tanh[x])/(2*Sqrt[2]*Sqrt[(a - b + (a + b)*Co
sh[2*x])*Sech[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(67) = 134.
time = 0.64, size = 276, normalized size = 3.25

method	result

derivativedivides	$-\frac{\sqrt{a+b\left(\tanh^2(x)\right)}\,\tanh(x)}{2}-\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\left(\tanh^2(x)\right)}\right)}{2\sqrt{b}}-\frac{\sqrt{b\left(\tanh(x)\right.}}{}$
default	$-\frac{\sqrt{a+b\left(\tanh^2(x)\right)}\,\tanh(x)}{2}-\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\left(\tanh^2(x)\right)}\right)}{2\sqrt{b}}-\frac{\sqrt{b\left(\tanh(x)\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)-1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*ta
nh(x)^2)^(1/2))-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)
*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+
1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+
2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x
))+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b
*(1+tanh(x))+a+b)^(1/2))-1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b
)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(67) = 134$.
time = 0.54, size = 4825, normalized size = 56.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 +
2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*
sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)
```

```
^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b
^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3
- 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*co
sh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*
(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(
a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)
^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 +
2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^
2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2
)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^
4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x
))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2
*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*
cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(
a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2
*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^
2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^
4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*s
inh(x)^5 + sinh(x)^6)) + ((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)
^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2
 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x)
 + a + 2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)
^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2
 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*
cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(
x) + 1)) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)
^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x)
+ b)*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
+ b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a
+ b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)
*sinh(x) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh
(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)
^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), 1/4*(2*((a + 2*b)*cosh(x)^4
+ 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)
^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3
 + (a + 2*b)*cosh(x))*sinh(x) + a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a
```

```
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a +
 b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)
^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x
)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*
cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)
^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2
 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x
)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5
+ (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 +
 a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 +
4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*
(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b
^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 +
6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x
)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4
 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*co
sh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tanh^2(x)}\,\tanh^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**2,x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(67) = 134.
time = 0.80, size = 554, normalized size = 6.52

$$-\frac{(a+2b)\arctan\left(-\frac{\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right) - \frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right) + \frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right) - \frac{2\left(\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^{3}(a+2b)+\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^{2}(3a-2b)\sqrt{a+b}+(3a^2-3ab-2b^2)\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)+(a^2-ab+2b^2)\sqrt{a+b}\right)}{\left(\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^{2}+2\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}+a-3b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="giac")
```

```
[Out] -(a + 2*b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*
sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*
a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/2*sqr
t(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs(-sqr
```

```
t(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) - sqrt(a + b))) - 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4
*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(a + 2*b) + (sqrt(a + b)*e^(2*x
) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*(3*a
 - 2*b)*sqrt(a + b) + (3*a^2 - 3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a
*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (a^2 - a*b + 2
*b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^
2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^2 \sqrt{b\tanh{(x)}^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^2*(a + b*tanh(x)^2)^(1/2), x)
```

3.212 $\int \tanh(x)\sqrt{a+b\tanh^2(x)}\, dx$

Optimal. Leaf size=44

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2
)
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3751, 455, 52, 65, 214}

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1-x^2}\,dx,x,\tanh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx,x,\tanh^2(x)\right)\\ &= -\sqrt{a+b\tanh^2(x)}+\frac{1}{2}(a+b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx,x,\tanh^2(x)\right)\\ &= -\sqrt{a+b\tanh^2(x)}+\frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\tanh^2(x)}\right)}{b}\\ &= \sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 1.00

$$\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^2],x]
```

[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]
^2]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(36) = 72$.
time = 0.69, size = 238, normalized size = 5.41

method	result
derivativedivides	$-\frac{\sqrt{b\left(1+\tanh\left(x\right)\right)^{2}-2b\left(1+\tanh\left(x\right)\right)+a+b}}{2}+\frac{\sqrt{b}\,\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b\left(1+\tanh\right.}\right.}{}$
default	$-\frac{\sqrt{b\left(1+\tanh\left(x\right)\right)^{2}-2b\left(1+\tanh\left(x\right)\right)+a+b}}{2}+\frac{\sqrt{b}\,\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b\left(1+\tanh\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x),x,method=_RETURNVERBOSE)

[Out] -1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/2*b^(1/2)*ln((b*(1+tanh(
x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))+1/2*(a+b)^(1/2)
*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))
+a+b)^(1/2))/(1+tanh(x)))-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1
/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+
b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tan
h(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(36) = 72$.
time = 0.43, size = 1543, normalized size = 35.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="fricas")

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*log(((a^3
 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sin
h(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*co
sh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a
^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x
)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)
^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3
*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*co
sh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 +
 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a
^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2
 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2
+ 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2
*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*c
osh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a
^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*si
nh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
 + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*log(-((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*c
osh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2)) - 4*sqrt(2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1), -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sq
rt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2
+ a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^
2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*co
sh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2
*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh
(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*arctan
(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(
((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b
)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)
^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)]
```

Sympy [A]
time = 1.27, size = 51, normalized size = 1.16

$$-\frac{2\left(\frac{b\sqrt{a+b\tanh^{2}(x)}}{2}+\frac{b(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^{2}(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x),x)
```

```
[Out] -2*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*tanh(x)**2)/sqrt
(-a - b))/(2*sqrt(-a - b)))/b
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(36) = 72.
time = 0.61, size = 349, normalized size = 7.93

$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)-\frac{4\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)b-\sqrt{a+b}b\right)}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^{2}+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}+a-3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) - 4*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*b - sqrt(a + b)*b)/((sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
 b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)
```

Mupad [B]
time = 1.69, size = 51, normalized size = 1.16

$$-\sqrt{b\tanh(x)^{2}+a}-2\operatorname{atan}\left(\frac{2\sqrt{b\tanh(x)^{2}+a}\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\sqrt{-\frac{a}{4}-\frac{b}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] - (a + b*tanh(x)^2)^(1/2) - 2*atan((2*(a + b*tanh(x)^2)^(1/2)*(- a/4 - b/4)
^(1/2))/(a + b))*(- a/4 - b/4)^(1/2)
```

3.213 $\int \sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=60

$$-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

```
[Out] -arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*b^(1/2)+arctanh((a+b)^(1/2)
*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3742, 399, 223, 212, 385}

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + Sqrt[a + b]*A
rcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
```

```
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\ &= -\left((-a-b)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\right) - b\text{Subst}\left(\int \frac{1}{\sqrt{a+b\right.\\ &= -\left((-a-b)\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right) - b\text{Subst}\left(\int \frac{}{1\right.\\ &= -\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\end{aligned}$$

Mathematica [A]
time = 0.22, size = 81, normalized size = 1.35

$$\sqrt{-a-b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\text{sech}^2(x)+\tanh(x)\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)+\sqrt{b}\,\log\left(-\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh^2(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[-a - b]*ArcTan[(Sqrt[b]*Sech[x]^2 + Tanh[x]*Sqrt[a + b*Tanh[x]^2])/Sqr
t[-a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Tanh[x]) + Sqrt[a + b*Tanh[x]^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(48) = 96$.
time = 0.76, size = 238, normalized size = 3.97

method	result
derivativedivides	$\frac{\sqrt{b\left(1+\tanh\left(x\right)\right)^{2}-2b\left(1+\tanh\left(x\right)\right)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b\left(1+\tanh\left(x\right.\right.}\right.}{2}$
default	$\frac{\sqrt{b\left(1+\tanh\left(x\right)\right)^{2}-2b\left(1+\tanh\left(x\right)\right)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b\left(1+\tanh\left(x\right.\right.}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(1+tanh(x
))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))-1/2*(a+b)^(1/2)*
ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+
a+b)^(1/2))/(1+tanh(x)))-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/
2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b
)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh
(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(48) = 96$.
time = 0.47, size = 3443, normalized size = 57.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*si
nh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2
+ 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(
x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^
```

```
3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3
- 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 -
 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*si
nh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)
^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3
*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cos
h(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*c
osh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 1
8*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3
*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^
2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3
*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cos
h(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh
(x)*sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a
+ 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*
(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*
cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*si
nh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh
(x)^3 + cosh(x))*sinh(x) + 1)) + 1/4*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*co
sh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), sqrt(
-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 1/4*
sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)
^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b
^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3
- 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*co
sh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*
(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(
a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)
^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 +
2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^
2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2
)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^
```

```
4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x
))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2
*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*
cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(
a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2
*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^
2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^
4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*s
inh(x)^5 + sinh(x)^6)) + 1/4*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)
*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*s
inh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), -1/2*sqrt(-a
- b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a -
b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tanh^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(48) = 96.
time = 0.56, size = 253, normalized size = 4.22

$$-\frac{2\,b\arctan\left(-\frac{\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}}{2\,\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{1}{2}\,\sqrt{a+b}\,\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right) - \frac{1}{2}\,\sqrt{a+b}\,\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right) + \frac{1}{2}\,\sqrt{a+b}\,\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*sqrt(a
 + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/2*sqrt(a +
b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs(-sqrt(a +
```

```
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
) - sqrt(a + b)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\tanh(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(x)^2)^(1/2),x)
[Out] int((a + b*tanh(x)^2)^(1/2), x)
```

3.214 $\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=56

$$-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

```
[Out] -arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))*a^(1/2)+arctanh((a+b*tanh(x)^2)^(1/
2)/(a+b)^(1/2))*(a+b)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3751, 457, 85, 65, 214}

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + Sqrt[a + b]*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a + b]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(1-x^2)}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(1-x)x}\,dx, x, \tanh^2(x)\right)\\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a
\end{aligned}$$

$$\begin{aligned}
&= \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx,\right.}{b}\\
&= -\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right.
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 1.00

$$-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + Sqrt[a + b]*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a + b]]
```

Maple [F]
time = 1.78, size = 0, normalized size = 0.00

$$\int \coth(x) \sqrt{a + b(\tanh^2(x))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(44) = 88$.
time = 0.45, size = 3467, normalized size = 61.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sin
h(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 +
 a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x
)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3
)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 +
 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 +
10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sin
h(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^
2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 +
3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sq
rt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh
(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*co
sh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18
```

```
*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*
a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*s
qrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3
 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 +
3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a
 + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(
3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*c
osh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(
x)^3 - cosh(x))*sinh(x) + 1)) + 1/4*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*co
sh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), sqrt(
-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 1/4*
sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^
7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*
b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 +
 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cos
h(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(
2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2
*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^
3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2
*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2
*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)
*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4
 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x)
)*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*
cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*c
osh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^
2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2
*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4
*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*si
```

```
nh(x)^5 + sinh(x)^6)) + 1/4*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)
*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*s
inh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)), -1/2*sqrt(-a
- b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a +
b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tanh^2{(x)}}\,\coth{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(44) =$ 88.
time = 0.58, size = 255, normalized size = 4.55

$$\frac{2\,a\arctan\left(-\frac{\sqrt{a+b}\,e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{1}{2}\,\sqrt{a+b}\,\log\left(\left|-\left(\sqrt{a+b}\,e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)+\frac{1}{2}\,\sqrt{a+b}\,\log\left(\left|-\sqrt{a+b}\,e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)-\frac{1}{2}\,\sqrt{a+b}\,\log\left(\left|-\sqrt{a+b}\,e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2*sqrt(a
+ b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/2*sqrt(a + b
)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)\,\sqrt{b\,{\tanh\left(x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)*(a + b*tanh(x)^2)^(1/2), x)
```

3.215 $\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=48

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\coth(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-coth(x)*(a+b
*tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3751, 486, 12, 385, 212}

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\coth(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - Coth[x]*
Sqrt[a + b*Tanh[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
```

```
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= -\coth(x)\sqrt{a+b\tanh^2(x)} + \text{Subst}\left(\int \frac{a+b}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh$$

$$= -\coth(x)\sqrt{a+b\tanh^2(x)} + (a+b)\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx,$$

$$= -\coth(x)\sqrt{a+b\tanh^2(x)} + (a+b)\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x,$$

$$= \sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \coth(x)\sqrt{a+b\tanh^2(x)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 42, normalized size = 0.88

$$-\coth(x)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{(a+b)\tanh^2(x)}{a+b\tanh^2(x)}\right)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, ((a + b)*Tanh[x]^2)/(a + b*Tanh[x
]^2)]*Sqrt[a + b*Tanh[x]^2])
```

Maple [F]
time = 1.49, size = 0, normalized size = 0.00

$$\int \left(\coth^2\left(x\right)\right)\sqrt{a+b\left(\tanh^2\left(x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(40) = 80$.
time = 0.39, size = 1539, normalized size = 32.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*log(-((a*
b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*si
nh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)
*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*c
osh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2
+ b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x
)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b +
3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*c
osh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*
b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^
2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 -
```

```
 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*
a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*s
inh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)
) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*log(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sin
h(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*c
osh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2)) - 4*sqrt(2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 - 1), -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sq
rt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2
- a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*
b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*co
sh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2
*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh
(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*arctan
(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(
((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b
)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)
^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^{2}{\left(x \right)}} \coth^{2}{\left(x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(40) = 80.
time = 0.62, size = 348, normalized size = 7.25

$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)+\frac{4\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)a+\sqrt{a+b}a\right)}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^{2}-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}-3a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) + 4*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*a + sqrt(a + b)*a)/((sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
 b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)^2 \sqrt{b \tanh(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2*(a + b*tanh(x)^2)^(1/2), x)
```

3.216 $\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=83

$$-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2}$$

```
[Out] -1/2*(2*a+b)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(1/2)+arctanh((a+b*ta
nh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-1/2*coth(x)^2*(a+b*tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 457, 101, 162, 65, 214}

$$-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -1/2*((2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + Sqrt[a +
b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^3(1-x^2)}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(1-x)x^2}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{\frac{1}{2}(2a+b)+\frac{bx}{2}}{(1-x)x\sqrt{a+bx}}\,dx, x, \text{ta}\right.$$

$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,d\right.$$

$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\right.}{b}$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\,\text{ta}}}{\sqrt{a+}}\right.$$

Mathematica [A]
time = 0.16, size = 83, normalized size = 1.00

$$-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -1/2*((2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2/Sqrt[a]])/Sqrt[a] + Sqrt[a +
b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/2
```

Maple [F]
time = 1.58, size = 0, normalized size = 0.00

$$\int \left(\coth^3(x)\right)\sqrt{a+b\left(\tanh^2(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 907 vs. $2(65) = 130$.
time = 0.51, size = 4891, normalized size = 58.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 +
2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)*
sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^
7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*
b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 +
 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cos
h(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(
2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2
*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^
3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2
*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2
*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)
*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4
 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x)
)*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*
cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*c
osh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^
2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2
*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4
*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*si
nh(x)^5 + sinh(x)^6)) + ((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^
3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2
- 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x)
```

```
+ 2*a + b)*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^
3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2
+ 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*c
osh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2
*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x
) + 1)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^
2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) +
 a)*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
+ b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a
+ b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)
*sinh(x) + a*sinh(x)^2 + a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)
^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a), 1/4*(2*((2*a + b)*cosh(x)^4
+ 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)
^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3
 - (2*a + b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a +
 b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)
^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x
)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*
cosh(x)^3 - a*cosh(x))*sinh(x) + a)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^
8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3
+ a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)
^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 +
 (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 +
6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4
*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(
2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^
3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 +
 b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)
^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4
+ 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cos
h(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 +...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^2(x)} \coth^3(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3*(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(65) = 130.
time = 0.75, size = 557, normalized size = 6.71

$$\frac{(2a+b)\arctan\left(-\frac{\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right) + \frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right) - \frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right) + \frac{2\left(\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^3(2a+b) + \left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^2(2a-3b)\sqrt{a+b} - (2a^2+3ab-3b^2)\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right) - (2a^2-ab+b^2)\sqrt{a+b}\right)}{\left(\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^2 - 2\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)\sqrt{a+b} - 3a + b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a + b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2*s
qrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/2*sqrt
(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs(-sqrt
(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b) - sqrt(a + b))) + 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(2*a + b) + (sqrt(a + b)*e^(2*x)
 - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*(2*a
- 3*b)*sqrt(a + b) - (2*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*
e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (2*a^2 - a*b +
b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^3 \sqrt{b \tanh(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3*(a + b*tanh(x)^2)^(1/2), x)
```

3.217 $\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=78

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a}-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/3*(3*a+b)*
coth(x)*(a+b*tanh(x)^2)^(1/2)/a-1/3*coth(x)^3*(a+b*tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 486, 597, 12, 385, 212}

$$\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - ((3*a +
b)*Coth[x]*Sqrt[a + b*Tanh[x]^2])/(3*a) - (Coth[x]^3*Sqrt[a + b*Tanh[x]^2])
/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4(1-x^2)}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{3}\text{Subst}\left(\int \frac{3a+b+2bx^2}{x^2(1-x^2)\sqrt{a+bx^2}}\,dx,\right.$$

$$= -\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} - $$

$$= -\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} - ($$

$$= -\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)} - ($$

$$= \sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2}}{3a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.23, size = 161, normalized size = 2.06

$$\frac{\cosh^4(x)\coth^3(x)\left(1+\frac{b\tanh^2(x)}{a}\right)\left(-\frac{\operatorname{sech}^4(x)\left(\operatorname{ArcSin}\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}+\sqrt{\cosh^2(x)+\frac{b\sinh^2(x)}{a}}\right)(a-2b\tanh^2(x))}{\sqrt{\cosh^2(x)+\frac{b\sinh^2(x)}{a}}}-\frac{4(a+b)\,{}_2F_1\left(2,2;\frac{3}{2};-\frac{(a+b)\sinh^2(x)}{a}\right)(a\tanh(x)+b\tanh^3(x))^2}{a^2}\right)}{3\sqrt{a+b\tanh^2(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] (Cosh[x]^4*Coth[x]^3*(1 + (b*Tanh[x]^2)/a)*(-((Sech[x]^4*(ArcSin[Sqrt[-(((a
 + b)*Sinh[x]^2)/a)]]*Sqrt[-(((a + b)*Sinh[x]^2)/a)] + Sqrt[Cosh[x]^2 + (b*
Sinh[x]^2)/a])*(a - 2*b*Tanh[x]^2))/Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a]) - (4
*(a + b)*Hypergeometric2F1[2, 2, 3/2, -(((a + b)*Sinh[x]^2)/a)]*(a*Tanh[x]
+ b*Tanh[x]^3)^2)/a^2))/(3*Sqrt[a + b*Tanh[x]^2])
```

Maple [F]
time = 1.66, size = 0, normalized size = 0.00

$$\int \left(\coth^4(x)\right)\sqrt{a+b\left(\tanh^2(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(64) = 128$.
time = 0.47, size = 2355, normalized size = 30.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4
 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x
)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(
a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a + b)*log(-((a*
b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*si
nh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)
*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*c
osh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2
+ b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x
)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b +
3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*c
osh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*
b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^
2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 -
 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*
a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^
```

3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*s inh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh (x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*c osh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a + b)*log(((a + b) *cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^ 2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)* sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh (x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*co sh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh (x)^2)) - 4*sqrt(2)*((4*a + b)*cosh(x)^4 + 4*(4*a + b)*cosh(x)*sinh(x)^3 + (4*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*a + b)*cosh(x)^2 - 2* a - b)*sinh(x)^2 + 4*((4*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 4* a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2* cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sin h(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^ 2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a) , -1/6*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^ 4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6* (a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos h(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*s inh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2* (a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + 3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x) ^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x) ^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 - 2*a *cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*((4*a + b)*cosh(x)^ 4 + 4*(4*a + b)*cosh(x)*sinh(x)^3 + (4*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((4*a + b)*cosh(x) ^3 - (2*a + b)*cosh(x))*sinh(x) + 4*a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x) ^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)

```
^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tanh^2(x)}\,\coth^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4*(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(64) = 128.
time = 0.82, size = 629, normalized size = 8.06

$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)+\frac{4\left(3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^5(2a+b)-3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^4(2a+3b)\sqrt{a+b}-2(10a^2+3ab-3b^2)\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3+6(6a^2+3ab+b^2)\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2\sqrt{a+b}+3(26a^3+9a^2b-4ab^2-3b^3)\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)+(34a^3-17a^2b+3b^3)\sqrt{a+b}\right)}{3\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}-3a+b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) + 4/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5*(2*a + b) - 3*(sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
 b))^4*(2*a + 3*b)*sqrt(a + b) - 2*(10*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^
(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3
+ 6*(6*a^2 + 3*a*b + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + 3*(26*a^3 + 9*a^2*b
- 4*a*b^2 - 3*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b)) + (34*a^3 - 17*a^2*b + 3*b^3)*sqrt(a + b))/
((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}(x)^4\,\sqrt{b\,\mathrm{tanh}\left(x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^4*(a + b*tanh(x)^2)^(1/2), x)
```

3.218 $\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal. Leaf size=121

$$-\frac{(8a^2+4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}}+\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{(4a+b)\coth^2(x$$

```
[Out] -1/8*(8*a^2+4*a*b-b^2)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arcta
nh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-1/8*(4*a+b)*coth(x)^2*(a+
b*tanh(x)^2)^(1/2)/a-1/4*coth(x)^4*(a+b*tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3751, 457, 101, 156, 162, 65, 214}

$$-\frac{(8a^2+4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}}+\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)}-\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -1/8*((8*a^2 + 4*a*b - b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/a^(3/2)
 + Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - ((4*a + b)*Coth
[x]^2*Sqrt[a + b*Tanh[x]^2])/(8*a) - (Coth[x]^4*Sqrt[a + b*Tanh[x]^2])/4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^5(1-x^2)}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(1-x)x^3}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{1}{2}(4a+b)+\frac{3bx}{2}}{(1-x)x^2\sqrt{a+bx}}\,dx, x, \text{t}\right.$$
$$= -\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} - $$
$$= -\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} - $$
$$= -\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)} + $$
$$= -\frac{(8a^2+4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}} + \sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{}}{}\right.$$

Mathematica [A]
time = 0.45, size = 111, normalized size = 0.92

$$\frac{(-8a^2-4ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\coth^2(x)\left(4a+b+2a\coth^2(x)\right)\sqrt{a+b\tanh^2(x)}\right)}{8a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((-8*a^2 - 4*a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(8
*a*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Coth[x]^2*(4*a
+ b + 2*a*Coth[x]^2)*Sqrt[a + b*Tanh[x]^2]))/(8*a^(3/2))
```

Maple [F]
time = 1.57, size = 0, normalized size = 0.00

$$\int \left(\coth^5(x)\right)\sqrt{a+b\left(\tanh^2(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(99) = 198$.
time = 0.67, size = 9642, normalized size = 79.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*c
osh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2
*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x
)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh
(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 +
9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5
 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*c
osh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*
(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*
sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh
(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(
x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x
)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3
 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^
3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 1
5*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b -
a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*
sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sin
h(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b
^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)
*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*
```

```
a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*
b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cos
h(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x
)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((8*a^2
+ 4*a*b - b^2)*cosh(x)^8 + 8*(8*a^2 + 4*a*b - b^2)*cosh(x)*sinh(x)^7 + (8*a
^2 + 4*a*b - b^2)*sinh(x)^8 - 4*(8*a^2 + 4*a*b - b^2)*cosh(x)^6 + 4*(7*(8*a
^2 + 4*a*b - b^2)*cosh(x)^2 - 8*a^2 - 4*a*b + b^2)*sinh(x)^6 + 8*(7*(8*a^2
+ 4*a*b - b^2)*cosh(x)^3 - 3*(8*a^2 + 4*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(
8*a^2 + 4*a*b - b^2)*cosh(x)^4 + 2*(35*(8*a^2 + 4*a*b - b^2)*cosh(x)^4 - 30
*(8*a^2 + 4*a*b - b^2)*cosh(x)^2 + 24*a^2 + 12*a*b - 3*b^2)*sinh(x)^4 + 8*(
7*(8*a^2 + 4*a*b - b^2)*cosh(x)^5 - 10*(8*a^2 + 4*a*b - b^2)*cosh(x)^3 + 3*
(8*a^2 + 4*a*b - b^2)*cosh(x))*sinh(x)^3 - 4*(8*a^2 + 4*a*b - b^2)*cosh(x)^
2 + 4*(7*(8*a^2 + 4*a*b - b^2)*cosh(x)^6 - 15*(8*a^2 + 4*a*b - b^2)*cosh(x)
^4 + 9*(8*a^2 + 4*a*b - b^2)*cosh(x)^2 - 8*a^2 - 4*a*b + b^2)*sinh(x)^2 + 8
*a^2 + 4*a*b - b^2 + 8*((8*a^2 + 4*a*b - b^2)*cosh(x)^7 - 3*(8*a^2 + 4*a*b
- b^2)*cosh(x)^5 + 3*(8*a^2 + 4*a*b - b^2)*cosh(x)^3 - (8*a^2 + 4*a*b - b^2
)*cosh(x))*sinh(x))*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)
*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*c
osh(x)^2 + 2*a - b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2
*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh
(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x
))*sinh(x) + 1)) + 4*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)
^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^
4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 3
0*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 -
 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*
cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a
^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cos
h(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a +
b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 2*sqrt(2)*((6*a^2 + a*b)*cosh(x)^6 + 6*(6*a^2 + a*b)*cosh(x)
*sinh(x)^5 + (6*a^2 + a*b)*sinh(x)^6 + (2*a^2 - a*b)*cosh(x)^4 + (15*(6*a^2
 + a*b)*cosh(x)^2 + 2*a^2 - a*b)*sinh(x)^4 + 4*(5*(6*a^2 + a*b)*cosh(x)^3 +
 (2*a^2 - a*b)*cosh(x))*sinh(x)^3 + (2*a^2 - a*b)*cosh(x)^2 + (15*(6*a^2 +
a*b)*cosh(x)^4 + 6*(2*a^2 - a*b)*cosh(x)^2 + 2*a^2 - a*b)*sinh(x)^2 + 6*a^2
 + a*b + 2*(3*(6*a^2 + a*b)*cosh(x)^5 + 2*(2*a^2 - a*b)*cosh(x)^3 + (2*a^2
- a*b)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^2*cosh(x)^8 + 8*a^2*cos
```

```
h(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*cosh(x)^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tanh^{2}(x)}\,\coth^{5}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5*(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 947 vs. 2(99) = 198.
time = 1.00, size = 947, normalized size = 7.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) + 1/4*(8*a^2 + 4*a*b - b^2)*arctan(-1/2*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a) + 1/2*((16*a^2 + 12*a*b + b^2)*
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^7 - (16*a^2 + 52*a*b + 7*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^
(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) - (48
*a^3 - 28*a^2*b - 109*a*b^2 - 21*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + (176*a^3 + 156*a^2*b
 - 115*a*b^2 - 35*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + (304*a^4 - 156*a^3*b -
317*a^2*b^2 + 130*a*b^3 + 35*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - (48*a^4 + 476*a^3*b - 37
9*a^2*b^2 + 94*a*b^3 + 21*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^
(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - (272*a^5 + 140*
a^4*b - 271*a^3*b^2 + 135*a^2*b^3 - 53*a*b^4 - 7*b^5)*(sqrt(a + b)*e^(2*x)
- sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (112*a
^5 - 116*a^4*b + 65*a^3*b^2 - 17*a^2*b^3 + 11*a*b^4 + b^5)*sqrt(a + b))/(((
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x
```

```
) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)^4*a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^5 \sqrt{b\tanh(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5*(a + b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)^5*(a + b*tanh(x)^2)^(1/2), x)
```

3.219 $\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=82

$$(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}$$

```
[Out] (a+b)^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b)*(a+b*tanh(x)^2
)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)-1/5*(a+b*tanh(x)^2)^(5/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 457, 81, 52, 65, 214}

$$-\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}-(a+b)\sqrt{a+b\tanh^2(x)}+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Sqrt[a +
 b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/3 - (a + b*Tanh[x]^2)^(5/2)/(5*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{x^3(a+bx^2)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{1-x}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b} + \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1-x}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b} + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{\sqrt{a}}{1}\right.$$
$$= -(a+b)\sqrt{a+b\tanh^2(x)} - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} - \frac{\left(a+b\tanh^2(x)\right)^{5}}{5b}$$
$$= -(a+b)\sqrt{a+b\tanh^2(x)} - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} - \frac{\left(a+b\tanh^2(x)\right)^{5}}{5b}$$
$$= (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\sqrt{a+b\tanh^2(x)} -$$

Mathematica [A]
time = 0.32, size = 86, normalized size = 1.05

$$(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{\sqrt{a+b\tanh^2(x)}\left(3a^2+20ab+15b^2+b(6a+5b)\tanh^2(x)+3b^2\tanh^4(x)\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh
[x]^2]*(3*a^2 + 20*a*b + 15*b^2 + b*(6*a + 5*b)*Tanh[x]^2 + 3*b^2*Tanh[x]^4
))/(15*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(66) = 132$.
time = 0.58, size = 488, normalized size = 5.95

method	result

derivativedivides	$-\frac{(a+b(\tanh^2(x)))^{\frac{5}{2}}}{5b}-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b\left(\tanh\left(x\right)-1\right.}}{4}\right.}{}$
default	$-\frac{(a+b(\tanh^2(x)))^{\frac{5}{2}}}{5b}-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b\left(\tanh\left(x\right)-1\right.}}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(a+b*tanh(x)^2)^(5/2)/b-1/6*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2
)-1/2*b*(1/4*(2*b*(tanh(x)-1)+2*b)/b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^
(1/2)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x
)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x
)-1)+a+b)^(1/2)+b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(
tanh(x)-1)+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2
)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1)))-1/6*(b*(1+tanh
(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+1/2*b*(1/4*(2*b*(1+tanh(x))-2*b)/b*(b*(1+
tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2)*ln((b*(
1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)))-1/2*(a+
b)*((b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-b^(1/2)*ln((b*(1+tanh(x))-b
)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+
2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/
2))/(1+tanh(x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2188 vs. $2(66) = 132$.
time = 0.59, size = 4941, normalized size = 60.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")

[Out] [1/60*(15*((a*b + b^2)*cosh(x)^10 + 10*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*b + b^2)*sinh(x)^10 + 5*(a*b + b^2)*cosh(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 + 10*(21*(a*b + b^2)*cosh(x)^4 + 14*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^6 + 4*(63*(a*b + b^2)*cosh(x)^5 + 70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b^2)*cosh(x))*sinh(x)^5 + 10*(a*b + b^2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh(x)^6 + 35*(a*b + b^2)*cosh(x)^4 + 15*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^4 + 40*(3*(a*b + b^2)*cosh(x)^7 + 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^3 + 5*(a*b + b^2)*cosh(x)^2 + 5*(9*(a*b + b^2)*cosh(x)^8 + 28*(a*b + b^2)*cosh(x)^6 + 30*(a*b + b^2)*cosh(x)^4 + 12*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*((a*b + b^2)*cosh(x)^9 + 4*(a*b + b^2)*cosh(x)^7 + 6*(a*b + b^2)*cosh(x)^5 + 4*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*((a*b + b^2)*cosh(x)^10 + 10*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*b + b^2)*sinh(x)^10 + 5*(a*b + b^2)*cosh(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 + 10*(21*(a*b + b^2)*cosh(x)^4 + 14*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^6 + 4*(63*(a*b + b^2)*cosh(x)^5 + 70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b^2)*cosh(x))*sinh(x)^5 + 10*(a*b + b^2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh(x)^6 + 35*(a*b + b^2)*cosh(x)^4 + 15*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*si

```
nh(x)^4 + 40*(3*(a*b + b^2)*cosh(x)^7 + 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b +
b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^3 + 5*(a*b + b^2)*cosh(x)^2 +
 5*(9*(a*b + b^2)*cosh(x)^8 + 28*(a*b + b^2)*cosh(x)^6 + 30*(a*b + b^2)*cos
h(x)^4 + 12*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*(
(a*b + b^2)*cosh(x)^9 + 4*(a*b + b^2)*cosh(x)^7 + 6*(a*b + b^2)*cosh(x)^5 +
 4*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a
+ b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2)) - 4*sqrt(2)*((3*a^2 + 26*a*b + 23*b^2)*cosh(x)^8 + 8*(3*a^2
+ 26*a*b + 23*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 + 26*a*b + 23*b^2)*sinh(x)^8
+ 4*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^6 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*co
sh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*sinh(x)^6 + 8*(7*(3*a^2 + 26*a*b + 23*b^
2)*cosh(x)^3 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(x))*sinh(x)^5 + 2*(9*a^2 +
54*a*b + 49*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^4 + 30
*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^2 + 9*a^2 + 54*a*b + 49*b^2)*sinh(x)^4 +
 8*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^5 + 10*(3*a^2 + 20*a*b + 12*b^2)*co
sh(x)^3 + (9*a^2 + 54*a*b + 49*b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 + 20*a*b
+ 12*b^2)*cosh(x)^2 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^6 + 15*(3*a^2
+ 20*a*b + 12*b^2)*cosh(x)^4 + 3*(9*a^2 + 54*a*b + 49*b^2)*cosh(x)^2 + 3*a^
2 + 20*a*b + 12*b^2)*sinh(x)^2 + 3*a^2 + 26*a*b + 23*b^2 + 8*((3*a^2 + 26*a
*b + 23*b^2)*cosh(x)^7 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^5 + (9*a^2 + 5
4*a*b + 49*b^2)*cosh(x)^3 + (3*a^2 + 20*a*b + 12*b^2)*cosh(x))*sinh(x))*sqr
t(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - ...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(70) = 140$.
time = 17.08, size = 175, normalized size = 2.13

$$-\frac{2a\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2}-\frac{2\left(\frac{b^3\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^3(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}{10}+\frac{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\left(-\frac{ab}{2}+\frac{b^2}{2}\right)}{3}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] -2*a*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a + b*tanh(x)*
*2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/2)/6)/b**2 -
2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(sqrt(a + b*tanh(x)**2)
/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(5/2)/10 + (a + b*t
anh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(66) = 132$.

time = 1.38, size = 1063, normalized size = 12.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)*l
og(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*
b*e^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 4/15*(15*(a^2 + 4*a*b
 + 3*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b))^9 + 15*(7*a^2 + 20*a*b + 9*b^2)*(sqrt(a + b)*e^(2*x)
 - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^8*sqrt(
a + b) + 20*(15*a^3 + 39*a^2*b + 21*a*b^2 + b^3)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^7 + 20*(21*a^
3 + 21*a^2*b - 57*a*b^2 - 65*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) + 2*(105*a^4 -
 210*a^3*b - 1860*a^2*b^2 - 1590*a*b^3 + 19*b^4)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 - 10*(21*a^
4 + 126*a^3*b + 288*a^2*b^2 - 390*a*b^3 - 349*b^4)*(sqrt(a + b)*e^(2*x) - s
qrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a +
b) - 20*(21*a^5 + 63*a^4*b - 18*a^3*b^2 - 378*a^2*b^3 - 235*a*b^4 + 19*b^5)
*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))^3 - 20*(15*a^5 + 21*a^4*b - 126*a^3*b^2 - 90*a^2*b^3 + 367*a*
b^4 + 325*b^5)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - 5*(21*a^6 + 24*a^5*b - 243*a^4*
b^2 + 280*a^3*b^3 + 815*a^2*b^4 - 944*a*b^5 - 1233*b^6)*(sqrt(a + b)*e^(2*x
) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (15*
a^6 - 165*a^4*b^2 + 920*a^3*b^3 - 1147*a^2*b^4 - 504*a*b^5 + 1713*b^6)*sqrt
(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4
*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^5
```

Mupad [B]

time = 10.99, size = 112, normalized size = 1.37

$$-\frac{\left(b\tanh(x)^2+a\right)^{5/2}}{5\,b}-\left(\frac{a+b}{3\,b}-\frac{a}{3\,b}\right)\,\left(b\tanh(x)^2+a\right)^{3/2}-(a+b)\,\left(\frac{a+b}{b}-\frac{a}{b}\right)\,\sqrt{b\tanh\left(x\right)^2+a}-\mathrm{atan}\left(\frac{(a+b)^{3/2}\,\sqrt{b\tanh\left(x\right)^2+a}\,\,1\mathrm{i}}{a^2+2\,a\,b+b^2}\right)\,(a+b)^{3/2}\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3*(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] - (a + b*tanh(x)^2)^(5/2)/(5*b) - ((a + b)/(3*b) - a/(3*b))*(a + b*tanh(x)^
2)^(3/2) - atan(((a + b)^(3/2)*(a + b*tanh(x)^2)^(1/2)*1i)/(2*a*b + a^2 + b
^2))*(a + b)^(3/2)*1i - (a + b)*((a + b)/b - a/b)*(a + b*tanh(x)^2)^(1/2)
```

3.220 $\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=123

$$-\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}}+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{8}(5a+4b)\tan$$

```
[Out] (a+b)^(3/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/8*(3*a^2+1
2*a*b+8*b^2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/8*(5*
a+4*b)*(a+b*tanh(x)^2)^(1/2)*tanh(x)-1/4*b*(a+b*tanh(x)^2)^(1/2)*tanh(x)^3
```

Rubi [A]
time = 0.17, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3751, 488, 596, 537, 223, 212, 385}

$$-\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}}-\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -1/8*((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]
^2]])/Sqrt[b] + (a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh
[x]^2]] - ((5*a + 4*b)*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/8 - (b*Tanh[x]^3*Sqrt
[a + b*Tanh[x]^2])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{x^2(a+bx^2)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}\text{Subst}\left(\int \frac{x^2(-a(4a+3b)-b(5a}{(1-x^2)\sqrt{a+b}}\right.$$

$$= -\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x}$$

$$= -\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x}$$

$$= -\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x}$$

$$= -\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}} + (a+b)^{3/2}\tanh$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 5.06, size = 247, normalized size = 2.01

$$\frac{\left(4\sqrt{2}\,a(5a+4b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)-32\sqrt{2}\,a(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\,\Pi\left(\frac{b}{a+b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)-(15a^2+5ab+2b^2+4(5a^2+4ab-2b^2)\cosh(2x)+(5a^2+11ab+6b^2)\cosh(4x))\operatorname{sech}^4(x)\right)\tanh(x)}{32\sqrt{2}\sqrt{(a-b+(a+b)\cosh(2x))\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ((4*Sqrt[2]*a*(5*a + 4*b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*E
llipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1
] - 32*Sqrt[2]*a*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*El
lipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/
Sqrt[2]], 1] - (15*a^2 + 5*a*b + 2*b^2 + 4*(5*a^2 + 4*a*b - 2*b^2)*Cosh[2*x
] + (5*a^2 + 11*a*b + 6*b^2)*Cosh[4*x])*Sech[x]^4)*Tanh[x])/(32*Sqrt[2]*Sqr
t[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(101) = 202$.
time = 0.60, size = 529, normalized size = 4.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)-3/4*a*(1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)
+1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2)))-1/6*(b*(tanh(x)-1
)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-1)+2*b)/b*(b*(tanh(
x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2)*ln((b*(tan
h(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)))-1/2*(a+b)*
((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((b*(tanh(x)-1)+b)/b
^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b
+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))
/(tanh(x)-1)))+1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)-1/2*b*(1/4*(
2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/8*(4*b
*(a+b)-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+
tanh(x))+a+b)^(1/2)))+1/2*(a+b)*((b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2
)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b
)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x
))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2167 vs. 2(101) = 202.
time = 0.71, size = 10046, normalized size = 81.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*((a*b + b^2)*cosh(x)^8 + 8*(a*b + b^2)*cosh(x)*sinh(x)^7 + (a*b +
b^2)*sinh(x)^8 + 4*(a*b + b^2)*cosh(x)^6 + 4*(7*(a*b + b^2)*cosh(x)^2 + a*b
 + b^2)*sinh(x)^6 + 8*(7*(a*b + b^2)*cosh(x)^3 + 3*(a*b + b^2)*cosh(x))*sin
h(x)^5 + 6*(a*b + b^2)*cosh(x)^4 + 2*(35*(a*b + b^2)*cosh(x)^4 + 30*(a*b +
b^2)*cosh(x)^2 + 3*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a*b + b^2)*cosh(x)^5 + 10
*(a*b + b^2)*cosh(x)^3 + 3*(a*b + b^2)*cosh(x))*sinh(x)^3 + 4*(a*b + b^2)*c
osh(x)^2 + 4*(7*(a*b + b^2)*cosh(x)^6 + 15*(a*b + b^2)*cosh(x)^4 + 9*(a*b +
 b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^2 + a*b + b^2 + 8*((a*b + b^2)*cosh(x)
^7 + 3*(a*b + b^2)*cosh(x)^5 + 3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x
```

```
))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cos
h(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*
(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3
)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2
 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 +
 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh
(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh
(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*
cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 +
a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x
)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*
b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 -
3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x
)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2
 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*si
nh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 -
3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (
a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) +
 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 +
 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((3*a^2 + 12*a*b + 8*b^2)*cosh(x)^8 +
8*(3*a^2 + 12*a*b + 8*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 + 12*a*b + 8*b^2)*sin
h(x)^8 + 4*(3*a^2 + 12*a*b + 8*b^2)*cosh(x)^6 + 4*(7*(3*a^2 + 12*a*b + 8*b^
2)*cosh(x)^2 + 3*a^2 + 12*a*b + 8*b^2)*sinh(x)^6 + 8*(7*(3*a^2 + 12*a*b + 8
*b^2)*cosh(x)^3 + 3*(3*a^2 + 12*a*b + 8*b^2)*cosh(x))*sinh(x)^5 + 6*(3*a^2
+ 12*a*b + 8*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 12*a*b + 8*b^2)*cosh(x)^4 + 30
*(3*a^2 + 12*a*b + 8*b^2)*cosh(x)^2 + 9*a^2 + 36*a*b + 24*b^2)*sinh(x)^4 +
8*(7*(3*a^2 + 12*a*b + 8*b^2)*cosh(x)^5 + 10*(3*a^2 + 12*a*b + 8*b^2)*cosh(
x)^3 + 3*(3*a^2 + 12*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 + 12*a*b +
8*b^2)*cosh(x)^2 + 4*(7*(3*a^2 + 12*a*b + 8*b^2)*cosh(x)^6 + 15*(3*a^2 + 12
*a*b + 8*b^2)*cosh(x)^4 + 9*(3*a^2 + 12*a*b + 8*b^2)*cosh(x)^2 + 3*a^2 + 12
*a*b + 8*b^2)*sinh(x)^2 + 3*a^2 + 12*a*b + 8*b^2 + 8*((3*a^2 + 12*a*b + 8*b
^2)*cosh(x)^7 + 3*(3*a^2 + 12*a*b + 8*b^2)*cosh(x)^5 + 3*(3*a^2 + 12*a*b +
8*b^2)*cosh(x)^3 + (3*a^2 + 12*a*b + 8*b^2)*cosh(x))*sinh(x))*sqrt(b)*log(-
((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4
+ 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2
*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a +
 b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*
b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(
x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 4*((a*b + b^2)
*cosh(x)^8 + 8*(a*b + b^2)*cosh(x)*sinh(x)^7 + (a*b + b^2)*sinh(x)^8 + 4*(a
*b + b^2)*cosh(x)^6 + 4*(7*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^6 + 8
*(7*(a*b + b^2)*cosh(x)^3 + 3*(a*b + b^2)*cosh(x))*sinh(x)^5 + 6*(a*b + b^2
)*cosh(x)^4 + 2*(35*(a*b + b^2)*cosh(x)^4 + 30*(a*b + b^2)*cosh(x)^2 + 3*a*
```

```
b + 3*b^2)*sinh(x)^4 + 8*(7*(a*b + b^2)*cosh(x)^5 + 10*(a*b + b^2)*cosh(x)^
3 + 3*(a*b + b^2)*cosh(x))*sinh(x)^3 + 4*(a*b + b^2)*cosh(x)^2 + 4*(7*(a*b
+ b^2)*cosh(x)^6 + 15*(a*b + b^2)*cosh(x)^4 + 9*(a*b + b^2)*cosh(x)^2 + a*b
 + b^2)*sinh(x)^2 + a*b + b^2 + 8*((a*b + b^2)*cosh(x)^7 + 3*(a*b + b^2)*co
sh(x)^5 + 3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x))*sqrt(a +
b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2
 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*((5*a*b + 6*b^2)*cosh(x)^6 + 6*(5*a*b +
6*b^2)*cosh(x)*sinh(x)^5 + (5*a*b + 6*b^2)*sinh(x)^6 + (5*a*b - 2*b^2)*cosh
(x)^4 + (15*(5*a*b + 6*b^2)*cosh(x)^2 + 5*a*b -...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(x\right)\right)^{\frac{3}{2}}\tanh^2\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(3/2)*tanh(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 949 vs. 2(101) = 202.
time = 1.15, size = 949, normalized size = 7.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2)*
log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) - sqrt(a + b))) - 1/4*(3*a^2 + 12*a*b + 8*b^2)*arctan(-
1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)
*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/2*((
5*a^2 + 20*a*b + 16*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^7 + (35*a^2 + 76*a*b + 16*b^2)*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))^6*sqrt(a + b) + (105*a^3 + 153*a^2*b - 28*a*b^2 - 48*b^3)*(sqrt(a +
```

```
b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
))^5 + (175*a^3 - 25*a^2*b - 260*a*b^2 - 176*b^3)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b
) + (175*a^4 - 110*a^3*b - 417*a^2*b^2 + 60*a*b^3 + 304*b^4)*(sqrt(a + b)*e
^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3
 + (105*a^4 - 210*a^3*b - 55*a^2*b^2 + 484*a*b^3 + 48*b^4)*(sqrt(a + b)*e^(
2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*s
qrt(a + b) + (35*a^5 - 79*a^4*b + 53*a^3*b^2 + 195*a^2*b^3 - 308*a*b^4 - 27
2*b^5)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*
b*e^(2*x) + a + b)) + (5*a^5 - 17*a^4*b + 51*a^3*b^2 - 19*a^2*b^3 - 44*a*b^
4 + 112*b^5)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a
*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a
- 3*b)^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^2 \left(b\tanh(x)^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2*(a + b*tanh(x)^2)^(3/2),x)
[Out] int(tanh(x)^2*(a + b*tanh(x)^2)^(3/2), x)
```

3.221 $\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=63

$$(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}$$

```
[Out] (a+b)^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b)*(a+b*tanh(x)^2
)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {3751, 455, 52, 65, 214}

$$-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Sqrt[a +
 b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx &= \text{Subst}\left(\int \frac{x(a+bx^2)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1-x}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}+\frac{1}{2}(a+b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \tanh^2(x)\right.\\
&= -(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}+\frac{1}{2}(a+b)^2\text{Subst}\Bigg(\\
&= -(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}+\frac{(a+b)^2\text{Subst}\Big(\int}{}\\
&= (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\sqrt{a+b\tanh^2(x)}-
\end{aligned}$$

Mathematica [A]

time = 0.11, size = 59, normalized size = 0.94

$$(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{1}{3}\sqrt{a+b\tanh^2(x)}\,\left(4a+3b+b\tanh^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh
[x]^2]*(4*a + 3*b + b*Tanh[x]^2))/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(51) = 102$.
time = 0.52, size = 473, normalized size = 7.51

method	result
derivativedivides	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b\left(\tanh\left(x\right)-1\right)^2+2b\left(\tanh\left(x\right)-\right.}}{4b}\right.}{}$
default	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b\left(\tanh\left(x\right)-1\right)^2+2b\left(\tanh\left(x\right)-\right.}}{4b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-1
)+2*b)/b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/
b^(3/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^
(1/2)))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((
b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)
^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh
(x)-1)+a+b)^(1/2))/(tanh(x)-1)))-1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^
(3/2)+1/2*b*(1/4*(2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a
+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+
tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)))-1/2*(a+b)*((b*(1+tanh(x))^2-2*b*(1+
tanh(x))+a+b)^(1/2)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2
*b*(1+tanh(x))+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^
(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(51) = 102$.
time = 0.45, size = 2385, normalized size = 37.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)
^6 + 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*(5
*(a + b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3
*(5*(a + b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)
*cosh(x)^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a
 + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a
^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14
*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*
a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4
 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3
+ a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 +
 a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^
3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(
a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b
^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*
cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(
5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x
)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)
^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*c
osh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b
^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(
x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^
5 + sinh(x)^6)) + 3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a +
```

```
 b)*sinh(x)^6 + 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(
x)^4 + 4*(5*(a + b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*co
sh(x)^2 + 3*(5*(a + b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 +
 6*((a + b)*cosh(x)^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a
+ b)*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
 + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*((a + b)*cosh(x)^4 + 4*(
a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + (2*a + b)*cosh(x)^2 + (6*(a
+ b)*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 2*(2*(a + b)*cosh(x)^3 + (2*a + b)*co
sh(x))*sinh(x) + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^6 + 6*cosh(x)*sinh
(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh
(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2
+ 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1), -1/6*(3
*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 + 3*(
a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*(5*(a + b)
*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a +
 b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)
^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*a
rctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqr
t(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cos
h(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6
*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 3*((
a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 + 3*(a +
 b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*(5*(a + b)*co
sh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)
*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5
+ 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arct
an(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqr
t(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a +
 b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(
x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 8*sqrt(2
)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + (2
*a + b)*cosh(x)^2 + (6*(a + b)*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 2*(2*(a + b
)*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + a + ...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(54) = 108$.

time = 10.01, size = 128, normalized size = 2.03

$$-\frac{2a\left(\frac{b\sqrt{a+b\tanh^{2}(x)}}{2}+\frac{b(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^{2}(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b}-\frac{2\left(\frac{b^{2}\sqrt{a+b\tanh^{2}(x)}}{2}+\frac{b^{2}(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^{2}(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b(a+b\tanh^{2}(x))^{\frac{3}{2}}}{6}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] -2*a*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*tanh(x)**2)/sq
rt(-a - b))/(2*sqrt(-a - b)))/b - 2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(
a + b)*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a +
b*tanh(x)**2)**(3/2)/6)/b
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(51) = 102.
time = 1.03, size = 662, normalized size = 10.51

$$\frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)-\frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)-\frac{(a^{2}+2ab+b^{2})\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}-\frac{8\left(3(ab+b^{2})\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^{5}+3(3ab+b^{2})\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^{4}\sqrt{a+b}+2(3a^{2}b-6ab^{2}-5b^{3})\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^{3}-6(a^{2}b+4ab^{2}+3b^{3})\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^{2}\sqrt{a+b}-3(3a^{3}b+a^{2}b^{2}-15ab^{3}-13b^{4})\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)-(3a^{3}b-9a^{2}b^{2}+5ab^{3}+17b^{4})\sqrt{a+b}\right)}{3\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^{2}+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}+a-3b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)*l
og(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*
b*e^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 8/3*(3*(a*b + b^2)*(s
qrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b))^5 + 3*(3*a*b + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^
(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + 2*(3*a^2*b - 6*
a*b^2 - 5*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))^3 - 6*(a^2*b + 4*a*b^2 + 3*b^3)*(sqrt(a + b)*e^
(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*
sqrt(a + b) - 3*(3*a^3*b + a^2*b^2 - 15*a*b^3 - 13*b^4)*(sqrt(a + b)*e^(2*x
) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (3*a
^3*b - 9*a^2*b^2 + 5*a*b^3 + 17*b^4)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - s
qrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))*sqrt(a + b) + a - 3*b)^3
```

Mupad [B]

time = 3.71, size = 64, normalized size = 1.02

$$\operatorname{atanh}\left(\frac{(a+b)^{3/2}\sqrt{b\tanh\left(x\right)^{2}+a}}{a^{2}+2\,a\,b+b^{2}}\right)\,(a+b)^{3/2}-(a+b)\,\sqrt{b\tanh\left(x\right)^{2}+a}\,-\frac{\left(b\tanh(x)^{2}+a\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] atanh(((a + b)^(3/2)*(a + b*tanh(x)^2)^(1/2))/(2*a*b + a^2 + b^2))*(a + b)^
(3/2) - (a + b)*(a + b*tanh(x)^2)^(1/2) - (a + b*tanh(x)^2)^(3/2)/3
```

3.222 $\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=88

$$-\frac{1}{2}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b$$

```
[Out] (a+b)^(3/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/2*(3*a+2*b
)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*b^(1/2)-1/2*b*(a+b*tanh(x)
^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3742, 427, 537, 223, 212, 385}

$$-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])
 + (a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - (b*
Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx &= \text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-a(2a+b)-b(3a+2b)x^2}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \text{t}\right.\\
&= -\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)} + (a+b)^2\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \text{ta}\right.\\
&= -\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)} + (a+b)^2\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\text{ta}}{\sqrt{a+b}}\right.\\
&= -\frac{1}{2}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\text{t}}{\sqrt{a+b\,\text{ta}}}\right.
\end{aligned}
$$

Mathematica [**A**]

time = 0.45, size = 110, normalized size = 1.25

$$\frac{1}{2}\left(-2(-a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\text{sech}^2(x)+\tanh(x)\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)+\sqrt{b}\,(3a+2b)\log\left(-\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh^2(x)}\right)-b\tanh(x)\sqrt{a+b\tanh^2(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (-2*(-a - b)^(3/2)*ArcTan[(Sqrt[b]*Sech[x]^2 + Tanh[x]*Sqrt[a + b*Tanh[x]^2
])/Sqrt[-a - b]] + Sqrt[b]*(3*a + 2*b)*Log[-(Sqrt[b]*Tanh[x]) + Sqrt[a + b*
Tanh[x]^2]] - b*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(70) = 140$.
time = 0.72, size = 473, normalized size = 5.38

method	result
derivativedivides	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b\left(\tanh\left(x\right)-1\right)^2+2b\left(\tanh\left(x\right)\right.}}{4b}\right.}{}$
default	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b\left(\tanh\left(x\right)-1\right)^2+2b\left(\tanh\left(x\right)\right.}}{4b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-1
)+2*b)/b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/
b^(3/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^
(1/2)))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((
b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)
^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh
(x)-1)+a+b)^(1/2))/(tanh(x)-1)))+1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^
(3/2)-1/2*b*(1/4*(2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a
+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+
tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)))+1/2*(a+b)*((b*(1+tanh(x))^2-2*b*(1+
tanh(x))+a+b)^(1/2)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2
*b*(1+tanh(x))+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^
(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(70) = 140.
time = 0.55, size = 4841, normalized size = 55.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
+ 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a*b^2
+ b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x
)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(
x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^
3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3
+ (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(
x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*
b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*
cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 -
b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a
*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b
+ 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)
^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(x)*sinh(x)^3 + (3*a + 2*b)*sinh
(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a + 2*b)*cosh(x)^2 + 3*a + 2*b)*s
inh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a + 2*b)*cosh(x))*sinh(x) + 3*a +
```

```
2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a
 + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a -
2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*
sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x)
)*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*co
sh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a + b
)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a + b)*co
sh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 +
 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)
^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(
x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1),
 1/4*(2*((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(x)*sinh(x)^3 + (3*a + 2
*b)*sinh(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a + 2*b)*cosh(x)^2 + 3*a
+ 2*b)*sinh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a + 2*b)*cosh(x))*sinh(x)
+ 3*a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*co
sh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*co
sh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x
) + a + b)) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*si
nh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 +
 4*((a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-
((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3
)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 +
b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^
3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*
b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*c
osh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*co
sh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*
b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^
3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6
+ 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(70) = 140.
time = 0.85, size = 584, normalized size = 6.64

$$-\frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)+\frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)-\frac{(3ab+2b^2)\arctan\left(-\frac{\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{(a^2+2ab+b^2)\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\sqrt{a+b}}-\frac{2\left((ab+2b^2)\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3+(3ab-2b^2)\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2\sqrt{a+b}+(3a^2b-3ab^2-2b^3)\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)+(a^2b-ab^2+2b^3)\sqrt{a+b}\right)}{\left(\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}+a-3b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2)*
log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) - sqrt(a + b))) - (3*a*b + 2*b^2)*arctan(-1/2*(sqrt(a +
 b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sq
rt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 2*((a*b + 2*b^2)*(s
qrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b))^3 + (3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^
(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + (3*a^2*b - 3*a*
b^2 - 2*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b)) + (a^2*b - a*b^2 + 2*b^3)*sqrt(a + b))/((sqrt(a +
 b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tanh}\left(x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int((a + b*tanh(x)^2)^(3/2), x)
```

3.223 $\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=71

$$-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-b\sqrt{a+b\tanh^2(x)}$$

```
[Out] -a^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))+(a+b)^(3/2)*arctanh((a+b*ta
nh(x)^2)^(1/2)/(a+b)^(1/2))-b*(a+b*tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3751, 457, 86, 162, 65, 214}

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)\right)-b\sqrt{a+b\tanh^2(x)}+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + (a + b)^(3/2)*ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - b*Sqrt[a + b*Tanh[x]^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx &= \text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(1-x)x}\,dx, x, \tanh^2(x)\right)\\
&= -b\sqrt{a+b\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-a^2+(-2a-b)bx}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -b\sqrt{a+b\tanh^2(x)} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) + \frac{1}{2}(\\
&= -b\sqrt{a+b\tanh^2(x)} + \frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b} + \\
&= -a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2}}{\sqrt{a+b}}\right.
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 71, normalized size = 1.00

$$-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-b\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + (a + b)^(3/2)*ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - b*Sqrt[a + b*Tanh[x]^2]
```

Maple [F]
time = 1.41, size = 0, normalized size = 0.00

$$\int \coth(x)\left(a+b\left(\tanh^2(x)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*coth(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(57) = 114$.
time = 0.54, size = 4039, normalized size = 56.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 +
a + b)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*s
inh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3
```

```
 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh
(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b
^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3
 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5
+ 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*s
inh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x
)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3
+ 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 +
sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*co
sh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*
cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 +
18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(
3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))
*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a
^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3
+ 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*co
sh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6)) + 2*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sin
h(x)^2 + a)*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)
^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2
 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*
cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(
x) + 1)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)
^2 + a + b)*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)
^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x
)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*s
qrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*b*sqrt(((a + b)*co
sh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), 1/4*(4*(a*cosh(x)^
2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(
x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + ((a + b)*cosh(x)^2 + 2*(a + b)*
cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(a + b)*log(((a^3 + a^2*b)
*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 +
2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2
)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*si
```

```
nh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cos
h(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh
(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a
^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 +
 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b
- a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x
)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*s
inh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b -
 b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^
2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (
3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2
+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(c
osh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh
(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a +
b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(
a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^2\left(x\right)\right)^{\frac{3}{2}}\coth\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(3/2)*coth(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(57) = 114.
time = 0.94, size = 433, normalized size = 6.10

$$\frac{2\,a^2\arctan\left(-\frac{\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)-\frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)-\frac{(a^2+2\,ab+b^2)\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\sqrt{a+b}}-\frac{4\left(\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)b^2-\sqrt{a+b}\,b^2\right)}{\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)\sqrt{a+b}+a-3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) + 1/2*(a +
b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)*log(abs(-s
qrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a + b)*e
```

```
^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(
a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 4*((sqrt(a + b)*e^(2*x) - sqrt
(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*b^2 - sqrt(a +
 b)*b^2)/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4
*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)\,\left(b\tanh(x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)*(a + b*tanh(x)^2)^(3/2), x)
```

3.224 $\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=77

$$-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

```
[Out] -b^(3/2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))+(a+b)^(3/2)*arctanh
((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-a*coth(x)*(a+b*tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.353$, Rules used = {3751, 485, 537, 223, 212, 385}

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + (a + b)^(3/2)
*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - a*Coth[x]*Sqrt[a +
b*Tanh[x]^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
```

```
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= -a\coth(x)\sqrt{a+b\tanh^2(x)} + \text{Subst}\left(\int \frac{a(a+2b)+b^2x^2}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan$$

$$= -a\coth(x)\sqrt{a+b\tanh^2(x)} - b^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$

$$= -a\coth(x)\sqrt{a+b\tanh^2(x)} - b^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x}{\sqrt{a+b\tanh}}\right.$$

$$= -b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tan}{\sqrt{a+b\tanh}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 2.03, size = 197, normalized size = 2.56

$$-\frac{a\left((a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)-\sqrt{2}\,(a+2b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)+\sqrt{2}\,(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\,\Pi\left(\frac{b}{a+b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\tanh(x)}{\sqrt{2}\,\sqrt{(a-b+(a+b)\cosh(2x))\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -((a*((a - b + (a + b)*Cosh[2*x])*Csch[x]^2 - Sqrt[2]*(a + 2*b)*Sqrt[((a -
b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b
)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + Sqrt[2]*(a + b)*Sqrt[((a - b + (a
 + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b +
(a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])*Tanh[x])/(Sqrt[2]*Sqrt[(a -
b + (a + b)*Cosh[2*x])*Sech[x]^2]))
```

Maple [F]
time = 1.37, size = 0, normalized size = 0.00

$$\int \left(\coth^2(x)\right)\left(a+b\left(\tanh^2(x)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*coth(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 634 vs. 2(63) = 126.
time = 0.51, size = 3913, normalized size = 50.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
a - b)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*
sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^
2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cos
h(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*
b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^
3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5
 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*
sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(
x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 -
 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 +
 sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*c
osh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2
*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 -
 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*
(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x)
)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*
b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 -
 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 - b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x
)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^
2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)
*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh
(x) + 1)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x
)^2 - a - b)*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)
^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x
)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*s
qrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*a*sqrt(((a + b)*co
sh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), 1/4*(4*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(
x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + ((a + b)*cosh(x)^2 + 2*(a + b)*
cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(a + b)*log(-((a*b^2 + b^3
```

```
)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 -
 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^
2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*s
inh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*co
sh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 +
 b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6
- 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4
*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(
x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*
sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b -
3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b
^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 -
(a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2
 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(
cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin
h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt
(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(63) = 126.
time = 0.91, size = 430, normalized size = 5.58

$$-\frac{2\,b^{2}\arctan\left(-\frac{\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}}{2\,\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{1}{2}\,(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)+\frac{1}{2}\,(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)-\frac{(a^{2}+2\,ab+b^{2})\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\,\sqrt{a+b}}+\frac{4\left(\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)a^{2}+\sqrt{a+b}\,a^{2}\right)}{\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)^{2}-2\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)\sqrt{a+b}-3\,a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*b^2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a +
 b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2)*log(abs(-
```

```
sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x
) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a + b)*
e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*
(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 4*((sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*a^2 + sqrt(a
+ b)*a^2)/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^2 \left(b\tanh(x)^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2*(a + b*tanh(x)^2)^(3/2), x)
```

3.225 $\int \sqrt{1+\tanh^2(x)}\,dx$

Optimal. Leaf size=31

$$-\sinh^{-1}(\tanh(x))+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)$$

```
[Out] -arcsinh(tanh(x))+arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3742, 399, 221, 385, 212}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)-\sinh^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] -ArcSinh[Tanh[x]] + Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{1+x^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= 2\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{1+x^2}}\,dx, x, \tanh(x)\right) - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)\\
&= -\sinh^{-1}(\tanh(x)) + 2\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)\\
&= -\sinh^{-1}(\tanh(x)) + \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 1.65

$$\frac{\left(\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh(x)\right) - \tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)\cosh(x)\sqrt{1+\tanh^2(x)}}{\sqrt{\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cosh
[x]*Sqrt[1 + Tanh[x]^2])/Sqrt[Cosh[2*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(25) = 50$.
time = 0.93, size = 97, normalized size = 3.13

method	result
derivativedivides	$\frac{\sqrt{(1+\tanh(x))^2-2\tanh(x)}}{2}-\operatorname{arcsinh}(\tanh(x))-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2\tanh(x))\cdots}{4\sqrt{(1+\tanh(x))^2\cdots}}\right.\cdots}{2}$
default	$\frac{\sqrt{(1+\tanh(x))^2-2\tanh(x)}}{2}-\operatorname{arcsinh}(\tanh(x))-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2\tanh(x))\cdots}{4\sqrt{(1+\tanh(x))^2\cdots}}\right.\cdots}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((1+tanh(x))^2-2*tanh(x))^(1/2)-arcsinh(tanh(x))-1/2*2^(1/2)*arctanh(1/
4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))^2-2*tanh(x))^(1/2))-1/2*((tanh(x)-1)^2
+2*tanh(x))^(1/2)+1/2*2^(1/2)*arctanh(1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1
)^2+2*tanh(x))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tanh(x)^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(25) = 50.
time = 0.37, size = 679, normalized size = 21.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(
x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5
+ 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x
)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 +
30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 +
10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*
sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4
```

```
 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x
)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 +
 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqr
t(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/4*sqrt(2)*log(2*(cosh(x)^4 + 4*cosh(x)
*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cos
h(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x)
+ sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)
^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tanh^2(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(x)**2 + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(25) = 50.
time = 0.44, size = 104, normalized size = 3.35

$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt
(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*x) +
 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) +
 1))
```

Mupad [B]
time = 0.23, size = 68, normalized size = 2.19

$$\frac{\sqrt{2}\left(\ln\left(\tanh(x)+1\right)-\ln\left(\sqrt{2}\sqrt{\tanh(x)^2+1}-\tanh(x)+1\right)\right)}{2}-\mathrm{asinh}(\tanh(x))+\frac{\sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh(x)^2+1}+1\right)-\ln\left(\tanh(x)-1\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)^2 + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) +
1)))/2 - asinh(tanh(x)) + (2^(1/2)*(log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(
1/2) + 1) - log(tanh(x) - 1)))/2
```

3.226 $\int \sqrt{-1-\tanh^2(x)}\,dx$

Optimal. Leaf size=45

$$\text{ArcTan}\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)$$

```
[Out] arctan(tanh(x)/(-1-tanh(x)^2)^(1/2))-arctan(2^(1/2)*tanh(x)/(-1-tanh(x)^2)^
(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3742, 399, 223, 209, 385}

$$\text{ArcTan}\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Tanh[x]^2],x]
```

```
[Out] ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]] - Sqrt[2]*ArcTan[(Sqrt[2]*Tanh[x])/Sqr
t[-1 - Tanh[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
```

```
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1-\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{-1-x^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\ &= -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}\,(1-x^2)}\,dx, x, \tanh(x)\right)\right) + \text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,d\right.\\ &= -\left(2\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x,\right.\\ &= \tan^{-1}\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) - \sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.18

$$\frac{\left(\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,\sinh(x)\right)-\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)\cosh(x)\sqrt{-1-\tanh^2(x)}}{\sqrt{\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cosh
[x]*Sqrt[-1 - Tanh[x]^2])/Sqrt[Cosh[2*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(37) = 74.
time = 0.95, size = 142, normalized size = 3.16

method	result
derivativedivides	$\frac{\sqrt{-\left(1+\tanh \left(x\right)\right)^{2}+2 \tanh \left(x\right)}}{2}+\frac{\arctan \left(\frac{\tanh \left(x\right)}{\sqrt{-\left(1+\tanh \left(x\right)\right)^{2}+2 \tanh \left(x\right)}}\right)}{2}-\frac{\sqrt{2}\ \mathrm{a}}{}$
default	$\frac{\sqrt{-\left(1+\tanh \left(x\right)\right)^{2}+2 \tanh \left(x\right)}}{2}+\frac{\arctan \left(\frac{\tanh \left(x\right)}{\sqrt{-\left(1+\tanh \left(x\right)\right)^{2}+2 \tanh \left(x\right)}}\right)}{2}-\frac{\sqrt{2}\ \mathrm{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(1+tanh(x))^2+2*tanh(x))^(1/2)+1/2*arctan(tanh(x)/(-(1+tanh(x))^2+2*t
anh(x))^(1/2))-1/2*2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(1+tanh(x))^
2+2*tanh(x))^(1/2))-1/2*(-(tanh(x)-1)^2-2*tanh(x))^(1/2)+1/2*arctan(tanh(x)
/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+1/2*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^
(1/2)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-tanh(x)^2 - 1), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.37, size = 226, normalized size = 5.02

$$-\frac{1}{4}\sqrt{-2}\log\left(-\left(\sqrt{-2}\sqrt{-2e^{(4x)}-2}+2e^{(2x)}+2\right)e^{(-2x)}\right)+\frac{1}{4}\sqrt{-2}\log\left(\left(\sqrt{-2}\sqrt{-2e^{(4x)}-2}-2e^{(2x)}-2\right)e^{(-2x)}\right)+\frac{1}{4}\sqrt{-2}\log\left(-2\left(\sqrt{-2e^{(4x)}-2}\left(e^{(2x)}-2\right)+\sqrt{-2}e^{(4x)}-\sqrt{-2}e^{(2x)}+2\sqrt{-2}\right)e^{(-4x)}\right)-\frac{1}{4}\sqrt{-2}\log\left(-2\left(\sqrt{-2e^{(4x)}-2}\left(e^{(2x)}-2\right)-\sqrt{-2}e^{(4x)}+\sqrt{-2}e^{(2x)}-2\sqrt{-2}\right)e^{(-4x)}\right)+\frac{1}{2}i\log\left(-4\left(i\sqrt{-2e^{(4x)}-2}+e^{(2x)}-1\right)e^{(-2x)}\right)-\frac{1}{2}i\log\left(-4\left(-i\sqrt{-2e^{(4x)}-2}+e^{(2x)}-1\right)e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-2)*log(-(sqrt(-2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)
) + 1/4*sqrt(-2)*log((sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(2*x) - 2)*e^(-2*
x)) + 1/4*sqrt(-2)*log(-2*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + sqrt(-2)*e^
(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) - 1/4*sqrt(-2)*log(-2*(sqr
t(-2*e^(4*x) - 2)*(e^(2*x) - 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x) - 2*s
qrt(-2))*e^(-4*x)) + 1/2*I*log(-4*(I*sqrt(-2*e^(4*x) - 2) + e^(2*x) - 1)*e^
(-2*x)) - 1/2*I*log(-4*(-I*sqrt(-2*e^(4*x) - 2) + e^(2*x) - 1)*e^(-2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\tanh^2(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-tanh(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 104, normalized size = 2.31

$$-\frac{1}{2}i\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sq
rt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*x)
 + 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x)
 + 1))
```

Mupad [B]
time = 1.34, size = 43, normalized size = 0.96

$$-\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tanh(x)}{\sqrt{-\tanh(x)^2-1}}\right)-\ln\left(\tanh(x)-\sqrt{-\tanh(x)^2-1}\;1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- tanh(x)^2 - 1)^(1/2),x)
```

```
[Out] - log(tanh(x) - (- tanh(x)^2 - 1)^(1/2)*1i)*1i - 2^(1/2)*atan((2^(1/2)*tanh
(x))/(- tanh(x)^2 - 1)^(1/2))
```

3.227 $\int \left(1+\tanh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=50

$$-\frac{5}{2}\sinh^{-1}(\tanh(x))+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}$$

```
[Out] -5/2*arcsinh(tanh(x))+2*arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/2
)-1/2*(1+tanh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.600$, Rules used = {3742, 427, 537, 221, 385, 212}

$$-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)-\frac{5}{2}\sinh^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tanh[x]^2)^(3/2),x]
```

```
[Out] (-5*ArcSinh[Tanh[x]])/2 + 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh
[x]^2]] - (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-3-5x^2}{\left(1-x^2\right)\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tanh(x)\right) + 4\text{Subst}\left(\right.$$

$$= -\frac{5}{2}\sinh^{-1}(\tanh(x)) - \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} + 4\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, -\right.$$

$$= -\frac{5}{2}\sinh^{-1}(\tanh(x)) + 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right) - \frac{1}{2}\tanh(x)\sqrt{1+\tan}$$

Mathematica [A]
time = 0.10, size = 74, normalized size = 1.48

$$-\frac{\left(-4\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)\cosh^3(x) + 5\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\cosh^3(x) + \cosh(x)\sqrt{\cosh(2x)}\sinh(x)\right)\left(1+\tanh^2(x)\right)^{3/2}}{2\cosh^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tanh[x]^2)^(3/2),x]
```

```
[Out] -1/2*((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/Sq
rt[Cosh[2*x]]]*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(1 + Tanh[x]^2)
^(3/2))/Cosh[2*x]^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(38) = 76$.
time = 0.76, size = 158, normalized size = 3.16

method	result
derivativedivides	$-\frac{\left((\tanh(x)-1)^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}-\frac{\tanh(x)\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{4}-\frac{5\,\mathrm{arcsinh}(\tanh(x))}{2}-\sqrt{(}$
default	$-\frac{\left((\tanh(x)-1)^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}-\frac{\tanh(x)\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{4}-\frac{5\,\mathrm{arcsinh}(\tanh(x))}{2}-\sqrt{(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((tanh(x)-1)^2+2*tanh(x))^(3/2)-1/4*tanh(x)*((tanh(x)-1)^2+2*tanh(x))^
(1/2)-5/2*arcsinh(tanh(x))-((tanh(x)-1)^2+2*tanh(x))^(1/2)+2^(1/2)*arctanh(
1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/2))+1/6*((1+tanh(x))
^2-2*tanh(x))^(3/2)-1/4*tanh(x)*((1+tanh(x))^2-2*tanh(x))^(1/2)+((1+tanh(x)
)^2-2*tanh(x))^(1/2)-2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))
^2-2*tanh(x))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((tanh(x)^2 + 1)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(38) = 76$.
time = 0.39, size = 1027, normalized size = 20.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4
 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(s
qrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*(cosh(x)^8 +
8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^
6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2
+ 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*
sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*
cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x
) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 +
3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sq
rt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18
*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt
(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2
))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(
x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
 + 2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 +
 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqr
t(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(2*(cosh(x)^4 + 4*c
osh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*
(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*si
nh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2)) - 5*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2)) + 5*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3
*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) +
 1)*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + si
nh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2)) - 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 - 1)*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*si
nh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\tanh^2(x) + 1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((tanh(x)**2 + 1)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(38) = 76.
time = 0.43, size = 202, normalized size = 4.04

$$-\frac{1}{4}\sqrt{2}\left(5\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(3\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{3}-\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}-1\right)}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}-2\sqrt{e^{(4x)}+1}+2e^{(2x)}-1\right)^{2}}+4\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+4\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-4\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(5*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sq
rt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(3*(sqrt(e^(4*x) + 1) - e^(2*
x))^3 - (sqrt(e^(4*x) + 1) - e^(2*x))^2 - sqrt(e^(4*x) + 1) + e^(2*x) - 1)/
((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)^2 +
 4*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + 4*log(sqrt(e^(4*x) + 1) - e^(2*x)
) - 4*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [B]
time = 0.29, size = 78, normalized size = 1.56

$$\sqrt{2}\left(\ln\left(\tanh(x)+1\right)-\ln\left(\sqrt{2}\sqrt{\tanh\left(x\right)^2+1}-\tanh(x)+1\right)\right)-\frac{5\,\mathrm{asinh}(\tanh(x))}{2}-\frac{\tanh(x)\sqrt{\tanh\left(x\right)^2+1}}{2}+\sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh\left(x\right)^2+1}+1\right)-\ln\left(\tanh(x)-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)^2 + 1)^(3/2),x)
```

```
[Out] 2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) + 1
)) - (5*asinh(tanh(x)))/2 - (tanh(x)*(tanh(x)^2 + 1)^(1/2))/2 + 2^(1/2)*(lo
g(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(1/2) + 1) - log(tanh(x) - 1))
```

3.228 $\int \left(-1-\tanh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=67

$$-\frac{5}{2}\mathrm{ArcTan}\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)+2\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)+\frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)}$$

```
[Out] -5/2*arctan(tanh(x)/(-1-tanh(x)^2)^(1/2))+2*arctan(2^(1/2)*tanh(x)/(-1-tanh
(x)^2)^(1/2))*2^(1/2)+1/2*(-1-tanh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3742, 427, 537, 223, 209, 385}

$$-\frac{5}{2}\mathrm{ArcTan}\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)+2\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)+\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)-1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Tanh[x]^2)^(3/2),x]
```

```
[Out] (-5*ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]])/2 + 2*Sqrt[2]*ArcTan[(Sqrt[2]*Tan
h[x])/Sqrt[-1 - Tanh[x]^2]] + (Tanh[x]*Sqrt[-1 - Tanh[x]^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{\left(-1-x^2\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-3-5x^2}{\sqrt{-1-x^2}\,(1-x^2)}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \tanh(x)\right) + 4\text{Subs}$$

$$= \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) +$$

$$= -\frac{5}{2}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + 2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + \frac{1}{2}\tanh$$

Mathematica [A]
time = 0.05, size = 76, normalized size = 1.13

$$-\frac{\left(-4\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)\cosh^3(x)+5\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\cosh^3(x)+\cosh(x)\sqrt{\cosh(2x)}\sinh(x)\right)\left(-1-\tanh^2(x)\right)^{3/2}}{2\cosh^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Tanh[x]^2)^(3/2),x]
```

```
[Out] -1/2*((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/Sq
rt[Cosh[2*x]]]*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(-1 - Tanh[x]^2
)^(3/2))/Cosh[2*x]^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(53) = 106$.
time = 0.82, size = 211, normalized size = 3.15

method	result
derivativedivides	$\frac{\left(-(1+\tanh(x))^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}+\frac{\tanh(x)\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{}{\sqrt{-(1+}}\right.}{}$
default	$\frac{\left(-(1+\tanh(x))^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}+\frac{\tanh(x)\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{}{\sqrt{-(1+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-(1+tanh(x))^2+2*tanh(x))^(3/2)+1/4*tanh(x)*(-(1+tanh(x))^2+2*tanh(x))
^(1/2)-5/4*arctan(tanh(x)/(-(1+tanh(x))^2+2*tanh(x))^(1/2))-(-(1+tanh(x))^2
+2*tanh(x))^(1/2)+2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(1+tanh(x))^2
+2*tanh(x))^(1/2))-1/6*(-(tanh(x)-1)^2-2*tanh(x))^(3/2)+1/4*tanh(x)*(-(tanh
(x)-1)^2-2*tanh(x))^(1/2)-5/4*arctan(tanh(x)/(-(tanh(x)-1)^2-2*tanh(x))^(1/
2))+(-(tanh(x)-1)^2-2*tanh(x))^(1/2)-2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1
/2)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-tanh(x)^2 - 1)^(3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.37, size = 361, normalized size = 5.39

$$\frac{2\left(\sqrt{-2}\,e^{(4x)}+2\sqrt{-2}\,e^{(2x)}+\sqrt{-2}\right)\log\left(2\left(\sqrt{-2}\sqrt{-2\,e^{(4x)}-2}+2\,e^{(2x)}+2\right)e^{(-2x)}\right)-2\left(\sqrt{-2}\,e^{(4x)}+2\sqrt{-2}\,e^{(2x)}+\sqrt{-2}\right)\log\left(-2\left(\sqrt{-2}\sqrt{-2\,e^{(4x)}-2}-2\,e^{(2x)}-2\right)e^{(-2x)}\right)-5\left(i\,e^{(4x)}+2i\,e^{(2x)}+i\right)\log\left(-4\left(i\sqrt{-2\,e^{(4x)}-2}+e^{(2x)}-1\right)e^{(-2x)}\right)-5\left(-i\,e^{(4x)}-2i\,e^{(2x)}-i\right)\log\left(-4\left(-i\sqrt{-2\,e^{(4x)}-2}+e^{(2x)}-1\right)e^{(-2x)}\right)-2\left(\sqrt{-2}\,e^{(4x)}+2\sqrt{-2}\,e^{(2x)}+\sqrt{-2}\right)\log\left(4\left(\sqrt{-2\,e^{(4x)}-2}\left(e^{(2x)}-2\right)+\sqrt{-2}\,e^{(4x)}-\sqrt{-2}\,e^{(2x)}+2\sqrt{-2}\right)e^{(-4x)}\right)+2\left(\sqrt{-2}\,e^{(4x)}+2\sqrt{-2}\,e^{(2x)}+\sqrt{-2}\right)\log\left(4\left(\sqrt{-2\,e^{(4x)}-2}\left(e^{(2x)}-2\right)-\sqrt{-2}\,e^{(4x)}+\sqrt{-2}\,e^{(2x)}-2\sqrt{-2}\right)e^{(-4x)}\right)+2\sqrt{-2\,e^{(4x)}-2}\left(e^{(2x)}-1\right)}{4\left(e^{(4x)}+2\,e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(2*(sqrt(-2)*s
qrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x) + 2*sq
rt(-2)*e^(2*x) + sqrt(-2))*log(-2*(sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(2*x
) - 2)*e^(-2*x)) - 5*(I*e^(4*x) + 2*I*e^(2*x) + I)*log(-4*(I*sqrt(-2*e^(4*x
) - 2) + e^(2*x) - 1)*e^(-2*x)) - 5*(-I*e^(4*x) - 2*I*e^(2*x) - I)*log(-4*(
-I*sqrt(-2*e^(4*x) - 2) + e^(2*x) - 1)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x) + 2*
sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + sq
rt(-2)*e^(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) + 2*(sqrt(-2)*e^(
4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*(e^(2*x)
- 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x) - 2*sqrt(-2))*e^(-4*x)) + 2*sqrt
(-2*e^(4*x) - 2)*(e^(2*x) - 1))/(e^(4*x) + 2*e^(2*x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\tanh^{2}\left(x\right)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((-tanh(x)**2 - 1)**(3/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 204, normalized size = 3.04

$$-\frac{1}{4}\sqrt{2}\left(-5i\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(-3i\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{3}+i\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}+i\sqrt{e^{(4x)}+1}-i\,e^{(2x)}+i\right)}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}-2\sqrt{e^{(4x)}+1}+2\,e^{(2x)}-1\right)^{2}}-4i\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)-4i\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)+4i\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-5*I*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/
(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(-3*I*(sqrt(e^(4*x) + 1) -
 e^(2*x))^3 + I*(sqrt(e^(4*x) + 1) - e^(2*x))^2 + I*sqrt(e^(4*x) + 1) - I*e
^(2*x) + I)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2
*x) - 1)^2 - 4*I*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) - 4*I*log(sqrt(e^(4*x
) + 1) - e^(2*x)) + 4*I*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(-\tanh(x)^2 - 1\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- tanh(x)^2 - 1)^(3/2),x)
```

```
[Out] int((- tanh(x)^2 - 1)^(3/2), x)
```

3.229 $\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+\frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)+(a-b)*(a+b*tanh(x)^2
)^(1/2)/b^2-1/3*(a+b*tanh(x)^2)^(3/2)/b^2
```

Rubi [A]
time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 457, 90, 65, 214}

$$-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}+\frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] + ((a - b)*Sqrt[a +
b*Tanh[x]^2])/b^2 - (a + b*Tanh[x]^2)^(3/2)/(3*b^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{x^5}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a-b}{b\sqrt{a+bx}} + \frac{1}{(1-x)\sqrt{a+bx}} - \frac{\sqrt{a+bx}}{b}\right) dx, x, \tanh^2(x)\right)$$

$$= \frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\right.$$

$$= \frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2} - \frac{\left(a+b\tanh^2(x)\right)^{3/}}{3b^2}$$

Mathematica [A]

time = 0.36, size = 68, normalized size = 0.97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+\frac{(a-b+(a-2b)\cosh(2x))\text{sech}^2(x)\sqrt{a+b\tanh^2(x)}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] + ((a - b + (a - 2*b
)*Cosh[2*x])*Sech[x]^2*Sqrt[a + b*Tanh[x]^2])/(3*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(58) = 116$.
time = 0.78, size = 164, normalized size = 2.34

method	result
derivativedivides	$-\frac{(\tanh^2(x))\sqrt{a+b(\tanh^2(x))}}{3b}+\frac{2a\sqrt{a+b(\tanh^2(x))}}{3b^2}-\frac{\sqrt{a+b(\tanh^2(x))}}{b}+\frac{\ln\left(\frac{2a}{\cdots}\right.}{\cdots}$
default	$-\frac{(\tanh^2(x))\sqrt{a+b(\tanh^2(x))}}{3b}+\frac{2a\sqrt{a+b(\tanh^2(x))}}{3b^2}-\frac{\sqrt{a+b(\tanh^2(x))}}{b}+\frac{\ln\left(\frac{2a}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*tanh(x)^2/b*(a+b*tanh(x)^2)^(1/2)+2/3*a/b^2*(a+b*tanh(x)^2)^(1/2)-(a+b
*tanh(x)^2)^(1/2)/b+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/
2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)^(1/2
)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x)
)+a+b)^(1/2))/(1+tanh(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. $2(58) = 116$.
time = 0.52, size = 2827, normalized size = 40.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*c
osh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(x)^
2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cosh(x)
)*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(
x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2
*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*
cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 +
 b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x
)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x
))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*co
sh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*
a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^
2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*
a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^
4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh
(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*
(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*
a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)
^5 + b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4
+ 3*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^
2*cosh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2
*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4
+ 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a
 + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
 b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b
*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 8
```

```
*sqrt(2)*((a^2 - a*b - 2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*b^2)*cosh(x)*sin
h(x)^3 + (a^2 - a*b - 2*b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^
2 - a*b - 2*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - a*b - 2*b^2 + 4*(
(a^2 - a*b - 2*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(((a + b)
*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)))/((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (
a*b^2 + b^3)*sinh(x)^6 + 3*(a*b^2 + b^3)*cosh(x)^4 + 3*(a*b^2 + b^3 + 5*(a*
b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + 3*(a*b^2 +
 b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 + 3*(a*b^2 + b^3)*cosh(x)^2 + 3*(5*(
a*b^2 + b^3)*cosh(x)^4 + a*b^2 + b^3 + 6*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2
 + 6*((a*b^2 + b^3)*cosh(x)^5 + 2*(a*b^2 + b^3)*cosh(x)^3 + (a*b^2 + b^3)*c
osh(x))*sinh(x)), -1/6*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*si
nh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cos
h(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4
 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x
)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*
cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2
 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/
((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh
(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*
a^2 + a*b - b^2)*cosh(x))*sinh(x))) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh
(x)^5 + b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)
^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5
*b^2*cosh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5
+ 2*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b
)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 4*sqrt(2)*((a^2 - a*b -
2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - a*b - 2
*b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\sqrt{a + b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**5/sqrt(a + b*tanh(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(58) = 116.
time = 0.76, size = 592, normalized size = 8.46

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}-\frac{8\left(3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^5+3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^4\sqrt{a+b}+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(3a-5b)+6\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2\sqrt{a+b}(a-3b)-3(3a^2+6ab-13b^2)\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)-(9a^2-22ab+17b^2)\sqrt{a+b}\right)}{3\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}+a-3b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))/sqrt(a + b) - 8/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + 3*(sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqr
t(a + b) + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b))^3*(3*a - 5*b) + 6*(sqrt(a + b)*e^(2*x) - sqrt(a*e
^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b)*(a -
 3*b) - 3*(3*a^2 + 6*a*b - 13*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (9*a^2 - 22*a*b + 17*b^2)
*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^3
```

Mupad [B]
time = 2.17, size = 65, normalized size = 0.93

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh\left(x\right)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{{\left(b\tanh(x)^2+a\right)}^{3/2}}{3\,b^2}-\left(\frac{a+b}{b^2}-\frac{2\,a}{b^2}\right)\,\sqrt{b\tanh\left(x\right)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2) - (a + b*tanh(x)
^2)^(3/2)/(3*b^2) - ((a + b)/b^2 - (2*a)/b^2)*(a + b*tanh(x)^2)^(1/2)
```

3.230 $\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=88

$$\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

```
[Out] 1/2*(a-2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh(
(a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)-1/2*(a+b*tanh(x)^2)^
(1/2)*tanh(x)/b
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 490, 537, 223, 212, 385}

$$\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}}-\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((a - 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(2*b^(3/2)) +
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Tanh[x]
*Sqrt[a + b*Tanh[x]^2])/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{x^4}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b} + \frac{\text{Subst}\left(\int \frac{a+(-a+2b)x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{2b}$$

$$= -\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{2b} + \text{Su}$$

$$= -\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b}$$

$$= \frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \tanh($$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 3.23, size = 208, normalized size = 2.36

$$\frac{\left(\sqrt{2}a(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)-2\sqrt{2}ab\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\Pi\left(\frac{b}{a+b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)-(a+b)(a-b+(a+b)\cosh(2x))\text{sech}^2(x)\right)\tanh(x)}{2\sqrt{2}b(a+b)\sqrt{(a-b+(a+b)\cosh(2x))\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((Sqrt[2]*a*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Ellipti
cF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - 2*
Sqrt[2]*a*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a
 + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]
- (a + b)*(a - b + (a + b)*Cosh[2*x])*Sech[x]^2)*Tanh[x])/(2*Sqrt[2]*b*(a +
 b)*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(70) = 140.
time = 0.80, size = 178, normalized size = 2.02

method	result
derivativedivides	$-\frac{\sqrt{a+b\left(\tanh^2(x)\right)}\,\tanh(x)}{2b}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\left(\tanh^2(x)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\right.}{}$
default	$-\frac{\sqrt{a+b\left(\tanh^2(x)\right)}\,\tanh(x)}{2b}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\left(\tanh^2(x)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)/b+1/2*a/b^(3/2)*ln(b^(1/2)*tanh(x)+(a+b*
tanh(x)^2)^(1/2))-ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))/b^(1/2)+1/2/(a+
b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(ta
nh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x
))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(70) = 140$.
time = 0.60, size = 5494, normalized size = 62.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh
(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*
cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^
3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^
6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2
```

```
 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4
*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a
*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3
)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3
)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2
*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x
)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*
sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^
6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x
)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*
cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b
 - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(
x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x
)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)
^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((a^2 - a*b - 2*b^2)*cosh(x)^4 +
 4*(a^2 - a*b - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - a*b - 2*b^2)*sinh(x)^4 +
2*(a^2 - a*b - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - a*b - 2*b^2)*cosh(x)^2 + a^2
- a*b - 2*b^2)*sinh(x)^2 + a^2 - a*b - 2*b^2 + 4*((a^2 - a*b - 2*b^2)*cosh(
x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))*sqrt(b)*log(-((a + 2*b)*cosh(x
)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cos
h(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (
a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*
((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4
*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^
2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (b^2*cosh(x)^4 + 4*b^2*cosh(x)*
sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sin
h(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cos
h(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a +
b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 2*sqrt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sin
h(x) + (a*b + b^2)*sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b^2 +
 b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2 + b^3)*sinh(x)
^4 + a*b^2 + b^3 + 2*(a*b^2 + b^3)*cosh(x)^2 + 2*(a*b^2 + b^3 + 3*(a*b^2 +
b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2 + b^3)*cosh
(x))*sinh(x)), -1/4*(2*((a^2 - a*b - 2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*b^
2)*cosh(x)*sinh(x)^3 + (a^2 - a*b - 2*b^2)*sinh(x)^4 + 2*(a^2 - a*b - 2*b^2
)*cosh(x)^2 + 2*(3*(a^2 - a*b - 2*b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(
```

```
x)^2 + a^2 - a*b - 2*b^2 + 4*((a^2 - a*b - 2*b^2)*cosh(x)^3 + (a^2 - a*b -
2*b^2)*cosh(x))*sinh(x))*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2
+ a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 +
4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cos
h(x))*sinh(x) + a + b)) - (b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*si
nh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(
b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(
x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b
^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh
(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^
5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4
 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4
+ 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2
*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 +
2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\sqrt{a + b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**4/sqrt(a + b*tanh(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(70) = 140.
time = 0.70, size = 559, normalized size = 6.35

$$\frac{(a-2b)\arctan\left(-\frac{\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}\,b}-\frac{\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\sqrt{a+b}}-\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}+\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}-\frac{2\left(\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(a+2b)+\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2(3a-2b)\sqrt{a+b}+(3a^2-3ab-2b^2)\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)+(a^2-ab+2b^2)\sqrt{a+b}\right)}{\left(\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}+a-3b\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a - 2*b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b) - 1
/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
 - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/2*
log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a + b)*
e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) -
```

```
sqrt(a + b)))/sqrt(a + b) - 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(a + 2*b) + (sqrt(a + b)*e^(
2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*(
3*a - 2*b)*sqrt(a + b) + (3*a^2 - 3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (a^2 - a*b
+ 2*b^2)*sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^
(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3
*b)^2*b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\sqrt{b\tanh{(x)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^4/(a + b*tanh(x)^2)^(1/2), x)
```

3.231 $\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2
)/b
```

Rubi [A]
time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3751, 457, 81, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x]
^2]/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{x^3}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{\sqrt{a+b\tanh^2(x)}}{b} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= -\frac{\sqrt{a+b\tanh^2(x)}}{b} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{a+b\tanh^2(x)}}{b}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 47, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2],x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x]
^2]/b

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(39) = 78$.
time = 0.73, size = 129, normalized size = 2.74

method	result
derivativedivides	$-\frac{\sqrt{a+b\left(\tanh^2(x)\right)}}{b}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b\left(\tanh(x)-1\right)^2+2b\left(\tanh(x\right.}}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$
default	$-\frac{\sqrt{a+b\left(\tanh^2(x)\right)}}{b}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b\left(\tanh(x)-1\right)^2+2b\left(\tanh(x\right.}}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -(a+b*tanh(x)^2)^(1/2)/b+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b
)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)
^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+ta
nh(x))+a+b)^(1/2))/(1+tanh(x)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^3/sqrt(b*tanh(x)^2 + a), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(39) = 78$.
time = 0.44, size = 1625, normalized size = 34.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a + b)*log
(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*
b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a
^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2
*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a
^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*c
osh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2
*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2
*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6
*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6
 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cos
h(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (
3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a
^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^
2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a
+ b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5
+ (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(
x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20
*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(
x)^6)) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a + b)*
log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 -
 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b + b^
2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a*b
+ b^2), -1/2*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a
 - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a +
 b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a
*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)
^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b
```

```
+ b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x)))
 + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a - b)*arcta
n(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a +
b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x
)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)
*(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cos
h(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a*b + b^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**3/sqrt(a + b*tanh(x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(39) = 78.
time = 0.62, size = 345, normalized size = 7.34

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\sqrt{a+b}}+\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}-\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}-\frac{4\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right)}{\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)\sqrt{a+b}+a-3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))/sqrt(a + b) - 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/((sqrt(a + b)*e
^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2
 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)
```

Mupad [**B**]

time = 1.69, size = 39, normalized size = 0.83

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh\left(x\right)^{2}+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{b\tanh\left(x\right)^{2}+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2) - (a + b*tanh(x)
^2)^(1/2)/b
```

3.232 $\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=60

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

```
[Out] -arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)+arctanh((a+b)^(1/2)
*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3751, 494, 223, 212, 385}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)+\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tan\right.\\ &= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)+\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x,\right.\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.29, size = 101, normalized size = 1.68

$$-\frac{a\coth(x)\Pi\left(\frac{b}{a+b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)\sqrt{(a-b+(a+b)\cosh(2x))\text{sech}^2(x)}}{b(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -((a*Coth[x]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])
*Csch[x]^2)/b]/Sqrt[2]], 1]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])/(b
*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(48) = 96$.
time = 0.70, size = 137, normalized size = 2.28

method	result
derivativedivides	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\left(\tanh^2(x)\right)}\right)}{\sqrt{b}}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b\left(\tanh(x)-1\right.}}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$
default	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\left(\tanh^2(x)\right)}\right)}{\sqrt{b}}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b\left(\tanh(x)-1\right.}}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))/b^(1/2)+1/2/(a+b)^(1/2)*ln((2*a+
2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/
2))/(tanh(x)-1))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*
(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(48) = 96$.
time = 0.50, size = 3361, normalized size = 56.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(a + b)*b*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(a + b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + sqrt(a + b)*b*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b + b^2), 1/4*(4*(a + b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + sqrt(a + b)*b*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6

```
 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (
a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^
3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(
14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b +
4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^
3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2
+ 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b
^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 +
4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(
x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^
2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b
 - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*s
inh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^
2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 +
 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 +
6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*
cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*b*log
(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((
a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a*b + b^2), -1/2*(sqrt(-a - b)*b*arctan(sqrt(2)*(b*cosh(
x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b
)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\sqrt{a + b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**2/sqrt(a + b*tanh(x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(48) = 96.
time = 0.59, size = 252, normalized size = 4.20

$$-\frac{2\arctan\left(-\frac{\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\sqrt{a+b}}-\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}+\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*log(abs(
-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/2*log(abs(-sq
rt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + s
qrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b
)))/sqrt(a + b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^2}{\sqrt{b\tanh\left(x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^2/(a + b*tanh(x)^2)^(1/2), x)
```

3.233 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3751, 455, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(23) = 46$.
time = 0.91, size = 114, normalized size = 3.93

method	result
derivativedivides	$\frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\,\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+}{}\right.}{}$
default	$\frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\,\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-
2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(t
anh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(
x)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(23) = 46$.
time = 0.42, size = 1361, normalized size = 46.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*si
nh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3
+ a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(
x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3
+ 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 +
```

```
 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*si
nh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)
^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 +
 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cos
h(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*c
osh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 1
8*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3
*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^
3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 +
 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cos
h(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh
(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cos
h(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
- 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))
*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a + b), -1
/2*(sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh
(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 +
 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b
^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x)
)*sinh(x))) + sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(
a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x
))*sinh(x) + a + b)))/(a + b)]
```

Sympy [A]
time = 0.69, size = 31, normalized size = 1.07

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] -atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/sqrt(-a - b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(23) = 46.
time = 0.50, size = 188, normalized size = 6.48

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\,\sqrt{a+b}}+\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\,\sqrt{a+b}}-\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))/sqrt(a + b)
```

Mupad [B]
time = 1.62, size = 23, normalized size = 0.79

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh\left(x\right)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2)
```

3.234 $\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3742, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(25) = 50.
time = 0.79, size = 114, normalized size = 3.68

method	result
derivativedivides	$\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} - \frac{\ln\left(\frac{2a+}{}\right.}{}$
default	$\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} - \frac{\ln\left(\frac{2a+}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+
2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1
+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tan
h(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(25) = 50.
time = 0.44, size = 1287, normalized size = 41.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*s
inh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2
 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh
(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b
^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3
 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5
- 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*s
inh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x
)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 -
3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 +
sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*co
sh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*
cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 -
18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(
3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))
*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b
^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 -
3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*co
sh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cos
```

```
h(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cos
h(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
+ 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))
*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a + b), -1
/2*(sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh
(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 +
 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b
^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x)
)*sinh(x))) + sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x
)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a +
 b)))/(a + b)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \tanh^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tanh(x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(25) = 50.
time = 0.51, size = 188, normalized size = 6.06

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\sqrt{a+b}}-\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}+\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))/sqrt(a + b)
```

Mupad [B]
time = 1.57, size = 25, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{\tanh(x)\,\sqrt{a+b}}{\sqrt{b\tanh\left(x\right)^2+a}}\right)}{\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tanh(x)^2)^(1/2),x)
[Out] atanh((tanh(x)*(a + b)^(1/2))/(a + b*tanh(x)^2)^(1/2))/(a + b)^(1/2)
```

3.235 $\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=56

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[Out] -arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(1/2)+arctanh((a+b*tanh(x)^2)^(1/
2)/(a+b)^(1/2))/(a+b)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3751, 457, 88, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{tan}\right.\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\text{tanh}}\right.}{b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 56, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Maple [F]
time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a+b\left(\tanh^2(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(44) = 88$.
time = 0.48, size = 3527, normalized size = 62.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*a*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*
sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^
```

```
3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cos
h(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 +
b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^
3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5
 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*
sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(
x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3
 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 +
 sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*c
osh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2
*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 +
 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*
(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x)
)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*
a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3
 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(a + b)*sqrt(a)*log(-((2*a + b)*cosh(x)^4
 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x
)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a +
 b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2
*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 +
 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + sqrt(a + b)*a*log(-((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(a^2 + a*b), 1/4*(4*sqrt(-a)*(a + b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)
^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2
+ 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b
)*cosh(x))*sinh(x) + a + b)) + sqrt(a + b)*a*log(((a^3 + a^2*b)*cosh(x)^8 +
 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a
^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6
+ 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6
*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a
^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(1
4*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b
 - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a
^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 +
```

```
a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5
+ a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4
*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x
)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2
 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*
b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
 a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
+ b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6
*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*c
osh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*a*log(
-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b
*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a
+ b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((
a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a^2 + a*b), -1/2*(a*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(
x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b
)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a + b\tanh^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*tanh(x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(44) = 88.
time = 0.59, size = 254, normalized size = 4.54

$$\frac{2\arctan\left(-\frac{\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2*log(abs(-
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/2*log(abs(-sqr
```

```
t(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)
))/sqrt(a + b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)}{\sqrt{b\tanh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)/(a + b*tanh(x)^2)^(1/2), x)
```

3.236 $\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)-coth(x)*(a+b*tanh(x)^2)^(1/2)/a

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 491, 12, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Coth[x]*Sqrt[a + b*Tanh[x]^2])/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x^2(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a} + \frac{\text{Subst}\left(\int \frac{a}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a}\\
&= -\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a} + \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a} + \text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 3.97, size = 123, normalized size = 2.41

$$\frac{\left(\frac{(a+b)^2(a-b+(a+b)\cosh(2x))^2\,{}_2F_1\left(2,2;\frac{5}{2};-\frac{(a+b)\sinh^2(x)}{a}\right)}{a^3}+3\text{ArcSin}\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)(2b+a\coth^2(x))\operatorname{csch}^2(x)\sqrt{-\frac{(a+b)\,(b+a\coth^2(x))\sinh^4(x)}{a^2}}\right)\tanh(x)}{3(a+b)\sqrt{a+b\tanh^2(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((((a + b)^2*(a - b + (a + b)*Cosh[2*x])^2*Hypergeometric2F1[2, 2, 5/2, -((
(a + b)*Sinh[x]^2)/a)])/a^3 + 3*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*(2*b
 + a*Coth[x]^2)*Csch[x]^2*Sqrt[-(((a + b)*(b + a*Coth[x]^2)*Sinh[x]^4)/a^2)
])*Tanh[x])/(3*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Maple [F]
time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\sqrt{a+b\left(\tanh^2(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(43) = 86.
time = 0.42, size = 1565, normalized size = 30.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a + b)*log
(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b
```

```
^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2
+ b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*
b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(
a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)
*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*
cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^
2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 +
b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(
a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^
6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*co
sh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 -
(a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^
2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b
^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5
 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh
(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 2
0*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh
(x)^6)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a + b)
*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
 (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*
b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2
- a*b), -1/2*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a
 - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a -
 b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b
^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)
^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b
+ b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x)))
 + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a - b)*arcta
n(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) + 2*sqrt(2)*(a + b)*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (
a^2 + a*b)*sinh(x)^2 - a^2 - a*b)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)**2/sqrt(a + b*tanh(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(43) = 86.
time = 0.59, size = 343, normalized size = 6.73

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}+\frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right)}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}-3a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a + b
)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
 - sqrt(a + b)))/sqrt(a + b) + 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/((sqrt(a + b)*e
^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2
 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)^2}{\sqrt{b\tanh(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2/(a + b*tanh(x)^2)^(1/2), x)
```

3.237 $\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal. Leaf size=88

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a}$$

```
[Out] -1/2*(2*a-b)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arctanh((a+b*ta
nh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-1/2*coth(x)^2*(a+b*tanh(x)^2)^(1/2)
/a
```

Rubi [A]
time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 457, 105, 162, 65, 214}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] -1/2*((2*a - b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/a^(3/2) + ArcTanh[S
qrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x^3(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x^2\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2a+b)-\frac{bx}{2}}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2a}$$
$$= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right) + \cdot$$
$$= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{b} + \frac{(2a}{}$$
$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\text{coth}}{}$$

Mathematica [A]
time = 0.34, size = 107, normalized size = 1.22

$$\frac{(-2a^2-ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+\sqrt{a}\left(2a\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}\right)}{2a^{3/2}(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

```
[Out] ((-2*a^2 - a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(2*a
*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Coth[x]^2
*Sqrt[a + b*Tanh[x]^2]))/(2*a^(3/2)*(a + b))
```

Maple [F]
time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a+b\left(\tanh^2(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^3/sqrt(b*tanh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(70) = 140$.
time = 0.64, size = 5711, normalized size = 64.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh
(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*
cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b
)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6
 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 +
a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b
- a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b -
a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)
*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)
*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b -
b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)
^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*s
inh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6
 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)
^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*c
osh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b
+ b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x
))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)
^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^
3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh
(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x
)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((2*a^2 + a*b - b^2)*cosh(x)^4 +
```

4*(2*a^2 + a*b - b^2)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2)*sinh(x)^4 - 2
*(2*a^2 + a*b - b^2)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(x)^2 - 2*a^2
 - a*b + b^2)*sinh(x)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(x
)^3 - (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-((2*a + b)*cosh(x)
^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh
(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a
 + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(
(2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*
cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2
 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + (a^2*cosh(x)^4 + 4*a^2*cosh(x)*s
inh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh
(x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a
 + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cos
h(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a +
b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sin
h(x) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)
*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + a
^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b)*sinh(x)
^4 + a^3 + a^2*b - 2*(a^3 + a^2*b)*cosh(x)^2 - 2*(a^3 + a^2*b - 3*(a^3 + a^
2*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 - (a^3 + a^2*b)*cosh
(x))*sinh(x)), 1/4*(2*((2*a^2 + a*b - b^2)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2
)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2)*sinh(x)^4 - 2*(2*a^2 + a*b - b^2)
cosh(x)^2 + 2(3*(2*a^2 + a*b - b^2)*cosh(x)^2 - 2*a^2 - a*b + b^2)*sinh(x
)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(x)^3 - (2*a^2 + a*b -
 b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3
*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh
(x))*sinh(x) + a + b)) + (a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sin
h(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 4*(a
^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)
^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3
 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x
)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5
+ (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 +
 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 +
4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a
^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*
(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 ...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)**3/sqrt(a + b*tanh(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(70) = 140.
time = 0.70, size = 565, normalized size = 6.42

$$\frac{(2a-b)\arctan\left(-\frac{\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\sqrt{a+b}}+\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}-\frac{\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}+\frac{2\left(\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(2a+b)+\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2(2a-3b)\sqrt{a+b}-(2a^2+3ab-3b^2)\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)-(2a^2-ab+b^2)\sqrt{a+b}\right)}{\left(\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}-3a+b\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a - b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a) - 1
/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
 - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/2*
log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b)*
e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) -
 sqrt(a + b)))/sqrt(a + b) + 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(2*a + b) + (sqrt(a + b)*e^(
2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*(
2*a - 3*b)*sqrt(a + b) - (2*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (2*a^2 - a*
b + b^2)*sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^
(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a +
 b)^2*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^3}{\sqrt{b\tanh(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*tanh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3/(a + b*tanh(x)^2)^(1/2), x)
```

3.238 $\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\sqrt{a+b\tanh^2(x)}}{b^2}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)-a^2/b^2/(a+b)/(a+b*t
anh(x)^2)^(1/2)-(a+b*tanh(x)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 457, 89, 65, 214}

$$-\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\sqrt{a+b\tanh^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) - a^2/(b^2*(a + b)
*Sqrt[a + b*Tanh[x]^2]) - Sqrt[a + b*Tanh[x]^2]/b^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{x^5}{(1-x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b(a+b)(a+bx)^{3/2}} - \frac{1}{b\sqrt{a+bx}} - \frac{1}{(a+b)(-1+x)\sqrt{a+bx}}\right)\right.$$

$$= -\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\sqrt{a+b\tanh^2(x)}}{b^2} - \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\right.}{2(a+b)}$$

$$= -\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\sqrt{a+b\tanh^2(x)}}{b^2} - \frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{b(a+b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\sqrt{a+b\tanh^2(x)}}{b^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 67, normalized size = 0.93

$$\frac{-b^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tanh^2(x)}{a+b}\right)-(a+b)\left(2a-b+b\tanh^2(x)\right)}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (-(b^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) - (a + b
)*(2*a - b + b*Tanh[x]^2))/(b^2*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(62) = 124.
time = 0.69, size = 322, normalized size = 4.47

method	result
derivativedivides	$-\frac{\tanh^2(x)}{b\sqrt{a+b\left(\tanh^2(x)\right)}}-\frac{2a}{b^2\sqrt{a+b\left(\tanh^2(x)\right)}}+\frac{1}{b\sqrt{a+b\left(\tanh^2(x)\right)}}-\frac{}{2(a+b)\sqrt{b(1}}$
default	$-\frac{\tanh^2(x)}{b\sqrt{a+b\left(\tanh^2(x)\right)}}-\frac{2a}{b^2\sqrt{a+b\left(\tanh^2(x)\right)}}+\frac{1}{b\sqrt{a+b\left(\tanh^2(x)\right)}}-\frac{}{2(a+b)\sqrt{b(1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -tanh(x)^2/b/(a+b*tanh(x)^2)^(1/2)-2*a/b^2/(a+b*tanh(x)^2)^(1/2)+1/b/(a+b*t
anh(x)^2)^(1/2)-1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-b/(a+
b)*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))
+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1
+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b)/(b*(tanh(x)-
1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*
b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*
b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)
)/(tanh(x)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1713 vs. $2(62) = 124$.
time = 0.59, size = 3991, normalized size = 55.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2
 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2
+ b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3
)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 + (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2
 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 + 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2
+ 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b
^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a
^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(
x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^
3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*
b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^
2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 +
b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*
b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cos
h(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^
2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(
x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cos
h(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a
^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*
a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*co
sh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cos
h(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh
(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*
sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*si
nh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a*b^2 + b^3)*cosh(x)^6 + 6*
(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)
*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*
(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3
+ (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 +
```

```
 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*
(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*l
og(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 -
2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2)) - 4*sqrt(2)*((2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^
4 + 4*(2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (2*a^3 + 4*a^2*
b + 3*a*b^2 + b^3)*sinh(x)^4 + 2*a^3 + 4*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 +
 2*a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(2*a^3 + 2*a^2*b - a*b^2 - b^3 + 3*(2
*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 4*a^2*b
+ 3*a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 2*a^2*b - a*b^2 - b^3)*cosh(x))*sinh(
x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^
6 + 6*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^5 + (a^3*b^2 +
3*a^2*b^3 + 3*a*b^4 + b^5)*sinh(x)^6 + a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5
+ (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + (3*a^3*b^2 + 5*a^2*b^3
+ a*b^4 - b^5 + 15*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)
^4 + 4*(5*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*
a^2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4
- b^5)*cosh(x)^2 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5 + 15*(a^3*b^2 + 3*a
^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^4 + 6*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)
*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^
5 + 2*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*a^2*
b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)), -1/2*(((a*b^2 + b^3)*cosh(x)^6 + 6*(a
*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)*c
osh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 +
(3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 + 6
*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*(3
*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(-a - b)*ar
ctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt
(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh
(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(3/2),x)
```

[Out] Integral(tanh(x)**5/(a + b*tanh(x)**2)**(3/2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(62) = 124$.
time = 0.68, size = 486, normalized size = 6.75

$$-\frac{\frac{(a^4 b + a^3 b^2)e^{(2x)}}{a^3 b^3 + 2a^2 b^4 + ab^5} + \frac{a^4 b + a^3 b^2}{a^3 b^3 + 2a^2 b^4 + ab^5}}{\sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}} + \frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)} - \frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{3/2}} - \frac{4\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right)}{\left(\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^2 + 2\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)\sqrt{a+b} + a - 3b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")

[Out] -((a^4*b + a^3*b^2)*e^(2*x)/(a^3*b^3 + 2*a^2*b^4 + a*b^5) + (a^4*b + a^3*b^2)/(a^3*b^3 + 2*a^2*b^4 + a*b^5))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a^2 + 2*a*b + b^2) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a^2 + 2*a*b + b^2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a + b)^(3/2) - 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)*b)

Mupad [B]
time = 2.52, size = 70, normalized size = 0.97

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,(2a+2b)}{2(a+b)^{3/2}}\right)}{(a+b)^{3/2}} - \frac{\sqrt{b\tanh(x)^2+a}}{b^2} - \frac{a^2}{b^2\,(a+b)\,\sqrt{b\tanh(x)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5/(a + b*tanh(x)^2)^(3/2),x)

[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(2*a + 2*b))/(2*(a + b)^(3/2)))/(a + b)^(3/2) - (a + b*tanh(x)^2)^(1/2)/b^2 - a^2/(b^2*(a + b)*(a + b*tanh(x)^2)^(1/2))

3.239 $\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] -arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh((a+b)^(1/2)
*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+a*tanh(x)/b/(a+b)/(a+b*tanh(x)^
2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 481, 537, 223, 212, 385}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}}+\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/b^(3/2)) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (a*Tanh[x])/(b*(a +
 b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{x^4}{(1-x^2)\,(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{a+(-a-b)x^2}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{b(a+b)}$$

$$= \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{b} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b} + \frac{\text{Subst}\cdots}{\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{a\tanh\cdots}{b(a+b)\sqrt{a\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 1.69, size = 188, normalized size = 2.24

$$-\frac{a\left(-2a-2b+\sqrt{2}(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\,F\left(\operatorname{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)+\sqrt{2}\,b\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\,\Pi\left(\frac{b}{a+b};\operatorname{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\tanh(x)}{\sqrt{2}\,b(a+b)^2\sqrt{(a-b+(a+b)\cosh(2x))\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -((a*(-2*a - 2*b + Sqrt[2]*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x
]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sq
rt[2]], 1] + Sqrt[2]*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Elli
pticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sq
rt[2]], 1])*Tanh[x])/(Sqrt[2]*b*(a + b)^2*Sqrt[(a - b + (a + b)*Cosh[2*x])*
Sech[x]^2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(70) = 140.
time = 0.72, size = 328, normalized size = 3.90

method	result
derivativedivides	$\frac{\tanh(x)}{b\sqrt{a+b\left(\tanh^2(x)\right)}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\left(\tanh^2(x)\right)}\right)}{b^{\frac{3}{2}}}-\frac{\tanh(x)}{a\sqrt{a+b\left(\tanh^2(x)\right)}}$
default	$\frac{\tanh(x)}{b\sqrt{a+b\left(\tanh^2(x)\right)}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\left(\tanh^2(x)\right)}\right)}{b^{\frac{3}{2}}}-\frac{\tanh(x)}{a\sqrt{a+b\left(\tanh^2(x)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] tanh(x)/b/(a+b*tanh(x)^2)^(1/2)-1/b^(3/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2
)^(1/2))-tanh(x)/a/(a+b*tanh(x)^2)^(1/2)+1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+
tanh(x))+a+b)^(1/2)+b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+t
anh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tan
h(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)
))-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh
(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1
/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2
*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1434 vs. 2(70) = 140.
time = 0.61, size = 6973, normalized size = 83.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2
 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b^2 - b^
3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*
b^2 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*
(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^
3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4
*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3
- a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 -
a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(
a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*
b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 -
3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*
b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*
cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b
^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5
*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2
 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 -
a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3
*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 +
b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b
^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*co
sh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh
(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 2*((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)
^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^
3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(
x))*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a
+ 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2
*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*s
qrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))
*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1))
 + ((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2 +
b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b^2 - b^3 +
 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2
 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*co
sh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^
2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh
(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a^2*
```

```
b + a*b^2 - (a^2*b + a*b^2)*cosh(x)^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) -
 (a^2*b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2 + 3*a^2*b^3 +
3*a*b^4 + b^5 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^4 + 4*(a^3*b^
2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (a^3*b^2 + 3*a^2*b^3 + 3
*a*b^4 + b^5)*sinh(x)^4 + 2*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*cosh(x)^2 + 2
*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5 + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)
*cosh(x)^2)*sinh(x)^2 + 4*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^3
+ (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)), 1/4*(4*((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*s
inh(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^
2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))
*sinh(x))*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(
x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) +
 a + b)) + ((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (
a*b^2 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 - b^3)*cosh(x)^2 + 2*(a*b^2
 - b^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 +...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(70) = 140.
time = 0.66, size = 410, normalized size = 4.88

$$\frac{\frac{(a^3b^2+a^2b^3)e^{(2x)}}{a^3b^3+2a^2b^4+ab^5}-\frac{a^3b^2+a^2b^3}{a^3b^3+2a^2b^4+ab^5}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a^2+2ab+b^2)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}-\frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^3*b^2 + a^2*b^3)*e^(2*x)/(a^3*b^3 + 2*a^2*b^4 + a*b^5) - (a^3*b^2 + a^2
*b^3)/(a^3*b^3 + 2*a^2*b^4 + a*b^5))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
```

```
x) - 2*b*e^(2*x) + a + b) - 1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) -
 sqrt(a + b)*(a - b)))/(a^2 + 2*a*b + b^2) - 1/2*sqrt(a + b)*log(abs(-sqrt(
a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) + sqrt(a + b)))/(a^2 + 2*a*b + b^2) + 1/2*sqrt(a + b)*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
+ b) - sqrt(a + b)))/(a^2 + 2*a*b + b^2) - 2*arctan(-1/2*(sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt
(a + b))/sqrt(-b))/(sqrt(-b)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^4/(a + b*tanh(x)^2)^(3/2), x)
```

3.240 $\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)+a/b/(a+b)/(a+b*tanh(
x)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3751, 457, 79, 65, 214}

$$\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sqr
t[a + b*Tanh[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x^3}{(1-x^2)\left(a+bx^2\right)^{3/2}}\,dx,x,\tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^{3/2}}\,dx,x,\tanh^2(x)\right)\\
&= \frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx,x,\tanh^2(x)\right)}{2(a+b)}\\
&= \frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\tanh^2(x)}\right)}{b(a+b)}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sqr
t[a + b*Tanh[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(44) = 88$.
time = 0.64, size = 287, normalized size = 5.52

method	result
derivativedivides	$\frac{1}{b\sqrt{a+b\left(\tanh^2(x)\right)}}-\frac{1}{2(a+b)\sqrt{b\left(1+\tanh(x)\right)^2-2b\left(1+\tanh(x)\right)+a+b}}-\frac{}{(a+b)(4}$
default	$\frac{1}{b\sqrt{a+b\left(\tanh^2(x)\right)}}-\frac{1}{2(a+b)\sqrt{b\left(1+\tanh(x)\right)^2-2b\left(1+\tanh(x)\right)+a+b}}-\frac{}{(a+b)(4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(a+b*tanh(x)^2)^(1/2)-1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(
1/2)-b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b*(
1+tanh(x))+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(
1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b)/(b
*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*
b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*
ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+
a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(3/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(44) = 88$.
time = 0.44, size = 2525, normalized size = 48.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")

```
[Out] [1/4*(((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2
)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 + 2*(3*(a*b + b^2)*cosh(x)^2 + a*b -
b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(x)^3 + (a*b - b^2)*cosh(x)
)*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(
x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2
*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*
cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 +
 b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x
)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x
))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*co
sh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*
a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^
2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*
a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^
4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh
(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*
(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*
a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*c
osh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 + 2*(3*(
a*b + b^2)*cosh(x)^2 + a*b - b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b + b^2)*co
sh(x)^3 + (a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4
 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 -
```

```
b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a
*b)*sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*cosh(x)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)*sinh
(x)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sinh(x)^4 + a^3*b + 3*a^2*b^2 +
 3*a*b^3 + b^4 + 2*(a^3*b + a^2*b^2 - a*b^3 - b^4)*cosh(x)^2 + 2*(a^3*b + a
^2*b^2 - a*b^3 - b^4 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^2)*sin
h(x)^2 + 4*((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^3 + (a^3*b + a^2*b^
2 - a*b^3 - b^4)*cosh(x))*sinh(x)), -1/2*(((a*b + b^2)*cosh(x)^4 + 4*(a*b +
 b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 +
 2*(3*(a*b + b^2)*cosh(x)^2 + a*b - b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b +
b^2)*cosh(x)^3 + (a*b - b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*
(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3
 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*c
osh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a
*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + ((a*b + b^2)*cosh(
x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + 2*(a*b - b
^2)*cosh(x)^2 + 2*(3*(a*b + b^2)*cosh(x)^2 + a*b - b^2)*sinh(x)^2 + a*b + b
^2 + 4*((a*b + b^2)*cosh(x)^3 + (a*b - b^2)*cosh(x))*sinh(x))*sqrt(-a - b)*
arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*sq
rt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*
sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*cosh(x)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)*sinh(x)^
3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sinh(x)^4 + a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4 + 2*(a^3*b + a^2*b^2 - a*b^3 - b^4)*cosh(x)^2 + 2*(a^3*b + a^2*b
^2 - a*b^3 - b^4 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^2)*sinh(x)
^2 + 4*((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cosh(x)^3 + (a^3*b + a^2*b^2 -
a*b^3 - b^4)*cosh(x))*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b\tanh^2(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(44) = 88.
time = 0.55, size = 313, normalized size = 6.02

$$\frac{\frac{(a^3+a^2b)e^{(2x)}}{a^3b+2a^2b^2+ab^3}+\frac{a^3+a^2b}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^3 + a^2*b)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) + (a^3 + a^2*b)/(a^3*b +
 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a^2 + 2*a*
b + b^2) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a^2 + 2*a*b
 + b^2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a + b)
^(3/2)
```

Mupad [B]
time = 2.06, size = 45, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{(b^2+a\,b)\,\sqrt{b\tanh(x)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(3/2) + a/((a*b + b^2)
*(a + b*tanh(x)^2)^(1/2))
```

3.241 $\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}-\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)-tanh(x)/(a+b
)/(a+b*tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3751, 482, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}-\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) - Tanh[x
]/((a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a+b}\\
&= -\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{a+b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 112 vs. 2(53) = 106.

time = 1.29, size = 112, normalized size = 2.11

$$\frac{\tanh(x)\left(\tanh^{-1}\left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{1+\frac{b\tanh^2(x)}{a}}}\right)\left(b+a\coth^2(x)\right)\sqrt{\frac{(a+b)\tanh^2(x)}{a}}-(a+b)\sqrt{1+\frac{b\tanh^2(x)}{a}}\right)}{(a+b)^2\sqrt{a+b\tanh^2(x)}\sqrt{1+\frac{b\tanh^2(x)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (Tanh[x]*(ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a]]*(b
 + a*Coth[x]^2)*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*Sqrt[1 + (b*Tanh[x]^2
)/a]))/((a + b)^2*Sqrt[a + b*Tanh[x]^2]*Sqrt[1 + (b*Tanh[x]^2)/a])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(45) = 90$.
time = 0.65, size = 289, normalized size = 5.45

method	result
derivativedivides	$-\frac{\tanh(x)}{a\sqrt{a+b\left(\tanh^2(x)\right)}}-\frac{1}{2(a+b)\sqrt{b\left(\tanh(x)-1\right)^2+2b\left(\tanh(x)-1\right)+a+b}}+$
default	$-\frac{\tanh(x)}{a\sqrt{a+b\left(\tanh^2(x)\right)}}-\frac{1}{2(a+b)\sqrt{b\left(\tanh(x)-1\right)^2+2b\left(\tanh(x)-1\right)+a+b}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -tanh(x)/a/(a+b*tanh(x)^2)^(1/2)-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)
+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^
2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*
(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(
a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+b/(a+b)*(2*b*(1+tanh(x))-2
*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)
^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+ta
nh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(45) = 90.
time = 0.46, size = 2281, normalized size = 43.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
+ 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a*b^2
+ b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x
)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(
x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^
3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3
+ (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(
x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*
b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*
cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 -
b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a
*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b
+ 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)
^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh
(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 -
a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
 - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*co
sh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a + b)*cosh(x
```

```
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
 - 4*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(
x)^2 - a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x)), -1/2*(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)
^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b +
b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 +
 a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x
)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*s
inh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3
*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh
(x))*sinh(x) + a + b)) + 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*s
inh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sin
h(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2
- b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*s
inh(x))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(45) = 90.
time = 0.55, size = 319, normalized size = 6.02

$$-\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+2a^2b^2+ab^3}-\frac{a^2b+ab^2}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b + a*b^2)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) - (a^2*b + a*b^2)/(a^
3*b + 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a^2 +
 2*a*b + b^2) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a^2 +
2*a*b + b^2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a
 + b)^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^2}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^2/(a + b*tanh(x)^2)^(3/2), x)
```

3.242 $\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)-1/(a+b)/(a+b*tanh(x)
^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3751, 455, 53, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) - 1/((a + b)*Sqrt[
a + b*Tanh[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x}{(1-x^2)\left(a+bx^2\right)^{3/2}}\,dx,x,\tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx,x,\tanh^2(x)\right)\\
&= -\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx,x,\tanh^2(x)\right)}{2(a+b)}\\
&= -\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\tanh^2(x)}\right)}{b(a+b)}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 41, normalized size = 0.84

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tanh^2(x)}{a+b}\right)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*Sqrt[
a + b*Tanh[x]^2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(41) = 82.
time = 0.66, size = 273, normalized size = 5.57

method	result
derivativedivides	$-\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}-\frac{b(2b(1}{(a+b)(4b(a+b)-4b^2)\sqrt{b(1+\tanh}}$
default	$-\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}-\frac{b(2b(1}{(a+b)(4b(a+b)-4b^2)\sqrt{b(1+\tanh}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-b/(a+b)*(2*b*(1+tanh
(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/2
/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b
*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(
x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x
)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-
1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(tanh(x)/(b*tanh(x)^2 + a)^(3/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(41) = 82$.
time = 0.45, size = 2277, normalized size = 46.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
+ 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a^3 + a
^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)
^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh
(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x
))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b
)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)
*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 +
 (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b
^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x
)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a
^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*c
osh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a
^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*
a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b
 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^
3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 +
 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x
))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3
*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
```

```
 - 4*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(
x)^2 + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x)), -1/2*(((a + b
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)
^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 +
 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x
)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*s
inh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3
*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh
(x))*sinh(x) + a + b)) + 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*s
inh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sin
h(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2
- b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*s
inh(x))]
```

Sympy [**A**]
time = 11.06, size = 51, normalized size = 1.04

$$-\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] -1/((a + b)*sqrt(a + b*tanh(x)**2)) - atan(sqrt(a + b*tanh(x)**2)/sqrt(-a -
 b))/(sqrt(-a - b)*(a + b))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(41) = 82.
time = 0.55, size = 318, normalized size = 6.49

$$-\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+2a^2b^2+ab^3}+\frac{a^2b+ab^2}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b + a*b^2)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) + (a^2*b + a*b^2)/(a^
3*b + 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a^2 +
 2*a*b + b^2) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a^2 +
2*a*b + b^2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a
 + b)^(3/2)
```

Mupad [B]
time = 1.94, size = 41, normalized size = 0.84

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\sqrt{b\tanh(x)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(3/2) - 1/((a + b)*(a
+ b*tanh(x)^2)^(1/2))
```

3.243 $\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+b*tanh(x)/a/
(a+b)/(a+b*tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3742, 390, 385, 212}

$$\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[x]^2)^(-3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Tan
h[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a+b}\\
&= \frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{a+b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.02, size = 223, normalized size = 3.98

$$
-\frac{\sinh^2(x)\left(\frac{15}{4}a(3a-2b+(3a+2b)\cosh(2x))\text{csch}(x)\text{sech}(x)\left((a-b)\text{ArcSin}\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)+(a+b)\text{ArcSin}\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)\cosh(2x)-2a\sqrt{-\frac{(a+b)\left(b+a\coth^2(x)\right)\sinh^4(x)}{a^2}}\right)+\sqrt{2}\,a^2(a+b)\,{}_2F_1\left(2,2;\frac{7}{2};-\frac{(a+b)\sinh^2(x)}{a}\right)\left(-\frac{(a+b)(a-b+(a+b)\cosh(2x))\sinh^2(x)}{a^2}\right)^{3/2}\tanh(x)\right)}{15a^4\left(-\frac{(a+b)\sinh^2(x)}{a}\right)^{3/2}\sqrt{\cosh^2(x)+\frac{b\sinh^2(x)}{a}}\sqrt{a+b\tanh^2(x)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tanh[x]^2)^(-3/2),x]
```

```
[Out] -1/15*(Sinh[x]^2*((15*a*(3*a - 2*b + (3*a + 2*b)*Cosh[2*x])*Csch[x]*Sech[x]
*((a - b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]] + (a + b)*ArcSin[Sqrt[-(((
a + b)*Sinh[x]^2)/a)]]*Cosh[2*x] - 2*a*Sqrt[-(((a + b)*(b + a*Coth[x]^2)*Si
nh[x]^4)/a^2)]))/4 + Sqrt[2]*a^2*(a + b)*Hypergeometric2F1[2, 2, 7/2, -(((a
 + b)*Sinh[x]^2)/a)]*(-(((a + b)*(a - b + (a + b)*Cosh[2*x])*Sinh[x]^2)/a^2
))^(3/2)*Tanh[x]))/(a^4*(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[Cosh[x]^2 + (
b*Sinh[x]^2)/a]*Sqrt[a + b*Tanh[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(48) = 96$.
time = 0.72, size = 272, normalized size = 4.86

method	result
derivativedivides	$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(t}{(a+b)(4b(a+b)-4b^2)\sqrt{b(\tanh(x)}$
default	$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(t}{(a+b)(4b(a+b)-4b^2)\sqrt{b(\tanh(x)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x
)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2
/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b
*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tan
h(x))+a+b)^(1/2)+b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh
(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x
))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(-3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(48) = 96$.
time = 0.46, size = 2509, normalized size = 44.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b
)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 -
a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x)
)*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh
(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(
a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)
*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2
+ 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 +
6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(
x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(
x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*c
osh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a
^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)
^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b
^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3
*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)
^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2
+ 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sin
h(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3
*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a
^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) +
15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 +
6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*
cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 + 2*(3*
(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*c
osh(x)^3 + (a^2 - a*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4
 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 +
a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*sqrt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b
^2)*sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4 + 3*a^3*b + 3*a^2*
b^2 + a*b^3)*cosh(x)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh
```

```
(x)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^4 + a^4 + 3*a^3*b + 3*a
^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 2*(a^4 + a^3
*b - a^2*b^2 - a*b^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sin
h(x)^2 + 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b -
a^2*b^2 - a*b^3)*cosh(x))*sinh(x)), -1/2*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 +
 a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 +
 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 +
a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*
(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt
(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3
 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*c
osh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b
^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + ((a^2 + a*b)*cosh(
x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 - a
*b)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a
*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*
arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*sq
rt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*
sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4 + 3*a^3*b + 3*a^2*b^2
+ a*b^3)*cosh(x)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^
3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^4 + a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 2*(a^4 + a^3*b -
 a^2*b^2 - a*b^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)
^2 + 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b - a^2*
b^2 - a*b^3)*cosh(x))*sinh(x))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \tanh^{2}{\left(x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(-3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(48) = 96.

time = 0.53, size = 314, normalized size = 5.61

$$\frac{\frac{(ab^2+b^3)e^{(2x)}}{a^3b+2a^2b^2+ab^3}-\frac{ab^2+b^3}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((a*b^2 + b^3)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) - (a*b^2 + b^3)/(a^3*b +
 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a^2 + 2*a*
b + b^2) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a^2 + 2*a*b
 + b^2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a + b)
^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\tanh(x)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*tanh(x)^2)^(3/2), x)
```

3.244 $\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

```
[Out] -arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arctanh((a+b*tanh(x)^2)^(1/
2)/(a+b)^(1/2))/(a+b)^(3/2)+b/a/(a+b)/(a+b*tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3751, 457, 87, 162, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/a^(3/2)) + ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + b/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)$$

$$= \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{-a-b+bx}{(1-x)x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2a(a+b)}$$

$$= \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tanh^2(x)\right)}{2a} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tanh^2(x)}\right)}{ab} + \frac{\text{Sub}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{}{a(a+b)\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.06, size = 70, normalized size = 0.90

$$\frac{-a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tanh^2(x)}{a+b}\right)+(a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\tanh^2(x)}{a}\right)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] (-(a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b)*
Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tanh[x]^2)/a])/(a*(a + b)*Sqrt[a + b
*Tanh[x]^2])
```

Maple [F]

time = 1.55, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\left(a+b\left(\tanh^2(x)\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. $2(64) = 128$.
time = 0.67, size = 6955, normalized size = 89.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 +
 a^2*b)*sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*cosh(x)^2 + 2*(a^3 - a^2*
b + 3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 + (a^
3 - a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(
a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b
)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*
(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3
 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 +
 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a
^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a
*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 +
 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2
*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*c
osh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^
2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*
a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2
+ (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b -
b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x
)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2
*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^
```

```
3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cos
h(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(
x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 2*((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^
3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b
^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3
 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x
))*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a
 + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a -
b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sq
rt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*
sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh
(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1))
+ ((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2
*b)*sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*cosh(x)^2 + 2*(a^3 - a^2*b +
3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 + (a^3 -
a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*co
sh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^
2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*
sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh
(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a^2*
b + a*b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) +
 (a^2*b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 3*a^4*b + 3*a^3*
b^2 + a^2*b^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 4*(a^5 +
3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^3 + (a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*sinh(x)^4 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3)*cosh(x)^2 + 2
*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + 3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)
*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3
+ (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*cosh(x))*sinh(x)), 1/4*(4*((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*s
inh(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^
2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))
*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(
x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) +
 a + b)) + ((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (
a^3 + a^2*b)*sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*cosh(x)^2 + 2*(a^3 -
```

```
a^2*b + 3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(coth(x)/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(64) = 128.
time = 0.65, size = 411, normalized size = 5.27

$$\frac{\frac{(a^3b^2+a^2b^3)e^{(2x)}}{a^5b+2a^4b^2+a^3b^3}+\frac{a^3b^2+a^2b^3}{a^5b+2a^4b^2+a^3b^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a^2+2ab+b^2)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}+\frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^3*b^2 + a^2*b^3)*e^(2*x)/(a^5*b + 2*a^4*b^2 + a^3*b^3) + (a^3*b^2 + a^2
*b^3)/(a^5*b + 2*a^4*b^2 + a^3*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b) - 1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) -
 sqrt(a + b)*(a - b)))/(a^2 + 2*a*b + b^2) + 1/2*sqrt(a + b)*log(abs(-sqrt(
a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) + sqrt(a + b)))/(a^2 + 2*a*b + b^2) - 1/2*sqrt(a + b)*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
+ b) - sqrt(a + b)))/(a^2 + 2*a*b + b^2) + 2*arctan(-1/2*(sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt
(a + b))/sqrt(-a))/(sqrt(-a)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)}{\left(b\tanh(x)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)/(a + b*tanh(x)^2)^(3/2), x)
```

3.245 $\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+b*coth(x)/a/
(a+b)/(a+b*tanh(x)^2)^(1/2)-(a+2*b)*coth(x)*(a+b*tanh(x)^2)^(1/2)/a^2/(a+b)
```

Rubi [A]

time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 483, 597, 12, 385, 212}

$$-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Cot
h[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]) - ((a + 2*b)*Coth[x]*Sqrt[a + b*Tan
h[x]^2])/(a^2*(a + b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right) \\
&= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{-a-2b+2bx^2}{x^2(1-x^2)\sqrt{a+bx^2}}\,dx, x, \tanh(x)\right)}{a(a+b)} \\
&= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots} \\
&= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots} \\
&= \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+2b)\coth(x)\cdots}{a^2(\cdots}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 7.22, size = 230, normalized size = 2.71

$$
-\frac{\left((a+b)\left(a^2-2b^2+\left(a^2+2ab+2b^2\right)\cosh(2x)\right)\operatorname{csch}^2(x)-\sqrt{2}\,a^2(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)+\sqrt{2}\,a^3\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\,\Pi\left(\frac{b}{a+b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\operatorname{sech}^2(x)\sinh(2x)}{2\sqrt{2}\,a^2(a+b)^2\sqrt{(a-b+(a+b)\cosh(2x))\operatorname{sech}^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

```
[Out] -1/2*(((a + b)*(a^2 - 2*b^2 + (a^2 + 2*a*b + 2*b^2)*Cosh[2*x])*Csch[x]^2 -
Sqrt[2]*a^2*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Ellipti
cF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + Sq
rt[2]*a^3*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a +
 b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])*S
ech[x]^2*Sinh[2*x])/(Sqrt[2]*a^2*(a + b)^2*Sqrt[(a - b + (a + b)*Cosh[2*x])
*Sech[x]^2])
```

Maple [F]
time = 1.69, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\left(a+b\left(\tanh^2(x)\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(coth(x)^2/(b*tanh(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. 2(75) = 150.
time = 0.57, size = 3929, normalized size = 46.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
[Out] [1/4*(((a^3 + a^2*b)*cosh(x)^6 + 6*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 +
 a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 +
a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b
)*cosh(x))*sinh(x)^3 - a^3 - a^2*b - (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 +
 a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b + 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2
+ 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2
*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2
+ b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh
(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a
*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b +
 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 +
 b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6
*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2
 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*co
```

```
sh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)
^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh
(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*co
sh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*
b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2
*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*c
osh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*co
sh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cos
h(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5
*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*s
inh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^3 + a^2*b)*cosh(x)^6 + 6
*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 + a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b
)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5
*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b)*cosh(x))*sinh(x)^3 - a^3 - a^2*b
 - (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 + a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b
+ 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2
*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*
log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2)) - 4*sqrt(2)*((a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^
4 + 4*(a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*b
+ 4*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3 + 2*(a^3 + a
^2*b - 2*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - 2*a*b^2 - 2*b^3 + 3*(a
^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b +
4*a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 + a^2*b - 2*a*b^2 - 2*b^3)*cosh(x))*sinh(
x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^
6 + 6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^5 + (a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^6 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3
+ (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*cosh(x)^4 + (a^5 - a^4*b - 5*a^3*b^
2 - 3*a^2*b^3 + 15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)
^4 + 4*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (a^5 - a^4*b -
5*a^3*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^
2*b^3)*cosh(x)^2 - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3 - 15*(a^5 + 3*a^4*b
 + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 - 6*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)
*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^
5 + 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*cosh(x)^3 - (a^5 - a^4*b - 5*a^
3*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x)), -1/2*(((a^3 + a^2*b)*cosh(x)^6 + 6*(a
^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 + a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b)*c
osh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b)*cosh(x))*sinh(x)^3 - a^3 - a^2*b -
```

```
(a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 + a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b + 6
*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2*(a
^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*ar
ctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt
(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh
(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a+b\tanh^2(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(coth(x)**2/(a + b*tanh(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 485 vs. 2(75) = 150.
time = 0.68, size = 485, normalized size = 5.71

$$-\frac{\frac{(a^2b^3+ab^4)e^{(2x)}}{a^5b+2a^4b^2+a^3b^3}-\frac{a^2b^3+ab^4}{a^5b+2a^4b^2+a^3b^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)}-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}+\frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right)}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)\sqrt{a+b}-3a+b\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b^3 + a*b^4)*e^(2*x)/(a^5*b + 2*a^4*b^2 + a^3*b^3) - (a^2*b^3 + a*b^
4)/(a^5*b + 2*a^4*b^2 + a^3*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))
)/(a^2 + 2*a*b + b^2) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt
(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))
/(a^2 + 2*a*b + b^2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a -
 b)))/(a + b)^(3/2) + 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/(((sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sq
rt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b))*sqrt(a + b) - 3*a + b)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^2}{\left(b\tanh(x)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*tanh(x)^2)^(3/2),x)
[Out] int(coth(x)^2/(a + b*tanh(x)^2)^(3/2), x)
```

3.246 $\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{a(a+}{b^2(a+b)^2\sqrt{}}$$

[Out] -arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(5/2)+arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+a*(a+2*b)*tanh(x)/b^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*a*tanh(x)^3/b/(a+b)/(a+b*tanh(x)^2)^(3/2)

Rubi [A]
time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3751, 481, 592, 537, 223, 212, 385}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}}+\frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/b^(5/2)) + ArcTanh[(Sqrt
[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (a*Tanh[x]^3)/(3*b*
(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (a*(a + 2*b)*Tanh[x])/(b^2*(a + b)^2*Sqr
t[a + b*Tanh[x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{x^6}{(1-x^2)\,(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{a\tanh^3(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{x^2(3a-3(a+b)x^2)}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3b(a+b)}\\
&= \frac{a\tanh^3(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{3a(a+2b)}{(1-x^2)\sqrt{}}\right.}{3}\\
&= \frac{a\tanh^3(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b}}\right.}{}\\
&= \frac{a\tanh^3(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,d\right.}{}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a\,\mathrm{t}}{3b(a+b)\,(a}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 1.45, size = 231, normalized size = 1.96

$$\frac{\sqrt{(a-b+(a+b)\cosh(2x))\text{sech}^2(x)}\left(-\frac{3\sqrt{2}\,a\coth(x)\left((a^2+3ab+2b^2)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)+b^2\Pi\left(\frac{b}{a+b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)}{b\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}+\frac{a(a+b)(3a^2+2ab-7b^2+(3a^2+10ab+7b^2)\cosh(2x))\sinh(2x)}{(a-b+(a+b)\cosh(2x))^2}\right)}{3\sqrt{2}\,b^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]*((-3*Sqrt[2]*a*Coth[x]*((a^2 +
 3*a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^
2)/b]/Sqrt[2]], 1] + b^2*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b
)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]))/(b*Sqrt[((a - b + (a + b)*Cosh[2*
```

```
x])*Csch[x]^2)/b]) + (a*(a + b)*(3*a^2 + 2*a*b - 7*b^2 + (3*a^2 + 10*a*b +
7*b^2)*Cosh[2*x])*Sinh[2*x])/(a - b + (a + b)*Cosh[2*x])^2))/(3*Sqrt[2]*b^2
*(a + b)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(100) = 200$.
time = 0.73, size = 706, normalized size = 5.98 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*tanh(x)^3/b/(a+b*tanh(x)^2)^(3/2)-1/b*(-tanh(x)/b/(a+b*tanh(x)^2)^(1/2)
+1/b^(3/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2)))+1/2*tanh(x)/b/(a+b*ta
nh(x)^2)^(3/2)-1/2*a/b*(1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)+2/3/a^2*tanh(x)
/(a+b*tanh(x)^2)^(1/2))-1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)
/(a+b*tanh(x)^2)^(1/2)-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2
)+1/2*b/(a+b)*(2/3*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2
+2*b*(tanh(x)-1)+a+b)^(3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(tanh(x)-1)+2*b
)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-1/2/(a+b)*(1/(a+b)/(b*(tanh(
x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-2*b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+
b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*a
+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1
/2))/(tanh(x)-1)))+1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+1/
2*b/(a+b)*(2/3*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b
*(1+tanh(x))+a+b)^(3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(1+tanh(x))-2*b)/(b
*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))+1/2/(a+b)*(1/(a+b)/(b*(1+tanh(x)
)^2-2*b*(1+tanh(x))+a+b)^(1/2)+2*b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4
*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*a+2*b
-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))
/(1+tanh(x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^6/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4472 vs. $2(100) = 200$.
time = 1.13, size = 19265, normalized size = 163.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)^8 + 8*(a^2*b^3 + 2*a*b^4 + b^5)
*cosh(x)*sinh(x)^7 + (a^2*b^3 + 2*a*b^4 + b^5)*sinh(x)^8 + 4*(a^2*b^3 - b^5
)*cosh(x)^6 + 4*(a^2*b^3 - b^5 + 7*(a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)^2)*sin
h(x)^6 + 8*(7*(a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)^3 + 3*(a^2*b^3 - b^5)*cosh(
x))*sinh(x)^5 + a^2*b^3 + 2*a*b^4 + b^5 + 2*(3*a^2*b^3 - 2*a*b^4 + 3*b^5)*c
osh(x)^4 + 2*(3*a^2*b^3 - 2*a*b^4 + 3*b^5 + 35*(a^2*b^3 + 2*a*b^4 + b^5)*co
sh(x)^4 + 30*(a^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^2*b^3 + 2*a*b^4
 + b^5)*cosh(x)^5 + 10*(a^2*b^3 - b^5)*cosh(x)^3 + (3*a^2*b^3 - 2*a*b^4 + 3
*b^5)*cosh(x))*sinh(x)^3 + 4*(a^2*b^3 - b^5)*cosh(x)^2 + 4*(7*(a^2*b^3 + 2*
a*b^4 + b^5)*cosh(x)^6 + a^2*b^3 - b^5 + 15*(a^2*b^3 - b^5)*cosh(x)^4 + 3*(
3*a^2*b^3 - 2*a*b^4 + 3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a^2*b^3 + 2*a*b^4 +
 b^5)*cosh(x)^7 + 3*(a^2*b^3 - b^5)*cosh(x)^5 + (3*a^2*b^3 - 2*a*b^4 + 3*b^
5)*cosh(x)^3 + (a^2*b^3 - b^5)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 +
 b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)
^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh
(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x
))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3
)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)
*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 +
 (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b
^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x
)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*c
osh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b
^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*
b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b +
 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^
3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a
^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x
))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3
*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 6
*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^8 + 8*(
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^7
+ (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(x)^8 + 4*(
a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^6 + 4*(a^5 +
 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^
3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^3 + 3*(a^5 + 3*a^4*b
 + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x)^5 + a^5 + 5*a^4*
```

```
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^
2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^4 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2
 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^
3 + 5*a*b^4 + b^5)*cosh(x)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2
*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3
+ 7*a*b^4 + 3*b^5)*cosh(x))*sinh(x)^3 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^
2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^
2*b^3 + 5*a*b^4 + b^5)*cosh(x)^6 + a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4 - b^5 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*
cosh(x)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*c
osh(x)^2)*sinh(x)^2 + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5)*cosh(x)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^
5)*cosh(x)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*
cosh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)
)*sinh(x))*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^
3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2
+ a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)*c
osh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2
*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x
) + 1)) + 3*((a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)^8 + 8*(a^2*b^3 + 2*a*b^4 + b
^5)*cosh(x)*sinh(x)^7 + (a^2*b^3 + 2*a*b^4 + b^5)*sinh(x)^8 + 4*(a^2*b^3 -
b^5)*cosh(x)^6 + 4*(a^2*b^3 - b^5 + 7*(a^2*b^3 ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**6/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 829 vs. 2(100) = 200.
time = 0.77, size = 829, normalized size = 7.03

$$-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{\left(\left(\left(\frac{3\,a^9b^8+22\,a^8b^9+65\,a^7b^{10}+100\,a^6b^{11}+85\,a^5b^{12}+38\,a^4b^{13}+7\,a^3b^{14}}{a^8b^{10}+6\,a^7b^{11}+15\,a^6b^{12}+20\,a^5b^{13}+15\,a^4b^{14}+6\,a^3b^{15}+a^2b^{16}}e^{(2x)}+\frac{3\,(a^9b^8+2\,a^8b^9-9\,a^7b^{10}-36\,a^6b^{11}-49\,a^5b^{12}-30\,a^4b^{13}-7\,a^3b^{14})}{a^8b^{10}+6\,a^7b^{11}+15\,a^6b^{12}+20\,a^5b^{13}+15\,a^4b^{14}+6\,a^3b^{15}+a^2b^{16}}\right)e^{(2x)}-\text{[illegible]}\right)e^{(2x)}-\text{[illegible]}\right)}{3\,(ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b)^{\frac{3}{2}}}-\frac{2\arctan\left(-\frac{\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
+ b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*sqrt(a + b)*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
) - sqrt(a + b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/3*((((3*a^9*b^8 + 22*
a^8*b^9 + 65*a^7*b^10 + 100*a^6*b^11 + 85*a^5*b^12 + 38*a^4*b^13 + 7*a^3*b^
14)*e^(2*x)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 + 20*a^5*b^13 + 15*a^4*b^1
4 + 6*a^3*b^15 + a^2*b^16) + 3*(a^9*b^8 + 2*a^8*b^9 - 9*a^7*b^10 - 36*a^6*b
^11 - 49*a^5*b^12 - 30*a^4*b^13 - 7*a^3*b^14)/(a^8*b^10 + 6*a^7*b^11 + 15*a
^6*b^12 + 20*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^15 + a^2*b^16))*e^(2*x) - 3*(
a^9*b^8 + 2*a^8*b^9 - 9*a^7*b^10 - 36*a^6*b^11 - 49*a^5*b^12 - 30*a^4*b^13
- 7*a^3*b^14)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 + 20*a^5*b^13 + 15*a^4*b
^14 + 6*a^3*b^15 + a^2*b^16))*e^(2*x) - (3*a^9*b^8 + 22*a^8*b^9 + 65*a^7*b^
10 + 100*a^6*b^11 + 85*a^5*b^12 + 38*a^4*b^13 + 7*a^3*b^14)/(a^8*b^10 + 6*a
^7*b^11 + 15*a^6*b^12 + 20*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^15 + a^2*b^16))
/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3/2) - 2*arct
an(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^6}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^6/(a + b*tanh(x)^2)^(5/2), x)
```

3.247 $\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)+a*(a+2*b)/b^2/(a+b)^
2/(a+b*tanh(x)^2)^(1/2)-1/3*a^2/b^2/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3751, 457, 89, 65, 214}

$$-\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) - a^2/(3*b^2*(a +
b)*(a + b*Tanh[x]^2)^(3/2)) + (a*(a + 2*b))/(b^2*(a + b)^2*Sqrt[a + b*Tanh[
x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^5}{(1-x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b(a+b)(a+bx)^{5/2}} - \frac{a(a+2b)}{b(a+b)^2(a+bx)^{3/2}} - \frac{1}{(a+b)^2(-1+x)\sqrt{a} \cdots}\right.\right.$$

$$= -\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{\cdots}{(-1+\cdots}\right.}{\cdots}$$

$$= -\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{\cdots}{-1-\cdots}\right.}{\cdots}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a \cdots}{b^2(a+b)^2\sqrt{a \cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 68, normalized size = 0.81

$$\frac{-b^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tanh^2(x)}{a+b}\right)+(a+b)\left(2a+b+3b\tanh^2(x)\right)}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (-(b^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a +
b)*(2*a + b + 3*b*Tanh[x]^2))/(3*b^2*(a + b)*(a + b*Tanh[x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(72) = 144$.
time = 0.72, size = 599, normalized size = 7.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] tanh(x)^2/b/(a+b*tanh(x)^2)^(3/2)+2/3*a/b^2/(a+b*tanh(x)^2)^(3/2)+1/3/b/(a+
b*tanh(x)^2)^(3/2)-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/
2*b/(a+b)*(2/3*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b
*(tanh(x)-1)+a+b)^(3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(tanh(x)-1)+2*b)/(b
*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-1/2/(a+b)*(1/(a+b)/(b*(tanh(x)-1
)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-2*b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4
*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*a+2*b
+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))
/(tanh(x)-1)))-1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)-1/2*b/
(a+b)*(2/3*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+
tanh(x))+a+b)^(3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(1+tanh(x))-2*b)/(b*(1+
tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))-1/2/(a+b)*(1/(a+b)/(b*(1+tanh(x))^2-
2*b*(1+tanh(x))+a+b)^(1/2)+2*b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2
)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*a+2*b-2*b
*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+
tanh(x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

[Out] integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(5/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3234 vs. $2(72) = 144$.

time = 0.67, size = 7033, normalized size = 83.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")

```
[Out] [1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^8 + 8*(a^2*b^2 + 2*a*b^3 + b^4)
*cosh(x)*sinh(x)^7 + (a^2*b^2 + 2*a*b^3 + b^4)*sinh(x)^8 + 4*(a^2*b^2 - b^4
)*cosh(x)^6 + 4*(a^2*b^2 - b^4 + 7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^2)*sin
h(x)^6 + 8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^3 + 3*(a^2*b^2 - b^4)*cosh(
x))*sinh(x)^5 + 2*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x)^4 + 2*(35*(a^2*b^2
+ 2*a*b^3 + b^4)*cosh(x)^4 + 3*a^2*b^2 - 2*a*b^3 + 3*b^4 + 30*(a^2*b^2 - b^
4)*cosh(x)^2)*sinh(x)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 + 2*a*b^3
 + b^4)*cosh(x)^5 + 10*(a^2*b^2 - b^4)*cosh(x)^3 + (3*a^2*b^2 - 2*a*b^3 + 3
*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 - b^4)*cosh(x)^2 + 4*(7*(a^2*b^2 + 2*
a*b^3 + b^4)*cosh(x)^6 + 15*(a^2*b^2 - b^4)*cosh(x)^4 + a^2*b^2 - b^4 + 3*(
3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^2*b^2 + 2*a*b^3 +
 b^4)*cosh(x)^7 + 3*(a^2*b^2 - b^4)*cosh(x)^5 + (3*a^2*b^2 - 2*a*b^3 + 3*b^
4)*cosh(x)^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^
2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^
8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(
x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x)
)*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)
*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*
sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 +
(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)
^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*co
sh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^
2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a
*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b
- b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3
 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x
)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 +
4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x)
)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*
sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3*
((a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^8 + 8*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)*
```

```
sinh(x)^7 + (a^2*b^2 + 2*a*b^3 + b^4)*sinh(x)^8 + 4*(a^2*b^2 - b^4)*cosh(x)
^6 + 4*(a^2*b^2 - b^4 + 7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^2)*sinh(x)^6 +
8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*cosh(x)^3 + 3*(a^2*b^2 - b^4)*cosh(x))*sinh(
x)^5 + 2*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x)^4 + 2*(35*(a^2*b^2 + 2*a*b^3
 + b^4)*cosh(x)^4 + 3*a^2*b^2 - 2*a*b^3 + 3*b^4 + 30*(a^2*b^2 - b^4)*cosh(x
)^2)*sinh(x)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*c
osh(x)^5 + 10*(a^2*b^2 - b^4)*cosh(x)^3 + (3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cos
h(x))*sinh(x)^3 + 4*(a^2*b^2 - b^4)*cosh(x)^2 + 4*(7*(a^2*b^2 + 2*a*b^3 + b
^4)*cosh(x)^6 + 15*(a^2*b^2 - b^4)*cosh(x)^4 + a^2*b^2 - b^4 + 3*(3*a^2*b^2
 - 2*a*b^3 + 3*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^2*b^2 + 2*a*b^3 + b^4)*cos
h(x)^7 + 3*(a^2*b^2 - b^4)*cosh(x)^5 + (3*a^2*b^2 - 2*a*b^3 + 3*b^4)*cosh(x
)^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4
 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 -
b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) +
8*sqrt(2)*((a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)^6 + 6*(a^4 + 5*a^3
*b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)*sinh(x)^5 + (a^4 + 5*a^3*b + 7*a^2*b^2 +
3*a*b^3)*sinh(x)^6 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x)^4 + 3*(a^4
 + 3*a^3*b + a^2*b^2 - a*b^3 + 5*(a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cosh
(x)^2)*sinh(x)^4 + a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3 + 4*(5*(a^4 + 5*a^3*
b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)^3 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*co
sh(x))*sinh(x)^3 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x)^2 + 3*(5*(a^
4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)^4 + a^4 + 3*a^3*b + a^2*b^2 - a*
b^3 + 6*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 +
5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cosh(x)^5 + 2*(a^4 + 3*a^3*b + a^2*b^2 - a*b
^3)*cosh(x)^3 + (a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*cosh(x))*sinh(x))*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7)*cosh(x)^8 + 8*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b
^6 + b^7)*cosh(x)*sinh(x)^7 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^
5 + 5*a*b^6 + b^7)*sinh(x)^8 + a^5*b^2 + 5*a^4*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\left(a + b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 735 vs. 2(72) = 144.
time = 0.61, size = 735, normalized size = 8.75

$$-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{2\left(\left(\left(\frac{(a^9+8\,a^8b+25\,a^7b^2+40\,a^6b^3+35\,a^5b^4+16\,a^4b^5+3\,a^3b^6)e^{(2x)}}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}+\frac{3\,(a^9+6\,a^8b+13\,a^7b^2+12\,a^6b^3+3\,a^5b^4-2\,a^4b^5-a^3b^6)}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)e^{(2x)}+\frac{3\,(a^9+6\,a^8b+13\,a^7b^2+12\,a^6b^3+3\,a^5b^4-2\,a^4b^5-a^3b^6)}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)e^{(2x)}+\frac{a^9+8\,a^8b+25\,a^7b^2+40\,a^6b^3+35\,a^5b^4+16\,a^4b^5+3\,a^3b^6}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)}{3\,(ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
+ b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/2*sqrt(a + b)*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
) - sqrt(a + b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2/3*((((a^9 + 8*a^8*b +
 25*a^7*b^2 + 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 + 3*a^3*b^6)*e^(2*x)/(a^
8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*
b^8) + 3*(a^9 + 6*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 3*a^5*b^4 - 2*a^4*b^5 -
 a^3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a
^3*b^7 + a^2*b^8))*e^(2*x) + 3*(a^9 + 6*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 3
*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*
b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + (a^9 + 8*a^8*b + 25*a^7*
b^2 + 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 + 3*a^3*b^6)/(a^8*b^2 + 6*a^7*b^
3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3/2)
```

Mupad [B]
time = 4.01, size = 92, normalized size = 1.10

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh\left(x\right)^2+a}\,\left(2\,a^2+4\,a\,b+2\,b^2\right)}{2\,{\left(a+b\right)}^{5/2}}\right)}{{\left(a+b\right)}^{5/2}}-\frac{\frac{a^2}{3\,\left(a+b\right)}-\frac{\left(a^2+2\,b\,a\right)\,\left(b\,{\mathrm{tanh}\left(x\right)}^2+a\right)}{{\left(a+b\right)}^2}}{b^2\,{\left(b\,{\mathrm{tanh}\left(x\right)}^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))/
(a + b)^(5/2) - (a^2/(3*(a + b)) - ((2*a*b + a^2)*(a + b*tanh(x)^2))/(a + b
)^2)/(b^2*(a + b*tanh(x)^2)^(3/2))
```

3.248 $\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)-1/3*(a+4*b)*
tanh(x)/b/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*a*tanh(x)/b/(a+b)/(a+b*tanh(x)^
2)^(3/2)
```

Rubi [A]

time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 481, 541, 12, 385, 212}

$$-\frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (a*Tan
h[x])/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((a + 4*b)*Tanh[x])/(3*b*(a +
 b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^4}{(1-x^2)\,(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{a\tanh(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+(-a-3b)x^2}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3b(a+b)}$$

$$= \frac{a\tanh(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{}{(1-x^2)}\right.}{}$$

$$= \frac{a\tanh(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{}{(1-x^2)}\right.}{}$$

$$= \frac{a\tanh(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a\tanh(x)}{3b(a+b)\,\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b}{3b(a+b)^2\sqrt{}}$$

Mathematica [A]
time = 2.29, size = 132, normalized size = 1.47

$$\frac{\tanh^3(x)\left(3\tanh^{-1}\left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{1+\frac{b\tanh^2(x)}{a}}}\right)\left(b+a\coth^2(x)\right)^2\sqrt{\frac{(a+b)\tanh^2(x)}{a}} - (a+b)\left(a+4b+3a\coth^2(x)\right)\sqrt{1+\frac{b\tanh^2(x)}{a}}\right)}{3(a+b)^3\left(a+b\tanh^2(x)\right)^{3/2}\sqrt{1+\frac{b\tanh^2(x)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (Tanh[x]^3*(3*ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a]
]*(b + a*Coth[x]^2)^2*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*(a + 4*b + 3*a*
Coth[x]^2)*Sqrt[1 + (b*Tanh[x]^2)/a]))/(3*(a + b)^3*(a + b*Tanh[x]^2)^(3/2)
*Sqrt[1 + (b*Tanh[x]^2)/a])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(76) = 152$.
time = 0.72, size = 642, normalized size = 7.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tanh(x)/b/(a+b*tanh(x)^2)^(3/2)-1/2*a/b*(1/3*tanh(x)/a/(a+b*tanh(x)^2)^
(3/2)+2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/3*tanh(x)/a/(a+b*tanh(x)^2)^
(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)+1/6/(a+b)/(b*(1+tanh(x))^2-2*b*
(1+tanh(x))+a+b)^(3/2)+1/2*b/(a+b)*(2/3*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*
b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2
*(2*b*(1+tanh(x))-2*b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))+1/2/(a+
b)*(1/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+2*b/(a+b)*(2*b*(1+t
anh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-
1/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*
b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tan
h(x)-1)+a+b)^(3/2)+1/2*b/(a+b)*(2/3*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)
/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*
b*(tanh(x)-1)+2*b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-1/2/(a+b)*(
1/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-2*b/(a+b)*(2*b*(tanh(x)
-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/(a
+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(t
anh(x)-1)+a+b)^(1/2))/(tanh(x)-1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2545 vs. $2(76) = 152$.
time = 0.69, size = 5719, normalized size = 63.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sin
h(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^
```

```
2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2
 + 3*a^2 - 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 1
0*(a^2 - b^2)*cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a
^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*c
osh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2
 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5
 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (
a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 1
4*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a
*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^
4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2
 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2
 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a
^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*
(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*
b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2
*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*
(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sin
h(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(
x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x
)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*
cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*
b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh
(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)
^5 + sinh(x)^6)) + 3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^
6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 +
 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a
*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2
)*cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*c
osh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh
(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(
a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sin
h(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^
2)*cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sin
h(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
 + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x
```

```
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*((a^2 + 2*a*b + b^2)*cos
h(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh
(x)^6 - 3*(a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a*b
- b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 - 3*(a*b + b^2)*cosh(
x))*sinh(x)^3 + 3*(a*b + b^2)*cosh(x)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^
4 - 6*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 6*
((a^2 + 2*a*b + b^2)*cosh(x)^5 - 2*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh
(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*cosh(x)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^
3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*sinh(x)^8 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3
- 3*a*b^4 - b^5)*cosh(x)^6 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a
*b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*co
sh(x)^2)*sinh(x)^6 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^
4 + b^5)*cosh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b
^5)*cosh(x))*sinh(x)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
+ b^5 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(
x)^4 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\left(a + b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 708 vs. 2(76) = 152.
time = 0.62, size = 708, normalized size = 7.87

$$-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}-\frac{4\left(\left(\left(\frac{(a^7b^2+5\,a^6b^3+10\,a^5b^4+10\,a^4b^5+5\,a^3b^6+a^2b^7)e^{(2x)}}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}-\frac{3\,(a^6b^3+4\,a^5b^4+6\,a^4b^5+4\,a^3b^6+a^2b^7)}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)e^{(2x)}+\frac{3\,(a^6b^3+4\,a^5b^4+6\,a^4b^5+4\,a^3b^6+a^2b^7)}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)e^{(2x)}-\frac{a^7b^2+5\,a^6b^3+10\,a^5b^4+10\,a^4b^5+5\,a^3b^6+a^2b^7}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)}{3\,(ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
```

```
+ b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*sqrt(a + b)*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
) - sqrt(a + b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 4/3*((((a^7*b^2 + 5*a^6
*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*e^(2*x)/(a^8*b^2 + 6*
a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8) - 3*(
a^6*b^3 + 4*a^5*b^4 + 6*a^4*b^5 + 4*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3
 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + 3
*(a^6*b^3 + 4*a^5*b^4 + 6*a^4*b^5 + 4*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b
^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) -
 (a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)/(a^8
*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b
^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b*tanh(x)^2)^(5/2),x)
[Out] int(tanh(x)^4/(a + b*tanh(x)^2)^(5/2), x)
```

3.249 $\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)-1/(a+b)^2/(a+b*tanh(
x)^2)^(1/2)+1/3*a/b/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 457, 79, 53, 65, 214}

$$\frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) + a/(3*b*(a + b)*(
a + b*Tanh[x]^2)^(3/2)) - 1/((a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^3}{(1-x^2)\,(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)$$

$$= \frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}$$

$$= \frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a \cdots}}\right.}{2(\cdots}$$

$$= \frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,d\cdots\right.}{b(\cdots}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 63, normalized size = 0.85

$$\frac{a(a+b) - 3b\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tanh^2(x)}{a+b}\right)\left(a+b\tanh^2(x)\right)}{3b(a+b)^2\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (a*(a + b) - 3*b*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]
*(a + b*Tanh[x]^2))/(3*b*(a + b)^2*(a + b*Tanh[x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(62) = 124.
time = 0.66, size = 565, normalized size = 7.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b/(a+b*tanh(x)^2)^(3/2)-1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)
^(3/2)-1/2*b/(a+b)*(2/3*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(
x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(1+tanh(x)
)-2*b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))-1/2/(a+b)*(1/(a+b)/(b*(
1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+2*b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*
b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/(a+b)^(3/2)*ln
((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+
b)^(1/2))/(1+tanh(x))))-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/
2)+1/2*b/(a+b)*(2/3*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^
2+2*b*(tanh(x)-1)+a+b)^(3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(tanh(x)-1)+2*
b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-1/2/(a+b)*(1/(a+b)/(b*(tanh
(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-2*b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a
+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*
a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(
1/2))/(tanh(x)-1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3028 vs. $2(62) = 124$.
time = 0.71, size = 6621, normalized size = 89.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^8 + 8*(a^2*b + 2*a*b^2 + b^3)*cos
h(x)*sinh(x)^7 + (a^2*b + 2*a*b^2 + b^3)*sinh(x)^8 + 4*(a^2*b - b^3)*cosh(x
)^6 + 4*(a^2*b - b^3 + 7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 8*(
7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*sinh(x)^5 +
2*(3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^4 + 2*(35*(a^2*b + 2*a*b^2 + b^3)*cos
h(x)^4 + 3*a^2*b - 2*a*b^2 + 3*b^3 + 30*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^4
+ 8*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^5 + 10*(a^2*b - b^3)*cosh(x)^3 + (3*
a^2*b - 2*a*b^2 + 3*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 2*a*b^2 + b^3 + 4*(a^
2*b - b^3)*cosh(x)^2 + 4*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^6 + 15*(a^2*b -
 b^3)*cosh(x)^4 + a^2*b - b^3 + 3*(3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^2)*si
nh(x)^2 + 8*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^7 + 3*(a^2*b - b^3)*cosh(x)^5
```

```
+ (3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh(x))*s
qrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7
 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b
 + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 +
3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh
(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2
*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*
a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3
 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*
(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*
b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*
(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4
+ 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))
*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*c
osh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*co
sh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a
+ b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2
*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*
b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*
sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6)) + 3*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^8 + 8*(a^2*b + 2*
a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a^2*b + 2*a*b^2 + b^3)*sinh(x)^8 + 4*(a^2
*b - b^3)*cosh(x)^6 + 4*(a^2*b - b^3 + 7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^2)
*sinh(x)^6 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(
x))*sinh(x)^5 + 2*(3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^4 + 2*(35*(a^2*b + 2*
a*b^2 + b^3)*cosh(x)^4 + 3*a^2*b - 2*a*b^2 + 3*b^3 + 30*(a^2*b - b^3)*cosh(
x)^2)*sinh(x)^4 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)^5 + 10*(a^2*b - b^3)
*cosh(x)^3 + (3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 2*a*b
^2 + b^3 + 4*(a^2*b - b^3)*cosh(x)^2 + 4*(7*(a^2*b + 2*a*b^2 + b^3)*cosh(x)
^6 + 15*(a^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 3*(3*a^2*b - 2*a*b^2 + 3*b^
3)*cosh(x)^2)*sinh(x)^2 + 8*((a^2*b + 2*a*b^2 + b^3)*cosh(x)^7 + 3*(a^2*b -
 b^3)*cosh(x)^5 + (3*a^2*b - 2*a*b^2 + 3*b^3)*cosh(x)^3 + (a^2*b - b^3)*cos
h(x))*sinh(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh
(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sin
h(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a +
b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((a^3 - a^2*b -
 5*a*b^2 - 3*b^3)*cosh(x)^6 + 6*(a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cosh(x)*sin
h(x)^5 + (a^3 - a^2*b - 5*a*b^2 - 3*b^3)*sinh(x)^6 + 3*(a^3 - a^2*b - a*b^2
 + b^3)*cosh(x)^4 + 3*(a^3 - a^2*b - a*b^2 + b^3 + 5*(a^3 - a^2*b - 5*a*b^2
 - 3*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cosh(
x)^3 + 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 - a^2*b - 5*a
*b^2 - 3*b^3 + 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 3*(5*(a^3 - a^2*b
```

```
- 5*a*b^2 - 3*b^3)*cosh(x)^4 + a^3 - a^2*b - a*b^2 + b^3 + 6*(a^3 - a^2*b -
 a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^3 - a^2*b - 5*a*b^2 - 3*b^3)*cos
h(x)^5 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + (a^3 - a^2*b - a*b^2 + b
^3)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5*b + 5*a^4*b^2 + 10*a^3*
b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^8 + 8*(a^5*b + 5*a^4*b^2 + 10*a^3
*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)*sinh(x)^7 + (a^5*b + 5*a^4*b^2 +
 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*sinh(x)^8 + 4*(a^5*b + 3*a^4*b^2
+ 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cosh(x)^6 + 4*(a^5*b + 3*a^4*b^2 +
 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6 + 7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3
+ 10*a^2*b^4 + 5*a*b^5 + b^6)*cosh(x)^2)*sinh(x...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\left(a + b\tanh^2(x)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 749 vs. 2(62) = 124.
time = 0.63, size = 749, normalized size = 10.12

$$-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{\left(\left(\frac{(a^8b+2\,a^7b^2-5\,a^6b^3-20\,a^5b^4-25\,a^4b^5-14\,a^3b^6-3\,a^2b^7)e^{(2x)}}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}+\frac{3\,(a^8b+2\,a^7b^2-a^6b^3-4\,a^5b^4-a^4b^5+2\,a^3b^6+a^2b^7)}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)e^{(2x)}+\frac{3\,(a^8b+2\,a^7b^2-a^6b^3-4\,a^5b^4-a^4b^5+2\,a^3b^6+a^2b^7)}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)e^{(2x)}+\frac{a^8b+2\,a^7b^2-5\,a^6b^3-20\,a^5b^4-25\,a^4b^5-14\,a^3b^6-3\,a^2b^7}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}}{3\,(ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
+ b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/2*sqrt(a + b)*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
) - sqrt(a + b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/3*((((a^8*b + 2*a^7*b
^2 - 5*a^6*b^3 - 20*a^5*b^4 - 25*a^4*b^5 - 14*a^3*b^6 - 3*a^2*b^7)*e^(2*x)/
(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a
^2*b^8) + 3*(a^8*b + 2*a^7*b^2 - a^6*b^3 - 4*a^5*b^4 - a^4*b^5 + 2*a^3*b^6
+ a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*
a^3*b^7 + a^2*b^8))*e^(2*x) + 3*(a^8*b + 2*a^7*b^2 - a^6*b^3 - 4*a^5*b^4 -
a^4*b^5 + 2*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b
```

```
^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + (a^8*b + 2*a^7*b^2 - 5*a^
6*b^3 - 20*a^5*b^4 - 25*a^4*b^5 - 14*a^3*b^6 - 3*a^2*b^7)/(a^8*b^2 + 6*a^7*
b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3/2)
```

Mupad [B]
time = 3.82, size = 82, normalized size = 1.11

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh\left(x\right)^2+a}\,\left(2\,a^2+4\,a\,b+2\,b^2\right)}{2\,{\left(a+b\right)}^{5/2}}\right)}{{\left(a+b\right)}^{5/2}}+\frac{\frac{a}{3\,\left(a+b\right)}-\frac{b\,\left(b\,\tanh\left(x\right)^2+a\right)}{{\left(a+b\right)}^2}}{b\,{\left(b\,\tanh\left(x\right)^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))/
(a + b)^(5/2) + (a/(3*(a + b)) - (b*(a + b*tanh(x)^2))/(a + b)^2)/(b*(a + b
*tanh(x)^2)^(3/2))
```

3.250 $\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)-1/3*(2*a-b)*
tanh(x)/a/(a+b)^2/(a+b*tanh(x)^2)^(1/2)-1/3*tanh(x)/(a+b)/(a+b*tanh(x)^2)^(
3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 482, 541, 12, 385, 212}

$$-\frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) - Tanh[x
]/(3*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((2*a - b)*Tanh[x])/(3*a*(a + b)^2*
Sqrt[a + b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1+2x^2}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3(a+b)}$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int -\frac{}{(1-}\right.}{}$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{}{(1-x}\right.}{}$$

$$= -\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{}{1-(a}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)}{3a(a+b)^2\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 5.56, size = 193, normalized size = 2.19

$$\frac{\coth(x)\left(-12(a+b)^3\,{}_2F_1\left(2,2;\frac{9}{2};-\frac{(a+b)\sinh^2(x)}{a}\right)\sinh^4(x)\tanh^2(x)\left(a+b\tanh^2(x)\right)-\frac{35a\cosh^2(x)\left(-5a-2b\tanh^2(x)\right)\left(3\text{ArcSin}\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)\left(a+b\tanh^2(x)\right)^2-a\,\text{sech}^2(x)\sqrt{-\frac{(a+b)\cosh^2(x)\sinh^2(x)\left(a+b\tanh^2(x)\right)}{a^2}}\left(3a+(a+4b)\tanh^2(x)\right)\right)}{\sqrt{-\frac{(a+b)\cosh^2(x)\sinh^2(x)\left(a+b\tanh^2(x)\right)}{a^2}}}\right)}{315a^3(a+b)^2\left(a+b\tanh^2(x)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (Coth[x]*(-12*(a + b)^3*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/
a)]*Sinh[x]^4*Tanh[x]^2*(a + b*Tanh[x]^2) - (35*a*Cosh[x]^2*(-5*a - 2*b*Tan
h[x]^2)*(3*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*(a + b*Tanh[x]^2)^2 - a*S
ech[x]^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)]*(3*a
+ (a + 4*b)*Tanh[x]^2)))/Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]
^2))/a^2)]))/(315*a^3*(a + b)^2*(a + b*Tanh[x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(74) = 148$.
time = 0.66, size = 584, normalized size = 6.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)+
1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+1/2*b/(a+b)*(2/3*(2*b
*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(
3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(1+tanh(x))-2*b)/(b*(1+tanh(x))^2-2*b*
(1+tanh(x))+a+b)^(1/2))+1/2/(a+b)*(1/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))
+a+b)^(1/2)+2*b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x)
)^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*
(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))-1/6/
(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/2*b/(a+b)*(2/3*(2*b*(ta
nh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)
+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(tanh(x)-1)+2*b)/(b*(tanh(x)-1)^2+2*b*(tan
h(x)-1)+a+b)^(1/2))-1/2/(a+b)*(1/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b
)^(1/2)-2*b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+
2*b*(tanh(x)-1)+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b
)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2939 vs. $2(74) = 148$.
time = 0.72, size = 6507, normalized size = 73.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 4*(a^3 - a*b^2)*cosh(x
)^6 + 4*(a^3 - a*b^2 + 7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(
7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^5 +
```

```
2*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^4 + 2*(35*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^4 + 3*a^3 - 2*a^2*b + 3*a*b^2 + 30*(a^3 - a*b^2)*cosh(x)^2)*sinh(x)^4
+ 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(a^3 - a*b^2)*cosh(x)^3 + (3*
a^3 - 2*a^2*b + 3*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*a^2*b + a*b^2 + 4*(a^
3 - a*b^2)*cosh(x)^2 + 4*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(a^3 - a
*b^2)*cosh(x)^4 + a^3 - a*b^2 + 3*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^2)*si
nh(x)^2 + 8*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(a^3 - a*b^2)*cosh(x)^5
+ (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*cosh(x))*sinh(x))*s
qrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^
7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^
3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 -
 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cos
h(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(
a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a
*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^
3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2
*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2
 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)
*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4
 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x)
)*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*
cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*c
osh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*
b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2
 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4
*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*si
nh(x)^5 + sinh(x)^6)) + 3*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 4*(a^
3 - a*b^2)*cosh(x)^6 + 4*(a^3 - a*b^2 + 7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2
)*sinh(x)^6 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(a^3 - a*b^2)*cosh
(x))*sinh(x)^5 + 2*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^4 + 2*(35*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)^4 + 3*a^3 - 2*a^2*b + 3*a*b^2 + 30*(a^3 - a*b^2)*cosh
(x)^2)*sinh(x)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(a^3 - a*b^2
)*cosh(x)^3 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*a^2*
b + a*b^2 + 4*(a^3 - a*b^2)*cosh(x)^2 + 4*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^6 + 15*(a^3 - a*b^2)*cosh(x)^4 + a^3 - a*b^2 + 3*(3*a^3 - 2*a^2*b + 3*a*b
^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(a^3 -
a*b^2)*cosh(x)^5 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*co
sh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh
(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sin
h(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a +
```

```
b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((3*a^3 + 5*a^2
*b + a*b^2 - b^3)*cosh(x)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(x)*sin
h(x)^5 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(x)^6 + 3*(a^3 - a^2*b - a*b^2
 + b^3)*cosh(x)^4 + 3*(a^3 - a^2*b - a*b^2 + b^3 + 5*(3*a^3 + 5*a^2*b + a*b
^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(
x)^3 + 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 - 3*a^3 - 5*a^2*b -
 a*b^2 + b^3 - 3*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 3*(5*(3*a^3 + 5*a^
2*b + a*b^2 - b^3)*cosh(x)^4 - a^3 + a^2*b + a*b^2 - b^3 + 6*(a^3 - a^2*b -
 a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^3 + 5*a^2*b + a*b^2 - b^3)*cos
h(x)^5 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - (a^3 - a^2*b - a*b^2 + b
^3)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^6 + 5*a^5*b + 10*a^4*b^2
+ 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^8 + 8*(a^6 + 5*a^5*b + 10*a^4*b^2
 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)*sinh(x)^7 + (a^6 + 5*a^5*b + 10*
a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*sinh(x)^8 + 4*(a^6 + 3*a^5*b + 2*
a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*cosh(x)^6 + 4*(a^6 + 3*a^5*b + 2*a
^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5 + 7*(a^6 + 5*a^5*b + 10*a^4*b^2 + 10
*a^3*b^3 + 5*a^2*b^4 + a*b^5)*cosh(x)^2)*sinh(x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 752 vs. 2(74) = 148.
time = 0.63, size = 752, normalized size = 8.55

$$-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}\,e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}\,(a-b)\right|\right)}{2\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}\,e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\left(\left(\frac{\left(3a^7b^2+14a^6b^3+25a^5b^4+20a^4b^5+5a^3b^6-2a^2b^7-ab^8\right)e^{(2x)}}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}+\frac{3\left(a^7b^2+2a^6b^3-a^5b^4-4a^4b^5-a^3b^6+2a^2b^7+ab^8\right)}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)e^{(2x)}-\frac{3\left(a^7b^2+2a^6b^3-a^5b^4-4a^4b^5-a^3b^6+2a^2b^7+ab^8\right)}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)e^{(2x)}-\frac{3a^7b^2+14a^6b^3+25a^5b^4+20a^4b^5+5a^3b^6-2a^2b^7-ab^8}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}}{3\left(ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
+ b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*sqrt(a + b)*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
```

```
) - sqrt(a + b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/3*((((3*a^7*b^2 + 14*
a^6*b^3 + 25*a^5*b^4 + 20*a^4*b^5 + 5*a^3*b^6 - 2*a^2*b^7 - a*b^8)*e^(2*x)/
(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a
^2*b^8) + 3*(a^7*b^2 + 2*a^6*b^3 - a^5*b^4 - 4*a^4*b^5 - a^3*b^6 + 2*a^2*b^
7 + a*b^8)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*
a^3*b^7 + a^2*b^8))*e^(2*x) - 3*(a^7*b^2 + 2*a^6*b^3 - a^5*b^4 - 4*a^4*b^5
- a^3*b^6 + 2*a^2*b^7 + a*b^8)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b
^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) - (3*a^7*b^2 + 14*a^6*b^3 +
 25*a^5*b^4 + 20*a^4*b^5 + 5*a^3*b^6 - 2*a^2*b^7 - a*b^8)/(a^8*b^2 + 6*a^7*
b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^2}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*tanh(x)^2)^(5/2),x)
[Out] int(tanh(x)^2/(a + b*tanh(x)^2)^(5/2), x)
```

3.251 $\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)-1/(a+b)^2/(a+b*tanh(
x)^2)^(1/2)-1/3/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3751, 455, 53, 65, 214}

$$-\frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) - 1/(3*(a + b)*(a
+ b*Tanh[x]^2)^(3/2)) - 1/((a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{1}{3(a+b)(a+b\tanh^2(x))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}$$
$$= -\frac{1}{3(a+b)(a+b\tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a}\right.}{2(}$$
$$= -\frac{1}{3(a+b)(a+b\tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\right.}{b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{1}{3(a+b)(a+b\tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 43, normalized size = 0.61

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tanh^2(x)}{a+b}\right)}{3(a+b)(a+b\tanh^2(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*(
a + b*Tanh[x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(58) = 116.
time = 0.66, size = 550, normalized size = 7.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)-1/2*b/(a+b)*(2/3*(2*
b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^
(3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(1+tanh(x))-2*b)/(b*(1+tanh(x))^2-2*b
*(1+tanh(x))+a+b)^(1/2))-1/2/(a+b)*(1/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x)
)+a+b)^(1/2)+2*b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x
))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2
*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))-1/6
/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/2*b/(a+b)*(2/3*(2*b*(t
anh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2
)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(tanh(x)-1)+2*b)/(b*(tanh(x)-1)^2+2*b*(ta
nh(x)-1)+a+b)^(1/2))-1/2/(a+b)*(1/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+
b)^(1/2)-2*b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2
+2*b*(tanh(x)-1)+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+
b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2607 vs. $2(58) = 116$.
time = 0.66, size = 5779, normalized size = 82.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sin
h(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^
2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2
+ 3*a^2 - 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 1
0*(a^2 - b^2)*cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a
^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*c
osh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2
 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5
 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(a
 + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a
```

```
^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14
*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*
a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4
 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3
+ a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 +
 a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^
3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(
a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b
^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*
cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(
5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh
(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x
)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)
^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*c
osh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b
^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(
x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^
5 + sinh(x)^6)) + 3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*
cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6
 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 +
2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*
b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)
*cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*co
sh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(
x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a
^2 - b^2)*cosh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh
(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2
)*cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh
(x))*sqrt(a + b)*log(-((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
 + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a
 + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*((a^2 + 2*a*b + b^2)*cos
h(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh
(x)^6 + 3*(a^2 + a*b)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2
+ a*b)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + a*b)*cosh(
x))*sinh(x)^3 + 3*(a^2 + a*b)*cosh(x)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^
4 + 6*(a^2 + a*b)*cosh(x)^2 + a^2 + a*b)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 6*
((a^2 + 2*a*b + b^2)*cosh(x)^5 + 2*(a^2 + a*b)*cosh(x)^3 + (a^2 + a*b)*cosh
(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*cosh(x)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^
```

```
3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*sinh(x)^8 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3
- 3*a*b^4 - b^5)*cosh(x)^6 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a
*b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*co
sh(x)^2)*sinh(x)^6 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^
4 + b^5)*cosh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b
^5)*cosh(x))*sinh(x)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
+ b^5 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(
x)^4 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^...
```

Sympy [**A**]
time = 14.81, size = 73, normalized size = 1.04

$$-\frac{1}{3\left(a+b\right)\left(a+b\tanh^{2}\left(x\right)\right)^{\frac{3}{2}}}-\frac{1}{\left(a+b\right)^{2}\sqrt{a+b\tanh^{2}\left(x\right)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^{2}\left(x\right)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\left(a+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] -1/(3*(a + b)*(a + b*tanh(x)**2)**(3/2)) - 1/((a + b)**2*sqrt(a + b*tanh(x)
**2)) - atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(sqrt(-a - b)*(a + b)**2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(58) = 116.
time = 0.64, size = 707, normalized size = 10.10

$$-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{4\left(\left(\left(\frac{(a^7b^2+5a^6b^3+10a^5b^4+10a^4b^5+5a^3b^6+a^2b^7)e^{(2x)}}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}+\frac{3(a^7b^2+4a^6b^3+6a^5b^4+4a^4b^5+a^3b^6)}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)e^{(2x)}+\frac{3(a^7b^2+4a^6b^3+6a^5b^4+4a^4b^5+a^3b^6)}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)e^{(2x)}+\frac{a^7b^2+5a^6b^3+10a^5b^4+10a^4b^5+5a^3b^6+a^2b^7}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)}{3\left(ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
+ b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/2*sqrt(a + b)*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
) - sqrt(a + b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 4/3*((((a^7*b^2 + 5*a^6
*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*e^(2*x)/(a^8*b^2 + 6*
a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(
a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)/(a^8*b^2 + 6*a^7*b^3
 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + 3
```

```
*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)/(a^8*b^2 + 6*a^7*b
^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) +
 (a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)/(a^8
*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b
^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3/2)
```

Mupad [B]
time = 3.56, size = 76, normalized size = 1.09

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,(2\,a^2+4\,a\,b+2\,b^2)}{2\,(a+b)^{5/2}}\right)}{(a+b)^{5/2}}-\frac{\frac{1}{3\,(a+b)}+\frac{b\tanh(x)^2+a}{(a+b)^2}}{\left(b\tanh(x)^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))/
(a + b)^(5/2) - (1/(3*(a + b)) + (a + b*tanh(x)^2)/(a + b)^2)/(a + b*tanh(x
)^2)^(3/2)
```

3.252 $\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=93

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*b*(5*a+2
*b)*tanh(x)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*b*tanh(x)/a/(a+b)/(a+b*ta
nh(x)^2)^(3/2)
```

Rubi [A]

time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3742, 425, 541, 12, 385, 212}

$$\frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[x]^2)^(-5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (b*Tan
h[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(5*a + 2*b)*Tanh[x])/(3*a^
2*(a + b)^2*Sqrt[a + b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{b-3(a+b)+2bx^2}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}$$

$$= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{}{(1-x}\right.}{}$$

$$= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{}{(1-x}\right.}{}$$

$$= \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{}{1-(a}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+}{3a^2(a+b)^2\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.22, size = 943, normalized size = 10.14

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tanh[x]^2)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sinh[x]*(1575*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]] + (3150*(a +
b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^2)/a + (1575*(a + b)^2*Ar
cSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4)/a^2 + 2100*(-(((a + b)*Sinh
[x]^2)/a))^(3/2)*Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a] + 96*Hypergeometric2F1[2
, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Sqrt[C
osh[x]^2 + (b*Sinh[x]^2)/a] + 24*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -((
(a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Sqrt[Cosh[x]^2 + (b
*Sinh[x]^2)/a] + (2100*b*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^2)/
```

```
a + (4200*b*(a + b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^2*Tanh[x
]^2)/a^2 + (2100*b*(a + b)^2*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]
^4*Tanh[x]^2)/a^3 + (2800*b*(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[Cosh[x]^2
 + (b*Sinh[x]^2)/a]*Tanh[x]^2)/a + (168*b*Hypergeometric2F1[2, 2, 9/2, -(((
a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Sqrt[Cosh[x]^2 + (b*
Sinh[x]^2)/a]*Tanh[x]^2)/a + (48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -
(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Sqrt[Cosh[x]^2 +
(b*Sinh[x]^2)/a]*Tanh[x]^2)/a + (840*b^2*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/
a)]]*Tanh[x]^4)/a^2 + (1680*b^2*(a + b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a
)]]*Sinh[x]^2*Tanh[x]^4)/a^3 + (840*b^2*(a + b)^2*ArcSin[Sqrt[-(((a + b)*Si
nh[x]^2)/a)]]*Sinh[x]^4*Tanh[x]^4)/a^4 + (1120*b^2*(-(((a + b)*Sinh[x]^2)/a
))^(3/2)*Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a]*Tanh[x]^4)/a^2 + (72*b^2*Hyperge
ometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^
(7/2)*Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a]*Tanh[x]^4)/a^2 + (24*b^2*Hypergeome
tricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^
2)/a))^(7/2)*Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a]*Tanh[x]^4)/a^2 - 1575*Sqrt[-
(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)] - (2100*b*Tanh[x]^2*
Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)])/a - (840*b^2*
Tanh[x]^4*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)])/a^2
))/(315*a*(-(((a + b)*Sinh[x]^2)/a))^(5/2)*Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a
]*(a + b*Tanh[x]^2)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(79) = 158$.
time = 0.72, size = 550, normalized size = 5.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/2*b/(a+b)*(2/3*(2*
b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^
(3/2)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(tanh(x)-1)+2*b)/(b*(tanh(x)-1)^2+2*b
*(tanh(x)-1)+a+b)^(1/2))-1/2/(a+b)*(1/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1
)+a+b)^(1/2)-2*b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-
1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2
*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1)))+1/6
/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+1/2*b/(a+b)*(2/3*(2*b*(1
+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2
)+16/3*b/(4*b*(a+b)-4*b^2)^2*(2*b*(1+tanh(x))-2*b)/(b*(1+tanh(x))^2-2*b*(1+
tanh(x))+a+b)^(1/2))+1/2/(a+b)*(1/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+
b)^(1/2)+2*b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*b*(a+b)-4*b^2)/(b*(1+tanh(x))^2
-2*b*(1+tanh(x))+a+b)^(1/2)-1/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+
b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3152 vs. $2(79) = 158$.
time = 0.72, size = 6933, normalized size = 74.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)
*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 - a^2*b^2
)*cosh(x)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sin
h(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 - a^2*b^2)*cosh(
x))*sinh(x)^5 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^4 + 2*(35*(a^4 + 2*
a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 - 2*a^3*b + 3*a^2*b^2 + 30*(a^4 - a^2*b^
2)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a
^2*b^2)*cosh(x)^5 + 10*(a^4 - a^2*b^2)*cosh(x)^3 + (3*a^4 - 2*a^3*b + 3*a^2
*b^2)*cosh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*
b + a^2*b^2)*cosh(x)^6 + 15*(a^4 - a^2*b^2)*cosh(x)^4 + a^4 - a^2*b^2 + 3*(
3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2
*b^2)*cosh(x)^7 + 3*(a^4 - a^2*b^2)*cosh(x)^5 + (3*a^4 - 2*a^3*b + 3*a^2*b^
2)*cosh(x)^3 + (a^4 - a^2*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 +
 b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)
^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh
(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x
))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3
)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)
*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 +
 (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b
^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x
)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b
 + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*c
osh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b
^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*
b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b +
 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^
```

```
3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a
^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x
))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3
*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3
*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)
*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 - a^2*b^2)*cosh(x
)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x)^6 +
 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 - a^2*b^2)*cosh(x))*sinh
(x)^5 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b +
a^2*b^2)*cosh(x)^4 + 3*a^4 - 2*a^3*b + 3*a^2*b^2 + 30*(a^4 - a^2*b^2)*cosh(
x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*
cosh(x)^5 + 10*(a^4 - a^2*b^2)*cosh(x)^3 + (3*a^4 - 2*a^3*b + 3*a^2*b^2)*co
sh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*
b^2)*cosh(x)^6 + 15*(a^4 - a^2*b^2)*cosh(x)^4 + a^4 - a^2*b^2 + 3*(3*a^4 -
2*a^3*b + 3*a^2*b^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*co
sh(x)^7 + 3*(a^4 - a^2*b^2)*cosh(x)^5 + (3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(
x)^3 + (a^4 - a^2*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4
 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(
a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 +
a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) +
8*sqrt(2)*((3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)^6 + 6*(3*a^3*b + 7
*a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)*sinh(x)^5 + (3*a^3*b + 7*a^2*b^2 + 5*a*b^
3 + b^4)*sinh(x)^6 + 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x)^4 + 3*(a^3
*b - a^2*b^2 - 3*a*b^3 - b^4 + 5*(3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cosh
(x)^2)*sinh(x)^4 - 3*a^3*b - 7*a^2*b^2 - 5*a*b^3 - b^4 + 4*(5*(3*a^3*b + 7*
a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)^3 + 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*co
sh(x))*sinh(x)^3 - 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x)^2 + 3*(5*(3*
a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)^4 - a^3*b + a^2*b^2 + 3*a*b^3 +
b^4 + 6*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^3*
b + 7*a^2*b^2 + 5*a*b^3 + b^4)*cosh(x)^5 + 2*(a^3*b - a^2*b^2 - 3*a*b^3 - b
^4)*cosh(x)^3 - (a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*cosh(x))*sinh(x))*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^
2*b^5)*cosh(x)^8 + 8*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 +
 a^2*b^5)*cosh(x)*sinh(x)^7 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*
a^3*b^4 + a^2*b^5)*sinh(x)^8 + a^7 + 5*a^6*b + ...
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tanh^{2}\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(79) = 158.
time = 0.62, size = 738, normalized size = 7.94

$$-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)}+\frac{2\left(\left(\left(\frac{\left(3a^{6}b^{3}+16a^{5}b^{4}+35a^{4}b^{5}+40a^{3}b^{6}+25a^{2}b^{7}+8ab^{8}+b^{9}\right)e^{(2x)}}{a^{8}b^{2}+6a^{7}b^{3}+15a^{6}b^{4}+20a^{5}b^{5}+15a^{4}b^{6}+6a^{3}b^{7}+a^{2}b^{8}}+\frac{3\left(a^{6}b^{3}+2a^{5}b^{4}-3a^{4}b^{5}-12a^{3}b^{6}-13a^{2}b^{7}-6ab^{8}-b^{9}\right)}{a^{8}b^{2}+6a^{7}b^{3}+15a^{6}b^{4}+20a^{5}b^{5}+15a^{4}b^{6}+6a^{3}b^{7}+a^{2}b^{8}}\right)e^{(2x)}-\frac{3\left(a^{6}b^{3}+2a^{5}b^{4}-3a^{4}b^{5}-12a^{3}b^{6}-13a^{2}b^{7}-6ab^{8}-b^{9}\right)}{a^{8}b^{2}+6a^{7}b^{3}+15a^{6}b^{4}+20a^{5}b^{5}+15a^{4}b^{6}+6a^{3}b^{7}+a^{2}b^{8}}\right)e^{(2x)}-\frac{3a^{6}b^{3}+16a^{5}b^{4}+35a^{4}b^{5}+40a^{3}b^{6}+25a^{2}b^{7}+8ab^{8}+b^{9}}{a^{8}b^{2}+6a^{7}b^{3}+15a^{6}b^{4}+20a^{5}b^{5}+15a^{4}b^{6}+6a^{3}b^{7}+a^{2}b^{8}}\right)}{3\left(ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
+ b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*sqrt(a + b)*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
) - sqrt(a + b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2/3*((((3*a^6*b^3 + 16*
a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*a*b^8 + b^9)*e^(2*x)/(a^
8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*
b^8) + 3*(a^6*b^3 + 2*a^5*b^4 - 3*a^4*b^5 - 12*a^3*b^6 - 13*a^2*b^7 - 6*a*b
^8 - b^9)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a
^3*b^7 + a^2*b^8))*e^(2*x) - 3*(a^6*b^3 + 2*a^5*b^4 - 3*a^4*b^5 - 12*a^3*b^
6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*
b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) - (3*a^6*b^3 + 16*a^5*b^4
+ 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*a*b^8 + b^9)/(a^8*b^2 + 6*a^7*b^
3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x)
 + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{tanh}\left(x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*tanh(x)^2)^(5/2), x)
```

3.253 $\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{b(}{a^2(a+b)^2\sqrt{}}$$

[Out] -arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(5/2)+arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)+b*(2*a+b)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*b/a/(a+b)/(a+b*tanh(x)^2)^(3/2)

Rubi [A]

time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3751, 457, 87, 157, 162, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/(a + b*Tanh[x]^2)^(5/2),x]

[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/a^(5/2)) + ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) + b/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(2*a + b))/(a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^2])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\,(1-x^2)\,(a+bx^2)^{5/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^{5/2}}\,dx, x, \tanh^2(x)\right)$$
$$= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a-b+bx}{(1-x)x(a+bx)^{3/2}}\,dx, x, \tanh^2(x)\right)}{2a(a+b)}$$
$$= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a+b}{(1-x)x}\right.}{}$$
$$= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+}}\right.}{}$$
$$= \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{}{3a(a+b)\,(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 73, normalized size = 0.68

$$\frac{-a\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tanh^2(x)}{a+b}\right)+(a+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\tanh^2(x)}{a}\right)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (-(a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b)
*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tanh[x]^2)/a])/(3*a*(a + b)*(a + b
*Tanh[x]^2)^(3/2))
```

Maple [F]
time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(x\right)}{\left(a+b\left(\tanh^{2}\left(x\right)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4510 vs. $2(90) = 180$.
time = 1.23, size = 19305, normalized size = 178.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^8 + 8*(a^5 + 2*a^4*b + a^3*b^2)
*cosh(x)*sinh(x)^7 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^8 + 4*(a^5 - a^3*b^2
)*cosh(x)^6 + 4*(a^5 - a^3*b^2 + 7*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sin
h(x)^6 + 8*(7*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^3 + 3*(a^5 - a^3*b^2)*cosh(
x))*sinh(x)^5 + a^5 + 2*a^4*b + a^3*b^2 + 2*(3*a^5 - 2*a^4*b + 3*a^3*b^2)*c
osh(x)^4 + 2*(3*a^5 - 2*a^4*b + 3*a^3*b^2 + 35*(a^5 + 2*a^4*b + a^3*b^2)*co
sh(x)^4 + 30*(a^5 - a^3*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^5 + 2*a^4*b + a
^3*b^2)*cosh(x)^5 + 10*(a^5 - a^3*b^2)*cosh(x)^3 + (3*a^5 - 2*a^4*b + 3*a^3
*b^2)*cosh(x))*sinh(x)^3 + 4*(a^5 - a^3*b^2)*cosh(x)^2 + 4*(7*(a^5 + 2*a^4*
b + a^3*b^2)*cosh(x)^6 + a^5 - a^3*b^2 + 15*(a^5 - a^3*b^2)*cosh(x)^4 + 3*(
3*a^5 - 2*a^4*b + 3*a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^5 + 2*a^4*b + a^3
*b^2)*cosh(x)^7 + 3*(a^5 - a^3*b^2)*cosh(x)^5 + (3*a^5 - 2*a^4*b + 3*a^3*b^
2)*cosh(x)^3 + (a^5 - a^3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^
2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^
8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(
```

x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x)
)*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)
*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*
sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 +
(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)
^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^
2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*co
sh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^
2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a
*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b
- b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3
 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x
)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 +
4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x)
)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*
sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 6*
((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^8 + 8*(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^7 +
 (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(x)^8 + 4*(a
^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^6 + 4*(a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3
*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^3 + 3*(a^5 + 3*a^4*b
+ 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x)^5 + a^5 + 5*a^4*b
 + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2
 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^4 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2
+ 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
 + 5*a*b^4 + b^5)*cosh(x)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3
*a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*
a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*
b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 +
 7*a*b^4 + 3*b^5)*cosh(x))*sinh(x)^3 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2
*b^3 - 3*a*b^4 - b^5)*cosh(x)^2 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2
*b^3 + 5*a*b^4 + b^5)*cosh(x)^6 + a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3
*a*b^4 - b^5 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*c
osh(x)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*co
sh(x)^2)*sinh(x)^2 + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
+ b^5)*cosh(x)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*cosh(x)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*c
osh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))
*sinh(x))*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3
 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 +
 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2

```
+ 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*co
sh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*
(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x)
 + 1)) + 3*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^8 + 8*(a^5 + 2*a^4*b + a^3*b^
2)*cosh(x)*sinh(x)^7 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^8 + 4*(a^5 - a^3*b
^2)*cosh(x)^6 + 4*(a^5 - a^3*b^2 + 7*(a^5 + 2*a...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth{(x)}}{\left(a + b\tanh^{2}{(x)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(coth(x)/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 832 vs. 2(90) = 180.
time = 0.73, size = 832, normalized size = 7.70

$$-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^3+3a^2b+3ab^2+b^3\right)}+ \text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
+ b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/2*sqrt(a + b)*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
) - sqrt(a + b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/3*((((7*a^14*b^3 + 38
*a^13*b^4 + 85*a^12*b^5 + 100*a^11*b^6 + 65*a^10*b^7 + 22*a^9*b^8 + 3*a^8*b
^9)*e^(2*x)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^
6 + 6*a^11*b^7 + a^10*b^8) + 3*(7*a^14*b^3 + 30*a^13*b^4 + 49*a^12*b^5 + 36
*a^11*b^6 + 9*a^10*b^7 - 2*a^9*b^8 - a^8*b^9)/(a^16*b^2 + 6*a^15*b^3 + 15*a
^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))*e^(2*x) + 3*(
7*a^14*b^3 + 30*a^13*b^4 + 49*a^12*b^5 + 36*a^11*b^6 + 9*a^10*b^7 - 2*a^9*b
^8 - a^8*b^9)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*
b^6 + 6*a^11*b^7 + a^10*b^8))*e^(2*x) + (7*a^14*b^3 + 38*a^13*b^4 + 85*a^12
*b^5 + 100*a^11*b^6 + 65*a^10*b^7 + 22*a^9*b^8 + 3*a^8*b^9)/(a^16*b^2 + 6*a
^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))
```

```
/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3/2) + 2*arct
an(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)/(a + b*tanh(x)^2)^(5/2), x)
```

3.254 $\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=131

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}-\frac{(3a+2b)(a$$

```
[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*b*(7*a+4
*b)*coth(x)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)-1/3*(3*a+2*b)*(a+4*b)*coth(x)
*(a+b*tanh(x)^2)^(1/2)/a^3/(a+b)^2+1/3*b*coth(x)/a/(a+b)/(a+b*tanh(x)^2)^(3
/2)
```

Rubi [A]
time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3751, 483, 593, 597, 12, 385, 212}

$$-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2}+\frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}+\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (b*Cot
h[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(7*a + 4*b)*Coth[x])/(3*a^
2*(a + b)^2*Sqrt[a + b*Tanh[x]^2]) - ((3*a + 2*b)*(a + 4*b)*Coth[x]*Sqrt[a
+ b*Tanh[x]^2])/(3*a^3*(a + b)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a-4b+4bx^2}{x^2(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{(3a+}{x^2}\right.}{}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+}{}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+}{}$$

$$= \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+}{3a^2(a+b)^2\sqrt{}}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 6.89, size = 246, normalized size = 1.88

$$\frac{\sqrt{(a-b+(a+b)\cosh(2x))\text{sech}^2(x)}\left(\frac{3\sqrt{2}a^3\coth(x)\left((a+b)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)-a\Pi\left(\frac{b}{a+b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)}{b\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}-\frac{(a+b)\left(3(a+b)^2(a-b+(a+b)\cosh(2x))^2\coth(x)+2ab^3\sinh(2x)+b^2(9a+5b)(a-b+(a+b)\cosh(2x))\sinh(2x)\right)}{(a-b+(a+b)\cosh(2x))^2}\right)}{3\sqrt{2}a^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]*((3*Sqrt[2]*a^3*Coth[x]*((a +
b)*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]
```

```
], 1] - a*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Cs
ch[x]^2)/b]/Sqrt[2]], 1]))/(b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/
b]) - ((a + b)*(3*(a + b)^2*(a - b + (a + b)*Cosh[2*x])^2*Coth[x] + 2*a*b^3
*Sinh[2*x] + b^2*(9*a + 5*b)*(a - b + (a + b)*Cosh[2*x])*Sinh[2*x]))/(a - b
 + (a + b)*Cosh[2*x])^2))/(3*Sqrt[2]*a^3*(a + b)^3)
```

Maple [F]
time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\left(a+b\left(\tanh^2(x)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(b*tanh(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5021 vs. $2(113) = 226$.
time = 1.19, size = 10671, normalized size = 81.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^10 + 10*(a^5 + 2*a^4*b + a^3*b^
2)*cosh(x)*sinh(x)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^10 + (3*a^5 - 2*a^
4*b - 5*a^3*b^2)*cosh(x)^8 + (3*a^5 - 2*a^4*b - 5*a^3*b^2 + 45*(a^5 + 2*a^4
*b + a^3*b^2)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x
)^3 + (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x)^7 + 2*(a^5 - 2*a^4*b +
 5*a^3*b^2)*cosh(x)^6 + 2*(a^5 - 2*a^4*b + 5*a^3*b^2 + 105*(a^5 + 2*a^4*b +
 a^3*b^2)*cosh(x)^4 + 14*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^2)*sinh(x)^6
 + 4*(63*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^5 + 14*(3*a^5 - 2*a^4*b - 5*a^3*
b^2)*cosh(x)^3 + 3*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x))*sinh(x)^5 - a^5 - 2
*a^4*b - a^3*b^2 - 2*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^4 + 2*(105*(a^5 +
```

```
2*a^4*b + a^3*b^2)*cosh(x)^6 - a^5 + 2*a^4*b - 5*a^3*b^2 + 35*(3*a^5 - 2*a^
4*b - 5*a^3*b^2)*cosh(x)^4 + 15*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^2)*sinh
(x)^4 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^7 + 7*(3*a^5 - 2*a^4*b - 5*
a^3*b^2)*cosh(x)^5 + 5*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^3 - (a^5 - 2*a^4
*b + 5*a^3*b^2)*cosh(x))*sinh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^
2 + (45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^8 + 28*(3*a^5 - 2*a^4*b - 5*a^3*b
^2)*cosh(x)^6 - 3*a^5 + 2*a^4*b + 5*a^3*b^2 + 30*(a^5 - 2*a^4*b + 5*a^3*b^2
)*cosh(x)^4 - 12*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a
^5 + 2*a^4*b + a^3*b^2)*cosh(x)^9 + 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)
^7 + 6*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2
)*cosh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*l
og(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 +
 b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^
2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 +
2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70
*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^
3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3
)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*
a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2
+ b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3
*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x
)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*
cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3
- (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 -
a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6
*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt((
(a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)
^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*co
sh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 +
 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + si
nh(x)^6)) + 3*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^10 + 10*(a^5 + 2*a^4*b + a
^3*b^2)*cosh(x)*sinh(x)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^10 + (3*a^5 -
 2*a^4*b - 5*a^3*b^2)*cosh(x)^8 + (3*a^5 - 2*a^4*b - 5*a^3*b^2 + 45*(a^5 +
2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*c
osh(x)^3 + (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x)^7 + 2*(a^5 - 2*a^
4*b + 5*a^3*b^2)*cosh(x)^6 + 2*(a^5 - 2*a^4*b + 5*a^3*b^2 + 105*(a^5 + 2*a^
4*b + a^3*b^2)*cosh(x)^4 + 14*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x)^2)*sinh
(x)^6 + 4*(63*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^5 + 14*(3*a^5 - 2*a^4*b - 5
*a^3*b^2)*cosh(x)^3 + 3*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x))*sinh(x)^5 - a^
5 - 2*a^4*b - a^3*b^2 - 2*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^4 + 2*(105*(a
^5 + 2*a^4*b + a^3*b^2)*cosh(x)^6 - a^5 + 2*a^4*b - 5*a^3*b^2 + 35*(3*a^5 -
 2*a^4*b - 5*a^3*b^2)*cosh(x)^4 + 15*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^2)
*sinh(x)^4 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^7 + 7*(3*a^5 - 2*a^4*b
 - 5*a^3*b^2)*cosh(x)^5 + 5*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^3 - (a^5 -
```

```
2*a^4*b + 5*a^3*b^2)*cosh(x))*sinh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cos
h(x)^2 + (45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^8 + 28*(3*a^5 - 2*a^4*b - 5*
a^3*b^2)*cosh(x)^6 - 3*a^5 + 2*a^4*b + 5*a^3*b^2 + 30*(a^5 - 2*a^4*b + 5*a^
3*b^2)*cosh(x)^4 - 12*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 2*
(5*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^9 + 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*co
sh(x)^7 + 6*(a^5 - 2*a^4*b + 5*a^3*b^2)*cosh(x)^5 - 4*(a^5 - 2*a^4*b + 5*a^
3*b^2)*cosh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*cosh(x))*sinh(x))*sqrt(a +
 b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^
4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^
2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
 + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a + b\tanh^2(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(5/2),x)
```

```
[Out] Integral(coth(x)**2/(a + b*tanh(x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 922 vs. 2(113) = 226.
time = 0.80, size = 922, normalized size = 7.04

$$-\frac{\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} - \frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} + \frac{\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} - [illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
 + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x)
+ sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
+ b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*sqrt(a + b)*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
) - sqrt(a + b)))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/3*((((9*a^13*b^4 + 50
*a^12*b^5 + 115*a^11*b^6 + 140*a^10*b^7 + 95*a^9*b^8 + 34*a^8*b^9 + 5*a^7*b
^10)*e^(2*x)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b
^6 + 6*a^11*b^7 + a^10*b^8) + 3*(3*a^13*b^4 + 6*a^12*b^5 - 11*a^11*b^6 - 44
*a^10*b^7 - 51*a^9*b^8 - 26*a^8*b^9 - 5*a^7*b^10)/(a^16*b^2 + 6*a^15*b^3 +
15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))*e^(2*x) -
```

```
 3*(3*a^13*b^4 + 6*a^12*b^5 - 11*a^11*b^6 - 44*a^10*b^7 - 51*a^9*b^8 - 26*a
^8*b^9 - 5*a^7*b^10)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 1
5*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))*e^(2*x) - (9*a^13*b^4 + 50*a^12*b^5 +
115*a^11*b^6 + 140*a^10*b^7 + 95*a^9*b^8 + 34*a^8*b^9 + 5*a^7*b^10)/(a^16*b
^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^
10*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)^(3/2)
+ 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b) + sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) -
sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b
) - 3*a + b)*a^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^2}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*tanh(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)^2/(a + b*tanh(x)^2)^(5/2), x)
```

3.255 $\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3742, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.40

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\,\text{sech}(x)}{\sqrt{2}\sqrt{1+\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Tanh[x]^2],x]
[Out] (ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[1 + Tanh[x
]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.
time = 0.93, size = 62, normalized size = 2.48

method	result
derivativedivides	$-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4\sqrt{(1+\tanh(x))^2-2\tanh(x)}}\right)}{4}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2}}\right.}{4}$
default	$-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4\sqrt{(1+\tanh(x))^2-2\tanh(x)}}\right)}{4}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2}}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4*2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))^2-2*tanh(x))^(1/2))+1/4*2^(1/2)*arctanh(1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/2))

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(tanh(x)^2 + 1), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(20) = 40$.
time = 0.36, size = 543, normalized size = 21.72

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\left(\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + (28\cosh(x)^2-3)\sinh(x)^6 - 3\cosh(x)^6 + 2(28\cosh(x)^3-9\cosh(x))\sinh(x)^5 + 5(14\cosh(x)^4-9\cosh(x)^2+1)\sinh(x)^4 + 5\cosh(x)^4 + 4(14\cosh(x)^5-15\cosh(x)^3+5\cosh(x))\sinh(x)^3 + (28\cosh(x)^6-45\cosh(x)^4+30\cosh(x)^2-4)\sinh(x)^2 - 4\cosh(x)^2 + 2(4\cosh(x)^7-9\cosh(x)^5+10\cosh(x)^3-4\cosh(x))\sinh(x) + \left(\sqrt{2}\cosh(x)^6 + 6\sqrt{2}\cosh(x)\sinh(x)^5 + \sqrt{2}\sinh(x)^6 + 3(5\sqrt{2}\cosh(x)^2-\sqrt{2})\sinh(x)^4 - 3\sqrt{2}\cosh(x)^4 + 4(5\sqrt{2}\cosh(x)^3-3\sqrt{2}\cosh(x))\sinh(x)^3 + (15\sqrt{2}\cosh(x)^4-18\sqrt{2}\cosh(x)^2+4\sqrt{2})\sinh(x)^2 + 4\sqrt{2}\cosh(x)^2 + 2(3\sqrt{2}\cosh(x)^5-6\sqrt{2}\cosh(x)^3+4\sqrt{2}\cosh(x))\sinh(x) - 4\sqrt{2}\right)\sqrt{\frac{\cosh(x)^2+\sinh(x)^2}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}} + 4\right)}{\cosh(x)^6 + 6\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 + 20\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh(x)^4 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6}\right) + \frac{1}{8}\sqrt{2}\log\left(\frac{2\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + (6\cosh(x)^2+1)\sinh(x)^2 + \cosh(x)^2 + 2(2\cosh(x)^3+\cosh(x))\sinh(x) + \left(\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 + \sqrt{2}\right)\sqrt{\frac{\cosh(x)^2+\sinh(x)^2}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}} + 1\right)}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/8*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2)*log(2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\tanh^2(x) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(tanh(x)**2 + 1), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(20) = 40.
time = 0.42, size = 58, normalized size = 2.32

$$-\frac{1}{4}\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
- e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [**B**]
time = 0.17, size = 63, normalized size = 2.52

$$\frac{\sqrt{2}\left(\ln(\tanh(x)+1)-\ln\left(\sqrt{2}\sqrt{\tanh(x)^2+1}-\tanh(x)+1\right)\right)}{4}+\frac{\sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh(x)^2+1}+1\right)-\ln(\tanh(x)-1)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tanh(x)^2 + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) +
1)))/4 + (2^(1/2)*(log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(1/2) + 1) - log(t
anh(x) - 1)))/4
```

3.256 $\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx$

Optimal. Leaf size=27

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)}{\sqrt{2}}$$

[Out] 1/2*arctan(2^(1/2)*tanh(x)/(-1-tanh(x)^2)^(1/2))*2^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3742, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-1 - Tanh[x]^2],x]

[Out] ArcTan[(Sqrt[2]*Tanh[x])/Sqrt[-1 - Tanh[x]^2]]/Sqrt[2]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}\,(1-x^2)}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.37

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\,\text{sech}(x)}{\sqrt{2}\sqrt{-1-\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Tanh[x]^2],x]
[Out] (ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[-1 - Tanh[
x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.
time = 0.96, size = 66, normalized size = 2.44

method	result
derivativedivides	$-\frac{\sqrt{2}\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}+\frac{\sqrt{2}\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+\cdots}}\right)}{4}$
default	$-\frac{\sqrt{2}\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}+\frac{\sqrt{2}\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+\cdots}}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-1)^2-2*tanh(x))^(
1/2))+1/4*2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(1+tanh(x))^2+2*tanh(
x))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-tanh(x)^2 - 1), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.35, size = 175, normalized size = 6.48

$$\frac{1}{8}i\sqrt{2}\log\left(\frac{1}{2}\left(i\sqrt{2}\sqrt{-2e^{(4x)}-2}+2e^{(2x)}+2\right)e^{(-2x)}\right)-\frac{1}{8}i\sqrt{2}\log\left(\frac{1}{2}\left(-i\sqrt{2}\sqrt{-2e^{(4x)}-2}+2e^{(2x)}+2\right)e^{(-2x)}\right)-\frac{1}{8}i\sqrt{2}\log\left(\left(\sqrt{-2e^{(4x)}-2}\left(e^{(2x)}-2\right)+i\sqrt{2}e^{(4x)}-i\sqrt{2}e^{(2x)}+2i\sqrt{2}\right)e^{(-4x)}\right)+\frac{1}{8}i\sqrt{2}\log\left(\left(\sqrt{-2e^{(4x)}-2}\left(e^{(2x)}-2\right)-i\sqrt{2}e^{(4x)}+i\sqrt{2}e^{(2x)}-2i\sqrt{2}\right)e^{(-4x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*I*sqrt(2)*log(1/2*(I*sqrt(2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-
2*x)) - 1/8*I*sqrt(2)*log(1/2*(-I*sqrt(2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x)
+ 2)*e^(-2*x)) - 1/8*I*sqrt(2)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + I*
sqrt(2)*e^(4*x) - I*sqrt(2)*e^(2*x) + 2*I*sqrt(2))*e^(-4*x)) + 1/8*I*sqrt(2
)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) - I*sqrt(2)*e^(4*x) + I*sqrt(2)*e
^(2*x) - 2*I*sqrt(2))*e^(-4*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\tanh^{2}(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-tanh(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 58, normalized size = 2.15

$$\frac{1}{4}i\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
 - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [B]
time = 1.20, size = 22, normalized size = 0.81

$$\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \tanh(x)}{\sqrt{-\tanh\left(x\right)^2-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- tanh(x)^2 - 1)^(1/2),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*tanh(x))/(- tanh(x)^2 - 1)^(1/2)))/2
```

3.257 $\int \left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=89

$$\left(a^2+b^2\right)x+\frac{2ab\log(\cosh(c+dx))}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] (a^2+b^2)*x+2*a*b*ln(cosh(d*x+c))/d-b^2*tanh(d*x+c)/d-a*b*tanh(d*x+c)^2/d-1
/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3742, 1824, 647, 31}

$$-\frac{ab\tanh^2(c+dx)}{d}-\frac{(a+b)^2\log(1-\tanh(c+dx))}{2d}+\frac{(a-b)^2\log(\tanh(c+dx)+1)}{2d}-\frac{b^2\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] -1/2*((a + b)^2*Log[1 - Tanh[c + d*x]])/d + ((a - b)^2*Log[1 + Tanh[c + d*x
]])/(2*d) - (b^2*Tanh[c + d*x])/d - (a*b*Tanh[c + d*x]^2)/d - (b^2*Tanh[c +
 d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
```

```
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx^3)^2}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int\left(-b^2-2abx-b^2x^2-b^2x^4+\frac{a^2+b^2+2abx}{1-x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=-\frac{b^2\tanh(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$
$$=-\frac{b^2\tanh(c+dx)}{d}-\frac{ab\tanh^2(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$
$$=-\frac{(a+b)^2\log(1-\tanh(c+dx))}{2d}+\frac{(a-b)^2\log(1+\tanh(c+dx))}{2d}-\frac{b^2\tan}{}$$

Mathematica [A]
time = 0.50, size = 95, normalized size = 1.07

$$-\frac{15((a+b)^2\log(1-\tanh(c+dx))-(a-b)^2\log(1+\tanh(c+dx)))+30b^2\tanh(c+dx)+30ab\tanh^2(c+dx)+10b^2\tanh^3(c+dx)+6b^2\tanh^5(c+dx)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^3)^2,x]
[Out] -1/30*(15*((a + b)^2*Log[1 - Tanh[c + d*x]] - (a - b)^2*Log[1 + Tanh[c + d*
x]]) + 30*b^2*Tanh[c + d*x] + 30*a*b*Tanh[c + d*x]^2 + 10*b^2*Tanh[c + d*x]
^3 + 6*b^2*Tanh[c + d*x]^5)/d
```

Maple [A]
time = 0.32, size = 99, normalized size = 1.11

method	result
derivativedivides	$\frac{-\frac{b^2\left(\tanh^5(dx+c)\right)}{5}-\frac{b^2\left(\tanh^3(dx+c)\right)}{3}-ab\left(\tanh^2(dx+c)\right)-b^2\tanh(dx+c)-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^2-2ab+b^2\right)}{}}{d}$
default	$\frac{-\frac{b^2\left(\tanh^5(dx+c)\right)}{5}-\frac{b^2\left(\tanh^3(dx+c)\right)}{3}-ab\left(\tanh^2(dx+c)\right)-b^2\tanh(dx+c)-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^2-2ab+b^2\right)}{}}{d}$
risch	$a^2x-2abx+b^2x-\frac{4abc}{d}+\frac{2b\left(30a\,e^{8dx+8c}+45b\,e^{8dx+8c}+90a\,e^{6dx+6c}+90b\,e^{6dx+6c}+90a\,e^{4dx+4c}+140b\,e^{4dx+}\right.}{15d\left(1+e^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*b^2*tanh(d*x+c)^5-1/3*b^2*tanh(d*x+c)^3-a*b*tanh(d*x+c)^2-b^2*tan
h(d*x+c)-1/2*(a^2+2*a*b+b^2)*ln(tanh(d*x+c)-1)+1/2*(a^2-2*a*b+b^2)*ln(1+tan
h(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(85) = 170$.
time = 0.48, size = 194, normalized size = 2.18

$$\frac{1}{15}\,b^2\left(15\,x+\frac{15\,c}{d}-\frac{2\,\left(70\,e^{(-2\,dx-2\,c)}+140\,e^{(-4\,dx-4\,c)}+90\,e^{(-6\,dx-6\,c)}+45\,e^{(-8\,dx-8\,c)}+23\right)}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)+2\,ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x -
 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2074 vs. $2(85) = 170$.
time = 0.35, size = 2074, normalized size = 23.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^10 + 150*(a^2 - 2*a*b + b^2)
*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 15*(a^2 - 2*a*b + b^2)*d*x*sinh(d*x +
c)^10 + 15*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^8 + 15
*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 5*(a^2 - 2*a*b + b^2)*d*x +
4*a*b + 6*b^2)*sinh(d*x + c)^8 + 120*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x +
 c)^3 + (5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^7 + 30*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^6 +
30*(105*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 5*(a^2 - 2*a*b + b^2)*d*x
 + 14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^2 + 6*a*b +
 6*b^2)*sinh(d*x + c)^6 + 60*(63*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^5 +
14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^3 + 3*(5*(a^2
- 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(15
*(a^2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c)^4 + 10*(315*(a^2
- 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 105*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b
 + 6*b^2)*cosh(d*x + c)^4 + 15*(a^2 - 2*a*b + b^2)*d*x + 45*(5*(a^2 - 2*a*b
 + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^2 + 18*a*b + 28*b^2)*sinh(d*x +
```

```
c)^4 + 40*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 21*(5*(a^2 - 2*a*b
+ b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^5 + 15*(5*(a^2 - 2*a*b + b^2)*d*x
 + 6*a*b + 6*b^2)*cosh(d*x + c)^3 + (15*(a^2 - 2*a*b + b^2)*d*x + 18*a*b +
28*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 15*(a^2 - 2*a*b + b^2)*d*x + 5*(15
*(a^2 - 2*a*b + b^2)*d*x + 12*a*b + 28*b^2)*cosh(d*x + c)^2 + 5*(135*(a^2 -
 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 84*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b +
 6*b^2)*cosh(d*x + c)^6 + 90*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*co
sh(d*x + c)^4 + 15*(a^2 - 2*a*b + b^2)*d*x + 12*(15*(a^2 - 2*a*b + b^2)*d*x
 + 18*a*b + 28*b^2)*cosh(d*x + c)^2 + 12*a*b + 28*b^2)*sinh(d*x + c)^2 + 46
*b^2 + 30*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x + c)^9 + a*
b*sinh(d*x + c)^10 + 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x + c)^2 + a*b
)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x + c)^3 + a*
b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 + 14*a*b*cosh
(d*x + c)^2 + a*b)*sinh(d*x + c)^6 + 10*a*b*cosh(d*x + c)^4 + 4*(63*a*b*cos
h(d*x + c)^5 + 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)
^5 + 10*(21*a*b*cosh(d*x + c)^6 + 35*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x
+ c)^2 + a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b*cosh(d*x
+ c)^7 + 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))
*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 + 28*a*b*cosh(d*x + c)^6 + 30*a
*b*cosh(d*x + c)^4 + 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^2 + a*b +
10*(a*b*cosh(d*x + c)^9 + 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x + c)^5 + 4
*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c
)/(cosh(d*x + c) - sinh(d*x + c))) + 10*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*
x + c)^9 + 12*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^7 +
 18*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^5 + 4*(15*(a^
2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c)^3 + (15*(a^2 - 2*a*b
+ b^2)*d*x + 12*a*b + 28*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c
)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 5*d*cosh(d
*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*(21*d
*cosh(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*(63*d*cosh
(d*x + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 +
10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15
*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 + 7*d*cos
h(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*d
*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cos
h(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*cosh(d*x +
 c)^9 + 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d
*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A]
time = 0.16, size = 100, normalized size = 1.12

$$\begin{cases} a^2x + 2abx - \frac{2ab\log\left(\tanh\left(c+dx\right)+1\right)}{d} - \frac{ab\tanh^2\left(c+dx\right)}{d} + b^2x - \frac{b^2\tanh^5\left(c+dx\right)}{5d} - \frac{b^2\tanh^3\left(c+dx\right)}{3d} - \frac{b^2\tanh\left(c+dx\right)}{d} & \text{for}\ d \neq 0 \\ x\left(a + b\tanh^3\left(c\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c +
 d*x)**2/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b**2*tanh(c + d*x)**3/(
3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**3)**2, True))
```

Giac [A]
time = 0.43, size = 142, normalized size = 1.60

$$\frac{30\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)+15\,(a^2-2\,ab+b^2)(dx+c)+\frac{2\,(23\,b^2+15\,(2\,ab+3\,b^2)e^{(8\,dx+8\,c)}+90\,(ab+b^2)e^{(6\,dx+6\,c)}+10\,(9\,ab+14\,b^2)e^{(4\,dx+4\,c)}+10\,(3\,ab+7\,b^2)e^{(2\,dx+2\,c)})}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/15*(30*a*b*log(e^(2*d*x + 2*c) + 1) + 15*(a^2 - 2*a*b + b^2)*(d*x + c) +
2*(23*b^2 + 15*(2*a*b + 3*b^2)*e^(8*d*x + 8*c) + 90*(a*b + b^2)*e^(6*d*x +
6*c) + 10*(9*a*b + 14*b^2)*e^(4*d*x + 4*c) + 10*(3*a*b + 7*b^2)*e^(2*d*x +
2*c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.16, size = 91, normalized size = 1.02

$$x\,\left(a^2+2\,a\,b+b^2\right)-\frac{b^2\,\mathrm{tanh}\left(c+d\,x\right)}{d}-\frac{b^2\,{\mathrm{tanh}\left(c+d\,x\right)}^3}{3\,d}-\frac{b^2\,{\mathrm{tanh}\left(c+d\,x\right)}^5}{5\,d}-\frac{2\,a\,b\,\ln\left(\mathrm{tanh}\left(c+d\,x\right)+1\right)}{d}-\frac{a\,b\,{\mathrm{tanh}\left(c+d\,x\right)}^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^3)^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (b^2*tanh(c + d*x))/d - (b^2*tanh(c + d*x)^3)/(3*d)
 - (b^2*tanh(c + d*x)^5)/(5*d) - (2*a*b*log(tanh(c + d*x) + 1))/d - (a*b*ta
nh(c + d*x)^2)/d
```

3.258 $\int \frac{1}{1+\tanh^3(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{2}-\frac{2\text{ArcTan}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6(1+\tanh(x))}$$

```
[Out] 1/2*x-2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/6/(1+tanh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3742, 2099, 213, 632, 210}

$$-\frac{2\text{ArcTan}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x}{2}-\frac{1}{6(\tanh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tanh[x]^3)^(-1),x]
```

```
[Out] x/2 - (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(6*(1 + Tanh[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
```

```
PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\tanh^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x^2)\,(1+x^3)}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{6(1+x)^2}-\frac{1}{2\,(-1+x^2)}+\frac{1}{3\,(1-x+x^2)}\right)\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{6(1+\tanh(x))}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx\right.\\
&= \frac{x}{2}-\frac{1}{6(1+\tanh(x))}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\
&= \frac{x}{2}-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6(1+\tanh(x))}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 1.29

$$\frac{1}{36}\left(8\sqrt{3}\,\text{ArcTan}\left(\frac{-1+2\tanh(x)}{\sqrt{3}}\right)-9\log(1-\tanh(x))+9\log(1+\tanh(x))-\frac{6}{1+\tanh(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tanh[x]^3)^(-1),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 9*Log[1 - Tanh[x]] + 9*Log[1
+ Tanh[x]] - 6/(1 + Tanh[x]))/36
```

Maple [A]
time = 0.34, size = 41, normalized size = 1.08

method	result	size

derivativedivides	$-\frac{\ln(\tanh(x)-1)}{4}-\frac{1}{6(1+\tanh(x))}+\frac{\ln(1+\tanh(x))}{4}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2\tanh(x)-1)\sqrt{3}}{3}\right)}{9}$	41
default	$-\frac{\ln(\tanh(x)-1)}{4}-\frac{1}{6(1+\tanh(x))}+\frac{\ln(1+\tanh(x))}{4}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2\tanh(x)-1)\sqrt{3}}{3}\right)}{9}$	41
risch	$\frac{x}{2}-\frac{e^{-2x}}{12}+\frac{i\sqrt{3}\,\ln\left(e^{2x}+i\sqrt{3}\right)}{9}-\frac{i\sqrt{3}\,\ln\left(e^{2x}-i\sqrt{3}\right)}{9}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tanh(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(tanh(x)-1)-1/6/(1+tanh(x))+1/4*ln(1+tanh(x))+2/9*3^(1/2)*arctan(1/3
*(2*tanh(x)-1)*3^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.
time = 0.47, size = 73, normalized size = 1.92

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{2}x-\frac{1}{12}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
(2))) + 1/2*x - 1/12*e^(-2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.
time = 0.36, size = 95, normalized size = 2.50

$$\frac{18\,x\cosh(x)^2+36\,x\cosh(x)\sinh(x)+18\,x\sinh(x)^2-8\left(\sqrt{3}\cosh(x)^2+2\sqrt{3}\cosh(x)\sinh(x)+\sqrt{3}\sinh(x)^2\right)\arctan\left(-\frac{\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)-3}{36\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 - 8*(sqrt(3)*c
osh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sqrt
(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) - 3)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(36) = 72$.
time = 0.25, size = 102, normalized size = 2.68

$$\frac{9x\tanh(x)}{18\tanh(x)+18}+\frac{9x}{18\tanh(x)+18}+\frac{4\sqrt{3}\tanh(x)\operatorname{atan}\left(\frac{2\sqrt{3}\tanh(x)}{3}-\frac{\sqrt{3}}{3}\right)}{18\tanh(x)+18}+\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\tanh(x)}{3}-\frac{\sqrt{3}}{3}\right)}{18\tanh(x)+18}-\frac{3}{18\tanh(x)+18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)**3),x)
```

```
[Out] 9*x*tanh(x)/(18*tanh(x) + 18) + 9*x/(18*tanh(x) + 18) + 4*sqrt(3)*tanh(x)*a
tan(2*sqrt(3)*tanh(x)/3 - sqrt(3)/3)/(18*tanh(x) + 18) + 4*sqrt(3)*atan(2*s
qrt(3)*tanh(x)/3 - sqrt(3)/3)/(18*tanh(x) + 18) - 3/(18*tanh(x) + 18)
```

Giac [A]
time = 0.41, size = 25, normalized size = 0.66

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right)+\frac{1}{2}x-\frac{1}{12}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tanh(x)^3),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) + 1/2*x - 1/12*e^(-2*x)
```

Mupad [B]
time = 0.10, size = 38, normalized size = 1.00

$$\frac{\frac{x}{2}+\frac{\tanh(x)}{6}+\frac{x\tanh(x)}{2}}{\tanh(x)+1}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}(2\tanh(x)-1)}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tanh(x)^3 + 1),x)
```

```
[Out] (x/2 + tanh(x)/6 + (x*tanh(x))/2)/(tanh(x) + 1) + (2*3^(1/2)*atan((3^(1/2)*
(2*tanh(x) - 1))/3))/9
```

3.259 $\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)+\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tanh^4(x)}}\right)-\frac{1}{4}(2(a+$$

```
[Out] 1/2*(a+b)^(3/2)*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-
1/4*(3*a+2*b)*arctanh(b^(1/2)*tanh(x)^2/(a+b*tanh(x)^4)^(1/2))*b^(1/2)-1/4*
(a+b*tanh(x)^4)^(1/2)*(2*a+2*b+b*tanh(x)^2)-1/6*(a+b*tanh(x)^4)^(3/2)
```

Rubi [A]
time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3751, 1262, 749, 829, 858, 223, 212, 739}

$$-\frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2}-\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)}+\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tanh^4(x)}}\right)-\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*(a + b*Tanh[x]^4)^(3/2),x]
```

```
[Out] -1/4*(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]
]) + ((a + b)^(3/2)*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[
x]^4])])/2 - ((2*(a + b) + b*Tanh[x]^2)*Sqrt[a + b*Tanh[x]^4])/4 - (a + b*T
anh[x]^4)^(3/2)/6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{x(a+bx^4)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{1-x}\,dx, x, \tanh^2(x)\right)$$

$$= -\frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} - \frac{1}{2}\text{Subst}\left(\int \frac{(-a-bx)\sqrt{a+bx^2}}{1-x}\,dx, x, \tanh$$

$$= -\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)} - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} -$$

$$= -\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)} - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} -$$

$$= -\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)} - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2} -$$

$$= -\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\tanh^{-1}\Bigg($$

Mathematica [A]
time = 3.28, size = 166, normalized size = 1.34

$$\frac{1}{12}\left(-6\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)+6(a+b)^{3/2}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b\tanh^4(x)}\,\left(8a+6b+3b\tanh^2(x)+2b\tanh^4(x)\right)-\frac{3\sqrt{a}\,\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\tanh^2(x)}{\sqrt{a}}\right)\sqrt{a+b\tanh^4(x)}}{\sqrt{1+\frac{b\tanh^4(x)}{a}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*(a + b*Tanh[x]^4)^(3/2),x]
```

```
[Out] (-6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]] + 6*
(a + b)^(3/2)*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])
] - Sqrt[a + b*Tanh[x]^4]*(8*a + 6*b + 3*b*Tanh[x]^2 + 2*b*Tanh[x]^4) - (3*
Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a]]*Sqrt[a + b*Tanh[x]^4])
/Sqrt[1 + (b*Tanh[x]^4)/a])/12
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 2.74, size = 788, normalized size = 6.35

method	result	size

derivativedivides	Expression too large to display	788
default	Expression too large to display	788

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a+b*tanh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*tanh(x)^4*(a+b*tanh(x)^4)^(1/2)-1/4*b*tanh(x)^2*(a+b*tanh(x)^4)^(1/2
)-1/2*(4/3*a*b+b^2)/b*(a+b*tanh(x)^4)^(1/2)-1/2*(-5/3*a*b-b^2)/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*ta
nh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^
(1/2),I)-1/2*(3/2*a*b+b^2)*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/2))/
b^(1/2)-1/2*I*(-7/5*a*b-b^2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x
)^4)^(1/2)/b^(1/2)*(EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elliptic
E(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*(a^2+2*a*b+b^2)*(-1/2/(a+b)^(1/
2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)
*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^
(1/2))^(1/2)))-1/2*(5/3*a*b+b^2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)
^(1/2)*EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*I*(7/5*a*b+b^2)*a
^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)/b^(1/2)*(EllipticF(
tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(tanh(x)*(I/a^(1/2)*b^(1/2))^
(1/2),I))-1/2*(a^2+2*a*b+b^2)*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+
2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*t
anh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(
1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2646 vs. 2(101) = 202.
time = 0.55, size = 11528, normalized size = 92.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((3*a + 2*b)*cosh(x)^12 + 12*(3*a + 2*b)*cosh(x)*sinh(x)^11 + (3*a
 + 2*b)*sinh(x)^12 + 6*(3*a + 2*b)*cosh(x)^10 + 6*(11*(3*a + 2*b)*cosh(x)^2
 + 3*a + 2*b)*sinh(x)^10 + 20*(11*(3*a + 2*b)*cosh(x)^3 + 3*(3*a + 2*b)*cos
h(x))*sinh(x)^9 + 15*(3*a + 2*b)*cosh(x)^8 + 15*(33*(3*a + 2*b)*cosh(x)^4 +
 18*(3*a + 2*b)*cosh(x)^2 + 3*a + 2*b)*sinh(x)^8 + 24*(33*(3*a + 2*b)*cosh(
x)^5 + 30*(3*a + 2*b)*cosh(x)^3 + 5*(3*a + 2*b)*cosh(x))*sinh(x)^7 + 20*(3*
a + 2*b)*cosh(x)^6 + 4*(231*(3*a + 2*b)*cosh(x)^6 + 315*(3*a + 2*b)*cosh(x)
^4 + 105*(3*a + 2*b)*cosh(x)^2 + 15*a + 10*b)*sinh(x)^6 + 24*(33*(3*a + 2*b
)*cosh(x)^7 + 63*(3*a + 2*b)*cosh(x)^5 + 35*(3*a + 2*b)*cosh(x)^3 + 5*(3*a
+ 2*b)*cosh(x))*sinh(x)^5 + 15*(3*a + 2*b)*cosh(x)^4 + 15*(33*(3*a + 2*b)*c
osh(x)^8 + 84*(3*a + 2*b)*cosh(x)^6 + 70*(3*a + 2*b)*cosh(x)^4 + 20*(3*a +
2*b)*cosh(x)^2 + 3*a + 2*b)*sinh(x)^4 + 20*(11*(3*a + 2*b)*cosh(x)^9 + 36*(
3*a + 2*b)*cosh(x)^7 + 42*(3*a + 2*b)*cosh(x)^5 + 20*(3*a + 2*b)*cosh(x)^3
+ 3*(3*a + 2*b)*cosh(x))*sinh(x)^3 + 6*(3*a + 2*b)*cosh(x)^2 + 6*(11*(3*a +
 2*b)*cosh(x)^10 + 45*(3*a + 2*b)*cosh(x)^8 + 70*(3*a + 2*b)*cosh(x)^6 + 50
*(3*a + 2*b)*cosh(x)^4 + 15*(3*a + 2*b)*cosh(x)^2 + 3*a + 2*b)*sinh(x)^2 +
12*((3*a + 2*b)*cosh(x)^11 + 5*(3*a + 2*b)*cosh(x)^9 + 10*(3*a + 2*b)*cosh(
x)^7 + 10*(3*a + 2*b)*cosh(x)^5 + 5*(3*a + 2*b)*cosh(x)^3 + (3*a + 2*b)*cos
h(x))*sinh(x) + 3*a + 2*b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^8 + 8*(a + 2*b)*
cosh(x)*sinh(x)^7 + (a + 2*b)*sinh(x)^8 + 4*(a - 2*b)*cosh(x)^6 + 4*(7*(a +
 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^6 + 8*(7*(a + 2*b)*cosh(x)^3 + 3*(a - 2*
b)*cosh(x))*sinh(x)^5 + 6*(a + 2*b)*cosh(x)^4 + 2*(35*(a + 2*b)*cosh(x)^4 +
 30*(a - 2*b)*cosh(x)^2 + 3*a + 6*b)*sinh(x)^4 + 8*(7*(a + 2*b)*cosh(x)^5 +
 10*(a - 2*b)*cosh(x)^3 + 3*(a + 2*b)*cosh(x))*sinh(x)^3 + 4*(a - 2*b)*cosh
(x)^2 + 4*(7*(a + 2*b)*cosh(x)^6 + 15*(a - 2*b)*cosh(x)^4 + 9*(a + 2*b)*cos
h(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 +
sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - co
sh(x))*sinh(x) + 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4
*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a +
3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*s
inh(x)^3 + sinh(x)^4)) + 8*((a + 2*b)*cosh(x)^7 + 3*(a - 2*b)*cosh(x)^5 + 3
*(a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^8 + 8
*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^
6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2
+ 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*s
inh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*c
osh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)
) + 6*((a + b)*cosh(x)^12 + 12*(a + b)*cosh(x)*sinh(x)^11 + (a + b)*sinh(x)
^12 + 6*(a + b)*cosh(x)^10 + 6*(11*(a + b)*cosh(x)^2 + a + b)*sinh(x)^10 +
20*(11*(a + b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^9 + 15*(a + b)*cosh(x
)^8 + 15*(33*(a + b)*cosh(x)^4 + 18*(a + b)*cosh(x)^2 + a + b)*sinh(x)^8 +
```

```
24*(33*(a + b)*cosh(x)^5 + 30*(a + b)*cosh(x)^3 + 5*(a + b)*cosh(x))*sinh(x
)^7 + 20*(a + b)*cosh(x)^6 + 4*(231*(a + b)*cosh(x)^6 + 315*(a + b)*cosh(x)
^4 + 105*(a + b)*cosh(x)^2 + 5*a + 5*b)*sinh(x)^6 + 24*(33*(a + b)*cosh(x)^
7 + 63*(a + b)*cosh(x)^5 + 35*(a + b)*cosh(x)^3 + 5*(a + b)*cosh(x))*sinh(x
)^5 + 15*(a + b)*cosh(x)^4 + 15*(33*(a + b)*cosh(x)^8 + 84*(a + b)*cosh(x)^
6 + 70*(a + b)*cosh(x)^4 + 20*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 20*(11
*(a + b)*cosh(x)^9 + 36*(a + b)*cosh(x)^7 + 42*(a + b)*cosh(x)^5 + 20*(a +
b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 6*(a + b)*cosh(x)^2 + 6*(11*(
a + b)*cosh(x)^10 + 45*(a + b)*cosh(x)^8 + 70*(a + b)*cosh(x)^6 + 50*(a + b
)*cosh(x)^4 + 15*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 12*((a + b)*cosh(x)
^11 + 5*(a + b)*cosh(x)^9 + 10*(a + b)*cosh(x)^7 + 10*(a + b)*cosh(x)^5 + 5
*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a^
2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2
 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b
^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 +
 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^4 + 2
*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 + 2*a
*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)
*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cos
h(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*
(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + sqrt(2)*((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*co
sh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
+ (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^4 + (
a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b
)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tanh^4\left(x\right)\right)^{\frac{3}{2}} \tanh\left(x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*tanh(x)**4)**(3/2)*tanh(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")
```

[Out] integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)\,\left(b\tanh(x)^4+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)*(a + b*tanh(x)^4)^(3/2),x)

[Out] int(tanh(x)*(a + b*tanh(x)^4)^(3/2), x)

3.260 $\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\,\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tanh^4(x)}}\right)-\frac{1}{2}\sqrt{a+b\tanh^4(x)}$$

```
[Out] -1/2*arctanh(b^(1/2)*tanh(x)^2/(a+b*tanh(x)^4)^(1/2))*b^(1/2)+1/2*arctanh((
a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))*(a+b)^(1/2)-1/2*(a+b*tanh
(x)^4)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3751, 1262, 749, 858, 223, 212, 739}

$$-\frac{1}{2}\sqrt{a+b\tanh^4(x)}-\frac{1}{2}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\,\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tanh^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]*Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] -1/2*(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) + (Sqrt[a
 + b]*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/2 - S
qrt[a + b*Tanh[x]^4]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \text{Subst}\left(\int \frac{x\sqrt{a+bx^4}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1-x}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{1}{2}\sqrt{a+b\tanh^4(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-a-bx}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{1}{2}\sqrt{a+b\tanh^4(x)} - \frac{1}{2}(-a-b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh$$
$$= -\frac{1}{2}\sqrt{a+b\tanh^4(x)} - \frac{1}{2}b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) -$$
$$= -\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a+b\,\text{ta}}{\sqrt{a+b}\sqrt{a+}}\right.$$

Mathematica [A]
time = 0.04, size = 86, normalized size = 0.97

$$\frac{1}{2}\left(-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \sqrt{a+b}\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \sqrt{a+b\tanh^4(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] (-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) + Sqrt[a + b
]*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])] - Sqrt[a +
 b*Tanh[x]^4])/2
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 2.37, size = 333, normalized size = 3.74

method	result

derivativedivides	$-\frac{\sqrt{a+b\left(\tanh^4(x)\right)}}{2}-\frac{\sqrt{b}\,\ln\left(2\sqrt{b}\,(\tanh^2(x))+2\sqrt{a+b\left(\tanh^4(x)\right)}\right)}{2}-\frac{(a+b)\left(-\frac{\operatorname{arctanh}\left(\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{\sqrt{a+b\left(\tanh^4(x)\right)}}{2}-\frac{\sqrt{b}\,\ln\left(2\sqrt{b}\,(\tanh^2(x))+2\sqrt{a+b\left(\tanh^4(x)\right)}\right)}{2}-\frac{(a+b)\left(-\frac{\operatorname{arctanh}\left(\cdots\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a+b*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a+b*tanh(x)^4)^(1/2)-1/2*b^(1/2)*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x
)^4)^(1/2))-1/2*(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+
b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^
(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4
)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I
/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))-1/2*(a+b)*(-1/2/(a+b)^(
1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)
*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/
2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*
b^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(69) = 138$.
time = 0.51, size = 5136, normalized size = 57.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*si
nh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(b)*log(-(
(a + 2*b)*cosh(x)^8 + 8*(a + 2*b)*cosh(x)*sinh(x)^7 + (a + 2*b)*sinh(x)^8 +
 4*(a - 2*b)*cosh(x)^6 + 4*(7*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^6 + 8*
(7*(a + 2*b)*cosh(x)^3 + 3*(a - 2*b)*cosh(x))*sinh(x)^5 + 6*(a + 2*b)*cosh(
x)^4 + 2*(35*(a + 2*b)*cosh(x)^4 + 30*(a - 2*b)*cosh(x)^2 + 3*a + 6*b)*sinh
(x)^4 + 8*(7*(a + 2*b)*cosh(x)^5 + 10*(a - 2*b)*cosh(x)^3 + 3*(a + 2*b)*cos
h(x))*sinh(x)^3 + 4*(a - 2*b)*cosh(x)^2 + 4*(7*(a + 2*b)*cosh(x)^6 + 15*(a
- 2*b)*cosh(x)^4 + 9*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(
cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2
 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*sqrt(b)*sqrt(((a + b)
*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)
^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*((a + 2*b)*cos
h(x)^7 + 3*(a - 2*b)*cosh(x)^5 + 3*(a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))
*sinh(x) + a + 2*b)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cos
h(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5
 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(
x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4
+ 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3
*cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + si
nh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh
(x))*sinh(x) + 1)*sqrt(a + b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 +
 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 -
b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6
+ 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2
*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 +
30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2
*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*
cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cos
h(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a
^2 - b^2)*sinh(x)^2 + sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x
)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a
- b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*s
qrt(a + b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^
2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 -
```

```
 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x
)^4)) + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2
)*cosh(x)^5 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh
(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*s
inh(x)^3 + sinh(x)^4)) - 2*sqrt(2)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x
)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 +
 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*co
sh(x)*sinh(x)^3 + sinh(x)^4)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4
 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*si
nh(x) + 1), -1/4*(2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*
sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^
3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a -
b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqr
t(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2
 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 -
4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)
^4))/((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)
^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 +
2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*co
sh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^
4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2
+ 6*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 -
 b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*
cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 +
 9*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2
 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + 3*(a^2 + 2*
a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))) - (cosh(x)^4 + 4*cosh
(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4
*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(b)*log(-((a + 2*b)*cosh(x)^8 + 8*(
a + 2*b)*cosh(x)*sinh(x)^7 + (a + 2*b)*sinh(x)^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tanh^4(x)}\,\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**4)*tanh(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)\,\sqrt{b\tanh\left(x\right)^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b*tanh(x)^4)^(1/2),x)
```

```
[Out] int(tanh(x)*(a + b*tanh(x)^4)^(1/2), x)
```

3.261 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

[Out] 1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3751, 1262, 739, 212}

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/Sqrt[a + b*Tanh[x]^4],x]

[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a + b])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{(1-x^2)\sqrt{a+bx^4}}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^4],x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a +
b])
```

Maple [A]
time = 3.00, size = 37, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{2b\left(\tanh^2(x)\right)+2a}{2\sqrt{a+b}\,\sqrt{a+b\left(\tanh^4(x)\right)}}\right)}{2\sqrt{a+b}}$	37
default	$\frac{\operatorname{arctanh}\left(\frac{2b\left(\tanh^2(x)\right)+2a}{2\sqrt{a+b}\,\sqrt{a+b\left(\tanh^4(x)\right)}}\right)}{2\sqrt{a+b}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(32) = 64$.
time = 0.51, size = 1286, normalized size = 32.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sin
h(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^
2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2
+ 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 1
0*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a
^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*c
```

```
osh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + sqr
t(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a +
b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*sqrt(((a + b)*
cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^
2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2
 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 + 2*
a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(
x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/sq
rt(a + b), -1/2*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*
cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*
cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh
(x) + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(
a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*
b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sin
h(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^
6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 +
 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 2*a*b
 + b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*co
sh(x)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(
x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3
+ 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 -
b^2)*cosh(x)^4 + 9*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)
^5 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x)))/(a +
b)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a + b\tanh^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(1/2),x)
[Out] Integral(tanh(x)/sqrt(a + b*tanh(x)**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)}{\sqrt{b\tanh(x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*tanh(x)^4)^(1/2),x)
```

```
[Out] int(tanh(x)/(a + b*tanh(x)^4)^(1/2), x)
```

3.262 $\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}$$

```
[Out] 1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(3/2)+
1/2*(-a+b*tanh(x)^2)/a/(a+b)/(a+b*tanh(x)^4)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3751, 1262, 755, 12, 739, 212}

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*(a + b)^(
3/2)) - (a - b*Tanh[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tanh[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{x}{(1-x^2)\left(a+bx^4\right)^{3/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}} + \frac{\text{Subst}\left(\int \frac{a}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)}{2a(a+b)}$$
$$= -\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \tanh^2(x)\right)}{2(a+b)}$$
$$= -\frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)}$$
$$= \frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}$$

Mathematica [A]
time = 0.41, size = 73, normalized size = 0.99

$$\frac{1}{2}\left(\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{3/2}} - \frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(a + b)^(3/
2) - (a - b*Tanh[x]^2)/(a*(a + b)*Sqrt[a + b*Tanh[x]^4]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 2.43, size = 431, normalized size = 5.82

method	result
derivativedivides	$\frac{b\left(-\frac{\tanh^3(x)}{4a(a+b)}+\frac{\tanh^2(x)}{4a(a+b)}-\frac{\tanh(x)}{4a(a+b)}-\frac{1}{4(a+b)b}\right)}{\sqrt{\left(\tanh^4(x)+\frac{a}{b}\right)b}}-\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\left(\tanh^2(x)\right)+2a}{2\sqrt{a+b}\,\sqrt{a+b\left(\tanh^4(x)\right)}}\right)}{2\sqrt{a+b}}+\frac{\sqrt{1-\frac{i\sqrt{}}{}}}{}}{}$
default	$\frac{b\left(-\frac{\tanh^3(x)}{4a(a+b)}+\frac{\tanh^2(x)}{4a(a+b)}-\frac{\tanh(x)}{4a(a+b)}-\frac{1}{4(a+b)b}\right)}{\sqrt{\left(\tanh^4(x)+\frac{a}{b}\right)b}}-\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\left(\tanh^2(x)\right)+2a}{2\sqrt{a+b}\,\sqrt{a+b\left(\tanh^4(x)\right)}}\right)}{2\sqrt{a+b}}+\frac{\sqrt{1-\frac{i\sqrt{}}{}}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/4/a/(a+b)*tanh(x)^3+1/4/a/(a+b)*tanh(x)^2-1/4/a/(a+b)*tanh(x)-1/4/(a+
b)/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*
b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^
(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I
*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))+b*(
1/4/a/(a+b)*tanh(x)^3+1/4/a/(a+b)*tanh(x)^2+1/4/a/(a+b)*tanh(x)-1/4/(a+b)/b
)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*ta
nh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2
)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(
1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(63) = 126$.
time = 0.56, size = 3914, normalized size = 52.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cosh(x)^8 + 8*(a^2 + a*b)*cosh(x)*sinh(x)^7 + (a^2 + a*b
)*sinh(x)^8 + 4*(a^2 - a*b)*cosh(x)^6 + 4*(7*(a^2 + a*b)*cosh(x)^2 + a^2 -
a*b)*sinh(x)^6 + 8*(7*(a^2 + a*b)*cosh(x)^3 + 3*(a^2 - a*b)*cosh(x))*sinh(x
)^5 + 6*(a^2 + a*b)*cosh(x)^4 + 2*(35*(a^2 + a*b)*cosh(x)^4 + 30*(a^2 - a*b
)*cosh(x)^2 + 3*a^2 + 3*a*b)*sinh(x)^4 + 8*(7*(a^2 + a*b)*cosh(x)^5 + 10*(a
^2 - a*b)*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 4*(a^2 - a*b)*cosh
(x)^2 + 4*(7*(a^2 + a*b)*cosh(x)^6 + 15*(a^2 - a*b)*cosh(x)^4 + 9*(a^2 + a*
b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 8*((a^2 + a*b)*cosh(x)^7
+ 3*(a^2 - a*b)*cosh(x)^5 + 3*(a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x))*
sinh(x))*sqrt(a + b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b +
b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh
(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 +
 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 -
 b^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^
2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*
sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 +
15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)
*sinh(x)^2 + sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a
+ b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh
(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b
)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*
(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x
)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a
^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)
^5 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(cos
h(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4)) - 2*sqrt(2)*((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*si
nh(x)^3 + (a^2 - b^2)*sinh(x)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a
^2 - b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 -
b^2)*cosh(x)^3 + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x
)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*
a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^4 + 3*a^3*b + 3*a^2*
b^2 + a*b^3)*cosh(x)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh
(x)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^8 + 4*(a^4 + a^3*b - a^
```

```
2*b^2 - a*b^3)*cosh(x)^6 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3 + 7*(a^4 + 3*a^
3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*cosh(x)^3 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x))*sinh(x
)^5 + 6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 + 2*(35*(a^4 + 3*a^3*
b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 +
30*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2)*sinh(x)^4 + a^4 + 3*a^3*b + 3
*a^2*b^2 + a*b^3 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^5 + 10*
(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*cosh(x))*sinh(x)^3 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 4
*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^6 + 15*(a^4 + a^3*b - a^2*b
^2 - a*b^3)*cosh(x)^4 + a^4 + a^3*b - a^2*b^2 - a*b^3 + 9*(a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 3*a^3*b + 3*a^2*b^2 + a
*b^3)*cosh(x)^7 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^5 + 3*(a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh
(x))*sinh(x)), -1/2*(((a^2 + a*b)*cosh(x)^8 + 8*(a^2 + a*b)*cosh(x)*sinh(x)
^7 + (a^2 + a*b)*sinh(x)^8 + 4*(a^2 - a*b)*cosh(x)^6 + 4*(7*(a^2 + a*b)*cos
h(x)^2 + a^2 - a*b)*sinh(x)^6 + 8*(7*(a^2 + a*b)*cosh(x)^3 + 3*(a^2 - a*b)*
cosh(x))*sinh(x)^5 + 6*(a^2 + a*b)*cosh(x)^4 + 2*(35*(a^2 + a*b)*cosh(x)^4
+ 30*(a^2 - a*b)*cosh(x)^2 + 3*a^2 + 3*a*b)*sinh(x)^4 + 8*(7*(a^2 + a*b)*co
sh(x)^5 + 10*(a^2 - a*b)*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 4*(
a^2 - a*b)*cosh(x)^2 + 4*(7*(a^2 + a*b)*cosh(x)^6 + 15*(a^2 - a*b)*cosh(x)^
4 + 9*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 8*((a^2 +
a*b)*cosh(x)^7 + 3*(a^2 - a*b)*cosh(x)^5 + 3*(a^2 + a*b)*cosh(x)^3 + (a^2 -
 a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*
(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*
(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(
x))*sinh(x) + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)
^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 +
3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cos
h(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\left(a + b\tanh^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(3/2),x)
[Out] Integral(tanh(x)/(a + b*tanh(x)**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)}{\left(b\tanh\left(x\right)^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*tanh(x)^4)^(3/2),x)
```

```
[Out] int(tanh(x)/(a + b*tanh(x)^4)^(3/2), x)
```

3.263 $\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{5/2}} dx$

Optimal. Leaf size=118

$$\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}$$

```
[Out] 1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(5/2)+
1/6*(-3*a^2+b*(5*a+2*b)*tanh(x)^2)/a^2/(a+b)^2/(a+b*tanh(x)^4)^(1/2)+1/6*(-
a+b*tanh(x)^2)/a/(a+b)/(a+b*tanh(x)^4)^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3751, 1262, 755, 837, 12, 739, 212}

$$-\frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}-\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Tanh[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*(a + b)^(
5/2)) - (a - b*Tanh[x]^2)/(6*a*(a + b)*(a + b*Tanh[x]^4)^(3/2)) - (3*a^2 -
b*(5*a + 2*b)*Tanh[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x}{(1-x^2)\,(a+bx^4)^{5/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\,(a+bx^2)^{5/2}}\,dx, x, \tanh^2(x)\right)$$
$$= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a-2b+2bx}{(1-x)(a+bx^2)^{3/2}}\,dx, x, \tanh^2(x)\right)}{6a(a+b)}$$
$$= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}} + \frac{\text{Subst}\left(\int -\right.}{(1\cdot}$$
$$= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}} + \frac{\text{Subst}\left(\int -\right.}{(1\cdot}$$
$$= -\frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}} - \frac{\text{Subst}\left(\int \frac{}{a\cdot}\right.}{}$$
$$= \frac{\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{3a^2}{6a^2(}$$

Mathematica [A]
time = 0.60, size = 113, normalized size = 0.96

$$\frac{1}{6}\left(\frac{3\tanh^{-1}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{5/2}} + \frac{-a^2(4a+b)+3ab(2a+b)\tanh^2(x)-3a^2b\tanh^4(x)+b^2(5a+2b)\tanh^6(x)}{a^2(a+b)^2\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(5/2),x]
```

```
[Out] ((3*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/(a + b)
^(5/2) + (-(a^2*(4*a + b)) + 3*a*b*(2*a + b)*Tanh[x]^2 - 3*a^2*b*Tanh[x]^4
+ b^2*(5*a + 2*b)*Tanh[x]^6)/(a^2*(a + b)^2*(a + b*Tanh[x]^4)^(3/2)))/6
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 2.54, size = 637, normalized size = 5.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/b*tanh(x)^2-1/6/a/(a+b)/b*tanh(x
)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(tanh(x)^4+a/b)^2+b*(1/8*(3*a+b)/a^2
/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a+b)^2*tanh(x)^2+1/24/a^2*(11*a+5*b)
/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)^2*(-1/2
/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1
/2))-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*
(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/
a^(1/2)*b^(1/2))^(1/2)))-1/2*(1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/b*tanh(x)
^2+1/6/a/(a+b)/b*tanh(x)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(tanh(x)^4+a/
b)^2+b*(-1/8*(3*a+b)/a^2/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a+b)^2*tanh(
x)^2-1/24/a^2*(11*a+5*b)/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(x)^4+a/b)*b)
^(1/2)-1/2/(a+b)^2*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^
(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(
1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^
(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8210 vs. 2(102) = 204.
time = 1.41, size = 16463, normalized size = 139.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^16 + 16*(a^4 + 2*a^3*b + a^2*b^
2)*cosh(x)*sinh(x)^15 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^16 + 8*(a^4 - a^2
*b^2)*cosh(x)^14 + 8*(a^4 - a^2*b^2 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^
```

```
2)*sinh(x)^14 + 112*(5*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + (a^4 - a^2*b^2
)*cosh(x))*sinh(x)^13 + 4*(7*a^4 - 2*a^3*b + 7*a^2*b^2)*cosh(x)^12 + 4*(455
*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^4 + 7*a^4 - 2*a^3*b + 7*a^2*b^2 + 182*(a
^4 - a^2*b^2)*cosh(x)^2)*sinh(x)^12 + 16*(273*(a^4 + 2*a^3*b + a^2*b^2)*cos
h(x)^5 + 182*(a^4 - a^2*b^2)*cosh(x)^3 + 3*(7*a^4 - 2*a^3*b + 7*a^2*b^2)*co
sh(x))*sinh(x)^11 + 56*(a^4 - a^2*b^2)*cosh(x)^10 + 8*(1001*(a^4 + 2*a^3*b
 + a^2*b^2)*cosh(x)^6 + 1001*(a^4 - a^2*b^2)*cosh(x)^4 + 7*a^4 - 7*a^2*b^2 +
 33*(7*a^4 - 2*a^3*b + 7*a^2*b^2)*cosh(x)^2)*sinh(x)^10 + 16*(715*(a^4 + 2*
a^3*b + a^2*b^2)*cosh(x)^7 + 1001*(a^4 - a^2*b^2)*cosh(x)^5 + 55*(7*a^4 - 2
*a^3*b + 7*a^2*b^2)*cosh(x)^3 + 35*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^9 + 2*(
35*a^4 + 6*a^3*b + 35*a^2*b^2)*cosh(x)^8 + 2*(6435*(a^4 + 2*a^3*b + a^2*b^2
)*cosh(x)^8 + 12012*(a^4 - a^2*b^2)*cosh(x)^6 + 990*(7*a^4 - 2*a^3*b + 7*a^
2*b^2)*cosh(x)^4 + 35*a^4 + 6*a^3*b + 35*a^2*b^2 + 1260*(a^4 - a^2*b^2)*cos
h(x)^2)*sinh(x)^8 + 16*(715*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^9 + 1716*(a^4
 - a^2*b^2)*cosh(x)^7 + 198*(7*a^4 - 2*a^3*b + 7*a^2*b^2)*cosh(x)^5 + 420*(
a^4 - a^2*b^2)*cosh(x)^3 + (35*a^4 + 6*a^3*b + 35*a^2*b^2)*cosh(x))*sinh(x)
^7 + 56*(a^4 - a^2*b^2)*cosh(x)^6 + 56*(143*(a^4 + 2*a^3*b + a^2*b^2)*cosh(
x)^10 + 429*(a^4 - a^2*b^2)*cosh(x)^8 + 66*(7*a^4 - 2*a^3*b + 7*a^2*b^2)*co
sh(x)^6 + 210*(a^4 - a^2*b^2)*cosh(x)^4 + a^4 - a^2*b^2 + (35*a^4 + 6*a^3*b
 + 35*a^2*b^2)*cosh(x)^2)*sinh(x)^6 + 16*(273*(a^4 + 2*a^3*b + a^2*b^2)*cos
h(x)^11 + 1001*(a^4 - a^2*b^2)*cosh(x)^9 + 198*(7*a^4 - 2*a^3*b + 7*a^2*b^2
)*cosh(x)^7 + 882*(a^4 - a^2*b^2)*cosh(x)^5 + 7*(35*a^4 + 6*a^3*b + 35*a^2*
b^2)*cosh(x)^3 + 21*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^5 + 4*(7*a^4 - 2*a^3*b
 + 7*a^2*b^2)*cosh(x)^4 + 4*(455*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^12 + 200
2*(a^4 - a^2*b^2)*cosh(x)^10 + 495*(7*a^4 - 2*a^3*b + 7*a^2*b^2)*cosh(x)^8
 + 2940*(a^4 - a^2*b^2)*cosh(x)^6 + 35*(35*a^4 + 6*a^3*b + 35*a^2*b^2)*cosh(
x)^4 + 7*a^4 - 2*a^3*b + 7*a^2*b^2 + 210*(a^4 - a^2*b^2)*cosh(x)^2)*sinh(x)
^4 + a^4 + 2*a^3*b + a^2*b^2 + 16*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^13
 + 182*(a^4 - a^2*b^2)*cosh(x)^11 + 55*(7*a^4 - 2*a^3*b + 7*a^2*b^2)*cosh(x)
^9 + 420*(a^4 - a^2*b^2)*cosh(x)^7 + 7*(35*a^4 + 6*a^3*b + 35*a^2*b^2)*cosh
(x)^5 + 70*(a^4 - a^2*b^2)*cosh(x)^3 + (7*a^4 - 2*a^3*b + 7*a^2*b^2)*cosh(x
))*sinh(x)^3 + 8*(a^4 - a^2*b^2)*cosh(x)^2 + 8*(15*(a^4 + 2*a^3*b + a^2*b^2
)*cosh(x)^14 + 91*(a^4 - a^2*b^2)*cosh(x)^12 + 33*(7*a^4 - 2*a^3*b + 7*a^2*
b^2)*cosh(x)^10 + 315*(a^4 - a^2*b^2)*cosh(x)^8 + 7*(35*a^4 + 6*a^3*b + 35*
a^2*b^2)*cosh(x)^6 + 105*(a^4 - a^2*b^2)*cosh(x)^4 + a^4 - a^2*b^2 + 3*(7*a
^4 - 2*a^3*b + 7*a^2*b^2)*cosh(x)^2)*sinh(x)^2 + 16*((a^4 + 2*a^3*b + a^2*b
^2)*cosh(x)^15 + 7*(a^4 - a^2*b^2)*cosh(x)^13 + 3*(7*a^4 - 2*a^3*b + 7*a^2*
b^2)*cosh(x)^11 + 35*(a^4 - a^2*b^2)*cosh(x)^9 + (35*a^4 + 6*a^3*b + 35*a^2
*b^2)*cosh(x)^7 + 21*(a^4 - a^2*b^2)*cosh(x)^5 + (7*a^4 - 2*a^3*b + 7*a^2*b
^2)*cosh(x)^3 + (a^4 - a^2*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^2 + 2
*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*
a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*c
osh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a
^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*
(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 + 2*a*b +
```

```
3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh
(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^
2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^
2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + sqrt(2)*((a + b)*cosh
(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a
- b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^4 + (a + b
)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sin
h(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^
2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b
 + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 3*b^2)*cosh(
x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) - 4*sqrt(2)*((2*a^4
+ a^3*b - 5*a^2*b^2 - 5*a*b^3 - b^4)*cosh(x)^12 + 12*(2*a^4 + a^3*b - 5*a^2
*b^2 - 5*a*b^3 - b^4)*cosh(x)*sinh(x)^11 + (2*a^4 + a^3*b - 5*a^2*b^2 - 5*a
*b^3 - b^4)*sinh(x)^12 + 6*(2*a^4 + a^3*b + a^2*b^2 + 3*a*b^3 + b^4)*cosh(x
)^10 + 6*(2*a^4 + a^3*b + a^2*b^2 + 3*a*b^3 + b^4 + 11*(2*a^4 + a^3*b - 5*a
^2*b^2 - 5*a*b^3 - b^4)*cosh(x)^2)*sinh(x)^10 + 20*(11*(2*a^4 + a^3*b - 5*a
^2*b^2 - 5*a*b^3 - b^4)*cosh(x)^3 + 3*(2*a^4 + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(a + b\tanh^4(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(5/2),x)
[Out] Integral(tanh(x)/(a + b*tanh(x)**4)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="giac")
[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)}{{\left(b\,{\tanh\left(x\right)}^4+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*tanh(x)^4)^(5/2),x)
[Out] int(tanh(x)/(a + b*tanh(x)^4)^(5/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```